SHORT-RUN AND LONG-RUN EFFECTS OF BANKING IN A NEW KEYNESIAN MODEL

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Short-run and long-run effects of banking in a New Keynesian model

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Abstract

This paper introduces both endogenous capital accumulation and deposit-in-advance requirements for investment in the banking model of Goodfriend and McCallum (2007). Impulse response functions from technology and monetary shocks show some attenuation effect due to the procyclical behavior of the marginal finance cost. In addition, an adverse financial shock produces sizeable declines in output, inflation and interest rates. In the long-run analysis, we find the following effects of banking intermediation: (i) the stock of capital increases to take advantage of its collateral services, and (ii) consumption and labor fall in response to the finance cost attached to purchases of goods. Using the baseline calibrated model, we show how a 10% increase in banking efficiency would result in a permanent welfare gain equivalent to 0.3% of output.

Keywords: financial attenuator, financial shocks, welfare cost of banking.

JEL codes: E32, E43, E44.

1 Introduction

The latest financial crisis has triggered the need for a reformulation of the New Keynesian model in a way that incorporates banking elements. Recent papers, such as De Fiore and Tristani (2009), Nolan and Thoenissen (2009) and Cúrdia and Woodford (2010), examine the implications of adding a financial sector to the New Keynesian model. Also recently, the Journal of Monetary Economics has devoted two entire special issues to papers that study the financial crisis: the July 2009 issue, "Distress in Credit Markets: Theory, Empirics, and Policy", and the January 2010 issue, "Credit Market Turmoil: Implications for Policy". Earlier works

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by Goodfriend and McCallum (2007) and Christiano, Motto and Rostagno (2008) already discussed the introduction of financial aspects in the New Keynesian models months before the start of the financial crisis.

Most of these papers have in common the hypothesis of the "financial accelerator" that was put forth in the seminal paper by Bernanke, Gertler and Gilchrist (1999). The financial accelerator provides a connection between the financial sector and the real sector. It assumes that the finance cost of spending is affected by the stock of net wealth in the economy. In good economic times, the value of wealth tends to rise and banks can use it as collateral to produce more loans and cut the cost of borrowing. These better financial conditions amplify the demand expansion with an increase in purchases of consumption and investment goods. By contrast, when the economic scenario turns gloomy the value of collateral is likely to drop, banks reduce the amount of loans and the external finance cost rises. If the cut in loan production during the downturn economic phase is severe the economy may enter a credit crunch that leads to a demand contraction like the one lately observed in most industrialized countries. Therefore, the financial accelerator amplifies business cycle fluctuations.

This paper brings another contribution to the literature of New Keynesian models that incorporate a financial sector and banking elements. Thus, we extend the model structure developed by Goodfriend and McCallum (2007) with two novel features. First, there will be variable capital accumulation and capital adjustment costs in contrast with the constant-capital case assumed in Goodfriend and McCallum (2007). Secondly, the deposit-in-advance requirement that households must face in their optimizing program will include total spending on consumption goods and also a part of the spending on investment goods. These two extensions will have significant implications, documented throughout the paper, such as the substantial effects of variable capital accumulation in the business cycle analysis or the long-run effects on output and welfare of adding a financial constraint to investment spending.

The quantitative analysis of banking will be carried out from a twofold perspective. On the one hand, the dynamic equations of the New Keynesian model with and without banking elements will be compared and put into play through impulse response functions. This simulation exercise will give an idea of the quantitative implications of missing banking elements for business cycle analysis. On the other hand, the models will be solved in steady state for a variety of cases that differ in the level of banking activities. This second exercise will provide information about the long-run effects of banking elements on capital, output, consumption and welfare.

The rest of the paper is organized in five more sections. The description of the baseline model and the derivation of its dynamic equations are done in Section 2. Two variants of that model, one assuming constant capital and the other one dropping financial frictions and the banking sector, are briefly described in Section 3. The calibration of the parameters across models is carried out in Section 4 and it is used in Section 5 to compute the impulse-response functions in the short-run analysis. Section 6 is devoted to the long-run analysis by examining the implications of alternative banking scenarios in the steady-state solution of the models. Finally, Section 7 reviews the main conclusions reached in the paper.
2 A New Keynesian model with banking and variable capital

This section describes how to extend a standard New Keynesian model with the introduction of banking elements and endogenous capital accumulation. Most of the banking sector has been adapted from the model described by Goodfriend and McCallum (2007), henceforth referred to as GM (2007). In particular, there is a loan production technology, a deposit-in-advance constraint, and nominal interest rates for bonds, loans and interbank lending. One of the differences with respect to GM (2007) is the presence of variable capital and adjustment costs on capital changes. Thus, we introduce an investment function, used by Woodford (2003, chapter 5), that incorporates adjustment costs of changes in the stock of capital:

\[ x_t = I \left( \frac{k_{t+1}}{k_t} \right) k_t, \]

where \( x_t \) is the total amount of output spent in period \( t \) in order to increase the stock of capital from \( k_t \) to \( k_{t+1} \). Hence, \( I \left( \frac{k_{t+1}}{k_t} \right) \) is a generic convex function that determines the cost of installing next period’s capital, \( k_{t+1} \), per unit of the current stock of capital \( k_t \). In the detrended steady state, the adjustment cost function yields \( I(1) = \delta \), where \( \delta \) is the rate of capital depreciation. Moreover, the first and second derivatives of the adjustment cost function in the detrended steady state are \( I'(1) = 1 \) and \( I''(1) = \epsilon \), where \( \epsilon \) is the positive measure of the curvature on the convex adjustment costs. A very high \( \epsilon \) would be used to justify constant capital in the short-run as assumed in GM (2007), while a number approaching to 0 for \( \epsilon \) would result in a fully-flexible capital model without adjustment costs.

A second difference with respect to GM (2007) is that a fraction \( \tau \) of purchases of investment goods, \( x_t \), and all purchases of consumption goods, \( c_t \), must be carried out with nominal deposits, \( D_t \). Hence, there is a deposit-in-advance constraint that brings the amount of nominal deposits that must be turned over for financing nominal spending\(^2\)

\[ P_t c_t + \tau P_t x_t = V D_t, \quad (1) \]

where \( V \) is a constant velocity parameter and \( P_t \) is the aggregate price level. The limit case without any deposit requirement for purchases of investment goods (\( \tau = 0.0 \)) would be equivalent to the deposit-in-advance constraint assumed in GM (2007).

Meanwhile, the balance sheet of the typical bank defines total deposits simply as the sum of high-powered (base) money, \( H_t \), plus loans to households, \( L_t \), according to the following expression

\[ D_t = H_t + L_t. \]

Since base money is equal to bank reserves, the reserve coefficient \( rr \) determines the chosen fraction of total deposits kept as bank reserves, \( H_t = rr D_t \), which once substituted into the previous expression implies

\[ (1 - rr) D_t = L_t. \quad (2) \]

\(^1\)Sveen and Weinke (2007) also use this investment function.

\(^2\)Wang and Wen (2006) introduce a cash-in-advance constraint for purchases of both consumption and investment goods in a New Keynesian model. They find that having the cash-in-advance requirement extended to investment spending improves the hump-shaped pattern in the response of output to a monetary shock.
Combining (1) and (2) results in the following loan-in-advance constraint expressed in real terms

\[ c_t + r x_t = \frac{V}{1 - rt} \frac{L_t}{P_t} \]

Also following the GM (2007) model, the amount of loan production in real terms is provided by the banking technology

\[ \frac{L_t}{P_t} = L(.) = F(b_{t+1} + e^{A_3} v k_{t+1})^\alpha (e^{A_2} m_t^d)^{1-\alpha}, \]

where \( F > 0 \), \( 0 < v < 1 \) and \( 0 < \alpha < 1 \) are constant parameters and \( m_t^d \) denotes the demand for labor required to monitor the value of collateral at the bank. The parameter \( v \) penalizes the collateral service of capital relative to bonds due to the larger monitoring effort required to verify the physical condition and market value of the stock of capital. There are two loan-production shocks: \( A_2 \) shapes labor banking productivity, and \( A_3 \) affects the productivity of the stock of capital as collateral in loan production (which it could very well indicate situations of financial stress due to overvalued or undervalued capital).

Again as in GM (2007), households maximize intertemporal utility, that depends positively on consumption and leisure time, subject to both a conventional budget constraint and a loan-in-advance constraint to meet financial requirements. Household preferences are defined by a logarithmic utility function, separable between consumption and leisure, where future utility is brought to the current time by applying a constant discount factor per period, \( \beta \). Leisure is obtained by subtracting both types of labor from a normalized unit total time. In addition, households act as bankers: they can use the loan production technology to increase their deposits available for funding purchases. In turn, the optimizing program of the representative household is written as follows

\[
\max_{c_t, n_t, m_t, m_t^d, b_{t+1}, k_{t+1}} E_t \sum_{j=0}^{\infty} \beta^j [\phi \log c_{t+j} + (1 - \phi) \log (1 - n_{t+j} - m_{t+j})]
\]

subject to current and future budget constraints for \( j = 0, 1, 2, \ldots \)

\[
E_t \beta^j w_{t+j} (n_{t+j} + m_{t+j} - m_t^d) + r_t^k k_{t+j} + d_{t+j} + g_{t+j} - c_{t+j} - I \left( \frac{k_{t+1+j}}{k_{t+j}} \right) k_{t+j} - H_t / P_t + H_{t-1+j} / P_t - (1 + r_t^H)^{-1} b_{t+1+j} + b_{t+j} = 0,
\]

and to current and future deposit-in-advance constraints that incorporate loan production for \( j = 0, 1, 2, \ldots \)

\[
E_t \beta^j \left[ c_{t+j} + \tau I \left( \frac{k_{t+1+j}}{k_{t+j}} \right) k_{t+j} - \frac{V}{1 - rt} F(b_{t+1+j} + e^{A_3} v k_{t+1+j})^\alpha (e^{A_2} m_t^d)^{1-\alpha} \right] = 0.
\]

Let us introduce the new notation from the budget constraint. Households have two sources of labor income at the real wage rate \( w_t \): the amount \( w_t n_t \) from working in industrial firms, and the net amount \( w_t (m_t - m_t^d) \) from monitoring labor at the bank. As owners of the stock of capital, households receive the competitive real rental rate, \( r_t^k \), per unit of capital lent to the firms. Households are also owners of monopolistically competitive firms that will provide some real dividends, \( d_t \).

