INCENTIVES BEYOND THE MONEY: IDENTITY AND MOTIVATIONAL CAPITAL IN PUBLIC ORGANIZATIONS

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Incentives Beyond the Money: Identity and Motivational Capital in Public Organizations.

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Abstract

This paper explores optimality of contracts and incentives when the principal (public organisation) can undertake investments to change agents’ (public workers) identity. In the model, workers within the organisation can have different identities. We develop a principal-agent dynamical model with moral hazard, which captures the possibility of affecting this workers’ identity through contracts offered by the firm. In the model, identity is a motivation source which reduces agents’ disutility from effort.

We use the term identity to refer to a situation in which the worker shares the organisational objectives and views herself as a part of the organisation. Contrary, we use the term conflict to refer to a situation in which workers behave self-interested and frequently in the opposite way of the organisation. We assume that identity can be achieved when principal include mission-sense developing investments in contracts. By mission we mean a single culture that is shared by all the members of an organization.

We discuss the conditions under which spending resources in changing workers’ identity and invest in this kind of motivational capital is optimal for organisations. Our results may help to inform public firms’ managers about the optimal design of incentive schemes and policies. For instance, we conclude that investing in motivational capital is the best option in the long run whereas pure monetary incentives works better in the short run.

Keywords: contracts, moral hazard, identity, socialization, mission, motivational capital.

JEL Codes: D03, D86.
1 Introduction

The present work deals with the economic effects of public workers’ identity. Is workers’ identity another productive asset of public organisations? How should public organisations’ managers design incentive schemes in order to benefit from this Motivational Capital? Could identity be the key to avoid shirking in public organisations?

The motivation of workers at the public sector has been an issue in the recent economic literature\(^1\). Privately owned competitive firms do not performance optimally when the good that have to be supplied is a collective good such as education, health, civil and social safety, a common pool resource or a public good. In the provision of these collective goods the role played by competition and the optimal incentives may differ from the private competitive provision of them.

Organizations that provide collective goods pursue goals and objectives, which are not necessarily monetary profitable, and the motivation of the employees who work within these organizations goes beyond the expected monetary gain. People who work in the provision of collective goods sector are generally agents who have a strong self-view as a pro-social agents. They share organisational goals and objectives and thus joint with organisations and the managers they cohere in what Wilson [31] called mission.

Akerlof and Kranton [1] consider this sharing-goals behaviour of agents as Identity. In their words, identity, is “a way to motivate employees, different than incentives from monetary compensation” and also believe that “[...] a change in identity is the ideal motivator if, [...] the effort of a worker is either hard to observe or hard to reward”.

The present work analyzes the effects of workers’ identity in the public provision of collective goods such as education, health, civil safety, social work, etc. We incorporate into a principal-agent model, the possibility to influence public workers’ (agents) identity with

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\(^1\)See for instance, Sen [29], Wilson [31], Akerlof and Kranton [1, 2], Besley and Ghatak [8], Ghatak and Mueller [17] and Prendergast [24, 25].
The use of incentives. We assume that including some motivational investments in contracts, public organisations may affect positively their employees’ identity.

Developing a sense of mission an organization may avoid “vague objectives” and will define a set of accurate “critical tasks” or “operational goals”.

The “culture” of an organization is a way to see what these critical tasks are and how to deal with them. Wilson (1989, 93)

A “mission” is a single culture that is widely and enthusiastically shared by the members of the organization. Wilson (1989, 99)

In our model “mission” has to do with the associative benefits that come from being part of an organization. “Mission” is a culture shared by all the members within organisation and diminish the experienced disutility of making high effort at the workplace.

Bureaucrats have preferences. Among them is the desire to do the job. That desire may spring entirely out of a sense of duty, or it may arise out of a willingness to conform to the expectations of fellow workers and superiors even when there is no immediate financial advantage in doing so. Wilson (1989, 156).

Many of most productive firms have tried to substitute monetary incentives with the culture of mission.

In business where one might suppose that money incentives are the whole story, great efforts have been made by the most productive firms to supplement those incentives with a sense of mission based on a shared organizational culture. Wilson (1989, 157).

Another branch of the literature related with the concept of identity is Corporate Culture (CC). In a seminal work about CC Kreps [19], treats the corporate culture as a principle
that helps to identify a firm’s rule of behavior which helps setting a good reputation that can be communicated to potential future trading partners and also can be evaluated by them. Such a rule will be characterized by a principle of application. In words of D.M. Kreps [19],

I wish to identify corporate culture with the principle and with the means by which the principle is communicated. My understanding of corporate culture is that it accomplishes just what the principle should — it gives hierarchical inferiors an idea \textit{ex ante} how the organization will react to circumstances as they arise; in strong sense, it gives identity to the organization.

Other relevant works from CC literature are: Barney [4], Schein [28], Crémer [11], Lazear [20], Tirole [30], Carrillo and Gromb [9, 10], Hermalin [18] and, Rob and Zemsky [27].

Identity is related with person’s self-image. How people think about themselves in terms of social categories, and how people think that they and others should behave\textsuperscript{2}. In a organisation, identity, is the degree in which agents share organisational goals and objectives. In the public provision of collective goods, identity is a measure of how accurately workers identify themselves with the organisation mission of providing these social valuable goods. Identity is the internalization of a culture by all the members of an organisation and culture can be interpreted as the organisational goal or mission.

Identity in our model makes effort subjectively less costly to workers and therefore weakens incentive constraint and helps to overcome moral-hazard. For a worker with identity, exert high effort is the ideal way to behave given that she views herself as being part of the organization.

Workers’ identity in organisations may be altered as a result of socialisation. Socialisation is the process through which organisations can change workers’ identity. Socialisation acts making workers’ and organisation’s goals and objectives to be aligned. By contrast,

\textsuperscript{2}See for instance Sen [29] and Akerlof and Kranton [1, 2]
conflict will be the process that lead workers to be completely disagree with organisational goals or organisational mission. Socialisation can be launched by organisation’s managers carrying out certain investments which serve as a signal of a general shared mission.

In general, less incentives are needed to elicit effort when organisation have workers with identity. Thus we can consider the identity as another productive asset of the organization in which the principal could invest. Then, we can measure what Akerlof and Kranton [1] call as \textit{motivational capital}: the current value of the stream of the expected costs saved by the organisation when principal invests a given amount of resources to improve workers’ identity.

2 The Model

We want to analyze the optimality of contracts in a principal agent model in which the principal may provoke changes in agents’ identity. We want to capture in the model whether the possibility of changing agents’ identity may influence optimal incentive contracts. This question has been neglected by standard Contract Theory, which traditionally assume that economic agents are motivated only by material self-interest.

In the present section of the paper, first, we describe in a precise way the channel through which changes in identity happen and, second, we incorporate this channel into the model. Then, we define the game and finally we solve it and we make comparative statics to draw some interesting results.
2.1 **Players’ Preferences and Utilities.**

There are two players in the game: the agent $A$ and the principal $P$. We assume that $A$ can develop identity. We also restrict the analysis to linear contracts.

We model a finite period $t = 0, 1, \ldots, T, \ldots$ principal-agent dynamical game where the agents’ effort is private information. Agents’ behaviour is affected by identity. We incorporate identity into $A$’s utility function. Identity is a non-monetary source of motivation that affects agents’ preferences. Identity also can be altered or changed by principal’s choices.

