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D.T. 1003

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An Estimated New-Keynesian Model with Unemployment as Excess Supply of Labor*

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(September 2010)

Abstract

As one alternative to search frictions, wage stickiness is introduced in a New-Keynesian model to generate endogenous unemployment fluctuations due to mismatches between labor supply and labor demand. The effects on an estimated New-Keynesian model for the U.S. economy are: i) the Calvo-type probability on wage stickiness rises, ii) the labor supply elasticity falls, iii) the implied second-moment statistics of the unemployment rate provide a reasonable match with those observed in the data, and iv) wage-push shocks, demand shifts and monetary policy shocks are the three major determinants of unemployment fluctuations.

Key words: sticky wages, unemployment, business cycles, New-Keynesian models.

JEL classification numbers: C32, E30.

*The authors would like to acknowledge financial support from the Spanish government (research projects ECO2008-02641, ECO2009-11151 and SEJ2007-66592-C03-01/ECON from Ministerio de Ciencia e Innovación, respectively). The first author also thanks Fundación Ramón Areces (VII Concurso Investigación en Economía) for financial support. We thank Francisco Galera for his comments and suggestions.

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The New-Keynesian macro model has been extended in recent years to incorporate the endogenous determination of unemployment fluctuations in the labor market.[1] Taking the search frictions approach, Walsh (2005) and Trigari (2009) introduced unemployment as the gap between job creation and destruction that results in a labor market with real rigidities *à la* Mortensen and Pissarides (1994). Alternatively, Casares (2007, 2010) and Galí (2010) assume nominal rigidities on wage setting to produce mismatches between labor supply and labor demand that delivers unemployment fluctuations.

This paper presents novel theoretical and empirical contributions. On the theoretical side, we discuss the effects of introducing unemployment as excess supply of labor on the inflation and real wage equations of a closed-economy New-Keynesian model with sticky prices, sticky wages and variable capital. Such model combines most of the nominal and real rigidities of fully-fledged New-Keynesian models – Calvo-type price stickiness, consumption habits, investment adjustment costs, variable capital utilization, etc. –, with a labor market similar to that of Casares (2010). In that regard, we replace the common way of introducing wage rigidities on labor contracts set by households (which follows the seminal paper by Erceg, Herdenson and Levin, 2000) for a labor market structure in which unemployment fluctuations are caused by sticky wages. A non-trivial difference with respect to the model derived in Casares (2010) is the existence of a rental market for capital that allows for labor-capital reallocations at the firm level.

On the empirical front, we provide a comparison between the proposed New-Keynesian model with unemployment driven by sticky wages and the model of Smets and Wouters (2007), which is a well-known reference model in the DSGE literature.[2] We follow a Bayesian econometric

[1]Referential New-Keynesian models without unemployment are Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2007) and all the model variants collected in Woodford (2003). They belong to the family of dynamic stochastic general equilibrium (DSGE) models.

[2]Gertler, Sala and Trigari (2008) and Christiano, Trabandt and Walentin (2010) estimate New-Keynesian models with unemployment, but in their setting unemployment is due to search frictions. Meanwhile, Blanchard and Galí

strategy to estimate the two models using U.S. quarterly data during the period 1984:1-2009:3. The estimation results provide a good fit to the data since both models capture most of the business cycle statistics. In the comparison across models, we find similar estimates of most structural model parameters, with three main differences. First, wage stickiness is significantly higher in the model with unemployment while price stickiness is nearly the same across models. As a consequence, the introduction of unemployment as excess supply of labor raises the average length of labor contracts (4.76 quarters with unemployment and 2.44 quarters without unemployment). Second, the labor supply curve is significantly more inelastic in the model with unemployment. Finally, the elasticity of capital adjustment costs is lower in the model with unemployment.

With our estimated New-Keynesian model, we analyze unemployment fluctuations driven by sticky wages. The model captures the volatility, countercyclicality and persistence of the quarterly U.S. unemployment rate. In addition, the impulse-response functions provide reasonable reactions of unemployment to technology innovations, demand shocks, monetary shocks and cost-push shocks. In the variance-decomposition analysis, model results indicate that the driving forces of unemployment fluctuations are wage inflation shocks, risk premium (demand-side) shocks, and monetary shocks, with little influence from technology shocks. Besides, the model provides a good matching of the lead-lag comovement between the unemployment rate and output growth.

The paper proceeds as follows. Section 1 describes the model with sticky wages, unemployment as excess supply of labor and variable capital. Section 2 introduces the estimation procedure and discusses the estimation results. Section 3 presents the empirical fit of the two models along three important dimensions (second-moment statistics, variance decomposition and impulse-response functions) and also compares some of the model-implied dynamic cross-correlations with those in the data. Section 4 concludes.

(2010) derive the optimal monetary policy in a model of this kind.

1. A model with unemployment as excess supply of labor and variable capital

This section introduces unemployment in a New-Keynesian model with endogenous capital accumulation. Thus, we borrow most of the elements of the New-Keynesian model described in Smets and Wouters (2007) except for the labor market and wage setting behavior. On that dimension, we extend Casares (2010), with the addition of variable capital accumulation, to incorporate unemployment as excess supply of labor. In contrast to Smets and Wouters (2007), employment variability is determined only by the extensive margin of labor (number of employees), assuming that the number of hours per worker is inelastically supplied as in Hansen (1985).[3] Hence, there is a representative household that supplies a variable number of workers for all differentiated types of labor while each firm demands one specific kind.[4] Following Bénassy (1995), and, more recently, Casares (2007, 2010), labor contracts are revised only if firms and households can get down together to set the labor-clearing nominal wage at the firm level. If labor contracts are not revised on a given period, the nominal wage will be raised as indicated by the application of an indexation rule. Using wage stickiness *à la* Calvo (1983), the labor-clearing nominal wage in firm i is set at the value that results from the intertemporal equilibrium condition:

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left(l_{t+j}^s(i) - l_{t+j}^d(i) \right) = 0, \tag{1}$$

where $\overline{\beta}$ is the discount rate that incorporates detrending from long-run growth, ξ_w is the Calvo (1983)-type constant probability of not experiencing a labor contract revision, $E_t^{\xi_w}$ is the rational expectations operator conditional on the lack of revisions in the future, whereas $l_{t+j}^s(i)$ and $l_{t+j}^d(i)$ represent the log deviations from their respective steady-state levels of the labor supply of workers

[3] This assumption relies on the generally accepted view that most variability of total hours worked in modern economies is explained by changes in the number of employed people whereas fluctuations of the number of hours at work have significantly less influence (Cho and Cooley, 1994; Mulligan, 2001).

[4] Woodford (2003, chapter 3) uses this labor market scenario for fluctuations of the intensive margin of labor (hours), claiming that the existence of heterogeneous labor services is more adequate for sticky-price models than the common assumption of an homogeneous labor market.

and the labor demand for jobs of type$-i$ labor in period $t+j$. If wage stickiness is eliminated ($\xi_w = 0.0$), the wage setting condition (1) would determine a perfect matching between labor supply and labor demand, $l_t^s(i) = l_t^d(i)$.[5] Hence, nominal rigidities on the setting of labor-clearing wages bring about gaps between the amounts of supply of labor (workers provided by the household) and the demand for labor (jobs demanded by the firm).

The actual value of nominal wages depends on how labor supply and labor demand enter in (1). Adapting the household optimizing program of Smets and Wouters (2007), the first order condition on type$-i$ labor supply implies a positive relationship between the specific nominal wage and the labor supply.[6] Taking logs and aggregating across all types of labor services yields

$$l_t^s(i) = \tfrac{1}{\sigma_l}\widetilde{W}_t(i) + l_t^s, \tag{2}$$

where $\widetilde{W}_t(i) = \log W_t(i) - \log W_t = \log W_t(i) - \int_0^1 \log W_t(i)di$ is the relative nominal wage and $l_t^s = \int_0^1 l_t^s(i)di$ is the log deviation of aggregate labor supply from the steady-state level.

Regarding firm-level labor demand, we also borrow the production technology and factor markets used in Smets and Wouters (2007) to derive the optimality condition that makes the ratio of marginal products (capital and labor) equal to the ratio of factor prices (real wage and rental rate)

$$\frac{1-\alpha}{\alpha}\frac{K_t^d(i)}{L_t^d(i)} = \frac{\frac{W_t(i)}{P_t}}{r_t^k},$$

where α is the capital share in the Cobb-Douglas production technology, $K_t^d(i)$ and $L_t^d(i)$ are the levels of capital and labor demanded by firm i, the aggregate price level is P_t and r_t^k is the real rental rate on capital goods. The loglinear version brings the labor demand equation[7]

$$l_t^d(i) = k_t^d(i) - \log W_t(i) + p_t + \log r_t^k. \tag{3}$$

[5]This is the case of an economy with heterogeneous labor and flexible wages described in Woodford (2003, chapter 3).

[6]See the technical appendix -section 1- for the optimizing program of the representative household and the derivation of the first order conditions.

[7]Lower-case variables denote the log deviations with respect to the steady-state levels.