\(^3\) Goodfriend and McCallum (2007) integrate the production activities of the firms into the household optimizing program. In our setup, however, there are separate firms that make decisions on price setting, labor demand and capital demand. Provided the same production technology, market structure and pricing conditions both setups deliver identical dynamic equations for output, consumption, investment, capital, labor, inflation and the interest rates.
government transfers, $g_t$. Income is spent on purchases of consumption goods, $c_t$, on purchases of investment goods, $I_t \frac{k_{t+1}}{k_t}$, on net increases of real money, $H_t/P_t - H_{t-1}/P_t$, and on net purchases of government bonds, $(1 + r^B_t)^{-1} b_{t+1} - b_t$, where $b_{t+1}$ is the amount of bonds in real terms that are bought in period $t$ to be reimbursed in $t + 1$ with a real interest rate $r^B_t$. The first order conditions are

\begin{align*}
\phi - \lambda_t + \xi_t &= 0, \\
- \frac{1}{1 - \phi} - \lambda_t w_t &= 0, \\
- \lambda_t w_t - \xi_t \left( \frac{1 - \alpha (c_t + \tau x_t)}{m_t^d} \right) &= 0, \\
- \lambda_t (1 + r^B_t)^{-1} + \beta E_t \lambda_{t+1} - \xi_t \left( \frac{\alpha (c_t + \tau x_t)}{b_{t+1} + \tau c_{t+1} + \tau k_{t+1}} \right) &= 0, \\
- \lambda_t ((1 + r^B_t)^{-1}) + \beta E_t \lambda_{t+1} \left[ r^k_{t+1} + I' \left( \frac{k_{t+2}}{k_{t+1}} \right) \left( \frac{k_{t+2}}{k_{t+1}} \right) - I \left( \frac{k_{t+2}}{k_{t+1}} \right) \right] + \xi_t \left[ \tau I' \left( \frac{k_{t+2}}{k_{t+1}} \right) \left( \frac{k_{t+2}}{k_{t+1}} \right) + \tau I \left( \frac{k_{t+2}}{k_{t+1}} \right) \right] &= 0,
\end{align*}

where the partial derivatives of the loan production function $\frac{\partial L()}{\partial m_t^d}$, $\frac{\partial L()}{\partial b_{t+1}}$, and $\frac{\partial L()}{\partial k_{t+1}}$ were used respectively in the first order conditions for the optimal values of $m_t^d$, $b_{t+1}$, and $k_{t+1}$. There are two Lagrange multipliers, $\lambda_t$ and $\xi_t$, respectively attached to the budget constraint and the deposit-in-advance constraint. This model implies a relationship between these Lagrange multipliers, found by rearranging terms in the first order condition of $m_t^d$, which yields

$$
\xi_t = - \lambda_t \left( \frac{w_t m_t^d}{(1 - \alpha)(c_t + \tau x_t)} \right),
$$

that can be substituted in the first order condition of $c_t$ to obtain

$$
\lambda_t = \frac{\phi}{1 + \left( \frac{w_t m_t^d}{(1 - \alpha)(c_t + \tau x_t)} \right)}.
$$

The interpretation of (3) is clarifying for the role of banking in the model. As the shadow value of one unit of consumption, $\lambda_t$ is the consumption marginal utility divided by one plus the amount of output required to provide loan production that finances one extra unit of consumption. Hence, additional consumption requires more deposits, which may be raised through some increase in loan production, which can be made by employing more banking labor at the real wage rate. All is collected in the marginal finance cost, $\chi$, computed as follows

$$
\chi_t = w_t \frac{\partial m_t^d}{\partial L()} \frac{\partial D_t}{\partial c_t} = \frac{w_t m_t^d}{(1 - \alpha)(c_t + \tau x_t)},
$$

and included in (3) as part of the denominator of $\lambda_t$.

Both the stock of capital and the level of bonds are used as inputs in the loan production technology. Thus, the collateral services of bonds are included in the first order condition of $b_{t+1}$ listed above; where inserting $\xi_t = - \lambda_t \left( \frac{w_t m_t^d}{(1 - \alpha)(c_t + \tau x_t)} \right)$, we get

$$
\beta E_t \lambda_{t+1} = \lambda_t \left( \frac{1}{1 + r^B_t} - \frac{\alpha w_t m_t^d}{(1 - \alpha)(b_{t+1} + \tau c_{t+1}) + \tau k_{t+1}} \right).
$$

The optimality condition for the supply of banking labor, $m_t$, is not included because it is an identical expression to that for the supply of industrial labor, $n_t$. Using the loan production function, the deposit-in-advance constraint (1), and the reserve condition (2), the computation of the marginal finance cost gives $w_t \frac{\partial m_t^d}{\partial L()} \frac{\partial D_t}{\partial c_t} = w_t \frac{V m_t^d}{(1 - \tau)(1 - \alpha)(c_t + \tau x_t)} \frac{1 - \tau r}{P_t} = \frac{w_t m_t^d}{(1 - \alpha)(c_t + \tau x_t)}.$
As discussed in GM (2007), the marginal financial services of bonds can be measured by the savings of real income that can be obtained with the use of bonds in loan production. GM (2007) refer to this as the "liquidity service yield on bonds" and denote it as $LSY^B_t$. In formal terms, it would be

$$LSY^B_t = w_t \frac{\partial m_t^B}{\partial L_t} \frac{\partial L_t}{\partial b_{t+1}} = \frac{\alpha w_t m_t^B e^{\alpha B_{t+1}}}{(1-\alpha)(b_{t+1} + e^{\alpha r_{t+1}})}.$$  \ (6)

The value of $LSY^B_t$ implied by (6) can be used to rewrite equation (5) as follows

$$\beta E_t \lambda_{t+1} = \lambda_t \left( \frac{1}{1+\rho_t} - LSY^B_t \right).$$  \ (7)

Inserting (3) and using (4), and also their analogous expressions for period $t+1$, (7) is transformed into the following expression

$$\beta E_t \left[ \frac{\phi}{\alpha (1+\chi_{t+1})} \right] = \left( \frac{\phi}{\alpha (1+\chi_t)} \right) \left( \frac{1}{1+\rho_t} - LSY^B_t \right).$$  \ (8)

The loglinear approximation to (8) yields

$$\tilde{c}_t = E_t \tilde{c}_{t+1} - (\chi_t - E_t \chi_{t+1}) - \left[ (r^B_t - r^B_t) + (LSY^B_t - LSY^B_t) \right],$$  \ (9)

where variables topped with a hat symbol denote log deviations from the reference values obtained in the steady-state solution of the model and terms with no time subscript represent such steady-state values.\footnote{6} Consumption dynamics are forward-looking and depend negatively on the difference between the current real interest rate of the bond and its steady-state rate, $r^B_t - r^B$, the difference between the current marginal finance cost of consumption and its expected next period’s value, $\chi_t - E_t \chi_{t+1}$, and the difference between marginal collateral service of bonds and its steady-state level, $LSY^B_t - LSY^B$. Remarkably, (9) collapses to the canonical equation for intertemporal consumption decisions, $\tilde{c}_t = E_t \tilde{c}_{t+1} - (r^B_t - r^B)$, when dropping banking-related terms ($\chi_t = \chi_{t+1} = LSY^B_t = 0$).

A semi-loglinear approximation to (4) can be used to obtain the steady-state deviation for the marginal finance cost

$$\chi_t - \chi = \chi \left( \tilde{w}_t + \tilde{m}_t - \frac{c_{t+1}}{c_t} \tilde{c}_t - \frac{c_{t+1}}{c_t} \tilde{c}_t \right).$$  \ (10)

Meanwhile, changes in $LSY^B_t$ can be explained by the semi-loglinearized expression obtained from equation (6), assuming that the stock of bonds is exogenous and fixed at a constant level $b$, which turns out as follows

$$LSY^B_t - LSY^B = LSY^B \left( \tilde{w}_t + \tilde{m}_t - \frac{vk}{b+k} \tilde{c}_{t+1} - \frac{vk}{b+k} A_{3t} \right).$$  \ (11)

For capital accumulation and investment dynamics, the relationships between the Lagrange multipliers $\xi_t = -\lambda_t \chi_t$ and $\xi_{t+1} = -\lambda_{t+1} \chi_{t+1}$, and the definition of the liquidity service yield on capital $LSY^k_t = w_t \frac{\partial m^k_t}{\partial L_t} \frac{\partial L_t}{\partial k_{t+1}} = \frac{\alpha w_t m^k_t e^{\alpha B_{t+1}}}{(1-\alpha)(b_{t+1} + e^{\alpha r_{t+1}})}$ can be substituted in the first order condition of next period’s stock of capital, $(k_{t+1})$, to obtain

$$-\lambda_t \left( 1 + \tau \chi_t \right) I' \left( \frac{k_{t+1}}{k_{t+1}} \right) + \beta E_t \lambda_{t+1} \left[ r^k_{t+1} + (1 + \tau \chi_{t+1}) \left( I' \left( \frac{k_{t+2}}{k_{t+1}} \right) \left( \frac{k_{t+2}}{k_{t+1}} \right) - I \left( \frac{k_{t+2}}{k_{t+1}} \right) \right) \right] + \lambda_t LSY^k_t = 0.$$  \ (12)

\footnote{6}{As a standard procedure, we used the approximation log(1+x) ≈ x when x is a small number. In addition, we took the approximation $\frac{1}{1+\rho_t} = LSY^B_t \approx \frac{1 - LSY^B_t}{1 + \rho_t}$, observing that $r^B_t LSY^B_t$ is negligible for being a very small number.}

\footnote{7}{This notation is continuously used throughout the rest of the paper.}
Next, equation (7) can be used in (12) to drop the lambdas and reach the following expression

\[-(1 + \tau \chi_t) I^c \left( \frac{k_{t+1}}{k_t} \right) + \left( \frac{1}{1 + r_t} - LSY_t^B \right) E_t \left[ \frac{k_{t+1}}{k_t} + (1 + \tau \chi_{t+1}) \left( I^c \left( \frac{k_{t+1}}{k_t} \right) \right) \right] + LSY_t^k = 0. \tag{13}\]

A log-linear approximation to (13) gives

\[\hat{k}_{t+1} = k_1 \hat{k}_t + (1 - k_1) E_t \hat{k}_{t+2} - k_2 \left[ (r^B_t - r^B) + (LSY^B_t - LSY^{B^*}_t) \right] + k_3 \left[ \beta E_t \left( \frac{k_{t+1}}{k_t} - r^B \right) + (LSY^k_t - LSY^k \right] - k_4 \left[ (\chi_t - \chi) - \beta (1 - \delta) E_t (\chi_{t+1} - \chi) \right], \tag{14}\]

with \(k_1 = \frac{1}{\delta}, k_2 = \frac{1 + \tau \chi - LSY^B}{\epsilon (1 + \tau \chi)(1 - \beta (1 - \delta))}, k_3 = \frac{1}{\epsilon (1 + \tau \chi)(1 - \beta (1 - \delta))}, k_4 = \frac{\tau}{\epsilon (1 - \beta (1 - \delta))}\). If the banking-related terms are dropped (\(\chi_t = \chi_{t+1} = LSY^B_t = LSY^k = 0\)), the capital accumulation (14) becomes that of Woodford (2003, chapter 5)’s model

\[\hat{k}_{t+1} = k_1 \hat{k}_t + (1 - k_1) E_t \hat{k}_{t+2} - \frac{k_1}{\epsilon} \left[ (r^B_t - r^B) - \beta E_t \left( \frac{k_{t+1}}{k_t} - r^B \right) \right].\]

It should be noticed that the liquidity service yield on capital, \(LSY^k_t\), is close to the homonymous on bonds as implied by the relationship \(LSY^k_t = LSY^{B^*}_t e^{A3_{it}}\). In semi-loglinear terms, it is obtained

\[LSY^k_t - LSY^k = LSY^k \left( \tilde{\omega}_t + \tilde{m}_t - \frac{\nu_k}{b + \delta} \hat{k}_{t+1} + \frac{b}{b + \delta} A3_t \right). \tag{15}\]

Fluctuations of investment are driven by changes in the stock of capital as indicated by the loglinear version of the investment definition, \(x_t = I \left( \frac{k_{t+1}}{k_t} \right) k_t\), which yields

\[\tilde{x}_t = \frac{1}{\delta} \hat{k}_{t+1} - \frac{1 - \delta}{\delta} \hat{k}_t. \tag{16}\]

Finally, output fluctuations are demand-determined as the weighted average of consumption and investment that is obtained in the log-linearized overall resources constraint\(^8\)

\[\tilde{y}_t = \frac{\alpha}{\beta} \tilde{c}_t + \frac{\delta_k}{\beta} \tilde{x}_t. \tag{17}\]

Summarizing, the Aggregate Demand block of the model depicts IS-style dynamic fluctuations of spending on consumption (equation 9) and on investment (equations 14 and 16) as the endogenous determinants of expenditure-driven output (equation 17). There are changes in the marginal finance cost of consumption (equation 10) and the marginal collateral services of either bonds (equation 11) or capital (equation 15) that affect private spending decisions. The marginal finance cost, \(\chi_t\), has a negative influence on current consumption and investment spending as a consequence of the cost of increasing the amount of loans to fund the additional purchases. It is also important the role of the liquidity services of asset holdings, \(LSY^B_t\) and \(LSY^k\), which increase the overall returns on bonds and capital respectively. Thus, \(LSY^B_t\) has a negative impact on purchases of both consumption (see equation 9) and capital accumulation (see equation 14) from the higher opportunity cost of buying bonds. By contrast, the liquidity service yield on capital, \(LSY^k\), rises

\(^8\)The overall resources constraint \(\gamma_t = c_t + x_t\) can be reached by inserting the government budget constraint, \(g_t = H_t / P_t + H_{t-1} / P_t - (1 + r^1_t)^{-1} b_{t+1} + b_t\), in the household budget constraint, substituting the aggregate dividends by the sum of profits across all firms, \(d_t = \int_0^1 f_t d_t(i) di = \int_0^1 (P_t(i) g_t(i)) / P_t - w_t m_t(i) - r^1_t k_t(i) di\), and also using the equilibrium condition on banking labor, \(m_t = n_t^d\).
the final return on the stock of capital and, therefore, enters the capital accumulation equation (14) with a positive sign.