2.1.1 **Principal**

First, assume that there is a performance measure $q_t$ that $P$ wants to maximize in each $t = 0, 1, 2, \ldots, T, \ldots$. The principal may use monetary incentives—“carrots and sticks”—or non-monetary incentives—“identity motivating”—to maximize $q_t$ or performance. We take $q_t$ as a target outcome for $P$ but it is not necessarily the only one for $A$, or even should be indirectly related to $A$’s mission. This condition of unrelatedness or disconnection may what makes necessary the use of incentives.

Let $R_t(q_t)$ be a function which assigns a monetary value to the performance level. $R_t(q_t)$ is positively correlated with the achieved social welfare, the total amount of the collective good of service delivered, and the sort of measures which are salient and observable by the electorate, tax payers and political advisors to evaluate the public supply of collective goods.

Performance $q_t$ is a function of $A$’s effort $e_t \in \{e, \bar{e}\}$. Assume that $q_t \in \{\bar{q}, q\}$ where $\bar{q} > q$; interpret $\bar{q}$ as $P$’s target on performance level and $q$ as a fail in this target. Let

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3 Often we use she and he to refer to the agent and the principal respectively, as conventionally the principal agent literature does.

4 Usually for the firm this monetary value is determined by the market price and the quantity sold. But in the case of the public provision of collective goods the absence of markets and market prices makes hard to measure the monetary value of $q_t$. We can interpret this function as one which calculate the opportunity cost of public supplying rather than market supplying.
\( p(q_t = \overline{q}|e_t) = \theta \) be the probability of high performance conditional to \( A \)’s effort choice. We use \( i = 0, 1 \) to label low and high effort: 0 means low effort \( \xi \) and 1 means high effort \( \xi \). Then \( p(q_t = \overline{q}|e_t = \xi) = \theta_1 \) will be the probability of high performance when the agent decides to exert high effort, and \( p(q_t = \overline{q}|e_t = \xi) = \theta_0 \) the probability of high performance when the agent decides to exert low effort. Alternatively \( p(q_t = \overline{q}|e_t = 1) = 1 - \theta_1 \) and \( p(q_t = \overline{q}|e_t = 0) = 1 - \theta_0 \) will be the probabilities of low performance when effort is high and low respectively. We assume that performance \( q_t \) is an informative but noisy signal of \( e_t \) which means that \( \theta_1 > \theta_0 \).

Let \( E[R_t(q_t)|\theta_i] \) be the expected material rewards achieved by the principal that depends on \( q_t \) that is conditional to \( \theta_i \). Let \( w_t(q_t) \) be the monetary payments, contingent to performance, offered by \( P \) to \( A \). Then, \( E[w_t(q_t)|\theta_i] \) will be the expected monetary payment of \( A \) and the expected monetary cost for \( P \) at the same time. Let \( s_0 \) be the total amount of resources invested to promote and change \( A \)’s identity. We assume that \( P \) makes this investment \( s_0 \) at the starting period of the game \( t = 0 \). Such initial investment generates a depreciation cost stream that we capture with the cost function \( C_t(s_0) \). Thus we will be able to write \( P \)’s problem as the profit maximizing problem described below.

Then \( P \)’s expected profit function \( \pi_t \) for each period \( t \) can be written as,

\[
\Pi_t = E[R_t(q_t)|\theta_i] - E[w_t(q_t)|\theta_i] - C_t(s_0) \tag{2.1}
\]

Where the investment cost function \( C_t(s_0) \) is such that, takes the value \( C_0(S) = S \) in \( t = 0 \) and an depreciation cost \( C_t(S) = \gamma S \) for every \( t \geq 1 \) at constant depreciation rate \( \gamma \).

### 2.1.2 Agent

We represent the \( A \)’s preferences with the following expected utility function.
\[ U_t = \underbrace{E[u_t(w_t(q_t))|\theta_i]}_{\text{Expected utility from income}} - \underbrace{\psi_t(e_t,v_t(s_0))}_{\text{Disutility from effort}} \]  \hspace{1cm} (2.2)

The first term on the right hand side of the above utility function, \( E[u_t(w_t(q_t))|\theta_i] \), is the expected utility from money. The agent is risk averse, \( u' > 0 \) and \( u'' < 0 \), and the parameter \( \theta_i \) is the probability of high or low performance.

The second term on the right hand side of the expected utility function represents the disutility from effort \( \psi_t(e_t,v_t(s_0)) \). The disutility from effort depends negatively on \( v_t(s_0) \) which is a function representing \( A \)'s identity and depends positively from effort. The properties of the disutility from effort are summed up in the following set of assumptions.

A1: *The function \( \psi_t(e_t,v_t(s_0)) \) is continuous in the interval \([\underline{v}, \overline{v}]\).*

A2: *The function \( \psi_t(e_t,v_t(s_0)) \) is strictly decreasing in its second argument when \( e_t = \overline{e} \). That is, \( \frac{\partial \psi_t(e_t,v_t)}{\partial v_t} |_{e_t=\overline{e}} < 0 \).*

A3: *When \( e_t = \underline{e} \), then \( \psi_t(\underline{e},v_t) = 0; \forall v_t \in [\underline{v}, \overline{v}] \).*

A4: *The function \( \psi_t(e_t,v_t(s_0)) \) is bounded below and above. Is bounded below when \( \psi_t(\underline{e},v_t) = 0 \) \( \forall v_t \), and \( \psi_t(\overline{e},v) = 0 \). The function is bounded above when \( \psi_t(\overline{e},v) = \Psi \), with \( \Psi \in \mathbb{R}^+ \).*

The above assumptions ensure that, when identity converge to its upper (lower) bound, then \( A \)'s disutility from doing high effort converges to zero (\( \Psi \)). That is, the agent does not suffer disutility from exerting high effort when she develops identity. Contrary, when she has no identity, \( A \) experiences the maximum disutility from effort and she only can diminish this disutility making low effort.
2.2 The Game

The game is a repeated game with two players, the agent $A$ and the principal $P$. We consider the game as a repeated dynamic recontracting game in which every period both players have to make new choices: $P$ must offer a new contract after updating his beliefs about $A$’s identity. Then, $A$ has to choose a new effort level. Thus, we analyze a repeated principal agent game with moral hazard, where the choices made by the $P$ affects $A$’s identity. Reciprocally these changes in identity and motivation affect the contracts and optimal choice of payments of $P$ in the next period.

2.2.1 Information

At the first period of the game, $P$ learns $A$’s type or identity\(^5\) probability distribution $F_0(v_0)$. $P$’s action over $s_0$ affects the $A$’s identity and therefore $P$ have to update the $A$’s identity distribution $F_t(v_t|s_0)$ in the subsequent of the periods of the game $t = 1, ..., T, ...$ where $t \in \mathbb{N}$.

2.2.2 Timing

Each period the game consists of three stages: stage 0, stage 1, and stage 2. The sequence of these stages in $t = 0, 1, 2, ...$ is graphically shown in figure 1.

The sequence of stages within each period $t = 0, 1, 2, ...$ is as described below:

**$(0)$**: The principal $P$ learns the distribution of $A$’s identity $F_0(v_0)$ in $t = 0$ or updates $A$’s identity distribution $F_t(v_t|s_0)$ in $t = 1, 2, ...$ conditional to the choice of $s_0 \in \{0, S\}$ in $t = 0$. Then taking into account the expected value of $A$’s identity, $P$ offers a contract. The contract will be a dupla of stochastic contingent payments $w_0(q_0) = \{w, \bar{w}\}$ joint with the choice to

\(^5\)We consider a continuum of types of $A$, $v_t \in [\underline{v}, \overline{v}]$. There is a possibility of switching $A$’s type or identity making an investment in the starting period of the game. For a precise description of the time evolution of the conditional distribution of types see the mathematical appendix.
Figure 1: The timing of the game in two separate sequences, \( t = 0 \) and \( t = 1, 2, \ldots \)

Invest or not in motivational capital \( s_0 \in \{0, S\} \) in \( t = 0 \): \( \{w_0(q_0), s_0\} \). In \( t = 1, 2, \ldots \) the contract will also be a dupla of stochastic contingent payments \( w_1(q_1) = \{\bar{w}, \underline{w}\} \) joint with the commitment of bearing the cost of depreciation of the \( s_0 \in \{0, S\} \) investment or \( C_t(s_0) = \gamma s_0: \{w_t(q_t), C_t(s_0)\} \).