As in Smets and Wouters (2007), the loglinearized production function, with technology shocks ε_t^a, is

$$y_t(i) = (1-\alpha)\, l_t^d(i) + \alpha k_t^d(i) + \varepsilon_t^a, \tag{4}$$

which determines the log of firm-specific capital demand

$$k_t^d(i) = \frac{1}{\alpha}\left(y_t(i) - (1-\alpha)\, l_t^d(i) - \varepsilon_t^a\right). \tag{5}$$

Substituting (5) into (3) and rearranging terms results in

$$l_t^d(i) = y_t(i) - \alpha\left(\log W_t(i) - p_t\right) + \alpha \log r_t^k - \varepsilon_t^a. \tag{6}$$

As discussed in Woodford (2003, p. 168), the Kimball (1995) scheme for the aggregation of goods –also used in the Smets and Wouters (2007)'s model–, yields a log approximation of demand-determined relative output that is inversely related to the relative price,

$$y_t(i) = y_t - \theta \widetilde{p}_t(i), \tag{7}$$

where $\theta > 0$ defines the elasticity of demand and the relative price is $\widetilde{p}_t(i) = \log P_t(i) - \log P_t = \log P_t(i) - \int_0^1 \log P_t(i) di$. Inserting (7) in (6) gives

$$l_t^d(i) = y_t - \theta \widetilde{p}_t(i) - \alpha\left(\log W_t(i) - p_t\right) + \alpha \log r_t^k - \varepsilon_t^a.$$

Summing up across all firms and taking the difference between firm-specific and aggregate values results in the firm-specific labor demand equation

$$l_t^d(i) = -\theta \widetilde{p}_t(i) - \alpha \widetilde{W}_t(i) + l_t, \tag{8}$$

which introduces l_t as the log deviation from steady state of demand-determined employment obtained from the aggregation of log deviations on firm-specific labor demand $l_t = \int_0^1 l_t^d(i) di$. Plugging expressions (2) and (8) in the loglinearized labor-clearing condition (1) for any $t+j$ period yields:

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left[\left(\sigma_l^{-1} + \alpha\right) \widetilde{W}_{t+j}(i) + l_{t+j}^s + \theta \widetilde{p}_{t+j}(i) - l_{t+j}\right] = 0. \tag{9}$$

For non-revised labor contracts, the nominal wage is adjusted with an indexation rule that gives a weight $0 < \iota_w < 1$ to lagged inflation and the complementary weight $1 - \iota_w$ to steady-state inflation plus the stochastic wage-push shock ε_t^w

$$W_t(j) = W_{t-1}(j) \left[(1 + \pi_{t-1})^{\iota_w} (1 + \pi + \varepsilon_t^w)^{1-\iota_w} \right]. \tag{10}$$

This indexation rule is very similar to the one assumed in Smets and Wouters (2007), with the only difference that we include the wage indexation shock ε_t^w to replace the lack of wage mark-up shocks. Then, assuming that the nominal wage attached to the i-type of labor is revised in period t, the conditional expectation of the weighted sum of relative wages is given by

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \widetilde{W}_{t+j}(i) = E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left(\log W_t^*(i) + \sum_{k=1}^{j} \iota_w \pi_{t+k-1} + (1 - \iota_w) \varepsilon_{t+k}^w - \log W_{t+j} \right) =$$

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left(\log W_t^*(i) + \sum_{k=1}^{j} \iota_w \pi_{t+k-1} + (1 - \iota_w) \varepsilon_{t+k}^w - \log W_{t+j} - \log W_t + \log W_t \right). \tag{11}$$

The definition of wage inflation as the log difference in the aggregate nominal wage, $\pi_t^w = \log W_t - \log W_{t-1}$, implies that $\log W_{t+j} = \log W_t + \sum_{k=1}^{j} \pi_{t+k}^w$, which can be inserted in (11) to obtain

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \widetilde{W}_{t+j}(i) = E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left(\widetilde{W}_t^*(i) + \sum_{k=1}^{j} \left(\iota_w \pi_{t+k-1} + (1 - \iota_w) \varepsilon_{t+k}^w - \pi_{t+k}^w \right) \right),$$

where $\widetilde{W}_t^*(i) = \log W_t^*(i) - \log W_t$ is the log difference between the labor-clearing nominal wage and the aggregate nominal wage, i.e. the relative labor-clearing nominal wage. After some algebra to extract the inner sum operator, the last expression yields

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \widetilde{W}_{t+j}(i) = E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \widetilde{W}_t^*(i) + E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_w^j \left(\iota_w \pi_{t+j-1} + (1 - \iota_w) \varepsilon_{t+k}^w - \pi_{t+j}^w \right),$$

which can be used in (9), together with the definition of the unemployment rate as the excess supply of labor,

$$u_{t+j} = l_{t+j}^s - l_{t+j}, \tag{12}$$

to obtain the relative nominal wage set in period t:

$$\widetilde{W}_t^*(i) = -\frac{1-\overline{\beta}\xi_w}{\sigma_l^{-1}+\alpha} E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left(u_{t+j} + \theta \widetilde{p}_{t+j}(i) \right) - E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_w^j \left(\iota_w \pi_{t+j-1} + (1 - \iota_w) \varepsilon_{t+k}^w - \pi_{t+j}^w \right). \tag{13}$$

Equation (13) shows that the value of the nominal wage newly set at the firm depends negatively on the stream of the economy-wide rate of unemployment and also negatively on the stream of relative prices. As in Casares (2010), let us introduce the following guess: relative optimal pricing and relative wage setting are related as follows:

$$\widetilde{p}_t^*(i) = \widetilde{p}_t^* + \tau_1 \widetilde{W}_{t-1}(i), \tag{14a}$$

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \widetilde{p}_t(i), \tag{14b}$$

where $\widetilde{p}_t^*(i) = \log P_t^*(i) - \log P_t$, $\widetilde{p}_t^* = \int_0^1 \log P_t^*(i) di$, $\widetilde{W}_t^* = \int_0^1 \log W_t^*(i) di$, and τ_1 and τ_2 are coefficients to be determined by equilibrium conditions. Using firm-specific relationships (14a)-(14b), and going through some algebra, equation (13) can be rewritten in the following way:[8]

$$\begin{aligned}(1+\Lambda)\,\widetilde{W}_t^*(i) = -\tfrac{\theta\left(1-\overline{\beta}\xi_w\right)}{\left(\sigma_l^{-1}+\alpha\right)\left(1-\overline{\beta}\xi_w\xi_p\right)}\widetilde{p}_t(i) - \tfrac{\left(1-\overline{\beta}\xi_w\right)}{\left(\sigma_l^{-1}+\alpha\right)} E_t \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j u_{t+j} \\ + (1+\Lambda)\, E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_w^j \left(\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1-\iota_w)\,\varepsilon_{t+j}^w\right),\end{aligned} \tag{15}$$

with $\Lambda = \frac{\tau_1 \overline{\beta}\xi_w \theta}{\sigma_l^{-1}+\alpha}\left(1 - \frac{\xi_p\left(1-\overline{\beta}\xi_w\right)}{1-\overline{\beta}\xi_w\xi_p}\right)$. Equation (15) proves right the proposed linear relation (14b), with the following solution for τ_2:

$$\tau_2 = \tfrac{\theta\left(1-\overline{\beta}\xi_w\right)}{\left(\sigma_l^{-1}+\alpha\right)(1-\overline{\beta}\xi_w\xi_p)(1+\Lambda)},$$

and the following expression for the average relative wage set in period t:

$$\widetilde{W}_t^* = -\tfrac{\left(1-\overline{\beta}\xi_w\right)}{\left(\sigma_l^{-1}+\alpha\right)(1+\Lambda)} E_t \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j u_{t+j} + E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_w^j \left(\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1-\iota_w)\,\varepsilon_{t+j}^w\right). \tag{16}$$

Calvo-type wage stickiness and the wage indexation rule (10) imply a proportional relationship between relative wages and the rate of wage inflation adjusted by the indexation factors:

$$\widetilde{W}_t^* = \tfrac{\xi_w}{1-\xi_w}\left(\pi_t^w - \iota_w \pi_{t-1} - (1-\iota_w)\,\varepsilon_t^w\right). \tag{17}$$

[8]The proof is shown in the technical appendix -section 2-.