On the supply side, we just follow the standard New Keynesian literature. Hence, monopolistically competitive firms operate by setting prices and supplying a differentiated good as in Dixit and Stiglitz (1977). Price stickiness is introduced assuming that the optimal price can be set only depending upon the outcome of a Calvo (1983)-type lottery. There is a constant probability, \( \pi \), which determines market conditions under which the firm cannot set the optimal price. As shown in Walsh (2003, chapter 5), changes in the rate of inflation from its steady-state rate are determined by the forward-looking New Keynesian Phillips curve

\[
\pi_t - \pi = \beta E_t (\pi_{t+1} - \pi) + \frac{(1-\beta)(1-\pi)}{\pi} \tilde{\psi}_t, \tag{18}
\]

where \( \tilde{\psi}_t \) represents loglinear fluctuations of the real marginal cost of production obtained in the following way

\[
\tilde{\psi}_t = \tilde{w}_t - (\tilde{y}_t - \tilde{k}_t). \tag{19}
\]

Output is produced with a Cobb-Douglas technology that uses labor and capital as inputs. With a labor-augmenting technology shock (denoted as \( A_{1_t} \)), the loglinearized production function of the model is

\[
\tilde{y}_t = \eta \tilde{k}_t + (1-\eta) \tilde{n}_t + (1-\eta) A_{1_t}, \tag{20}
\]

where \( \eta \) is the capital-share parameter. Firms demand labor and capital in competitive factor markets. The demand for capital makes the real rental rate of capital in equilibrium equal to the marginal product of capital multiplied by the real marginal cost, which in semi-loglinear terms implies

\[
\frac{1}{\pi} (r^k_t - r^k) = \tilde{\psi}_t + \left( \tilde{y}_t - \tilde{k}_t \right). \tag{21}
\]

Meanwhile, the equilibrium real wage can be determined in the labor supply curve. The supply of industrial labor services derived above implies that \( \lambda_t w_t = -\frac{1}{1-n_1-m_1} \) where the Lagrange multiplier is \( \lambda_t = \frac{\phi}{c_t} (1+\chi_t)^{-1} \). It leads to the following log-linear equation for fluctuations of the competitive real wage

\[
\tilde{w}_t = n \frac{n}{1-n-m} \tilde{n}_t + m \frac{m}{1-n-m} \tilde{m}_t + \tilde{c}_t + \chi_t, \tag{22}
\]

where \( n \) and \( m \) are respectively the steady-state shares of time spent on industrial labor and banking (monitoring) labor.

Hence, the Aggregate Supply sector of the model (equations 18-22) provides inflation dynamics driven by the standard forward-looking New Keynesian Phillips curve. Inflation evolves depending on current and expected future fluctuations of the real marginal cost. The only new element is the presence of the marginal finance cost, \( \chi_t \), in the labor supply curve (22). A rise in \( \chi_t \) would cut the amount of labor supplied by the household as a consequence of a higher cost on the funding required for purchases of consumption goods.

Finally, the Monetary block of the model contains equations that determine fluctuations of loans and the variety of nominal interest rates of the model. The deposit-in-advance constraint of the households can be loglinearized to obtain the demand for real loans

\[
\tilde{L}_t = \frac{c}{c+\tau_2} \tilde{c}_t + \frac{\tau_2}{c+\tau_2} \tilde{x}_t, \tag{23}
\]
where \( \hat{I}_t \) is provided by the log-linearized loan production function (assuming a constant level of bonds exogenously issued by the government)

\[
\hat{I}_t = \frac{\alpha k}{1 + \alpha} \hat{I}_{t+1} + \frac{\alpha k}{1 + \alpha} A3_t + (1 - \alpha) \hat{m}_t + (1 - \alpha) A2_t. \tag{24}
\]

The nominal interest rate of bonds is the corresponding real interest rate plus expected inflation from the Fisher-type relationship

\[
R^B_t = r^B_t + E_t \pi_{t+1}, \tag{25}
\]

while the nominal return on the physical capital is equal to the real rental rate on capital plus expected inflation

\[
R^k_t = r^k_t + E_t \pi_{t+1}. \tag{26}
\]

The central bank sets the interbank nominal interest rate, \( R^{IB}_t \), to stabilize inflation and the output gap. Following the rule included in GM (2007), we modify the famous Taylor (1993)'s rule to incorporate a component of interest-rate smoothing and the response to fluctuations of the real marginal cost as a proxy of the output gap

\[
R^{IB}_t - R^{IB} = (1 - \mu_3) \left[ \mu_1 (\pi_t - \pi) + \mu_2 \hat{\psi}_t \right] + \mu_3 \left( R^{IB}_{t-1} - R^{IB} \right) + \varepsilon_t, \tag{27}
\]

where \( \varepsilon_t \) is a white-noise monetary policy shock. As in GM (2007), a fictitious bond that does not provide collateral services represents the benchmark bond of a conventional New Keynesian model with no banking elements. Dropping the collateral services of bonds \( (LSY^B_t = 0) \) leaves the first order condition of bonds as

\[
-\lambda_t (1 + r^T_t)^{-1} + \beta E_t \lambda_{t+1} = 0
\]

where \( r^T_t \) is the real interest rate of such bond with no collateral capacity. If we compare this result with the actual optimality condition of bonds with collateral services (equation 7), it is easy to reach

\[
\frac{1}{1+r^T_t} = \frac{1}{1+r^B_t} - LSY^B_t,
\]

that can be fairly approximated by the expression

\[
\frac{1}{1+r^T_t} = \frac{1-LSY^B_t}{1+r^B_t}.
\]

Since the rates of return \( r^B_t, r^T_t \) and \( LSY^B_t \) are small numbers relative to one, we can find an intuitive expression that determines the real interest rate of a purely intertemporal security with no collateral power as the sum of the market real return of bonds plus their liquidity service yield

\[
r^T_t = r^B_t + LSY^B_t,
\]

where using the Fisher relation leads to the analogous expression in nominal terms

\[
R^T_t = R^B_t + LSY^B_t. \tag{28}
\]

The uncollateralized interest rate of loans must coincide with the rate of return of bonds that do not provide collateral services, \( R^T_t \). Quoting GM (2007): "This reflects a no-arbitrage condition between the loan market and the asset market". Therefore, \( R^T_t \) also represents the nominal interest rate on uncollateralized loans.
Next, it is assumed that, in their banking activity, households can borrow funds from the central bank at the interbank nominal interest rate, $R_{IB}^B$, that they could lend to other households. In the case of uncollateralized loans the interest rate on those loans, $R^T$, should take into account both the borrowing cost $R_{IB}^B$ and the cost of producing the additional loans. The latter is determined by the marginal cost of loan production $w_t \frac{dV}{dmd_t}$, that is proportional to the marginal finance cost $\chi_t$, $w_t \frac{dV}{dmd_t} = \frac{V}{1-\tau \gamma} (1-\alpha)(c_t+\tau x_t)$, that is proportional to the marginal finance cost $\chi_t$. Consequently, the equilibrium condition is

$$1 + R^T = (1 + R_{IB}^B) \left(1 + \frac{V}{1-\tau \gamma} \chi_t\right),$$

where taking logs and assuming that the rates of return are small relative to one, it is obtained

$$R^T_t = R_{IB}^B + \frac{(1-\alpha)V}{1-\tau \gamma} \chi_t. \tag{29}$$

The nominal interest rate on collateralized loans, $R^L$, must be lower than $R^T$ because borrowers provide collateral services as owners of bonds and capital. In that case, banking activity only employs the labor cost of producing the loan. Given the loan production function at hand (with constant returns to scale), the monitoring labor cost $w_t m_t$ is a constant share $(1-\alpha)$ of total cost of loan production. Thus, the marginal cost of loan production is cut by $(1-\alpha)$ and the nominal interest rate on collateralized loans, $R^L$, is determined by the marginal cost equal to marginal income condition

$$1 + R^L = (1 + R_{IB}^B) \left(1 + \frac{(1-\alpha)V}{1-\tau \gamma} \chi_t\right),$$

that, after taking a log approximation, results in

$$R^L_t = R_{IB}^B + \frac{(1-\alpha)V}{1-\tau \gamma} \chi_t. \tag{30}$$

The spread between the borrowing rate and the lending rate represents the collateralized external finance premium (CEFP) highlighted in the GM (2007) model

$$CEFP_t = R^L_t - R_{IB}^B = \frac{(1-\alpha)V}{1-\tau \gamma} \chi_t.$$

Remarkably, the CEFP is proportional to the marginal finance cost $\chi_t$; it reflects the proximity of both measures of the cost attached to financial intermediation.

In summary, our New Keynesian model with banking activities and variable capital includes these dynamic equations:

- seven equations (9-11 and 14-17) that belong to the Aggregate Demand sector,
- five equations (18-22) that belong to the Aggregate Supply sector,
- and eight equations (23-30) that belong to the Monetary block.

The system of twenty equations may provide solution paths for the following twenty endogenous variables: $y_t, c_t, x_t, k_{t+1}, \tilde{n}_t, \bar{n}_t, \tilde{m}_t, \bar{m}_t, \tilde{w}_t, \bar{w}_t, \tilde{\psi}_t, \bar{\psi}_t, \chi_t, LSY^B_t, LSY^k_t, \pi_t, v^B_t, R^B_t, v^k_t, R^k_t, R_{IB}^B, R^T_t, R^L_t$, and $10$. The stock of bonds is assumed to be exogenous and constant at the steady-state level $b$. In addition, there are two predetermined variables, $k_t$ and $R_{t-1}^IB$, and four exogenous variables, $A_{1t}, A_{2t}, A_{3t}$, and $\varepsilon_t$. 10
3 Alternative models

Within the class of New Keynesian models with sticky prices, we present now two more models for the quantitative analysis on the implications of variable capital, financial intermediation and banking activities. One model variant is the model of GM (2007). The comparison between the baseline model of Section 2 and the GM model explains the consequences of adding both variable capital accumulation and partial financial frictions on investment spending.