1. **A** accepts or refuses the contract. If she accepts, then choose an action over effort \( e_0 \in \{e, \tau\} \). Contrary, if she refuses then she gets her reservation utility \( \overline{U} \).

2. **Finally**, output is realized \( q_t \in \{\overline{q}, \underline{q}\} \). Stochastic contingent payment is realized \( w_t(q_t) = \{\bar{w}, \underline{w}\} \) and payoffs \( \pi_t \) and \( U_t \) are realized.

### 2.2.3 Identity and Socialization

Agents only differ in their identity. For all of them, their skills and qualification for work is the same. They are equally productive in the production of \( q_t \). Therefore, \( \mathcal{P} \) only deals with *moral hazard* because the differences in motivation degree does not involve any difference in agents’ abilities in the production of \( q \). **A**’s identity distribution is assumed to be known.
Incentives and contracts offered by the principal may change agent’s identity. Assume that agent’s identity can take a value within some closed real interval $\nu \in [\underline{\nu}, \overline{\nu}]$, with $\underline{\nu} < \overline{\nu}$ and $\nu \in \mathbb{R}^+$. Higher identity means lower disutility from effort. Thus, we can anticipate that more identity needs less monetary incentives in order to overcome moral hazard and incentivize high effort. However, if the principal does not make any investment in motivational capital and only use monetary incentives to elicit high effort, he will need to offer a higher amount money to $A$ because she will experience more disutility from effort.

What we want to capture with the socialization process\textsuperscript{6} is $\mathcal{P}$’s ability to change agents’ identity carrying out investments in the organization which signal support and awareness toward agents or workers. This behavior of $\mathcal{P}$ makes lead agents to share the organization’s goals. Thus if $\mathcal{P}$ chooses to invest $s_0 = S$ he will switch $A$’s identity to a higher level. But if he decides not to invest, $s_0 = 0$, then agents will switch to lower identity bringing all of them to conflict.

\textbf{2.2.4 Solving Principal’s Problem}

Assume that $A$ and $\mathcal{P}$ have to renegotiate contracts period after period. This assumption turns the game into a \textit{dynamic re-contracting game}. Then we solve the game implementing the spot contract in each period of the game. In order to make the vector of the spot contracts as the long term optimal solution we have to assume no availability to renegotiate in the short term. In this game the only way to agree upon a contract is playing the repeated game at every period $t = 0, 1, ..., T, ...$ as a new game.

Then we can write the $\mathcal{P}$’s problem as follows,

\textsuperscript{6}See Adler and Bryan [3]
\[
\max_{(w_t(q_t), s_0)} \mathbb{E}[R_t(q_t) | \theta_i] - \mathbb{E}[w_t(q_t) | \theta_i] - C_t(s_0)
\] (2.3)

Subject to

\[
\mathbb{E}[u_t(w_t(q_t) | \theta_i)] - \psi_t(\bar{r}, v_t) \geq \mathbb{E}[u_t(w_t(q_t) | \theta_i)] - \psi_t(\underline{q}, v_t) \quad \text{(ICC)}
\] (2.4)

\[
\mathbb{E}[u_t(w_t(q_t) | \theta_i)] - \psi_t(\bar{r}, v_t) \geq U \quad \text{(PC)}
\] (2.5)

\[
u_t(w) \geq 0 \iff w_t(q) \geq 0 \iff h(u_t(w)) \geq 0 \quad \text{(LLC)}
\] (2.6)

(4) is \(\mathcal{A}\)'s incentive compatibility constraint (ICC), and ensures that the agent will prefer to exert high effort. (5) is the \(\mathcal{A}\)'s participation constraint (PC), and ensures that the agent will prefer to participate and accept the contract. Finally, (6) is a limited liability constraint (LLC), and ensures that the low payment never falls below zero level.

\(\mathcal{P}\)'s problem is solved, for each \(t\). Solution in each period \(t\) consist in the following payment function \(w(q) : q \rightarrow w\)

\[
w(q) = \begin{cases} 
\bar{w} & \text{if } q = \bar{q} \\
w & \text{if } q = q 
\end{cases}
\]

where \(\bar{w} > w^7\). We analyze in the next section, the conditions under which to make investments in changing \(\mathcal{A}\)s’ identity is optimal for \(\mathcal{P}\). In the following we just show the pair of payments which solves the spot contracting problem. Let \(h : u \rightarrow w\) be the inverse function of the utility function,

\(^7\)How these contingent payments have been calculated is formally shown in the mathematical appendix, section B.3
\[ h(u) = \begin{cases} \overline{w} & \text{if } u = \overline{u} \\ w & \text{if } u = \underline{u} \end{cases} \]

Applying the variable change \( w = h(u(w(q))) = (u(w(q)))^{-1} \) we have the following payments,

\[ \overline{w}_t = h(u_t(\overline{w})) = \left( \overline{U} + \frac{(1 - \theta_0)}{\Delta \theta} \psi_t(\overline{\varphi}, \nu_t(s_0)) \right)^{-1} \tag{2.7} \]
\[ \underline{w}_t = h(u_t(\underline{w})) = \left( U - \frac{\theta_0}{\Delta \theta} \psi_t(\underline{\varphi}, \nu_t(s_0)) \right)^{-1}. \tag{2.8} \]

As it can be seen identity lowers \( \overline{w} \) and raises \( \underline{w} \). A more precise and consistent relation between identity and the money that the manager have to put in the pocket of the worker in order to incentivize high effort requires to analyze how identity and the disutility from effort interacts each with the other. Also we have to analyze how socialization affects workers’ identity and how these changes affects future stochastic contingent payments and by the way the expected costs of incentivize effort.

\( \mathcal{P} \) can not discriminate workers or agents attending their identity. \( \mathcal{P} \) only knows the distribution of identity. Then, he updates such distribution at every period taking into account his own past actions and knowing how socialization works. Once \( \mathcal{P} \) has updated this information about \( \mathcal{A}_s \)’s identity, he will be able to offer a contract based on the expected identity of agents\(^8\). Thus, at every period of the game, \( \mathcal{P} \) must offer a new pair of expected payments conditioned to agents’ expected identity,

\(^8\)This solution is suboptimal compared with the first best solution where effort level and identity are perfectly observable. Also is more far away from the first best compared with in a stronger sense of the second best solution where only the effort is unobservable but identity doesn’t play any role.
We write the Expected Cost Function for \( \mathcal{P} \) at each \( t \),

\[
ECF_t = (\theta_1 \overline{w}_t(E[v_t|s_0]) + (1 - \theta_1) \underline{w}_t(E[v_t|s_0])) + C_t(s_0)
\]  

(2.11)

Let us use the superscript \( s_0 \in \{0, S\} \) in \( ECF_t^{s_0} \), in order to differentiate the expected cost function when \( \mathcal{P} \) invests in identity \( s_0 = S \), from the no-investment case \( s_0 = 0 \). Then we have \( ECF_t^S \) and \( ECF_t^0 \).