Combining (16) and (17) yields

$$\pi_t^w = \iota_w \pi_{t-1} + (1-\iota_w)\varepsilon_t^w - \frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w\left(\sigma_l^{-1}+\alpha\right)(1+\Lambda)} E_t \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j u_{t+j}$$
$$+ \frac{1-\xi_w}{\xi_w} E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_w^j \left(\pi_{t+j}^w - \iota_w \pi_{t+j-1} - (1-\iota_w)\varepsilon_{t+j}^w\right).$$

Thus, $\pi_t^w - \overline{\beta}\xi_w E_t \pi_{t+1}^w$ can be expressed as:

$$\pi_t^w - \overline{\beta}\xi_w E_t \pi_{t+1}^w = \iota_w \pi_{t-1} - \overline{\beta}\xi_w \iota_w \pi_t + (1-\iota_w)\varepsilon_t^w - \overline{\beta}\xi_w (1-\iota_w) E_t \varepsilon_{t+1}^w$$
$$- \frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w\left(\sigma_l^{-1}+\alpha\right)(1+\Lambda)} u_t + \frac{1-\xi_w}{\xi_w}\overline{\beta}\xi_w E_t \left(\pi_{t+1}^w - \iota_w \pi_t - (1-\iota_w)\varepsilon_{t+1}^w\right),$$

which collapses to the wage inflation equation

$$\pi_t^w = \overline{\beta} E_t \pi_{t+1}^w + \iota_w \pi_{t-1} - \overline{\beta}\iota_w \pi_t - \frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w\left(\sigma_l^{-1}+\alpha\right)(1+\Lambda)} u_t + (1-\iota_w)\left(\varepsilon_t^w - \overline{\beta} E_t \varepsilon_{t+1}^w\right). \quad (18)$$

Therefore, wage inflation dynamics are inversely related to the rate of unemployment.[9] For the real wage equation, we can take the log difference to its definition, $w_t = \log\left(\frac{W_t}{P_t}\right)$, to obtain

$$w_t - w_{t-1} = \pi_t^w - \pi_t. \quad (19)$$

Using (18) in (19) and solving out for the log of the real wage leads to

$$w_t = w_1 w_{t-1} + (1-w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 u_t + w_5 \left(\varepsilon_t^w - \overline{\beta} E_t \varepsilon_{t+1}^w\right), \quad (20)$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}} \frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w\left(\sigma_l^{-1}+\alpha\right)(1+\Lambda)}$ and $w_5 = \frac{1-\iota_w}{1+\overline{\beta}}$.

Let us turn now to derive the New-Keynesian Phillips curve. The presence of unemployment as excess supply of labor is going to influence inflation dynamics through the effect of firm-specific wage setting on firm-specific real marginal costs. For its derivation, we start from the loglinearized equation for the optimal price in Smets and Wouters (2007):[10]

$$p_t^*(i) = \left(1-\overline{\beta}\xi_p\right) E_t^{\xi_p} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_p^j \left(A\left(mc_{t+j}(i) + \lambda_{t+j}^p\right) + p_{t+j} - \iota_p \sum_{k=1}^{j} \pi_{t+k-1}\right), \quad (21)$$

[9] The slope coefficient in the wage inflation equation (18) is different from the one found in Casares (2010) due to the presence of variable capital and a competitive rental market.

[10] This result is provided in the technical appendix of Smets and Wouters (2007), available at http://www.aeaweb.org/aer/data/june07/20041254_app.pdf .

where $p_t^*(i)$ is the log of the optimal price set by firm i, $A > 0$ is a constant parameter that depends upon the Kimball (1995) goods aggregator and the steady-state price mark-up,[11] and $E_t^{\xi_p}$ is the rational expectations operator conditional on the lack of optimal pricing after period t. The log of the optimal price depends on the evaluation of three factors: the log of the real marginal costs, $mc_{t+j}(i)$, exogenous price mark-up variations, λ_{t+j}^p, and the log of the aggregate price level adjusted by the indexation rule, $p_{t+j} - \iota_p \sum_{k=1}^{j} \pi_{t+k-1}$. Since $p_{t+j} = p_t + \sum_{k=1}^{j} \pi_{t+k}^p$, the following optimal relative price ($\widetilde{p}_t^*(i) = p_t^*(i) - p_t$) obtains:

$$\widetilde{p}_t^*(i) = A\left(1 - \overline{\beta}\xi_p\right) E_t^{\xi_p} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_p^j \left(mc_{t+j}(i) + \lambda_{t+j}^p\right) + E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_p^j \left(\pi_{t+j} - \iota_p \pi_{t+j-1}\right). \tag{22}$$

Unlike Smets and Wouters (2007), the real marginal cost is firm-specific in our model as a consequence of firm-specific nominal wages. Taking logs in the definition of the firm-specific real marginal cost gives[12]

$$mc_t(i) = (1 - \alpha)\left(\log W_t(i) - p_t\right) + \alpha r_t^k - z_t, \tag{23}$$

where z_t is the log of capital utilization. Summing up across all firms and subtracting the result from (23) leads to

$$mc_t(i) = mc_t + (1 - \alpha)\widetilde{W}_t(i). \tag{24}$$

Generalizing (25) for $t + j$ periods and inserting the resulting expressions in (22) yields

$$\widetilde{p}_t^*(i) = A\left(1 - \overline{\beta}\xi_p\right) E_t^{\xi_p} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_p^j \left(mc_{t+j} + (1 - \alpha)\widetilde{W}_{t+j}(i) + \lambda_{t+j}^p\right) + AE_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_p^j \left(\pi_{t+j} - \iota_p \pi_{t+j-1}^p\right). \tag{25}$$

Recalling the firm-specific relationships (14a) and (14b), and doing some algebra, equation (25)

[11] Concretely, $A = 1/((\Phi - 1)\varepsilon_p + 1)$ where ε_p is the curvature of the Kimball goods market aggregator and Φ is the steady-state price mark-up.

[12] The real marginal cost of firm i in period t is $MC_t(i) = \frac{\left(\frac{W_t(i)}{P_t}\right)^{1-\alpha}\left(R_t^k\right)^{\alpha}}{Z_t \alpha^{\alpha}(1-\alpha)^{1-\alpha}}$.

can be written as follows[13]

$$(1+\Theta)\,\widetilde{p}_t^*(i)=\tfrac{A(1-\alpha)\left(1-\overline{\beta}\xi_p\right)\xi_w}{1-\overline{\beta}\xi_w\xi_p}\widetilde{W}_{t-1}(i)+A\left(1-\overline{\beta}\xi_p\right)E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j}+\lambda_{t+j}^p\right)$$
$$+\,(1+\Phi)\,E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j}-\iota_p\pi_{t+j-1}\right), \quad (26)$$

where $\Theta=\tau_2A(1-\alpha)\left(1-\frac{\left(1-\overline{\beta}\xi_p\right)\xi_w}{1-\overline{\beta}\xi_p\xi_w}\right)$. Equation (26) validates (14a) with τ_1 given by

$$\tau_1=\tfrac{A(1-\alpha)\left(1-\overline{\beta}\xi_p\right)\xi_w}{\left(1-\overline{\beta}\xi_p\xi_w\right)(1+\Theta)}, \quad (27)$$

and also determines the average relative prices across all firms that were able to optimally set prices in period t

$$\widetilde{p}_t^*=\tfrac{A\left(1-\overline{\beta}\xi_p\right)}{1+\Theta}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j}+\lambda_{t+j}^p\right)+E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j}-\iota_p\pi_{t+j-1}\right). \quad (28)$$

Calvo pricing combined with the same price indexation rule as in Smets and Wouters (2007) determine, after loglinearization, that relative prices and the rate of inflation are related as follows

$$\widetilde{p}_t^*=\tfrac{\xi_p}{1-\xi_p}\left(\pi_t-\iota_p\pi_{t-1}\right),$$

which can be substituted into the left-hand side of (28) to obtain

$$\pi_t=\iota_p\pi_{t-1}+\tfrac{A\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p(1+\Theta)}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j}+\lambda_{t+j}^p\right)+\tfrac{1-\xi_p}{\xi_p}E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j}-\iota_p\pi_{t+j-1}\right). \quad (29)$$

Rewriting (30) one period ahead to compute $\overline{\beta}\xi_pE_t\pi_{t+1}$ and then subtracting it from (29) results in

$$\pi_t-\overline{\beta}\xi_pE_t\pi_{t+1}=\iota_p\pi_{t-1}-\overline{\beta}\xi_p\iota_p\pi_t^p+\tfrac{A\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p(1+\Theta)}\left(mc_t+\lambda_t^p\right)+\tfrac{1-\xi_p}{\xi_p}\overline{\beta}\xi_pE_t\left(\pi_{t+1}-\iota_p\pi_t\right).$$

Finally, we can put together terms on current and expected next period's inflation, re-scale the mark-up shock at $\varepsilon_t^p=\pi_3\lambda_t^p$ and -following the Smets and Wouters (2007) convention- introduce μ_t^p as the log deviation of the price mark-up ($mc_t=-\mu_t^p$), so that the inflation equation becomes

$$\pi_t=\pi_1\pi_{t-1}+\pi_2E_t\pi_{t+1}-\pi_3\mu_t^p+\varepsilon_t^p, \quad (30)$$

[13] The algebra involved is provided in the technical appendix -section 3-.

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, $\pi_3 = \frac{1}{1+\overline{\beta}\iota_p} \frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(\left(\phi_p-1\right)\varepsilon_p+1\right)(1+\Theta)}$. Interestingly, the inflation equation (30) is a hybrid New-Keynesian Phillips curve where the slope coefficient is affected by the presence of nominal rigidities on both the goods and labor market. More precisely, the slope of (30) depends on the value of the sticky-wage probability, ξ_w, that is contained in Θ, reflecting the complementarities between pricing and wage setting assumed in (14a) and (14b) that are absent in Smets and Wouters (2007). The slope coefficient is also analytically different from the model of Casares (2010) which features unemployment as excess supply of labor and constant capital.