The second alternative model abstracts from a banking sector because it assumes no financial frictions. We will refer to this model as the "New Neoclassical Synthesis" (NNS) model. For comparative purposes, the NNS model is equivalent to the cashless economy described by Woodford (2003, chapter 5) with variable capital accumulation and adjustment costs as incorporated to the baseline model of Section 2. Therefore, the comparison between the baseline model and the NNS model can be used to discuss the consequences of financial constraints and banking activities in one economy with sticky prices and variable capital.

3.1 The GM model

GM (2007) assume that capital is constant and no deposit holding is required to fund investment purchases. Hence, the GM model can be recovered from our baseline setup by assuming that \( \tau = 0 \) in the deposit-in-advance constraint and the stock of capital remains constant in the short-run at the steady-state level \( k \). Despite having a constant capital, investment spending is variable because the GM model contemplates variability in the relative price of capital goods, denoted by \( q \). The budget constraint and the deposit-in-advance constraint of the GM model can therefore be written as follows

\[
\begin{align*}
\phi_t - \lambda_t + \xi_t &= 0, \\
-\frac{1-\phi}{1-n_t-m_t} + \lambda_t w_t &= 0, \\
-\lambda_t w_t - \xi_t \frac{(1-\alpha) c_t}{m_t} &= 0, \\
-\lambda_t (1 + r_t^B)^{-1} + \beta E_t \lambda_{t+1} - \frac{\alpha c_t}{b_{t+1} + e^{At} v q_{t+1} k_{t+1}} &= 0, \\
-\lambda_t q_t + \beta E_t \lambda_{t+1} q_{t+1} (1 - \delta + r_{t+1}^k) - \frac{\alpha e^{At} v q_{t+1}^3}{b_{t+1} + e^{At} v q_{t+1} k_{t+1}} &= 0.
\end{align*}
\]

From the first order condition of \( m_t^d \), it is obtained a relationship between the Lagrange multipliers, \( \lambda_t \) and \( \xi_t \),

\[
\xi_t = -\lambda_t \frac{m_t^d}{(1-\alpha) c_t}.
\]

Such name for the model without banking is used in Goodfriend and McCallum (2007). There are no banking elements and purchases of consumption and investment goods do not require any holdings of deposits or loan production.
which it is substituted into the first order condition of capital to imply

$$\lambda_t = \frac{\phi}{1 + \frac{w_{t} m_{t}^{d}(1-\alpha)c_t}{c_t}}. \quad (3')$$

As in the model with variable capital, the shadow value of one unit of consumption, $\lambda_t$, is the consumption marginal utility divided by one plus the marginal finance cost. The difference is found when holding the case $\tau = 0$ in the computation of the marginal finance cost

$$\chi_t = w_{t} \frac{\partial m_{t}^{d}}{\partial L(.)} \frac{\partial D_{t}}{\partial c_t} = \frac{w_{t} m_{t}^{d}(1-\alpha)c_t}{c_t}. \quad (4')$$

The liquidity service yield on bonds, $LSY_t^B$, is also slightly different in the GM model which incorporates the relative price of capital $q_t$. Using its definition, $LSY_t^B = w_{t} \frac{\partial m_{t}^{d}}{\partial L(.)} \frac{\partial c_t}{\partial k_{t+1}}$, it gives:

$$LSY_t^B = \frac{\alpha w_t m_t^{d}}{(1-\alpha)(b_{t+1} + c_{t}^{\lambda_{t+1}}v_{q}k_{t+1})}. \quad (6')$$

The intertemporal allocation of consumption is determined by the consumption equation that results from the first order condition of bonds and equations (3') and (4'). After loglinearization, it results in the same equation (9) as in the baseline model with variable capital

$$\hat{c}_t = E_t \hat{c}_{t+1} - (\chi_t - E_t \chi_{t+1}) - \left[(r_t^B - r^B_t) + (LSY_t^B - LSY_t^B)\right], \quad (9)$$

with different definitions for changes in the marginal finance cost (which does not depend on investment)

$$\chi_t - \chi = \chi (\hat{w}_t + \hat{m}_t - \hat{c}_t), \quad (10')$$

and in the $LSY_t^B$ that now depends upon the relative price of capital

$$LSY_t^B - LSY^B = LSY^B \left(\frac{\hat{w}_t + \hat{m}_t - \frac{v_k}{b^r v_k} \hat{q}_t - \frac{v_k}{b^r v_k} A3_t}{(1-\alpha)(b_{t+1} + c_{t}^{\lambda_{t+1}}v_{q}k_{t+1})}\right), \quad (11')$$

while both the stock of capital and the stock of bonds are held at their constant steady-state levels.

The dynamic equation for the relative price of capital, $q_t$, can be reached by inserting $\xi_t = -\lambda_t \frac{w_t m_t^{d}}{(1-\alpha)c_t}$ in the first order condition of next-period capital, $(k_{t+1})$, and using the definition of the liquidity service yield on capital $LSY_t^k = \frac{\alpha w_t m_t^{d}}{(1-\alpha)(b_{t+1} + c_{t}^{\lambda_{t+1}}v_{q}k_{t+1})}$ to reach

$$-\lambda_t q_t + \beta E_t \lambda_{t+1} q_{t+1} (1 - \delta + r_{k_{t+1}}^k) + \lambda_t LSY_t^k = 0.$$  

Next, the first order condition of bonds and the definition of $LSY_t^B$ are plugged into the previous expression to obtain

$$-q_t + \left(\frac{1}{1+r_t^B} - LSY_t^B\right) E_t q_{t+1} (1 - \delta + r_{k_{t+1}}^k) + LSY_t^k = 0,$$

which results in the following equation for log fluctuations of $q$

$$\hat{q}_t = (1 - LSY_t^k) \left[E_t \hat{q}_{t+1} + E_t (r_{k_{t+1}}^k) - (r_t^B - r^B) - (LSY_t^B - LSY_t^B)\right] + LSY_t^k (LSY_t^k - LSY_k^k). \quad (14')$$

The relative price of capital rises with an increase in its expected next-period value, the expected next-period rental rate on capital and the liquidity service yield on capital. As opportunity costs, the market return on
bonds and the their liquidity service yield have a negative impact on the demand for capital and its relative price. Assuming constant capital and bonds, the semi-loglinear expression for fluctuations of $LSY_t^k$ is in the GM model

$$LSY_t^k - LSY^k = LSY^k \left( \hat{w}_t + \hat{m}_t - \frac{w_k}{b + v k} \hat{q}_t + \frac{b}{b + v k} A3_t \right). \quad (15')$$

Even though the stock of capital is held constant, the variability of $\hat{q}_t$ gives rise to fluctuations in investment spending,

$$x_t = q_t (k_{t+1} - (1 - \delta) k_t) = \delta k q_t.$$ After loglinearization, log fluctuations of investment become proportional to those of the relative price of capital

$$\hat{x}_t = \hat{q}_t. \quad (16')$$

In the end, the overall resources constraint of the GM model indicates that output is spend on either consumption or investment. The log-linear version of this overall resources constraint is

$$\hat{y}_t = \frac{c}{c + \delta k} \hat{c}_t + \frac{\delta k}{c + \delta k} \hat{q}_t, \quad (17')$$

recalling that, as in GM (2007), $q$ is equal to one in steady state.

Thus, the Aggregate Demand sector of the GM model consists of seven equations (9, 10', 11', 14'-17') that are comparable to the Aggregate Demand block of the model with variable capital. Under common assumptions defining production technology, market structure, price stickiness, monetary policy and loan production, the Aggregate Supply sector and the Monetary block of the GM model will be equivalent to that of the baseline model provided that $k$ is constant and $q$ is introduced in the loan production technology. The complete set of dynamic equations of the GM model is displayed in the technical appendix.

### 3.2 A New Keynesian model without banking (NNS model)

Both financial frictions and banking activities are dropped from the setup introduced in Section 2 to obtain a version of the canonical New Keynesian model with variable capital and adjustments costs (Woodford, 2003, chapter 5). Adopting the log utility function specification from above, the optimizing program of the household would be written as follows

$$\max_{c_t, n_t, b_{t+1}, k_{t+1}} E_t \sum_{j=0}^{\infty} \beta^j \left[ \phi \log c_{t+j} + (1 - \phi) \log (1 - n_{t+j}) \right]$$

subject to current and future budget constraints

$$\begin{align*}
E_t \beta^j [w_{t+j} n_{t+j} + r_{t+j}^k k_{t+j} + d_{t+j} + q_{t+j} - c_{t+j} - I \left( \frac{k_{t+1+j}^t}{k_{t+j}} \right) k_{t+j} - (1 + r_{t+j}^B)^{-1} b_{t+1+j} + b_{t+j}] &= 0, \text{ for } j = 0, 1, 2, \ldots
\end{align*}$$

There is no deposit-in-advance requirement. The set of first order conditions includes

$$\begin{align*}
\frac{\phi}{c_t} - \lambda_t &= 0, \quad (c_t) \\
-\frac{1 - \phi}{1 - n_t} + \lambda_t w_t &= 0, \quad (n_t) \\
-\lambda_t (1 + r_t^B)^{-1} + \beta E_t \lambda_{t+1} &= 0, \quad (b_t + 1) \\
-\lambda_t I' \left( \frac{k_{t+1}^t}{k_t^t} \right) + \beta E_t \lambda_{t+1} \left[ r_{t+1}^k + I' \left( \frac{k_{t+2}^t}{k_{t+1}^t} \right) \left( \frac{k_{t+2}^t}{k_{t+1}^t} \right) - I \left( \frac{k_{t+2}^t}{k_{t+1}^t} \right) \right] &= 0, \quad (k_t + 1)
\end{align*}$$

13
where $\lambda_t$ is the (only) Lagrange multiplier, attached to the budget constraint in period $t$. Obviously, the NNS model would abstract from the marginal finance cost and the liquidity service yield on either bonds or capital because it does not consider any deposit-in-advance constraint for private spending.

The supply-side equations are obtained from the optimizing behavior of a monopolistically competitive firm that faces a Calvo-type rigidity when setting prices, in a way described above for the baseline model. The Monetary block only brings one equation for the nominal interest rate decided by the central bank in application of a stabilizing monetary policy rule identical to (27). The complete set of equations of this NNS model is displayed in the technical appendix.

4 **Calibration**

The numerical calibration of parameters is required for the economic analysis carried out in the upcoming sections. In that regard, most of the numbers are borrowed from GM (2007), while some others are assigned at numbers that result in reasonable business cycle properties of the models. The calibration is made assuming that time units represent quarters. Table 1 provides the calibration of parameters across the three models used in the paper.

The value assigned to the subjective discount factor of the household, $\beta$, is jointly determined by the rate of intertemporal preference ($\rho$) and the rate of long-run economic growth ($\gamma$) as $\beta = [(1 + \rho)(1 + \gamma)]^{-1}$. Assuming a 2% long-run economic growth per year ($\gamma = 0.005$) and a 4% annual rate of intertemporal preference ($\rho = 0.01$) leads to a value of $\beta = 0.985$, also chosen in GM (2007). The parameter that determines the weight of the log utility of consumption in the utility function, $\phi$, is set at the value that is required to find that leisure takes one third of total time in the steady-state solution of the model. It turns out to have $\phi$ close to 0.40 in the three models.