\[
ECF_t^0 = (\theta_1 \overline{w}_t(E[v_t|0]) + (1 - \theta_1) \underline{w}_t(E[v_t|0]))
\]

\[
ECF_t^S = (\theta_1 \overline{w}_t(E[v_t|S]) + (1 - \theta_1) \underline{w}_t(E[v_t|S])) + c_t(S)
\]

Identity can be considered another productive asset of the organization. Another kind of capital of the organization that we call *Motivational Capital*. Then we can measure the return of such investment in *Motivational Capital* computing the present value of the stream of the expected costs saved by the health organization. This return can be measured with the following mathematical expression,

\[
CNV^m_k = \sum_{t=0}^{T} \delta^t [ECF_t^0 - ECF_t^S]
\]  

(2.12)
Where, \(\delta^t = (\frac{1}{1+r})^t\) is the discount factor, and \(r\) is the discount rate. We say that the principal has incentives to invest in motivational capital when \(CNV^{mk} \geq 0\) and we say that, there is no incentive to invest in motivational capital when \(CNV^{mk} < 0\).

3 Results

Now we calculate using equilibrium payments the spot contract’s cost, the organisation profits and agents utilities for every \(t = 0, 1, ..., T, ...\). We solve the model for the case in which \(P\) decides to invest in motivational capital and change identity of \(A_0 s_0 = S\) and for the case in which \(P\) decides to not invest in motivational capital at all \(s_0 = 0\). We calculate the solution for each case. In the last subsection we make comparative statics and derive necessary and sufficient conditions for investing in motivational capital. We also discuss on some conclusions.

3.1 Shifting Agents Toward identity: Investment in Motivational Capital

We will study the case in which the principal chooses to invest in motivational capital \(s_0 = S\). In this case spot payments for every \(t\) are,

\[
\overline{w}^S_t(E_t[v_t|S]) = h\left(U + \frac{(1 - \theta_0)}{\Delta \theta} \psi_t(\bar{e}, E_t[v_t|S])\right)
\]

(3.1)

\[
\underline{w}^S_t(E_t[v_t|S]) = h\left(U - \frac{\theta_0}{\Delta \theta} \psi_t(\bar{e}, E_t[v_t|S])\right)
\]

(3.2)

For the starting period \(t = 0\) payments will be,
\[
\bar{w}_t(E_0[v_0]) = h \left( U + \frac{(1 - \theta_0)}{\Delta \theta} \psi_0(\bar{v}, E_0[v_0]) \right) \\
\bar{w}_t(E_0[v_0]) = h \left( U - \frac{\theta_0}{\Delta \theta} \psi_0(\bar{v}, E_0[v_0]) \right)
\] (3.3)

These two payments of \( t = 0 \) will be exactly the same for the case in which \( P \) does not invest any amount in motivational capital because at the starting period of the game the socialization effect can not have occurred yet. For the case of \( s_0 = S \) we write the following expected cost function for health manager,

\[
ECF_t^S = \theta_1 \bar{w}_t^S(E_t[v_t|S]) + (1 - \theta_1)\bar{w}_t^S(E_t[v_t|S]) + C_t(S).
\] (3.5)

Now we can calculate the spot expected profit \( \Pi_t^S \) for \( P \) and the spot expected utility \( U_t^S \) for \( A \) given that the latter chooses \( e_t = \bar{v} \) and therefore the probability to achieve \( q_t = \bar{q} \) outcome is \( \theta_1 \).

\[
\Pi_t^S = E_t[R_t(q_t)|\theta_1] - [ECF_t^S]
\] (3.6)

\[
U_t^S = \theta_1 \left( U + \frac{(1-\theta_0)}{\Delta \theta} \psi_t(\bar{v}, E_t[v_t|S]) \right) \\
+ (1 - \theta_1) \left( U - \frac{\theta_0}{\Delta \theta} \psi_t(\bar{v}, E_t[v_t|S]) \right) - \psi_t(e_t, v_t(s_0))
\] (3.7)

Finally, we compute the present value of the sum of spot profits and the sum of the spot utilities, and also the expression which measures the present value of the total surplus \( T S^S \) when \( P \) action is \( s_0 = S \).

\[
\Gamma^S = \sum_{t=0}^{T} \delta^t \Pi_t^S = \sum_{t=0}^{T} \delta^t \left( E_t[R_t(q_t)|\theta_1] - [ECF_t^S] \right)
\] (3.8)
\[ TU^S = \sum_{t=0}^{T} \delta^t E_t[U^S_t|\theta_1] = \sum_{t=0}^{T} \delta^t \left[ \theta_1 \left( U + \frac{(1 - \theta_0)}{\Delta \theta} \psi_1(\bar{e}, E_t[v_t|S]) \right) \right] + \sum_{t=0}^{T} \delta^t \left[ (1 - \theta_1) \left( U - \frac{\theta_0}{\Delta \theta} \psi_1(\bar{e}, E_t[v_t|S]) \right) - \psi_1(\bar{e}, v_t) \right] \] (3.9)

\[ TS^S = \sum_{t=0}^{T} \delta^t (E_t[U^S_t|\theta_1] + E_t[\pi^S_t|\theta_1]) \] (3.10)

3.2 Agents in Conflict: No-investment in Motivational Capital

In this section we analyze the no investment case or \( s_0 = 0 \). In this case \( \mathcal{P} \) does not invest any amount of resources to promote \( \mathcal{A}'s \) identity. The mere use of monetary incentives to control \( \mathcal{A}'s \) behavior put agents into conflict toward organization. For this case spot payments are,

\[ w^0_0(E_t[v_t|0]) = h \left( U + \frac{(1 - \theta_0)}{\Delta \theta} \psi_1(\bar{e}, E_t[v_t|0]) \right) \] (3.11)

\[ w^0_0(E_t[v_t|0]) = h \left( U - \frac{\theta_0}{\Delta \theta} \psi_1(\bar{e}, E_t[v_t|0]) \right) \] (3.12)

Payments in \( t = 0 \) are exactly the same as those described in the previous subsection. We write the expected cost function for this incentive policy in every \( t = 0, 1, \ldots, T, \ldots \), that we call \( ECF^0_t \).

\[ ECF^0_t = \theta_1 w^0_t(E_t[v_t|0]) + (1 - \theta_1) w^0_t(E_t[v_t|0]) \] (3.13)

Now we can calculate the spot expected profit \( \Pi^0_t \) for \( \mathcal{P} \) and the spot expected utility \( \mathcal{U}^0_t \) for \( \mathcal{A} \), given the agent choice \( e_t = \bar{e} \) and probability \( \theta_1 \).
\[ \Pi_t^0 = E_t[R_t(q_t)\mid \theta_1] - ECF_t^0 \]  
(3.14)

\[ \mathcal{U}_t^0 = \theta_1 \left( U + \frac{(1 - \theta_0)}{\Delta \theta} \psi_t(\overline{v}, E_t[\nu_t][0]) \right) + (1 - \theta_1) \left( U - \frac{\theta_0}{\Delta \theta} \psi_t(\overline{v}, E_t[\nu_t][0]) \right) - \psi_t(\overline{v}, \nu_t(0)) \]  
(3.15)

Also for this case we complete the results showing the present value of the sum of spot profits and the sum of the spot utilities, and also the expression which measures the present value of the social welfare \( TS^0 \) under the incentive policy \( s_0 = 0 \).