Hence, equations (20) and (30) collect the effects of unemployment as excess supply of labor in the dynamics of the real wage and inflation. The demand-side equations, the monetary policy rule and all the stochastic elements of the model (except for the wage indexation shock) are borrowed from Smets and Wouters (2007) since all these equations can be reached with no influence of the wage setting behavior and unemployment fluctuations. Thus, the shock processes are: the AR(1) technology shock $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, the AR(1) risk premium disturbance that shifts the demand for purchases of consumption and investment goods $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, the exogenous spending shock driven by an AR(1) process with an extra term capturing the potential influence of technology innovations on exogenous spending $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a$, the AR(1) investment shock $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$, the AR(1) monetary policy shock: $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$, the ARMA(1,1) price mark-up shock: $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$, and the ARMA(1,1) wage shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$. The technical appendix -sections 4 and 5- displays the complete set of equations of the model and discusses the equation-to-equation comparison across models. From now on, we will refer to the model with unemployment as the CMV model while the model of Smets and Wouters (2007) will be the SW model, taking the initial letters of the authors' last names.

2. Estimation

We estimate both models with U.S. data from the first quarter of 1984 to the third quarter of 2009. Except for some of the last quarters of the sample, corresponding to the 2007-08 financial crises, this period is characterized by mild fluctuations (the so-called Great Moderation) of aggregate variables (see Stock and Watson, 2002, among others). Thus, the estimation exercises do not suffer from some potential miss-specification sources, such as parameter instability in both the private sector -for instance, Calvo probabilities (Moreno, 2004)- and the monetary policy reactions to inflation or output. Indeed, some authors argue that a sound monetary policy implementation is the main factor behind the low business cycle volatility in this period (Clarida Galí and Gertler, 1999).

Regarding the data set, we take as observable variables quarterly time series of the inflation rate, the Federal funds rate and the log differences of the real Gross Domestic Product (GDP), real consumption, real investment, civilian employment, and the real wage.[14] Thus, variables displaying a long-run trend enter the estimation procedure in log differences to extract their stationary business cycle component.[15] In the estimation of the CMV model, we add the quarterly unemployment rate as another observable variable and ignore the log difference of civilian employment in order to consider the same (number of) shocks in the two models. The data were retrieved from the Federal Reserve of St. Louis (FRED2) database.

The estimation procedure also follows Smets and Wouters (2007). Thus, we consider a two-step Bayesian procedure. In the first step, the log posterior function is maximized in a way that combines the prior information of the parameters with the empirical likelihood of the data. In a second step, we perform the Metropolis-Hastings algorithm to compute the posterior distribution of the parameter set.[16] It should be noted that in the estimation of the CMV model, the slope

[14]The rate of inflation is obtained as the first difference of (the log of) the implicit GDP deflator, whereas the real wage is computed as the ratio between nominal compensation per hour and the GDP price deflator. Smets and Wouters (2007) estimate their model with (the log of) hours in levels, whereas we estimate with (the log of) civilian employment in first differences, given that this series exhibits an upward trend. Nevertheless, estimation results are very similar if the log of employment is considered instead of the growth rate of employment.

[15]In this way, we avoid the well-known measurement error implied by standard filtering treatments.

[16]All estimation exercises are performed with Dynare. A sample of 250,000 draws was used (ignoring the first 20%

coefficients in the inflation and real wage equations were introduced as implicit functions of the undetermined coefficients τ_1 and τ_2. These coefficients can be analytically solved through a non-linear two-equation system. We choose the positive values associated with these solutions, as implied by theory.

In terms of the priors, we select the same prior distributions as Smets and Wouters (2007) for the estimation of the two models (see the first three columns in Tables 1A and 1B), and we also borrow their notation for the structural parameters. In the CMV model we have two additional parameters: the Dixit-Stiglitz elasticity of substitution across goods, θ, and the steady-state unemployment rate, u. The prior mean of these two parameters is set at 6.0, in line with previous studies.

Tables 1A and 1B show the posterior mean estimates together with the 5% and 95% quantiles of the posterior distribution of both SW and CMV model parameters.

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of draws). A step size of 0.3 resulted in an average aceptation rate of roughly 30% across the five Metropolis-Hastings blocks used.

Table 1A. Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
				CMV model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
φ	Normal	4.00	1.50	3.07	1.47	4.60	4.71	2.92	6.39
h	Beta	0.70	0.10	0.52	0.36	0.68	0.56	0.46	0.68
σ_c	Normal	1.50	0.37	0.82	0.57	1.11	0.88	0.60	1.14
σ_l	Normal	2.00	0.75	5.46	4.73	6.29	2.36	1.37	3.27
ξ_p	Beta	0.50	0.10	0.75	0.68	0.81	0.76	0.68	0.84
ξ_w	Beta	0.50	0.10	0.79	0.74	0.85	0.59	0.45	0.72
ι_w	Beta	0.50	0.15	0.38	0.17	0.56	0.49	0.25	0.73
ι_p	Beta	0.50	0.15	0.31	0.09	0.56	0.40	0.14	0.63
ψ	Beta	0.50	0.15	0.81	0.68	0.93	0.68	0.50	0.85
Φ	Normal	1.25	0.12	1.66	1.47	1.83	1.56	1.43	1.69
r_π	Normal	1.50	0.25	1.74	1.36	2.09	1.93	1.64	2.23
ρ	Beta	0.75	0.10	0.84	0.80	0.88	0.82	0.78	0.86
r_y	Normal	0.12	0.05	0.16	0.09	0.24	0.02	-0.01	0.05
$r_{\Delta y}$	Normal	0.12	0.05	0.25	0.19	0.31	0.20	0.15	0.24
π	Gamma	0.62	0.10	0.64	0.53	0.73	0.66	0.52	0.79
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	0.25	0.11	0.37	0.22	0.10	0.34
Δl	Normal	0.00	0.10	–	–	–	0.02	-0.01	0.05
γ	Normal	0.40	0.10	0.41	0.33	0.48	0.41	0.36	0.46
α	Normal	0.30	0.05	0.13	0.10	0.16	0.12	0.09	0.14
θ	Normal	6.00	1.50	6.87	4.60	9.26	–	–	–
u	Normal	6.00	2.00	6.19	5.60	6.83	–	–	–

Table 1B. Priors and estimated posteriors of the shock processes

	Priors			Posteriors					
				CMV model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a	Invgamma	0.10	2.00	0.45	0.36	0.54	0.34	0.30	0.38
σ_b	Invgamma	0.10	2.00	0.06	0.05	0.08	0.06	0.05	0.08
σ_g	Invgamma	0.10	2.00	0.40	0.35	0.45	0.36	0.32	0.40
σ_i	Invgamma	0.10	2.00	0.47	0.37	0.58	0.43	0.32	0.55
σ_R	Invgamma	0.10	2.00	0.13	0.11	0.15	0.13	0.11	0.15
σ_p	Invgamma	0.10	2.00	0.15	0.12	0.17	0.13	0.11	0.16
σ_w	Invgamma	0.10	2.00	1.63	0.91	2.09	0.31	0.25	0.38
ρ_a	Beta	0.50	0.20	0.98	0.97	0.99	0.94	0.91	0.98
ρ_b	Beta	0.50	0.20	0.92	0.89	0.96	0.89	0.83	0.95
ρ_g	Beta	0.50	0.20	0.96	0.93	0.99	0.97	0.95	0.99
ρ_i	Beta	0.50	0.20	0.66	0.52	0.80	0.67	0.52	0.81
ρ_R	Beta	0.50	0.20	0.28	0.15	0.41	0.32	0.19	0.44
ρ_p	Beta	0.50	0.20	0.54	0.16	0.45	0.72	0.51	0.96
ρ_w	Beta	0.50	0.20	0.74	0.57	0.92	0.97	0.95	0.99
μ_p	Beta	0.50	0.20	0.50	0.24	0.77	0.61	0.40	0.85
μ_w	Beta	0.50	0.20	0.40	0.20	0.61	0.71	0.55	0.91
ρ_{ga}	Beta	0.50	0.20	0.30	0.13	0.45	0.60	0.44	0.77

As the last three columns of Tables 1A and 1B show, our replication of the SW model -with a five-year longer sample period- confirms their estimates of the structural parameters. Across models, there are three main differences. First, the labor supply is significantly less elastic in the CMV model, where σ_l is 5.46, while it is 2.36 in the SW model. Second, the Calvo probability of wage stickiness is significantly higher in the CMV model, where ξ_w is 0.79, while it is 0.59 in the SW model. Therefore, the introduction of unemployment as excess supply of labor increases the estimated average length of labor contracts from $(1-0.59)^{-1} = 2.44$ quarters to $(1-0.79)^{-1} = 4.76$ quarters. Third, the elasticity of capital adjustment costs is higher in the SW model, where φ is 4.71, while it is 3.07 in the CMV model.