The production function is parameterized with a value of the capital share at $\eta = 0.36$ as assumed in the real business cycle literature (Kydland and Prescott, 1982; Cooley and Hansen, 1989). The rate of depreciation on capital is $\delta = 0.025$, which implies a 10% annualized capital depreciation as also typically assumed in the real business cycle literature. The elasticity in the adjustment cost function is set at $\epsilon = 3.0$, which is the value suggested in Woodford (2003, chapter 5).

The Dixit-Stiglitz elasticity in the demand curve is $\theta = 11.0$, so as to conveys a 10% mark-up of prices over marginal costs in steady state as assumed in GM (2007). Price stickiness is determined by the Calvo probability fixed at $\varpi = 0.75$, which leads to having an average frequency of posting optimal prices equal to one time a year as found empirically plausible by Blinder (1994) and Taylor (1999).\(^{10}\)

\(^{10}\)The Calvo probability at $\varpi = 0.75$ brings about a slope of the New Keynesian Phillips curve (18) equal to 0.087, not far from 0.05 in the benchmark calibration of Goodfriend and McCallum (2007).
The macroeconomic stabilizing role of the central bank was incorporated to the model through the Taylor (1993)-type monetary policy rule (27). Following the calibration of GM (2007), the reaction coefficients for deviations of the rate of inflation and output from their long-run values are the ones recommended by Taylor (1993), $\mu_1 = 1.5$ and $\mu_2 = 0.5$, with long interest-rate inertia setting the smoothing coefficient at $\mu_3 = 0.8$. Also as in the GM (2007) calibration, the stock of bonds in steady state represents 56% of consumption, $b = 0.56c$, in order to match the value observed in the US economy in the third quarter of 2005; while the bank reserve coefficient is $rr = 0.005$ to pick up the US ratio of total bank reserves to M3. The fraction $\tau$ of purchases of investment goods (specific from the baseline model) is fixed at $\tau = 0.81$ to replicate the average ratio of loans to investment observed in US data.$^{11}$ Such a high percentage of investment purchases subject to financial constraints is also suggested in Wang and Wen (2006). The velocity parameter $V$ takes the value consistent with the number chosen in GM (2007) in the particular case of lack of financial requirements for

$^{11}$The average ratio of the stock of real loans to quarterly investment is 9.95 in the US data over the period 1996-2009 and also in the steady-state solution of the baseline model. For US data, we took the series of "Total Loans and Investments at All Commercial Banks", the "GDP Implicit Price Deflator" and "Real Gross Private Domestic Investment". Source: FRED database elaborated by the Federal Reserve Bank of St. Louis.
investment \( (\tau = 0) \). It brings the formula \( V = 0.31 (1 + \tau x/e) \); which gives \( V = 0.31 \) in the GM model (as calibrated in GM, 2007) and \( V = 0.38 \) in the baseline model.

The three parameters of the loan production technology, \( \alpha, v \) and \( F \), are calibrated under the same criteria as in GM (2007). Therefore, they are jointly set to best approximate the following three conditions: (i) match the observed 1\% per year average short-term real "riskless rate" in the US with the model rates \( R^{LB} \) and \( R^B \) in steady state, (ii) match a 2\% average spread of the loan rate over the federal funds rate in postwar US data with \( CEFP \) in steady state, and (iii) a share of US total employment in depository credit intermediation of 1.6\% as reported by the Bureau of Labor Statistics with \( \frac{m}{n+m} = 0.016 \) in steady state. In turn, the baseline model takes the setting \( \alpha = 0.66, v = 0.22 \) and \( F = 8.50 \), not far from the values assigned in the GM model which are \( \alpha = 0.65, v = 0.20 \) and \( F = 9.21 \).

5 Impulse-response functions

The short-run analysis of the effects of banking in New Keynesian models is carried out by examining impulse response functions obtained from four sources of variability: the production technology shock, \( A_1t \), the monetary policy (interest-rate) shock, \( \varepsilon_t \), the banking labor productivity shock, \( A_2t \), and the shock that alters the collateral value of capital, \( A_3t \). Special attention will be devoted to the effects of the last shock because it may well represent the economic scenario of financial crisis that we have witnessed recently. For such impulse-response analysis, it is assumed that technology shocks in both output and loan production are strongly persistent by setting their coefficients of autocorrelation at 0.95, whereas the financial shock is slightly less persistent with a coefficient of autocorrelation set at 0.90 as in GM (2007). The interest-rate shock, \( \varepsilon_t \), is a white-noise perturbation in the Taylor-type monetary policy rule that already includes endogenous inertia through the lagged nominal interest rate. The size of the shock is in all cases a 1\% innovation.\footnote{In the case of the interest-rate shock, it actually is a 1\% annualized shock.}

The analysis includes a comparison between the three models described above: the baseline New Keynesian model with banking and variable capital, the GM model with banking and without variable capital and the NNS model as a New Keynesian model without banking and with variable capital. The responses reported in Figures 1-4 represent percentage deviations from steady state in the cases of output, consumption, investment, the real wage, labor and the stock of capital, while the responses of the marginal finance cost, the nominal interest rates and inflation are given in basis-point deviations from their steady-state rates.

5.1 Technology shock

Figure 1 displays the responses to a positive 1\% technology shock that rises labor productivity in the goods production function. An increase in productivity reduces the marginal cost of production and monopolistically competitive firms cut optimal prices when Calvo signal allows them to do so. In turn, economy-wide inflation falls. The reaction of the central bank to the inflation drop is announcing cuts in the interbank interest rates which are transmitted to lower interest rates on both loans and bonds. The fall in the interest
rates stimulates output through their positive influence on the demand components, consumption and investment, and also increases the demand for loans to dispose of the amount of deposits required for purchasing additional goods.

Comparing the responses in the baseline model (solid lines) with those in the NNS model (dotted lines), it is observed that the implications of financial frictions are quantitatively of little importance in the effects of technology shocks. Actually, the banking sector does not reproduce the financial accelerator of Bernanke et al. (1999) because the response of output is slightly lower when the banking elements are in place. Technology improvements increase the demand for loans, which raises the marginal finance cost, $\chi$, in a way that attenuates the response of output (via its demand components, consumption and investment). Hence, there is an "attenuation effect" explained by a procyclical external finance premium as already discussed in GM (2007). Meanwhile, inflation and the nominal interest rates present shorter falls in the frictionless
finance NNS model, which may be relevant for the correct implementation of a stabilizing Taylor-type rule. Likewise, if the central bank uses the interest rate that ignores financial frictions, $R_T$, the interest-rate cut would be of 8 annualized basis points (-0.02 in Figure 1) while that cut should be by something more than 15 annualized basis points (-0.038 in Figure 1) in terms of the interbank interest rate $R^{IB}$.

The constant-capital assumption of the GM model (dashed lines in Figure 1) also has significant implications in the responses to a technology shock. Investment barely rises to collect the effect of the increase in the capital relative price $q$, and leaving consumption as the main determinant of changes in aggregate demand. In turn, output rises less and inflation falls more than in the baseline model. Moreover, the reaction of labor is markedly different in the GM model compared with the models that incorporate variable capital (see box of "n" in Figure 1). The lack of capital adjustments makes labor fall to accommodate the technology shocks. In the models with variable capital, labor productivity further increases with higher capital and it makes the response of labor be positive after a technology shock. The marginal finance cost is also procyclical in the GM model which explains the financial attenuation mechanism that was already mentioned above. However, the increase in the marginal finance cost is much less significant than in the baseline model because the demand for loans does not depend upon investment spending.

5.2 Monetary policy shock

An unexpected increase in the interbank nominal interest rate set by the central bank can be induced in the model by having a positive value on the monetary policy shock to the Taylor-type rule (27). The impulse response functions obtained from an annualized 1% monetary policy shock ($\varepsilon = 0.25$) are shown in Figure 2.

The influence of financial frictions on the reactions of output and inflation is quantitatively small. Figure 2 displays similar-size drops on output and inflation in the baseline model as in the NNS model. In sticky-price economies, a sudden increase in the nominal interest rates raises the real interest rate, driving consumption and investment down in the Aggregate Demand sector. Such declines in desired spending reduce the demand for loans, banking labor, and the marginal finance cost. The procyclical response of the marginal finance cost helps to contain the drop in output as it provides some economic stimulus to both consumption and investment. The NNS model without financial frictions does not contemplate this financial mitigation, reporting slightly larger drops for output, investment and labor.

As firms cut production and employment falls, productivity rises and the real marginal cost moves downward. The subset of firms that can optimally adjust the price will charge a lower price in reaction to the decreasing marginal cost. Inflation drops as a result, by nearly 15 basis points in the three models. Since the central bank has an instantaneous reaction to inflation deviation in the application of (27), the actual increase in the interbank nominal interest rate is less than the 25 basis points embedded in the initial interest-rate shock.

The assumption of either constant or variable capital is crucial to observe different reactions between the baseline model and the GM model. Thus, the baseline model with endogenous capital accumulation shows a severe reduction of investment (nearly by -3%) which is much higher than that observed in the GM model with constant capital (less than -0.5% as $q$ adjusts moving downwards). Such different investment
Figure 2: Responses to a 1% annualized monetary policy shock. Baseline model (solid lines), GM model (dashed lines) and NNS model (dotted lines).
behavior is passed along to see larger declines of demand-determined output in the model with variable capital. Subsequently, the demand for loans and banking activity also suffers a larger contraction in the baseline model, which explains the deeper cut in the marginal finance cost $\chi$ and lower interest rates than in the GM model.

### 5.3 Labor banking productivity shock

Figure 3 provides the responses obtained in reaction to a 1% innovation in labor banking productivity, $A_{2t}$. There are no responses reported from the NNS model because it does not include loan production. As productivity of banking labor rises, the marginal finance cost falls and demand-determined output rises to pick up the expansionary effects of such lower finance cost on consumption and investment. Both industrial labor and the capital stock are raised to produce the additional units of output demanded in the goods
market. The increase in labor leads to a decline in industrial labor productivity, which raises the real marginal cost and subsequently the rate of inflation. Therefore, both output and inflation rise in response to the demand expansion triggered by the higher banking productivity and lower marginal finance cost. However, the quantitative effects of this shock are not very significant. Thus, output increases by less than 0.1% and inflation only rises in 2 basis points in the baseline model.

With constant capital (GM model), investment has a much smaller response which also reduces the reactions of output and labor in comparison to the baseline model. The Monetary block provides procyclical responses of all the nominal interest rates similar to those of the baseline model. Hence, the central bank responds to the inflation pressure with higher interbank rates that are transmitted to the interest rates on bonds and loans. The required increase of the interbank rate is in line with the increase of inflation (around 2 basis points) but the changes in the central-bank rate are more gradual and longer lasting. Both the bond and loan interest rates report smaller increases as a consequence of the lower marginal finance cost.

5.4 Financial shock

Both the baseline model and the GM model include an exogenous process $A_3_t$ that tunes up or down the collateral value of the stock of capital for loan production. We can refer to $A_3_t$ as a financial shock in the sense of a financial source of business cycle fluctuations. The collateral value of capital would increase with a positive financial shock. By contrast, an adverse financial shock would reduce the collateral value of capital, increasing the cost of loan production and thus raising the marginal finance cost. Figure 4 shows the results obtained after a -1% negative financial shock. The transmission channel from the financial shock to the real sector of the economy resembles that of the labor banking shock although now all the variables move on the opposite direction. Hence, the marginal finance cost, $\chi$, rises when the lower collateral capacity increases the marginal cost of loan production. The increase in the finance cost reduces the purchases of both consumption and investment goods. Consequently, output falls by one tenth of the shock (-0.1%) in the baseline model while the output drop is even smaller (-0.06%) in the GM model. Again, the constant-capital assumption makes investment barely move down (only for the drop in $q$) which explains the lower output decline with respect to that observed in the variable-capital baseline model. The labor responses in both models are similar in magnitudes and dynamic patterns to the ones of output.