\[ \Gamma^0 = \sum_{t=0}^{T} \delta^t E_t[\Pi_t^0 \mid \theta_1] = \sum_{t=0}^{T} \delta^t \left( E_t[R_t(q_t)\mid \theta_1] - [ECF_t^0] \right) \]  
(3.16)

\[ T\mathcal{U}^0 = \sum_{t=0}^{T} \delta^t E_t[\mathcal{U}_t^0 \mid \theta_1] = \sum_{t=0}^{T} \delta^t \left[ \theta_1 \left( U + \frac{(1 - \theta_0)}{\Delta \theta} \psi_t(\overline{v}, E_t[\nu_t][0]) \right) \right] \]

\[ + \sum_{t=0}^{T} \delta^t \left[ (1 - \theta_1) \left( U - \frac{\theta_0}{\Delta \theta} \psi_t(\overline{v}, E_t[\nu_t][0]) \right) - \psi_t(\overline{v}, \nu_t(0)) \right] \]  
(3.17)

\[ TS^0 = \sum_{t=0}^{T} \delta^t [\mathcal{U}_t^0 + \Pi_t^0] \]  
(3.18)

### 3.3 Comparative statics

Our model shows that an agent with identity within the firm or public organisation is willing to work hard at a high effort for a lower overall pay. This means that less variation in payment schedule is required to incentivize high effort. This less variation in payments results in additional cost savings for \( P \). In addition, when agents are risk averse, less
variation in payments means that they must be compensated with a lower risk premium and this constitutes another cost saving source for the organization management. When these cost advantages are high enough, it can be worthwhile for \( P \) to undertake a costly program to promote agents’ identity.

Comparative statics of our model establish under which conditions agents’ identity lead the organization to find profitable to invest in promoting identity among workers. If inculcating identity is cheap, if output and agents’ effort are weakly correlated (effort is hard to observe an hard to reward), if agents are especially risk averse, if high effort is critical to the organization’s output, then the use of an identity-oriented motivational incentive scheme, would be more profitable and more likely to be used.

3.3.1 Identity and Motivational Capital

One very first result that it is straightforward to set, comes from the comparison between the current value of the sum of spot profits for \( P \) when he takes \( S \) action and when he takes 0 action. That is, firstly calculating the Current Net Value of the \( A \)’s motivational capital \( (CNV^{mk}) \), and then checking if it is positive or negative. This first result is formally shown in proposition 1.

**Proposition 1.** Let \( T \) the number of periods of the game. For a given \( \delta \) large enough, and given \( \gamma \) small enough, there exists a threshold \( t^* \) such that,

\[
\sum_{t=0}^{t^*} \delta^t \left[ \theta_1 (w_0^T - w^S_T) + (1 - \theta_1) (w_0^0 - w^S_T) \right] = \sum_{t=0}^{t^*} \delta^t C_t(S)
\]

and for which

1. If \( t^* \leq T \) then \( CNV^{mk} \geq 0 \) and \( P \) finds profitable to invest in motivational capital and choose the \( s_0 = S \) strategy.
ii. If $t^* > T$ then $CNV^{mk} < 0$ and $P$ finds profitable not to invest in motivational capital and choose the $s_0 = 0$ strategy.

Figure 4 shows graphically this first result. The graph on the left side of the figure 4 shows jointly as a function of time $t$, either, $P$’s expected cost function in the case of $s_0 = 0$ and $P$’s expected cost function in the case of $s_0 = S$. The discounted sum of the difference between these two functions will be the measure of the current net value perceived by $P$ from an investment in motivational capital. In the graph two different cases – (i) and (ii) – are shown with the goal of being illustrative of the proposition 1 results.

The graph on the right side of the figure 4 shows the value of the $CNV^{mk}$ as a function of time $t$. In this graph also two cases are shown in the other graph. The $t^*$ threshold determines the critical point below which the best strategy for $P$ will be not to invest.

The motivational capital profitability threshold $t^*$ is the key variable for $P$’s optimal action choice. This threshold depends on several variables. The relations given between these variables and the motivational capital profitability threshold is what determines $P$’s
optimal decision in this contracting game. We will focus on the analysis of these relations in order to draw conditions under which one or another strategy, \( s_0 \in \{0, S\} \) is optimal.

Now let us compare the total surplus of each strategy \( s_0 \in \{0, S\} \) of \( P \), to analyze the cases in which all the members within an organization are better off. What we summarise in the following proposition is that the social optimum coincides with the optimal choice of \( P \). This is the case despite the socialization or conflict effects of incentive policies. It is so because incentive compatibility constraint (2) and participation constraint (3) ensure that for every choice \( s_0 \in \{0, S\} \) of \( P \), and for every \( t = 0, 1, \ldots, T \) the expected utility required by \( A \) to exert high effort is the same. Depending on the \( P \)'s choice over \( s_0 \in 0, S \) the only difference in the expected utility experienced by \( A \) is the source through this utility comes—monetary or identity—while the total of this expected utility remains constant.

Related to Proposition 1, we have the following result:

**Proposition 2.** Let \( CNV^{mk} = TS^S - TS^0 = \sum_{t=0}^{T} \beta^t \left[E C^0_t - E C^S_t \right] \). Let \((s_0, e_t)\) be the strategy profile that solves the game.

i. If \( CNV^{mk} \geq 0 \), then \((S, \bar{e})\) is a Pareto-Efficient strategy profile and Pareto-Dominates \((0, \bar{e})\). Therefore investing in Motivational Capital, \( s_0 = S \), is the optimal social choice.

ii. If \( CNV^{mk} < 0 \), then \((0, \bar{e})\) is a Pareto Efficient strategy profile and Pareto Dominates \((S, \bar{e})\). Therefore not investing in Motivational Capital, \( s_0 = 0 \) is the optimal social choice.

### 3.3.2 Investment Depreciation Pace and Motivational Capital

Another interesting insight that we can draw from the solution of the game relates the kind of the investment that the principal has to run and the profitability threshold \( t^* \). In other words, motivational investments that have higher depreciation rates are highly
expensive in relation with the costs that the principal can save from having workers with identity. Therefore these high depreciation cost featured investments will be less likely to be implemented. In the case of high depreciation of motivational investments, the returns for \( P \) may be either; small and will require a very high number of periods to turn profitable, or negative and never will turn profitable.

For the latter we can establish that there is a maximum depreciation rate \( \bar{\gamma} \) above of which the Current Net Value of motivational capital never will reach a positive value and investing in motivational capital will not be profitable at all. Proposition 3 summarize this result.

**Proposition 3.** Assume that all the agents within the organization have reached the upper value of identity \( \nu_t = \bar{\nu} \) at any given time period \( t = \hat{t} \). Taking \( S \) as constant, if \( \gamma S > \left[ \theta_1 (w_0^i - w_i^S) + (1 - \theta_1) (w_0^i - w_i^S) \right] \), then \( CNV_{mk} < 0 \) and decreasing for all \( t = 1, 2, ... \) and \( P \) never will find profitable to invest in motivational capital.

![Figure 3: Negative Current Net Value of the Motivational Capital, \( CNV_{mk} < 0 \) due to high cost of depreciation \( \gamma S \).](image)

The intuition behind proposition 3 as shown in the figure 3 is as follows: whenever \( \bar{A}s' \) identity is not large enough to cause an advantage in payments which offset the cost of
promoting identity \( E[w^0_t - w^S_t | \theta_1] < C_t(S) \), at any time \( t = 0, 1, 2, \ldots, T, \ldots \), then there is no reason to spend resources to change workers’ identity neither, in the short run nor in the long run.