In both models, the inverse of the elasticity of intertemporal substitution (σ_c) is quite low, around 0.85, and the presence of habit persistence is significant and moderate, as h is in the vicinity of 0.55. The Calvo probability of price stickiness is high and also very similar across models, as the estimates of ξ_p are 0.75 and 0.76, respectively. Both wage and price indexation parameters (ι_w and ι_p, respectively) are slightly higher in the SW model, although not significantly, while the elasticity of capital utilization adjustment cost ψ is 0.81 in the CMV model and somewhat lower at 0.68 in the SW model. Monetary policy parameters are similar across models, with a stabilizing interest reaction of inflation, r_π, between 1.74 and 1.93, a response to output growth, $r_{\Delta y}$, between 0.20 and 0.25, and a high policy rule persistence parameter, ρ, between 0.82 and 0.84. The only noticeable difference is that the response to the output gap, r_y, is not significantly different from zero in the SW model, whereas it is small, 0.16, but significant in the CMV model. The estimate of one plus the fixed-cost share, Φ, is slightly higher in the CMV model. The estimates of the steady-state parameter that determines the long-run rate of growth, γ, the real interest rate, $100(\beta^{-1}-1)$, and the rate of inflation, π, are similar in the two models, as well as the estimate of the capital share in the production function, α. Finally, the elasticity of substitution across goods, θ, and the steady state rate of unemployment, u, are only estimated in the CMV model, and are in line with the values chosen as priors.

Table 1B shows the standard deviations and autocorrelations of the seven structural shocks. The estimates of the standard deviations of the innovations look similar in both models. The only significant difference lies in the volatility of the wage-push innovation, which is significantly higher in the CMV model. As shown in the technical appendix, this is due to the fact that the wage inflation equation differs across models and the wage-push shock also has a different interpretation; it is a wage indexation shock in the CMV model while it is a wage mark-up shock in the SW model.[17] Again, the estimates of persistence and moving-average parameters are similar across models. The monetary policy shock is the one exhibiting the lowest first-order autocorrelation -around 0.30- in both models. Technology, risk premium and exogenous spending innovations are highly persistent across models. The remaining shocks show less persistence with the exception of wage shocks in the SW model.

3. Empirical Fit

This section compares the performance of the SW and CMV models along three dimensions in the first three subsections. First, we analyze the ability of the two models to reproduce second-moment statistics found in U.S. quarterly data. Second, we study the contribution of each structural shock in explaining the total variance decomposition of macroeconomic variables. Third, we carry out an impulse-response analysis. Finally, the fourth subsection analyzes the ability of the CMV model to replicate U.S. lead-lag comovements between the unemployment rate and the output growth rate and between the rate of inflation and the output growth rate.

3.1 Second-moment statistics

Table 2 shows second-moment statistics obtained from actual data, and the ones found in the estimated CMV and SW models.

[17] Moreover, the wage-push shock in the real wage equation of the CMV model appears multiplied by the coefficient w_5, which is estimated to be close to 0.25 while it is a unit coefficient in the real wage equation of the SW model. As a result, the effective size of the wage-push shock is similar across models

Table 2. Second-moment statistics

	Δy	Δc	Δi	Δw	Δl	u	R	π
U.S. data (1984:1-2009:3):								
Standard deviation (%)	0.63	0.61	2.26	0.69	0.42	1.17	0.63	0.26
Correlation with output growth	1.0	0.67	0.70	−0.03	0.61	−0.07	0.27	−0.05
Autocorrelation	0.35	0.30	0.61	0.05	0.69	0.97	0.98	0.51
Estimated CMV model:								
Standard deviation (%)	0.70	0.67	2.37	0.79	0.40	1.12	0.49	0.29
Correlation with output growth	1.0	0.66	0.52	0.25	0.37	−0.23	−0.07	−0.11
Autocorrelation	0.28	0.36	0.60	0.25	0.05	0.92	0.96	0.61
Estimated SW model:								
Standard deviation (%)	0.77	0.69	2.52	0.80	0.51	–	0.50	0.44
Correlation with output growth	1.0	0.74	0.59	0.27	0.73	–	−0.10	−0.28
Autocorrelation	0.38	0.48	0.64	0.32	0.34	–	0.96	0.81

In general, the two models do a good job in reproducing the cyclical features of the data. Thus, both models match quite well the historical volatility of output growth, consumption growth, investment growth, the real wage growth, employment growth, price inflation, and the nominal interest rate. The CMV model matches all these volatilities better except for the nominal interest rate. Importantly, the introduction of unemployment as excess supply of labor in the estimated CMV model reproduces the unemployment rate volatility very accurately.

The contemporaneous correlations between each variable and the output growth rate are also reported in Table 2 as a measure of their procyclical or countercyclical behavior. Both models provide the sign found in the data for these correlations except for the ones of the real wage growth and the nominal interest rate. In general, most of the model implied contemporaneous correlations are close to their data counterparts. The correlation coefficient of the growth rates of output and employment is significantly higher in the SW model. Finally, the two models do a reasonable job in replicating the first-order autocorrelation of all variables, with the exceptions of a low inertia of employment growth in the two models (in particular, in the CMV model) and an excessive inflation persistence in the SW model.

3.2 Variance decomposition

Tables 3 shows the total variance decomposition analysis for the CMV and SW models. In the CMV model, technology innovations, η^a, explain nearly half of the fluctuations in both output and consumption growth, as well as one third of changes in employment. Meanwhile, demand (risk-premium) shocks, η^b, drive around 80% of the variability of the nominal interest rate, more than one fifth of changes in employment and almost 30% of the variability in the rate of unemployment.[18] The influence of exogenous spending (fiscal/net exports) shocks, η^g, is rather low as it determines at most 16% of output growth fluctuations and lower shares of the other variables.[19] Innovations in the investment adjustment costs, η^i, only have a substantial impact on investment fluctuations (54%) whereas monetary policy shocks, η^R, explain between 14% and 21% of fluctuations of output growth, consumption growth, employment growth and the rate of unemployment. Meanwhile, inflation (price-push) shocks, η^p, are the main determinant of inflation variability (43%) but account for less than 10% of the variance share of the other variables. The wage-push (indexation) shock, η^w, exerts a strong influence on the real wage growth (77%) and the rate of unemployment (43%), while having a weak impact on the rest of the variables.

[18]Risk-premium shocks η^b shape private spending in an equivalent way to shocks that shift household utility.

[19]As in Smets and Wouters (2007), the role of the exogenous spending shock is to bring demand-determined changes in output that are not collected by either private consumption or private investment, such as fiscal shocks or exports/imports variations.

Table 3. Long-run variance decomposition[20]

Estimated CMV model:

Innovations	Δy	Δc	Δi	Δw	Δl	u	R	π
Technology, η^a	0.43	0.49	0.08	0.03	0.32	0.03	0.06	0.04
Risk premium, η^b	0.19	0.18	0.20	0.07	0.22	0.28	0.83	0.35
Fiscal/Net exports, η^g	0.16	0.13	0.00	0.00	0.21	0.01	0.02	0.01
Investment adj. costs, η^i	0.04	0.02	0.54	0.01	0.05	0.02	0.02	0.00
Interest-rate, η^R	0.14	0.15	0.12	0.04	0.16	0.21	0.03	0.07
Wage-push, η^w	0.01	0.02	0.02	0.77	0.02	0.43	0.03	0.08
Price-push, η^p	0.03	0.01	0.04	0.08	0.02	0.02	0.01	0.43

Estimated SW model:

Innovations	Δy	Δc	Δi	Δw	Δl	u	R	π
Technology, η^a	0.18	0.06	0.03	0.01	0.09	–	0.04	0.02
Risk premium, η^b	0.20	0.31	0.01	0.05	0.20	–	0.60	0.17
Fiscal/Net exports, η^g	0.13	0.04	0.00	0.00	0.14	–	0.01	0.01
Investment adj. costs, η^i	0.05	0.01	0.67	0.01	0.05	–	0.02	0.00
Interest-rate, η^R	0.14	0.16	0.07	0.03	0.13	–	0.05	0.05
Wage-push, η^w	0.21	0.36	0.13	0.77	0.32	–	0.25	0.45
Price-push, η^p	0.09	0.06	0.09	0.13	0.07	–	0.03	0.30

Therefore, the introduction of unemployment as excess supply of labor in an estimated New-Keynesian model implies that the main driving forces behind U.S. unemployment fluctuations are wage-push shocks (43%), demand shifts driven by risk-premium variations (28%) and also monetary policy shocks (21%). By contrast, the dynamics of output growth are evenly determined by technology shocks (43%) and a mix of demand-side perturbations (risk-premium shocks, ex-

[20]For a 100-period ahead forecast.

ogenous spending innovations and monetary policy shocks that jointly take 49% in its variance decomposition).