As shown in Figure 4 for both models, inflation and the nominal interest rates fall after the demand contraction that results from the adverse financial shock. The inflation response is connected to the increase in labor productivity (which lowers real marginal costs) as a result of less labor employed. The interbank interest rates go down to fight deflation as prescribed by the Taylor-type monetary policy rule (27). The quantitative reactions of the interbank rate reach their peak effects one quarter after the shock with a drop of -0.023% (9.2 annualized basis points) in the baseline model, and somewhat lower of -0.019% (7.6 basis points) in the GM model. Lower interbank rates are transmitted to the bond and loan markets through higher borrowing costs. Remarkably, the interbank rate, the bond rate and the collateralized loan rate ($R^B_t$, $R^B$ and $R^L$) report sharper declines than the one of the uncollateralized loan ($R^T$). Such spreads are explained by the increase observed in the marginal finance cost and the external finance premium.
Figure 4: Responses to a -1% financial shock. Baseline model (solid lines) and GM model (dashed lines).
The financial shock might help to illustrate the effects of the credit crunch that hit the United States and other industrialized economies in 2008. The origin of the crisis was the housing bubble that was followed by significant corrections in home prices and the stock market. Such scenario might be incorporated in the model as a large adverse financial shock that cuts the capital collateral value by 35% (i.e., multiplying by 35 the quantitative effects displayed in Figure 4). Table 2 gives the peak responses in the baseline model, in the GM model and in US data. The real effects predicted by the baseline model are coherent with what has been observed in the aftermath of the crisis: output suffers a 3.48% contraction, consumption falls by 2.11%, investment declines by 10.76% and labor falls by 5.44%. The only significant difference is the incapacity of the model to reproduce the investment slump that the US economy suffered in 2008 (-33%). The GM model delivers a lower drop of output, a (much) lower of investment and a higher of consumption as expected from an economy with constant capital.

Table 2 also reports significant effects on the financial variables, although the models underestimate the unprecedented cuts of the interbank rate and the bond rate observed in the data. The weakness of demand brings a drop of 3.02% in the annualized rate of inflation which partially resembles the temporary deflation episode that experience the US economy at the end of 2008 (-5.00%). The model also predicts significant cuts in all interest rates. The interbank rate reaches its minimum value at 3.17% below the initial rate as a result of applying the Taylor-type rule (27) in a context of rapid price corrections. The interest rates on bonds (-3.44%) and loans (-2.40%) provide similar cuts. In the GM model, the reductions of the interest rates are less significant. Neither model provides enough cuts in interest rates to match the data during this period of enormous financial turbulence (with the exception of the loan interest rate). The model could be re-calibrated in some dimension (lower price stickiness, higher responsiveness to inflation in the Taylor-type rule, lower interest-rate smoothing in the Taylor-type rule, etc.) or fully estimated with some modern econometric technique to improve the goodness of fit of these interest-rate reactions to the actual data.

13 According to Standard and Poor’s Case-Shiller Home Price Indices, the US 10-city Composite Index declined by 33.5% from the second quarter of 2006 to April 2009. The stock market also suffered a large correction. Hence, the Dow Jones Industrial Average fell by nearly 50% from the third quarter of 2007 to the first quarter of 2009.

14 Peak effects are obtained as the percent difference between the maximum level and the minimum level observed over the sample period from 2007:1 to 2009:4. In US data, Output is "Real Gross Domestic Product", Consumption is "Real Personal Consumption Expenditures", Investment is "Real Gross Private Domestic Investment", Labor is the "Total Nonfarm Payrolls: All Employees", Inflation is the annualized quarterly change in the "GDP Implicit Price Deflator", the Interbank interest rate is the "Fed’s Primary Credit Rate", the Bond interest rate is the "3-Month Treasury Bill: Secondary Market Rate" and the Loan interest rate is the "30-Year Fixed Rate Mortgage Average in the United States". Source: FRED database, Federal Reserve Bank of St. Louis, USA.
Table 2. Peak effects of a 35% decline in collateral value of capital

<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th>GM model</th>
<th>US data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-3.48%</td>
<td>-2.11%</td>
<td>-3.46%</td>
</tr>
<tr>
<td>Consumption</td>
<td>-1.55%</td>
<td>-2.42%</td>
<td>-2.41%</td>
</tr>
<tr>
<td>Investment</td>
<td>-10.76%</td>
<td>-0.98%</td>
<td>-33.31%</td>
</tr>
<tr>
<td>Labor</td>
<td>-5.44%</td>
<td>-3.29%</td>
<td>-5.93%</td>
</tr>
<tr>
<td>Inflation (annualized)</td>
<td>-3.02%</td>
<td>-2.47%</td>
<td>-5.60%</td>
</tr>
<tr>
<td>Interbank interest rate (annualized)</td>
<td>-3.17%</td>
<td>-2.61%</td>
<td>-5.03%</td>
</tr>
<tr>
<td>Bond interest rate (annualized)</td>
<td>-3.44%</td>
<td>-2.95%</td>
<td>-4.85%</td>
</tr>
<tr>
<td>Loan interest rate (annualized)</td>
<td>-2.40%</td>
<td>-1.89%</td>
<td>-1.76%</td>
</tr>
</tbody>
</table>

6 Steady-state analysis

With the purpose of examining the long-run properties of the models, we can solve them in a (detrended) steady state that abstracts from short-run variability and economic growth. The four exogenous processes are shut down with no variability. For convenience, we also follow GM (2007) when assuming that the steady-state rate of inflation is zero ($\pi = 0$) and, therefore, there is no difference between nominal and real interest rates. Hence, the steady-state solution of the baseline model can be obtained by solving the following non-linear system of equations:

- The capital accumulation equation from rewriting the optimality condition of capital, (equation 13), in steady state

$$\left(1 + \tau \chi \right) = \left( \frac{1}{1 + \tau \pi} - \text{LSY}^B \right) \left[ R^k + \left( 1 + \tau \chi \right) \left( 1 - \delta \right) \right] + \text{LSY}^k,$$

where we used the steady-state properties of the adjustment costs function, $I(1) = \delta$ and $I'(1) = 1$, and the coincidence between real and nominal interest rates in steady state.

- The definition of the marginal finance cost in steady state from equation (4)

$$\chi = \frac{\omega m}{(1 - \alpha)(c + \tau \delta k)},$$

where steady-state investment $x$ has been replaced with $\delta k$.

- The definition of the liquidity service yield on bonds in steady state from equation (6)

$$\text{LSY}^B = \frac{\omega m}{(1 - \alpha)(c + \tau \delta k)}.$$

- The proportionality between the liquidity service yield on capital and bonds in steady state

$$\text{LSY}^k = \nu \text{LSY}^B.$$

- The steady-state nominal interest rate on the fictitious bond that does not provide collateral services

$$1 + R^T = \beta^{-1},$$

which is obtained from its optimality condition $-\lambda_t (1 + \tau_t^T)^{-1} + \beta E_t \lambda_{t+1} = 0$, noticing that the Lagrange multiplier is constant in the detrended steady state and also recalling that the steady-state nominal and real interest rates coincide.
- Taking equation (7) in the detrended steady state, the discount factor collects both the market return on bonds and their liquidity service yield

\[ \beta = \frac{1}{1+\rho} - LSY^B. \] (36)

- As discussed above, \( R^T \) also defines the rate of interest applied to the uncollateralized loan offered by a bank that can borrow funds from the central bank. The marginal cost of that uncollateralized loan is obtained as the product of the cost of borrowing times the marginal cost of loan production. It gives the steady-state relationship

\[ 1 + R^T = \left( 1 + R^{IB} \right) \left( 1 + \frac{V}{1-\tau \chi} \right). \] (37)

- Analogously, the interest rate on loans that are collateralized is in equilibrium the product of the borrowing cost times the marginal cost of production exclusively attached to monitoring effort. In steady state, it says

\[ 1 + R^L = \left( 1 + R^{IB} \right) \left( 1 + \frac{(1-\alpha)V}{1-\tau \chi} \right), \] (38)

which leads to the definition of the collateralized external finance premium (CEFP) in steady state as the spread between the collateralized loan rate and the interbank rate, \( CEFP = R^L - R^{IB} \). Using (38), the CEFP in steady state is fairly approximated by the following expression

\[ CEFP = \frac{(1-\alpha)V}{1-\tau \chi}. \] (39)

- The firm’s first order condition on the demand for labor makes the equilibrium real wage in steady state equal to the product of the real marginal cost by the marginal product of labor

\[ w = \psi \frac{(1-\eta)y}{n}. \] (40)

- Meanwhile, the firm’s first order condition on the demand for capital implies that the rental rate in steady state is the product of the real marginal cost by the marginal product of capital

\[ R^k = \psi \frac{n\psi}{Y}. \] (41)

- From the firm’s first order condition on the selling price, we can derive the real marginal cost in steady state as the inverse of the constant mark-up between prices and the nominal marginal costs

\[ \psi = \frac{\theta - 1}{\sigma}. \] (42)

- The steady-state overall resources constraint is

\[ y = c + \delta k. \] (43)

- The Cobb-Douglas production function in the detrended steady state is

\[ y = k^n n^{1-\eta}. \] (44)

- The steady-state labor supply curve is

\[ \frac{1-\phi}{1-\eta} = \frac{\phi w}{c(1+\chi)}. \] (45)
- The amount of real loans in the detrended steady state is provided by the loan production technology

\[ l = F(b + \nu k)^{\alpha} m^{1-\alpha}. \]  

(46)

- As set in the model calibration, the amount of bonds is assumed to be at 56% of consumption in the detrended steady state

\[ b = 0.56c. \]  

(47)

- Finally, households must satisfy the following deposit-in-advance constraint

\[ c + \tau \delta k = \frac{V}{1+\tau} l. \]  

(48)

Hence, the steady-state solution of the baseline model can be reached by solving a nonlinear system of eighteen equations, (31)-(48), in order to find numerical values for the eighteen variables: \( k, y, c, w, n, m, b, l, \psi, \chi, R^k, R^B, R^T, R^{IB}, R^L, CEFP, LSY^B \) and \( LSY^k \).

The solution for the GM model can be found as the particular case of the (31)-(48) non-linear system in which we fix \( \tau = 0.0 \).

The NNS model with no banking elements can also be represented by the set of equations (31)-(48) under the assumption \( m = 0 \), that will drop banking-related variables (\( \chi = LSY^B = LSY^k = CEFP = l = 0 \)) and will leave all the interest rates fully determined by the subjective discount factor as \( R^T = R^B = R^{IB} = R^k - \delta = \beta^{-1} \).