The analogous case in which identity is large enough to overcome the cost of generating it \( E[w^0_{t'} - w^S_{t'} | \theta_1] > C_{t'}(S) \), at some time period \( t = t' \), and \( t' \in \{0, 1, \ldots\} \) is more of our interest. In this case organisation’s expected saving in payments due to agents’ identity when \( P \)’s choice is \( s_0 = S \) is strictly increasing and bounded. Taken together with the assumption of a constant depreciation cost, \( \gamma S \) we have that the optimality of investing in motivational capital becomes a matter of time. Then the time that the organisation has to wait in order to get profits from identity changes, \( CNV^{mk} > 0 \), will be a function of the depreciation rate value \( \gamma \). Proposition 4 shows such a relation.

**Proposition 4.** Let \( i \in \{l, h\} \) be two alternative ways of changing agents’ identity with \( \gamma_l \) and \( \gamma_h \) two different depreciation costs such that \( \gamma_l < \gamma_h \). Let \( t = t'_i, t'_i \in \{0, 1, \ldots\} \) be the time periods in which the change in agents’ identity reach a value such that \( E[w^0_{t'_i} - w^S_{t'_i} | \theta_1] \geq \gamma_i S \). Let \( t = t^*_i, t^*_i \in \{0, 1, \ldots\} \) be the number of time periods in which \( CNV^i_{mk} = 0 \). Then \( t'_l < t'_h \) and \( t^*_l < t^*_h \).

Figure 4 illustrates the result of the proposition 4. The intuition behind this proposition is as follows: think in situations in which investing in motivational capital through changing identity have a potential to be profitable. Then, those actions which require of investments with higher depreciation costs will turn profitable in more long time horizons. In other words, although investment in motivational capital is an optimal long run strategy, when the change in agents’ identity requires more resources (obstinate agents, distrustful agents, \ldots) the principal must be more patient or should has the security of maintaining workers with enough long contracts.
Figure 4: Current Net Value of Motivational Capital with two different depreciation rates $\gamma_l$ and $\gamma_h$ such that $\gamma_l < \gamma_h$.

Other way of looking at this problem is that, in organizations with high rotation rates investing in motivational capital is less likely to be an optimal strategy unless the change in identity could be achieved by promoting work stability for instance.

3.3.3 Effort Effectiveness and Motivational Capital

The model captures $A$'s effort effectiveness degree with the $\theta_i \in [0,1]$ parameter, where $i = 0, 1$ mean respectively $A$'s low effort action and high effort action. $\theta_i$ can be interpreted as the probability of high performance conditional to $A$'s effort choice. We say that $\theta_i$ is informative if $\theta_1 > \theta_0$. When the value of $\theta_0$ is closer to $\theta_1$ $P$ must offer higher incentives to incentivize $A$ to exert high effort.

Previous findings establish that pure monetary incentives results too expensive when the signal used to link payments and effort is hard to observe. In such cases, investing in motivational capital through changing agents’ identity, $s_0 = S$, will be the optimal choice for $P$ because effort is hard to observe and hard to reward. This is because although investing in motivational capital is costly $C_t(S)$, $P$ will reduce payment costs because workers with identity do not need as higher monetary incentives to exert high effort as workers without
identity. Higher $\theta_0$ implies higher monetary incentives, then potential savings from $s_0 = S$ strategy will be very high if it is the case and then investing in motivational capital becomes more likely to be optimal because the current net value, $CNV^{mk}$ becomes positive earlier.

Proposition 5 summarizes the above result:

**Proposition 5.** Let $p(q = \overline{q}|e = \varepsilon) = \theta'_0$ and $p(q = \overline{q}|e = \varepsilon) = \theta''_0$ be two alternative probabilities of high performance when $A$’s effort is low such that $\theta'_0 > \theta''_0$. Let $CNV^{mk}_{\theta'_0}$ and $CNV^{mk}_{\theta''_0}$ be the current net value of investing in motivational capital in the cases of $\theta'_0$ and $\theta''_0$ respectively. Let $t'^*_{\theta'_0}$ and $t''^*_{\theta''_0}$ be the threshold number of periods such that their respective current net values of investing in motivational capital, $CNV^{mk}_{\theta'_0}$ and $CNV^{mk}_{\theta''_0}$ become positive. For $\theta'_0, \theta''_0 \in [0, 1]$ such that $\theta'_0 > \theta''_0$, then $CNV^{mk}_{\theta'_0} > CNV^{mk}_{\theta''_0}$ and therefore $t'^*_{\theta'_0} < t''^*_{\theta''_0}$.

Proposition 5 shows that as higher is the probability of achieving high performance when the effort is low, $p(q = \overline{q}|e = \varepsilon) = \theta_0$, then more profitable will be for $P$ to invest in motivational capital $s_0 = S$.

### 3.3.4 Agents’ Risk Aversion and Motivational Capital

In the model, agents are risk-averse with respect to their monetary earnings. They perceive utility from Incentives which consists in contingent rewards linked to performance $q_t$. But agents also experience utility from identity and as mean as agent identity raises, fewer incentives are required in order to encourage him to exert high effort. Less variation in payments indicates that $A$ can be compensated with a lower risk premium, and this constitutes another cost-saving source for the organisation.

Proposition 6 formally states that investing in motivational capital is more profitable in the presence of risk-averse $A$:

---

9This result is consistent with what Akerlof and Kranton (2005) state about identity as an incentive for work motivation. […] a change in identity is the ideal motivator if, […] the effort of a worker is either hard to observe or hard to reward.(p. 10)
Proposition 6. Let $A_1$ and $A_2$ be a pair of agents with $v_1$ and $v_2$ identity respectively. If agents are risk-averse and $v_1 < v_2$, then the risk premium will be lower in the case of $A_2$ than in the case of $A_1$. Therefore $t^*_1 > t^*_2$ and organisations will find more profitable to invest in motivational capital when agents are more risk averse.

The intuition behind this result is that incentives must be greater in order to encourage high effort from agents without identity. Higher incentives required by lower identity raise the range between the low $w$ and the high $\bar{w}$ payments. Given that $A$ is risk averse, the risk premium that $P$ should offer to make the incentive contract attractive for $A$ will result higher. Analogously, agents with identity will require fewer incentives to exert high effort. Consequently, an agent with high identity has to bear a lower variance over payments and has to be compensated with a lower risk premium. The decrease in the risk premium thus involves an additional source of savings for $P$ that can be exploited by managing changes in the $A$’s identity.

4 Conclusion

We introduce the notion of identity in a model of principal agent with moral hazard. Incentives beyond the money can be an alternative option to money incentivization for encourage agents to exert high effort. The incorporation of identity has been done on the basis of an extense literature on identity\textsuperscript{10}. Our approach has to do with what Fehr et al. [16] summed up with the following quote:

\[\ldots\] This approach is a first step to developing richer models that may become part of “behavioral contract theory.”

Introducing the notion of identity in a principal agent model joint with the ability of the principal to manage agents’ identity and align their goals with their own, the present

\textsuperscript{10}See for instance Akerlof and Kranton [7, 1, 2] and Benabou and Tirole [6].
work has shown the conditions under which to spend resources to change agents’ identity is profitable for organisations.

These conditions are, for instance, the length of the contracts offered, the total cost of investing in changing agents’ identity for the principal, the informative value of the signal used to observe and incentivize effort, the degree of agents’ risk aversion.