The SW model estimated here confirms that technology innovations, η^a, are less influential on business cycle fluctuations than in the CMV model, affecting nearly one fifth of output changes and lower percentages of the rest of the variables. The risk-premium shocks, η^b, account for approximately 20% of the variability of output growth, employment growth and inflation, similarly to the CMV model, while their influence on the movements of the nominal interest rate is still high at 60% of total variability. As in the CMV model, the influence of exogenous spending shocks, η^g, is not substantial with only 13% of fluctuations of output growth, while the investment shock, η^i, mainly affects private investment (67% of its variability) and has a minor influence on the remaining variables. Monetary policy shocks, η^R, account for around 15% of the changes in output and employment; whereas inflation shocks, η^p, only explain significant fractions of variability of inflation (30%) and the changes in the real wage (13%). Wage-push shocks are more influential in the SW model than in the CMV model as they continue explaining over $3/4$ of all variations in the real wage growth, but in addition they also explain an important share of employment and consumption growth changes (32% and 36%), 45% of inflation variability and 21% of changes in the output growth.[21]

3.3. Impulse-response functions

Figures 1, 2 and 3 plot the impulse response functions obtained in the SW and CMV models to the seven -one standard deviation- structural shocks. Across figures, we observe that the responses are quite similar for both models in terms of sign, size and dynamics. Figure 1 shows that the technology shock increases output, consumption and investment, with the effects being more persistent for investment growth. The risk premium shock also raises these three variables whereas

[21] This result will be reflected in the relatively large reactions of these variables to the estimated wage-push shock displayed in the impulse-response analysis conducted below.

the investment shock increases output and investment at the expense of a drop in consumption due to the consequent monetary policy tightening -see Figure 3 below-. The fiscal-net exports (exogenous spending) shock increases output but crowds out investment and consumption, whereas the interest rate shock has a negative impact on these three variables, as expected. Models only disagree substantially in the effects of the wage-push shocks. These different effects do not come as a surprise since the interpretation of this shock in the two models, as explained above, is different. Thus, the CMV model provides a slight one-time increase in output while in the SW model we find a negative output growth response that still persists eight quarters after the shock. Price mark-up shocks are contractionary on output, consumption and investment, through the implied increase in interest rates in response to higher inflation.

Figure 2 shows the responses of the labor market variables to the structural shocks. While both the SW and CMV models include the log differences of the real wage and employment, the CMV model captures the response of the unemployment rate as well, in contrast to the SW model. The real wage rises after technology, risk premium, investment, fiscal-net exports and wage-push shocks, whereas it decreases after monetary policy and price-push shocks. Meanwhile, employment falls countercyclically in both models when there is a positive technology shock. This is a characteristic response in New-Keynesian models with sticky prices, as discussed in Galí (1999). By contrast, procyclical reactions of employment are always reported after demand-side disturbances such as risk-premium shocks, investment shocks, and fiscal-net exports shocks. Both models imply declines of employment in reaction to price and wage cost-push shocks. However, the fall of employment after a wage-push shock is much deeper and persistent in the SW model than in the CMV model (see Figure 2), which is consistent with the variance decomposition analysis conducted above.[22]

[22]This higher sensitivity to wage-push shocks in the SW model is the result of its particular labor market assumptions. Households must attend firm-specific relative labor demand as constraints in their optimizing programs. In turn, those households that apply the indexation rule with the positive wage shock will suffer from a significant employment

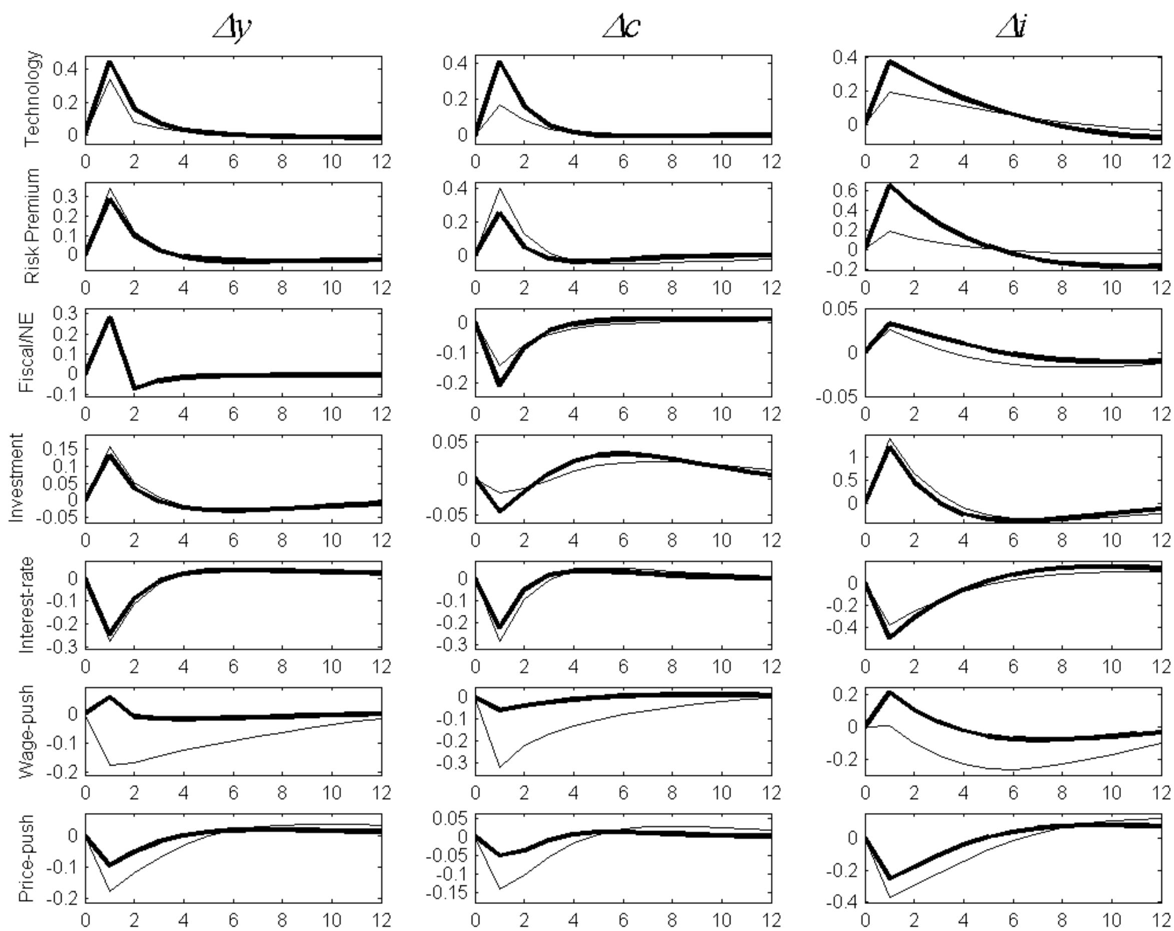

Figure 1: Impulse response functions of the log difference in output Δy, the log difference in consumption Δc, and the log difference in investment Δi. CMV model (thick line) and SW model (thin line).

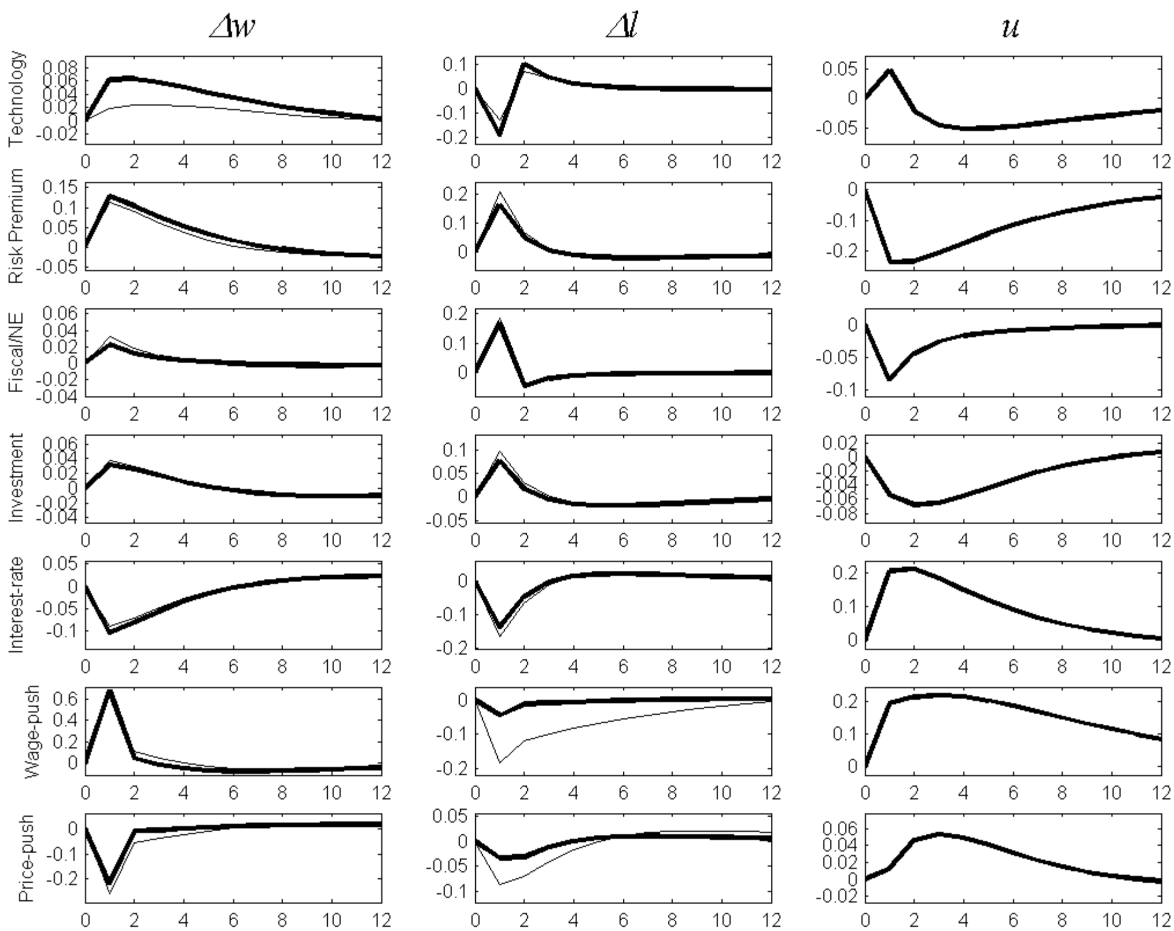

Figure 2: Impulse responses of the log difference of the real wage Δw, the log difference of employment Δl, and the rate of unemployment u. CMV model (thick line) and SW model (thin line).