Table 3 shows the numbers found in the steady-state solution of the three models:

<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th>GM model</th>
<th>NNS model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital stock</td>
<td>( k = 9.0810 )</td>
<td>( k = 9.2388 )</td>
<td>( k = 8.8962 )</td>
</tr>
<tr>
<td>Output</td>
<td>( y = 1.0823 )</td>
<td>( y = 1.0727 )</td>
<td>( y = 1.0873 )</td>
</tr>
<tr>
<td>Consumption</td>
<td>( c = 0.8553 )</td>
<td>( c = 0.8417 )</td>
<td>( c = 0.8649 )</td>
</tr>
<tr>
<td>Real wage</td>
<td>( w = 1.9249 )</td>
<td>( w = 1.9534 )</td>
<td>( w = 1.8981 )</td>
</tr>
<tr>
<td>Labor</td>
<td>( n = 0.3271 )</td>
<td>( n = 0.3195 )</td>
<td>( n = 0.3333 )</td>
</tr>
<tr>
<td>Banking labor</td>
<td>( m = 0.0062 )</td>
<td>( m = 0.0063 )</td>
<td>–</td>
</tr>
<tr>
<td>Real loans</td>
<td>( l = 2.7452 )</td>
<td>( l = 2.7017 )</td>
<td>–</td>
</tr>
<tr>
<td>Real marginal cost</td>
<td>( \psi = 0.9091 )</td>
<td>( \psi = 0.9091 )</td>
<td>( \psi = 0.9091 )</td>
</tr>
<tr>
<td>Marginal finance cost</td>
<td>( \chi = 0.0338 )</td>
<td>( \chi = 0.0418 )</td>
<td>–</td>
</tr>
<tr>
<td>Liquidity serv. yield on bonds</td>
<td>( LSY^B = 0.0094 )</td>
<td>( LSY^B = 0.0099 )</td>
<td>–</td>
</tr>
<tr>
<td>Liquidity serv. yield on capital</td>
<td>( LSY^k = 0.0021 )</td>
<td>( LSY^k = 0.0020 )</td>
<td>–</td>
</tr>
<tr>
<td>Coll. external finance premium</td>
<td>( CEFP = 0.0043 )</td>
<td>( CEFP = 0.0046 )</td>
<td>–</td>
</tr>
<tr>
<td>Interbank interest rate</td>
<td>( R^{IB} = 0.0022 )</td>
<td>( R^{IB} = 0.0020 )</td>
<td>( R = 0.0150 )</td>
</tr>
<tr>
<td>Bond interest rate</td>
<td>( R^B = 0.0056 )</td>
<td>( R^B = 0.0051 )</td>
<td>( R = 0.0150 )</td>
</tr>
<tr>
<td>Uncoll. loan interest rate</td>
<td>( R^T = 0.0150 )</td>
<td>( R^T = 0.0150 )</td>
<td>–</td>
</tr>
<tr>
<td>Coll. loan interest rate</td>
<td>( R^L = 0.0065 )</td>
<td>( R^L = 0.0065 )</td>
<td>–</td>
</tr>
<tr>
<td>Net capital interest rate</td>
<td>( R^k - \delta = 0.0140 )</td>
<td>( R^k - \delta = 0.0130 )</td>
<td>( R^k - \delta = 0.0150 )</td>
</tr>
</tbody>
</table>
As expected from their common calibration procedure, the steady-state solution of the baseline model and the GM model are quite similar. The only substantial difference is observed in the marginal finance cost $\chi$; the GM model has a higher $\chi$ because there are no financial requirement for investment spending ($\tau = 0$) which lowers the value of $\chi$ in (32). However, there are more noticeable differences between both banking models and the NNS model. The lack of financial frictions places the economy in a steady state with less capital, higher levels of output, consumption and labor, and higher interest rates in the bond and capital markets.

Interestingly, the solutions of the detrended steady state in the baseline and GM models (with banking) bring a stock of capital higher than the one reached in the NNS model (without banking elements). Why does capital increase when financial intermediation is considered? The answer to this questions is found in the determination of the optimal stock of capital. Combining equations (31) and (36), it is obtained

$$\frac{1 + R^k - \delta + \tau \chi (1 - \delta)}{1 + \tau \chi - LSY^k} = \beta^{-1},$$

where the left-hand side represents the steady-state total return on capital while the right-hand side indicates the (constant) rate of intertemporal preference required by the household to sacrifice one unit of current consumption for future consumption. The presence of banking elements in both the baseline and GM models gives rise to positive levels of the liquidity service yield on capital ($LSY^k$) and of the marginal finance cost ($\chi$), which are factors that determine the total return on capital (left-hand side of 49). Their influence is of opposite sign. Thus, a higher $LSY^k$ increases the total return on capital as the denominator of the left-hand side of (49) becomes smaller. By contrast, a higher $\chi$ reduces the marginal return on capital.$^{15}$ The final effect on the left-hand side of (49) is what results from balancing the positive impact of a higher $LSY^k$ with the negative impact that brings a higher $\chi$.

Unlike the baseline model, the GM model does not pick up the influence of $\chi$ on the steady-state capital stock because investment on capital accumulation is not subject to the financial constraint. Setting $\tau = 0.0$ in (49) leads to the optimal capital condition in the steady state solution of the GM model

$$\frac{1 + R^k - \delta}{1 - LSY^k} = \beta^{-1},$$

which implies that a positive $LSY^k$ rises the capital return with no effect from $\chi$.

In the NNS model, the capital stock provides no collateral service ($LSY^k = 0$) and the optimality condition that determines steady state capital becomes

$$1 + R^k - \delta = \beta^{-1},$$

which provides a lower total return on capital.

Two consequences emerge from the last three paragraphs:

(i) The steady-state capital stock in the baseline model with financial rigidity on investment ($\tau > 0$) is lower than the steady-state capital stock in the GM model without financial rigidity on investment ($\tau = 0$).

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$^{15}$The partial derivative of the left-hand side of (49) with respect to $\chi$ is $-\frac{\tau (R^k + (1 - \delta) LSY^k)}{(1 + \tau \chi - LSY^k)^2} < 0$. 

27
The negative impact of the investment financial constraint on the capital return requires a reduction of the capital stock to hold \( (49) \).

(ii) The steady-state capital stock in the GM model is lower than the one of the NNS model without any financial friction. A positive \( LSY_k \) rises the capital return which requires a lower stock of capital to hold \( (49') \).

The results reported in Table 3 are coherent with conclusions (i) and (ii). Using the same calibration criteria, the steady-state capital in the baseline model \( (k = 9.0810) \) is lower than the steady-state capital in the GM model \( (k = 9.2388) \). In addition, the steady-state solution of the NNS model gives a stock of capital lower than the numbers found in the other two models \( (k = 8.8962) \), which indicates that financial frictions have the long-run effect of increasing the capital stock to take advantage of its collateral yield.

By contrast, the long-run impact of finance requirements on labor is of negative sign. Table 3 informs that steady-state labor is lower in both banking models \( (n = 0.3271 \text{ in the baseline model and } n = 0.3195 \text{ in the GM model}) \) compared to the NNS model \( (n = 0.3333) \). The transmission channel form the banking intermediation to the labor market is through the labor supply curve \( (45) \). A positive marginal finance cost \( (\chi > 0) \) reduces the shadow value of consumption which makes the labor supply curve shift to the left. In turn, the real wage is higher and the amount of labor is lower in the detrended steady states of both banking models compared to that of the NNS model.

As for the long-run effect of banking on output, it can be obtained as the combination of the (positive) effect on capital and the (negative) effect on labor. The interaction of these two effects in the Cobb-Douglas production function \( (44) \) determines the final effect on output. The numbers reported in Table 3 indicate that output is higher in the NNS model (with no financial requirement) than in any of the other two models with a banking sector. Thus, the increase in labor would fully offset the reduction in the stock of capital, when moving towards an economy with no financial friction.

The long-run impact of banking on consumption is important for the welfare analysis. In steady state, consumption is obtained as the difference between output and capital replacement, \( c = y - \delta k \). We just mentioned that output is lower and capital is higher in both banking models in comparison to the NNS model. Therefore, consumption will be unambiguously lower. Table 3 show a steady-state consumption of 0.8553 in the baseline model and of 0.8417 in the GM model, both numbers lower than 0.8649 obtained in the NNS model.

How much long-run welfare gain can be reached from a more efficient banking technology? This measure can be obtained by examining the effects of altering the banking technology in a way that delivers more output of real loans with the same amount of inputs employed. Following GM (2007), the calibrated number of the scale parameter \( F \) can be raised (or lowered) to increase (or decrease) banking efficiency. We decided to make a 10\% adjustment of \( F \) in both directions, up and down. Then we did recalculate the steady-state solution in both banking models and compare the results with those of the baseline calibration. Welfare gains can be taken from the log utility function of the model. Following Lucas (2000), we assume that household’s utility is the measure of social welfare and we calculate the level of consumption that must be added (or deducted) in the economy with the new banking efficiency to reach the same steady-state utility as the one obtained with the initial level of banking efficiency. Such consumption equivalent is provided as a percentage.
of steady-state output in order to provide the estimates of welfare effects with straightforward economic interpretation. Table 4 contains the results in the cases of a ±10% change in banking efficiency:

Table 4. Steady-state effects of changes in banking efficiency.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta k$</th>
<th>$\Delta y$</th>
<th>$\Delta c$</th>
<th>$\Delta n$</th>
<th>$\Delta m$</th>
<th>$\Delta LSY^k$</th>
<th>$\Delta \chi$</th>
<th>Welfare gain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10% higher efficiency ($F' = 1.1 F$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline model</td>
<td>-0.36%</td>
<td>+0.23%</td>
<td>+0.38%</td>
<td>+0.57%</td>
<td>-23.6%</td>
<td>-25.2%</td>
<td>-24.1%</td>
<td>+0.30%</td>
</tr>
<tr>
<td>GM model</td>
<td>-1.25%</td>
<td>-0.11%</td>
<td>+0.20%</td>
<td>+0.53%</td>
<td>-22.1%</td>
<td>-23.0%</td>
<td>-22.8%</td>
<td>+0.12%</td>
</tr>
<tr>
<td><strong>10% lower efficiency ($F' = 0.9 F$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline model</td>
<td>+0.50%</td>
<td>-0.35%</td>
<td>-0.57%</td>
<td>-0.81%</td>
<td>+33.9%</td>
<td>+31.4%</td>
<td>+34.9%</td>
<td>-0.45%</td>
</tr>
<tr>
<td>GM model</td>
<td>+1.74%</td>
<td>+0.13%</td>
<td>-0.30%</td>
<td>-0.75%</td>
<td>+30.5%</td>
<td>+28.0%</td>
<td>+31.9%</td>
<td>-0.20%</td>
</tr>
</tbody>
</table>

Looking into Table 4, the most significant steady-state effects of changing banking efficiency are found in the variables of the banking sector: monitoring labor ($m$), the marginal finance cost ($\chi$) and the liquidity service yield on capital ($LSY^k$). The 10% improvement in banking efficiency is used to save substantial monitoring effort as the steady-state level of $m$ falls by 23.6% in the baseline model and 22.1% in the GM model. Such important reductions of banking effort are translated into similar percent declines in both $\chi$ and $LSY^k$. The new financial conditions are transmitted to the rest of the variables of the model through the labor supply and capital accumulation equations. Thus, a lower $\chi$ rises the shadow value of consumption which expands supply in the labor market. The banking models show positive changes in labor between 0.53% and 0.57%. In the meantime, a lower $LSY^k$ penalizes the overall capital return and reduces the steady-state capital. Such capital reduction is more moderate in the baseline model (-0.36%) than in the GM model (-1.25%) because in the former the lower $\chi$ has a positive impact on the capital return that partially compensates the negative effect of a lower $LSY^k$. As a matter of fact, output falls in the GM model because the negative impact of the capital decline in the Cobb-Douglas production function wipes out the positive effect coming from the increase in labor. Such output contraction after a banking technology improvement might be considered as some unrealistic steady-state feature of the GM model that is not observed in the baseline model.