Taking all into account, what we conclude from this work is: an initial investment in motivational capital using incentives beyond the money though costly at inception, will result more effective to control public organisations expenditure. Then, Governments, political advisors and public organisations should take into account and incorporate these findings to the policy design. For instance, from the proposition 1 a planner could conclude that monetary incentives are the best way to achieve a specific goal in the short term. However for the long term goals: quality, efficiency, effectiveness, research and development results, then proposition 1 establishes, that a change in identity and investments in motivational capital is the most profitable action for the organisation.

Finally, wherever the principal in the public organisation is politically designed, their time horizon will be the legislative time period and then it is more likely that they are focused in the short term goals. Thus, they will have a willingness to choose pure monetary rewards as incentive schemes, despite in the long term the best choice is the investment in motivational capital given that workers’ contracts are much longer than legislative piece of time. Anyway these conclusions are interesting future research questions which, should be tested and studied in depth in the future.
A Mathematical Appendix

A.1 Socialization: the Evolution of Identity Distribution.

Let $F(\upsilon_t|s_0)$ be the probability distribution function of the $A$'s identity $\upsilon_t$, where $\upsilon_t \in [\underline{\upsilon}, \overline{\upsilon}]$, $\underline{\upsilon} < \overline{\upsilon}$ and $\underline{\upsilon}, \overline{\upsilon} \in \mathbb{R}_+$. Assume that for any decision choice of $s_0$, $F_0(\upsilon_0|S) = F_0(\upsilon_0|0) = F_0(\upsilon_0)$. Socialization will reflect evolution of identity distribution through time, conditional to the choice of $s_0$.

We separate the socialization into two cases: socialization and conflict. The distribution of identity will evolve oppositely depending on the $P$'s $s_0$ investment strategy.

Thus for every value of $\upsilon_t = \upsilon^*$ when $s_0 = 0$ the distribution function at any period $t$ is stochastically dominated by the distribution function of the previous period $t - 1$. Alternatively for every value of $\upsilon_t$ when $s_0 = S$ the distribution function at any period $t$ dominates stochastically the distribution function of the previous period $t - 1$. This property is formally written as follows,

$$F_t(\upsilon_t = \upsilon^*|0) \geq F_{t-1}(\upsilon_{t-1} = \upsilon^*|0) \geq \cdots \geq F_0(\upsilon_0)$$

$$\geq \cdots \geq F_{t-1}(\upsilon_{t-1} = \upsilon^*|S) \geq F_t(\upsilon_t = \upsilon^*|S)$$
Figure 5: Identity. Stochastic Dominance.

Figure 6: Identity. Time Evolution of Densities.
Finally assume that $F_t(\upsilon_t|S)$ converges to put all the probability on the upper bound of the identity $\upsilon_t = \overline{\upsilon}$, and $F_t(\upsilon_t|0)$ converges to put all the probability on the lower bound of the identity $\upsilon_t = \underline{\upsilon}$.

$$
\lim_{t \to \infty} F_t(\upsilon_t|S) = \lambda \quad \text{where} \quad \lambda = \begin{cases} 
1 & \text{if } \upsilon = \overline{\upsilon} \\
0 & \text{otherwise}
\end{cases}
$$

and

$$
\lim_{t \to \infty} F_t(\upsilon_t|0) = 1, \text{ for every } \upsilon \in [\underline{\upsilon}, \overline{\upsilon}].
$$

Let $E_t[\upsilon_t|s_0]$ be the mathematical expectation in $t$ of the value of $\upsilon_t$ conditional to the incentive policy $s_0$. Implications of the $s_0$ conditioned stochastic dominance on $E_t[\upsilon_t|s_0]$:

$$
\forall t = 0, 1, ..., T, ... \quad E_t[\upsilon_{t+1}|0] < E_t[\upsilon_t|0]
$$

$$
\forall t = 0, 1, ..., T, ... \quad E_{t+1}[\upsilon_{t+1}|S] > E_t[\upsilon_t|S]
$$

$$
\forall t = 0, 1, ..., T, ... \quad E_t[\upsilon_t|0] < E_t[\upsilon_t|S]
$$

Where,

$$
E_t[\upsilon_t|s_0] = \int \overline{\upsilon} \upsilon f(\upsilon_t|s_0)d\upsilon_t
$$

A.2 Problem Solving

Let us now to simplify the notation in order to make algebraic operations easier. We relabel some variables of the model in order to do that. All changes are summarized in table 1.

Then we can rewrite the $P$’s problem as follows:
Utility from monetary payments: \( u_t(\overline{w}) = \overline{u} \); \( u_t(w) = u \)

Disutility from effort: \( \psi_t(\overline{v}, \nu_t(s_0)) = \psi_t \)

Intrinsic motivation \( \phi_t(\Delta w_t, m(s_t)) = \phi_t \)

\( \mathcal{P} \)'s revenue function: \( R_t(\overline{q}) = \overline{R} \); \( R_t(q) = R \)

Change of variables: \( \overline{w} = h(\overline{u}) \); \( w = h(u) \)

Probability variation: \( \Delta \theta = (\theta_1 - \theta_0) \)

Reservation utility:

<table>
<thead>
<tr>
<th>Table 1: Notational simplification</th>
</tr>
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\[
\text{Max}_{\{w_t(q_t), s_0\}} \theta_1 (\overline{R} - h(\overline{u})) - (1 - \theta_1)(\overline{R} - h(u)) - s_t \quad (A.1)
\]

Subject to

\[
\theta_1 \overline{u} + (1 - \theta_1)u - \psi_t + \phi_t \geq \theta_0 \overline{u} + (1 - \theta_0)u + \phi_t \quad \text{(ICC)} \quad (A.2)
\]

\[
\theta_1 \overline{u} + (1 - \theta_1)u - \psi_t + \phi_t \geq \overline{U} \quad \text{(PC)} \quad (A.3)
\]

Note that the \( \mathcal{P} \)'s objective function is now strictly concave in \( \overline{u} \) and \( u \), because \( h(\cdot) \) is strictly convex. The function \( u^{-1} = h(u) \) gives back ex post the monetary payments from utility levels. We have now linear constraints and a nonempty interior of the constrained set and therefore the problem is concave and the Kuhn-Tucker conditions are sufficient and necessary for characterizing optimality.

Letting \( \lambda \) and \( \mu \) be the non-negative multipliers associated respectively with the (ICC) and (PC) constraints. First-order conditions of this problem yield:

\[
\frac{1}{u'(\overline{w})} = \mu + \lambda \frac{\Delta \theta}{\theta_1} \quad (A.4)
\]

\[
\frac{1}{u'(w)} = \mu - \lambda \frac{\Delta \theta}{1 - \theta_1} \quad (A.5)
\]
The equations (9) and (10) jointly with (6) and (7) form a system of four equations with four variables \((\bar{w}, w, \mu, \lambda)\) which allows us to calculate the solution. Multiplying (9) by \(\theta_1\) and (10) by \((1 - \theta_1)\) and adding those two modified equations, we obtain,

\[
\mu = \frac{\theta_1}{u'(\bar{w})} + \frac{1 - \theta_1}{u'(\bar{w})} > 0 \tag{A.6}
\]

Hence, \(\mu > 0\) and the participation constraint (7) is binding. Using (11) and (9), we also obtain,

\[
\lambda = \frac{(1 - \theta_1)\theta_1}{\Delta \theta} \left( \frac{1}{u'(\bar{w})} - \frac{1}{u'(\bar{w})} \right) > 0 \tag{A.7}
\]

And the incentive compatibility constraint (6) is also binding. Thus we can obtain immediately the values of \(u(\bar{w})\) and \(u(w)\) by solving a system with two equations and two unknowns. The result is shown below,

\[
u_t(\bar{w}) = U - \phi_t(\Omega_t, m(s_0)) + \frac{(1 - \theta_0)}{\Delta \theta} \psi_t(\bar{v}, v_t(s_0)) \tag{A.8}
\]

\[
u_t(w) = U - \phi_t(\Omega_t, m(s_0)) - \frac{\theta_0}{\Delta \theta} \psi_t(\bar{v}, v_t(s_0)). \tag{A.9}
\]

### A.3 Proof of Proposition 1

Proof available from the authors upon request.
A.4 Proof of Proposition 2
Proof available from the authors upon request.