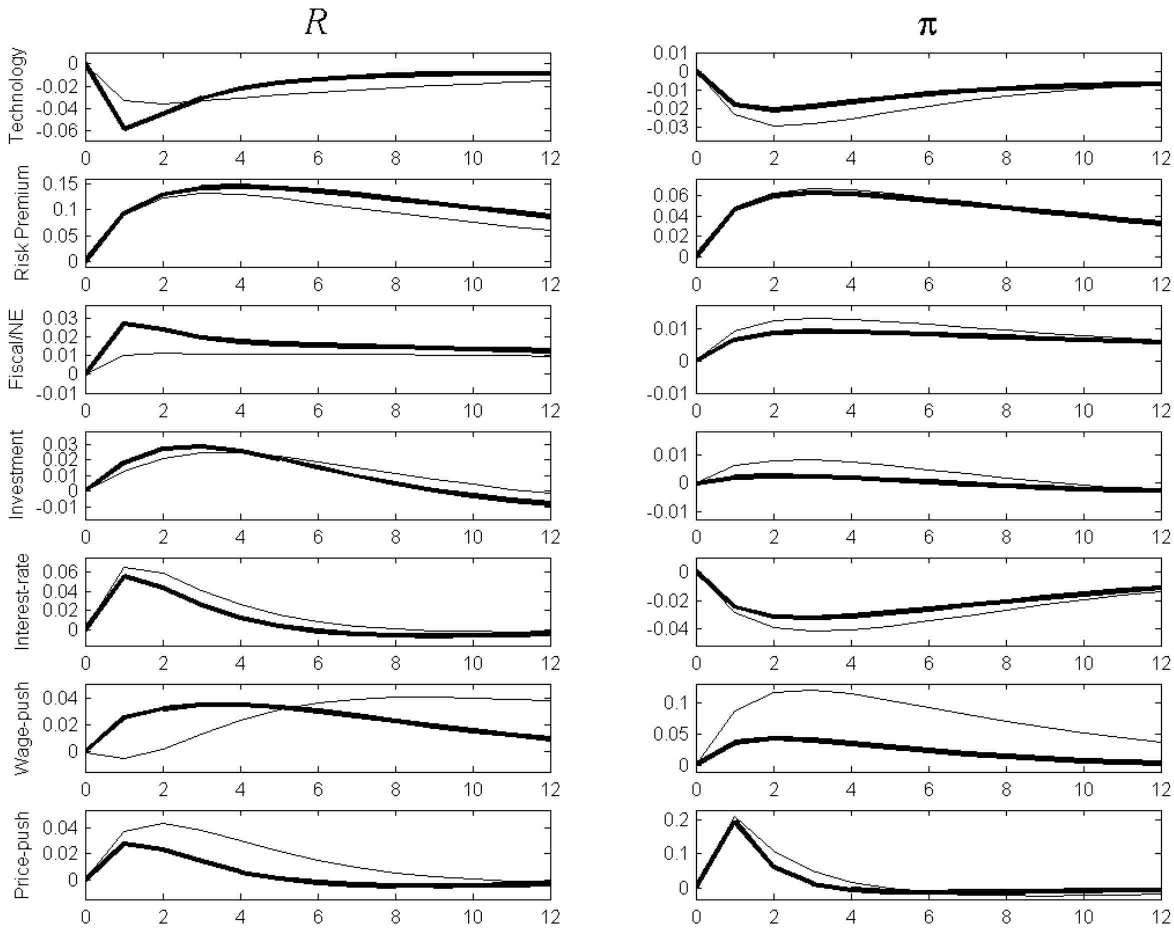

Figure 3: Impulse response functions of the nominal interest rate R and the rate of inflation π. CMV model (thick line) and SW model (thin line).

The reactions of unemployment –only present in the CMV model– are closely and inversely related to those of employment, as the influence of labor supply is small due to the low estimated labor supply elasticity. A positive technology shock increases unemployment only during the quarter of the shock. Expansionary demand shocks (risk premium, investment and fiscal-net exports) decrease unemployment whereas a contractionary monetary policy shock induces a persistent increase in the rate of unemployment. The unemployment rate also raises after wage and price cost-push shocks, since monetary policy reacts through higher interest rates to these supply-side disturbances.

Figure 3 shows the responses of the interest rate and inflation. The plots are again similar across models, although the reactions in the SW model show in general more amplitude. A positive technology shock lowers inflation and the nominal interest rate whereas the three positive demand shocks imply inflation and interest rate increases. The interest rate shock represents an unexpected monetary policy tightening that brings a realistic U-shaped decline in inflation (Romer and Romer, 2004). Finally, positive wage and price shocks increase inflation and, as a result, trigger a positive and persistent increase in the nominal interest rate.

3.4 Dynamic Cross-Correlation Functions

This section studies the ability of the CMV model to reproduce two important comovement patterns observed in U.S. business cycles. First, we examine the dynamic correlations between the rate of unemployment and the output growth rate in a model-to-data comparison (Figure 4). Then, we assess the capacity of the model to replicate the dynamic cross correlations between inflation and output growth rates (Figure 5). These figures compare the lead-lag correlation functions in the data with those implied by the CMV model. They also show the $\pm$ two-standard deviation confidence interval (CI) bands derived from simulated data obtained from 5,000 independent draws of the

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cut. This implies contractionary effects on consumption due to the non-separability between labor and consumption in the utility function, which justifies the difference in the response of output growth across models (see Figure 1).

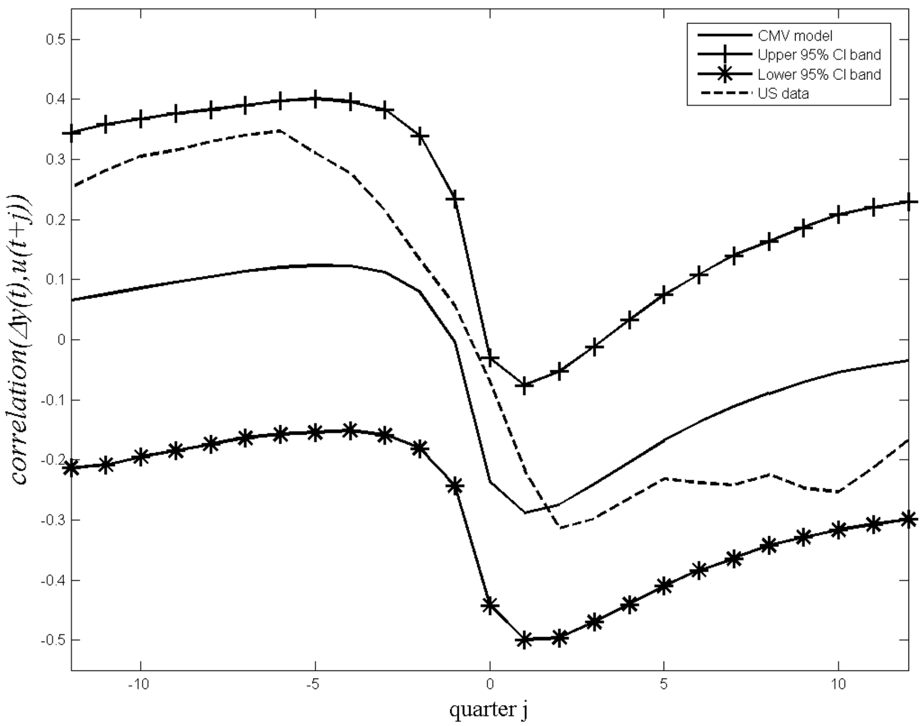

Figure 4: Dynamic cross-correlation between output growth, Δy_t, and unemployment rate, u_{t+j}.

seven innovations of the CMV model.

Figure 4 shows the lead and lag correlations between output growth and the rate of unemployment observed in actual U.S. data (dashed line) and the corresponding comovement implied by the CMV model (solid line) within the model-implied 95% confidence interval. The CMV model reproduces the actual comovement pattern of these two variables. Indeed, the model features the negative contemporaneous comovement between the rates of unemployment and output growth. In addition, the model replicates the positive correlation between lagged rates of unemployment and current output growth rate (i.e. cases with $j < 0$ in Figure 4) and the negative correlation between future unemployment rates and current output growth rate (i.e. cases with $j > 0$ in Figure 4).