The amounts of $n$, $m$ and $c$ obtained in the steady states with $F$ and $F' = 1.1 F$ are plugged in the log utility function to compare the level of welfare. Once the consumption equivalences are obtained, Table 4 reports that there is a welfare gain equivalent to 0.3% of output in the baseline model, and somewhat lower at 0.12% of output in the GM model. This difference is due to the larger steady-state influence of banking efficiency on consumption in the baseline model (+0.38%) than in the GM model (+0.20%).

Table 4 shows that when banking efficiency worsens by 10% all the steady-state effects flip their sign. So, banking labor, the liquidity service yield and the marginal finance cost rise while there are contractionary effects on labor and consumption, and a positive impact on capital. The diminishing marginal returns on

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16Steady-state leisure barely changes with different banking efficiency because banking labor and non-banking labor move in opposite directions, bringing similar contributions to leisure that mostly cancel out. In this case of a 10% higher efficiency, steady-state leisure slightly falls in both models (-0.06% in the baseline model and -0.05% in the GM model). The welfare gain is therefore explained by the more substantial increase in consumption.
monitoring labor explains that changes are quantitatively higher with a 10% lower banking efficiency than with a 10% higher efficiency. Across models, we observe that the increase in the capital stock is much more important in the GM model that ignores the effects of a higher marginal finance cost for capital accumulation. Actually, steady-state capital is 1.74% higher in the GM model after the 10% decline in banking efficiency which is more than three times the effect of the baseline model. When introducing both capital and labor in the production function, output rises +0.13% in the GM model and falls -0.35% in the baseline model.

Finally, the welfare effects of this 10% drop in banking efficiency are equivalent to permanent declines of output, -0.45% in the baseline model and -0.20% in the GM model. The difference between the two models is again explained by the more intense effect on consumption in the baseline model (-0.57%) than in the GM model (-0.30%).

7 Conclusions

The effects of banking activities have been examined in a New Keynesian model with Calvo-style sticky prices and endogenous capital where the need for banking intermediation stems from a deposit-in-advance requirement for all purchases of consumption goods and a fraction of the spending on acquiring investment goods. Following Goodfriend and McCallum (2007), households act as bankers by producing loans with monitoring labor and their stock of collateral. For such a banking model, we have derived dynamic semi-loglinear equations that determine short-run fluctuations of consumption and the stock of capital. The introduction of banking elements adds new terms on those equations that were absent in conventional New Keynesian models. Thus, consumption negatively depends on the marginal finance cost (it makes consumption more costly) and also on the liquidity services yield on bonds (it rises the opportunity cost). Investment dynamics are also influenced by the marginal finance cost (negatively) and on the liquidity services yield on capital (positively). These equations are compared to the alternative equations obtained in both the constant-capital model of Goodfriend and McCallum (2007), and in a standard New Keynesian model with no financial frictions and variable capital (Woodford 2003, chapter 5).

The impulse-response analysis shows that the introduction of banking features does not have significant effects for the reactions of macro variables to either technology innovations (Figure 1) or monetary shocks (Figure 2). Actually, there is some attenuating effect found in the responses of output (contrary to the financial accelerator hypothesis) as a result of the procyclical behavior of the marginal finance cost. For example, a demand contraction after an interest-rate shock comes with a lower financial cost that stimulates demand to partially compensate for the interest-rate hike. We have also examined the effects of two kinds of banking shocks on loan production: one shaping labor banking productivity and the other one affecting the collateral productivity of the stock of capital. We found sizeable effects of financial shocks on the real sector (Figures 3 and 4), as other possible source of business cycle fluctuations. Moreover, we did replicate an scenario of a financial crisis by producing a large drop in the collateral value of capital and the reactions found were significant falls of output, consumption, investment, labor, inflation and the interest rates (Table 2).

The model was solved in a detrended steady state for the long-run analysis of banking. The steady-state
results reported in Table 3 show how banking intermediation increases the stock of capital to take advantage of its collateral services; however, both output and consumption fall as a consequence of the labor supply shrink. Subsequently, there is a permanent welfare cost of banking activities. Our results in the baseline model show that a 10% improvement in banking efficiency results in a permanent welfare gain equivalent to 0.30% of output (Table 4). In the version of the Goodfriend and McCallum (2007) banking model used here, we found that steady-state capital is higher because there is no finance cost on capital accumulation, which acts as a compensating effect in the welfare analysis. Thus, the welfare gain after a 10% improvement of banking efficiency lowers to 0.12% of output.
References


Technical Appendix.

Complete log-linearized banking model with constant capital (GM model)

- Aggregate Demand sector, seven equations:

\[ \hat{c}_t = E_t \hat{c}_{t+1} - (\chi_t - E_t \chi_{t+1}) - \left[ (r_t^B - r^B) + (LSY_t^B - LSY_B) \right], \quad (A1) \]

\[ \chi_t - \chi = \chi (\hat{w}_t + \hat{\nu}_t - \hat{c}_t), \quad (A2) \]

\[ LSY_t^B - LSY_B = LSY_B \left( \hat{w}_t + \hat{\nu}_t - \frac{v_k}{b + v_k} \hat{q}_t - \frac{v_k}{b + v_k} A3_t \right), \quad (A3) \]

\[ \hat{q}_t = (1 - LSY^k) (E_t \hat{q}_{t+1} + E_t (r_{t+1}^k - r^k) - (r_t^B - r^B) - (LSY_t^B - LSY_B)) + LSY^k (LSY_t^k - LSY^k), \quad (A4) \]

\[ LSY_t^k - LSY^k = LSY^k \left( \hat{w}_t + \hat{\nu}_t - \frac{v_k}{b + v_k} \hat{q}_t + \frac{b}{b + v_k} A3_t \right), \quad (A5) \]

\[ \hat{x}_t = \hat{q}_t, \quad (A6) \]

\[ \hat{\gamma}_t = \frac{c}{c + \delta k} \hat{c}_t + \frac{\delta k}{c + \delta k} \hat{q}_t. \quad (A7) \]

- Aggregate Supply sector, five equations:

\[ \pi_t - \pi = \beta E_t (\pi_{t+1} - \pi) + \frac{(1 - \beta \pi)(1 - \pi)}{\pi} \hat{\psi}_t, \quad (A8) \]

\[ \hat{\psi}_t = \hat{w}_t - (\hat{\gamma}_t - \hat{\nu}_t), \quad (A9) \]

\[ \hat{\gamma}_t = (1 - \eta) \hat{\nu}_t + (1 - \eta) A1_t, \quad (A10) \]

\[ \frac{1}{\pi} (r_t^k - r^k) = \hat{\psi}_t + (1 - \eta) \hat{\nu}_t + (1 - \eta) A1_t - \hat{q}_t, \quad (A11) \]

\[ \hat{w}_t = \frac{m}{1 - n - m} \hat{\nu}_t + \frac{n}{1 - n - m} \hat{m}_t + \hat{c}_t + \chi_t. \quad (A12) \]

- Monetary block, eight equations:

\[ \hat{\iota}_t = \hat{c}_t, \quad (A13) \]

\[ \hat{l}_t = \frac{av_k}{b + v_k} \hat{q}_t + \frac{av_k}{b + v_k} A3_t + (1 - \alpha) \hat{m}_t + (1 - \alpha) A2_t, \quad (A14) \]

\[ R_t^B = r_t^B + E_t \pi_{t+1}, \quad (A15) \]

\[ R_t^k = r_t^k + E_t \pi_{t+1}, \quad (A16) \]

\[ R_t^{IB} - R_t^{IB} = (1 - \mu_3) \left[ \mu_1 (\pi_t - \pi) + \mu_2 \hat{\psi}_t \right] + \mu_3 \left( R_{t-1}^{IB} - R_t^{IB} \right) + \varepsilon_t, \quad (A17) \]

\[ R_t^B = R_t^B - LSY_t^B; \quad (A18) \]

\[ R_t^L = R_t^B \left\{ \frac{1}{1 - r^{IP}} \chi_t, \quad (A19) \right. \]

\[ R_t^L = R_t^L \left\{ \frac{1 - \alpha}{1 - r^{IP}} \chi_t. \quad (A20) \right. \]

The set contains twenty equations, (A1)-(A20), that may provide solution paths for the following twenty variables: \( \hat{y}_t, \hat{c}_t, \hat{x}_t, \hat{q}_t, \hat{\nu}_t, \hat{m}_t, \hat{\iota}_t, \hat{w}_t, \hat{\psi}_t, \chi_t, LSY_t^B, LSY_t^k, \pi_t, r_t^B, R_t^B, r_t^k, R_t^k, R_t^{IB}, R_t^L \), and \( R_t^{L-1} \). The stock of bonds is assumed to be exogenous and constant at the steady-state level. There is one predetermined variable \( R_{t-1}^{IB} \), and four exogenous variables, \( A1_t, A2_t, A3_t, \) and \( \varepsilon_t \).
Complete log-linearized model with variable capital and no financial friction (NNS model)

- Aggregate Demand sector, four equations:

\[ \hat{c}_t = E_t \hat{c}_{t+1} - (r_t - r), \quad (A21) \]

\[ \hat{k}_{t+1} = \frac{1}{1 + \beta (1 - \sigma)} \hat{k}_t + \frac{\beta (1 - \delta)}{1 + \beta (1 - \sigma)} E_t \hat{k}_{t+2} - \frac{1}{\sigma (1 + \beta (1 - \sigma))} \left( (r_t - r) - \beta E_t (r_{t+1}^k - r^k) \right), \quad (A22) \]

\[ \bar{x}_t = \frac{1}{\sigma} \hat{k}_{t+1} - \frac{1}{\sigma} \hat{k}_t, \quad (A23) \]

\[ \hat{y}_t = \frac{\gamma}{\rho} \hat{c}_t + \frac{\delta k}{y} \bar{x}_t. \quad (A24) \]

- Aggregate Supply sector, five equations:

\[ \pi_t - \pi = \beta E_t (\pi_{t+1} - \pi) + \frac{(1 - \beta \varpi) (1 - \varpi)}{\varpi} \hat{\psi}_t, \quad (A25) \]

\[ \hat{\psi}_t = \hat{w}_t - (\hat{y}_t - \hat{n}_t), \quad (A26) \]

\[ \hat{y}_t = \eta \hat{k}_t + (1 - \eta) \hat{n}_t + (1 - \eta) A1_t, \quad (A27) \]

\[ \frac{1}{\tau^\pi} (r_t^k - r^k) = \hat{\psi}_t + (\hat{y}_t - \hat{k}_t), \quad (A29) \]

\[ \hat{w}_t = \frac{n}{1 - \gamma} \hat{n}_t + \hat{c}_t. \quad (A29) \]

- Monetary block, two equations:

\[ R_t - R = (1 - \mu_3) \left[ \mu_1 (\pi_t - \pi) + \mu_2 \hat{\psi}_t \right] + \mu_3 (R_{t-1} - R) + \varepsilon_t, \quad (A30) \]

\[ R_t = r_t + E_t \pi_{t+1}. \quad (A31) \]

The set contains eleven equations, (A21)-(A31), that can serve to reach solution paths for the following eleven endogenous variables: \( \hat{y}_t, \hat{c}_t, \bar{x}_t, \hat{k}_{t+1}, \hat{n}_t, \hat{w}_t, \hat{\psi}_t, \pi_t, r_t, R_t, \) and \( r_t^k. \) The stock of bonds is assumed to be exogenous and constant at the steady-state level. There are two predetermined variables, \( \hat{k}_t \) and \( R_{t-1}, \) and four exogenous variables, \( A1_t, A2_t, A3_t, \) and \( \varepsilon_t. \)