A.5 Proof of Proposition 3
Proof available from the authors upon request.

A.6 Proof of Proposition 4
Proof available from the authors upon request.

A.7 Effort effectiveness: some analysis on $\theta_i$.

Preliminary assumptions over $\theta_i$:

\[
P(q_t = \overline{q} | e_t = \overline{e}) = \theta_i \quad P(q_t = \overline{q} | e_t = \varepsilon) = \theta_0 \]
\[
P(q_t = q | e_t = \overline{e}) = 1 - \theta_i \quad P(q_t = q | e_t = \varepsilon) = 1 - \theta_0
\]

Assume also that performance is an informative signal about effort, $\theta_1 > \theta_0$. Further, results show that in any case, benchmark case or standard economic model, investment in motivational capital, and no-investment in motivational capital, the stochastic parameter $\theta_i$ only affects the stochastic payments $\tilde{w}_i(\tilde{q}_t)$.

Let us to take the standard model’s stochastic payments $\tilde{w}_t$ as start point. We analyze the impact of $\theta_0$ on both, $w = h(U - \frac{\theta_0}{\theta_i - \theta_0} \Psi)$ and $\overline{w} = h(U + \frac{1 - \theta_0}{\theta_i - \theta_0} \Psi)$. By definition $h'(\cdot) > 0$ and $h''(\cdot) > 0$.
\[
\frac{dw(q)}{d\theta_0} = h'(u) \frac{\partial(U - \frac{\theta_0}{\theta_1 - \theta_0} \Psi)}{\partial \theta_0} = -h'(u) \left[ \frac{\Psi \theta_1}{(\theta_1 - \theta_0)^2} \right] < 0
\]

The sign of this first derivative of \( h(u(q)) \) from \( \theta_0 \) is negative for any value \( \theta_0 \in [0, \theta_1] \). Then the low stochastic payment depends negatively from \( \theta_0 \).

Now we calculate the second derivative of \( w \) from \( \theta_0 \),

\[
\frac{d^2 w(q)}{d\theta_0^2} = \left[ -h''(u) \cdot \left( \frac{\Psi \theta_1}{(\theta_1 - \theta_0)^2} \right)^2 \right] + \left[ -h'(u) \cdot \left( \frac{2 \Psi \theta_1}{(\theta_1 - \theta_0)^3} \right) \right] < 0
\]

The second derivative is negative. Then, the value of the utility experienced from the low payment, decreases more quickly on \( \theta_0 \) as mean as the latter increases. Is straightforward to see that in the limit the low payment \( w \) converges to \( -\infty \) when \( \theta_0 \) goes to \( \theta_1 \)

\[
\lim_{\theta_0 \to \theta_1} h\left(U - \frac{\theta_0 \Psi}{\theta_1 - \theta_0}\right) = -\infty
\]

On the other hand, the first derivative on \( \theta_0 \) of the high payment is as follows,

\[
\frac{d\bar{w}(\bar{q})}{d\theta_0} = h'(\bar{u}) \cdot \frac{\partial u(\bar{w}(\bar{q}))}{\partial \theta_0} = h'(\bar{u}) \cdot \frac{\partial(U + \frac{(1-\theta_0)\Psi}{\theta_1 - \theta_0})}{\partial \theta_0} = h'(\bar{u}) \cdot \Psi \cdot \frac{(1 - \theta_1)}{(\theta_1 - \theta_0)^2} > 0
\]

The sign of the first derivative in the case of high payment, is positive. Then, as mean as the value of \( \theta_0 \) increases, the high payment also increases. The sign of the second derivative show whether the payment increases faster or slower as mean as \( \theta_0 \) increases.
Figure 7: Payments and the informative value of the signal

\[
\frac{d^2 \pi(\theta)}{d\theta_0^2} = \left[ h''(\bar{u}) \cdot \frac{\partial \left( \frac{(1-\theta_0)\Psi}{\theta_1-\theta_0} \right)}{\partial \theta_0} \right] \cdot \left( \Psi \frac{(1-\theta_1)}{(\theta_1-\theta_0)^2} \right) + \left[ h'(\bar{u}) \cdot \frac{2\Psi}{(\theta_1-\theta_0)^3} \right] = \]

\[
= h''(\bar{u}) \cdot \left( \Psi \frac{(1-\theta_1)}{(\theta_1-\theta_0)^2} \right)^2 + h'(\bar{u}) \cdot \frac{2\Psi}{(\theta_1-\theta_0)^3} > 0
\]

It is straightforward to see that the high payment is increasing in \( \theta_0 \) and this positive relation is also increasing in \( \theta_0 \). Then, when \( \theta_0 \) value converges to \( \theta_1 \), the high stochastic optimal payment converges to \( \infty \).

\[
\lim_{\theta_0 \to \theta_1} h \left( \bar{U} + \frac{(1-\theta_0)\Psi}{\theta_1-\theta_0} \right) = \infty
\]

Figure

Finally, we analyze how \( \Omega = \bar{w} - \underline{w} \) evolves with the informative value of the signal. That is, how the informative value of the signal, \( \Delta \theta = \theta_1 - \theta_0 \) affects the range and the amount of the incentives. As it is shown, as more close to \( \theta_1 \) is \( \theta_0 \), then lower will be \( \underline{w} \) and higher will be \( \bar{w} \), and therefore \( \Omega = \Delta w \) must be greater. Analogously, as more separate is \( \theta_0 \) from \( \theta_1 \), then higher will be \( \underline{w} \), lower will be \( \bar{w} \), and therefore \( \Omega = \Delta w \) must be lower. In conclusion, signal informativeness makes monetary incentives less necessary.

Now, assume that intrinsic motivation and identity plays a role in the agent behaviour.
Then, as it is shown in the section 3.2.4. of this paper, the problem solving payments are,

\[
\bar{w} = h(u(\bar{w})) = h \left( \bar{U} + \frac{(1 - \theta_0)}{\Delta \theta} \psi(\bar{e}, \nu(s_0)) \right)
\]

\[
\bar{w} = h(u(\bar{w})) = h \left( \bar{U} - \frac{\theta_0}{\Delta \theta} \psi(\bar{e}, \nu(s_0)) \right)
\].

Higher identity means more alignment between organizational or \(P\)'s goals and \(A\)'s goals. In the model more identity means that \(A\) experience a lower disutility from effort, which will be zero in the limit when agent achieve full identity.

\[
\psi_t(\bar{e}, \bar{\nu}) = 0 \quad (A.10)
\]

Thus, the incentives to elicit \(A\) to exert effort will not depend on the effectiveness of the signal \(\theta\), because the agent will choose \(e = \bar{e}\) action in every \(t\) due to his high identity and thus to link higher expected payments to the choice of \(e = \bar{e}\) action is not necessary.

A.8 Proof of Proposition 5

Proof available from the authors upon request.

A.9 Proof of Proposition 6

Proof available from the authors upon request.
References