Similarly, Figure 5 compares the dynamic comovement patterns between inflation and output

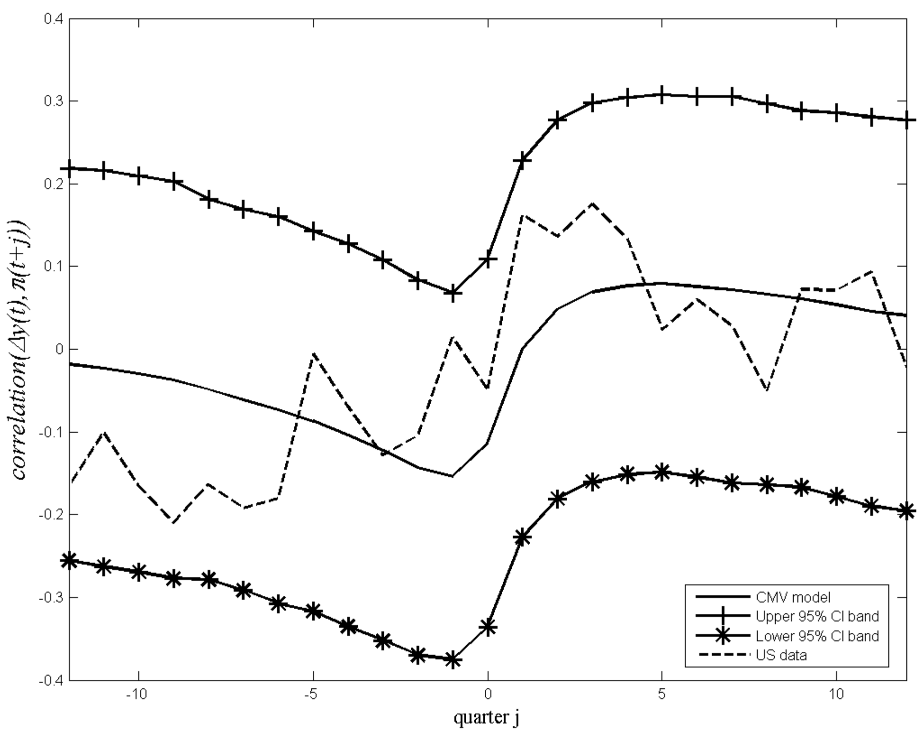

Figure 5: Dynamic cross-correlation between output growth, Δy_t, and inflation, π_{t+j}.

growth found in the data (dashed line) with those implied by the CMV model (solid line). Once again the CMV model does a good job in reproducing the lead-lag pattern shape displayed by actual data.[23] Thus, the model reproduces two stylized facts. First, higher lagged inflation anticipates a lower current output growth rate (i.e. for $j < 0$). Second, higher current output growth anticipates higher future inflation 1-4 quarters ahead.

4. Conclusions

This paper introduces a model with both sticky prices and sticky wages that combines elements of Smets and Wouters (2007) and Casares (2010) in a way that incorporates unemployment as excess supply of labor in a medium-scale New-Keynesian model. The alternative labor market assumptions have implications for the real wage equation (where the real wage is inversely related to the rate of unemployment) and also for the New Keynesian Phillips curve (where the slope coefficient depends upon the level of wage stickiness).

The structural model parameters were estimated with Bayesian techniques and then compared to the estimates of the benchmark New-Keynesian model of Smets and Wouters (2007). Most parameter estimates are quite similar across models. The only substantial differences are that in the model with unemployment the labor supply curve is less elastic, wages are stickier with a longer average in the length of labor contracts, and the elasticity of capital adjustment costs is lower. The empirical comparison also shows that the two models do a similar job in reproducing many of the features characterizing the recent U.S. business cycles. More important, the model with unemployment is able to explain the most salient features -volatility, cyclical correlation and persistence- characterizing U.S. unemployment rate fluctuations. The impulse-response functions show that the rate of unemployment reacts in a countercyclical way to demand shocks and price-push shocks, whereas the response is initially procyclical and later countercyclical after productivity innovations

[23] Smets and Wouters (2007) also report a good matching to dynamic cross correlations between inflation and Hodrick-Prescott filtered output in their model without unemployment.

and clearly procyclical after wage-push innovations.

Our results also indicate that fluctuations in the unemployment rate are mostly driven by wage-push shocks and by demand-side shocks such as risk-premium disturbances and monetary policy shocks, while technology shocks play a more secondary role. Regarding output growth variability, changes in output growth are evenly driven by technology innovations (nearly 50% of total variability) and demand shocks in the model with unemployment. The model without unemployment gives less influence to technology shocks and more to cost-push shocks. Finally, the estimated model with unemployment is able to provide a good match of the U.S. dynamic cross correlation between unemployment and output growth rates.

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Technical Appendix. (Not for publication).

1. Labor supply of type i.

Households maximize intertemporal utility subject to a budget constraint. Unlike Smets and Wouters (2007), there is a representative household that provides all types of labor services. Thus, instantaneous utility is

$$\left[\frac{1}{1-\sigma_c}\left(C_t-\lambda C_{t-1}\right)^{1-\sigma_c}\right]\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\int_0^1\left(L_t^s(i)\right)^{1+\sigma_l}di\right).$$

The first order conditions for consumption and labor supply of type i that result from the household optimizing program are

$$\left(C_t-\lambda C_{t-1}\right)^{-\sigma_c}\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\int_0^1\left(L_t^s(i)\right)^{1+\sigma_l}di\right)-\Xi_t=0, \qquad (C_t^{foc})$$

$$-\left(\sigma_c-1\right)L_t(i)^{\sigma_l}\left[\frac{1}{1-\sigma_c}\left(C_t-\lambda C_{t-1}\right)^{1-\sigma_c}\right]\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\int_0^1\left(L_t^s(i)\right)^{1+\sigma_l}di\right)+\Xi_t\frac{W_t(i)}{P_t}=0, \qquad (L_t^{foc}(i))$$

where Ξ_t is the Lagrange multiplier of the budget constraint in period t. Inserting (C_t^{foc}) in ($L_t^{foc}(i)$) and rearranging terms leads to the optimal supply of i-type labor

$$L_t^s(i)=\left(\frac{\frac{W_t(i)}{P_t}}{C_t-\lambda C_{t-1}}\right)^{1/\sigma_l}.$$

2. Derivation of equation (15): The dynamics of relative labor-clearing wages.

The equation of the relative nominal wage, (13) in the main text, is:

$$\widetilde{W}_t^*(i) = -\frac{1-\overline{\beta}\xi_w}{\sigma_l^{-1}+\alpha} E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \left(u_{t+j} + \theta \widetilde{p}_{t+j}(i)\right) - E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_w^j \left(\iota_w \pi_{t+j-1} + (1-\iota_w)\varepsilon_{t+k}^w - \pi_{t+j}^w\right). \tag{A1}$$

We want to express the expected stream of relative prices, $E_t^{\xi_w} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_w^j \widetilde{p}_{t+j}(i)$, as a function of the relative price current value in order to have an expression for $\widetilde{W}_t^*(i)$ consistent with (14b). Beginning with $\widetilde{p}_{t+1}(i)$, the Calvo aggregation scheme implies

$$E_t^{\xi_w} \widetilde{p}_{t+1}(i) = \xi_p \left(\widetilde{p}_t(i) + \iota_p \pi_t - E_t \pi_{t+1}\right) + \left(1-\xi_p\right) E_t^{\xi_w} \widetilde{p}_{t+1}^*(i), \tag{A2}$$

where the second term is $E_t^{\xi_w} \widetilde{p}_{t+1}^*(i) = E_t \widetilde{p}_{t+1}^* + \tau_1 \widetilde{W}_t^*(i)$ using (14a) in $t+1$ conditional on having a labor-clearing wage contract set in t. Using that information in (A2) yields

$$E_t^{\xi_w} \widetilde{p}_{t+1}(i) = \xi_p \left(\widetilde{p}_t(i) + \iota_p \pi_t - E_t \pi_{t+1}\right) + \left(1-\xi_p\right) \left(E_t \widetilde{p}_{t+1}^* + \tau_1 \widetilde{W}_t^*(i)\right). \tag{A3}$$

The Calvo aggregation scheme implies $\widetilde{p}_{t+1}^* = \frac{\xi_p}{1-\xi_p}\left(\pi_{t+1} - \iota_p \pi_t\right)$. Taking rational expectations and substituting in (A3), this equation becomes:

$$E_t^{\xi_w} \widetilde{p}_{t+1}(i) = \xi_p \widetilde{p}_t(i) + \tau_1 \left(1-\xi_p\right) \widetilde{W}_t^*(i). \tag{A4}$$

Analogously to (A3), $E_t^{\xi_w} \widetilde{p}_{t+2}(i)$ is a linear combination of non-adjusted relative prices and optimal relative prices:

$$E_t^{\xi_w} \widetilde{p}_{t+2}(i) = \xi_p \left(E_t^{\xi_w} \widetilde{p}_{t+1}(i) + \iota_p E_t \pi_{t+1} - E_t \pi_{t+2}\right) + \left(1-\xi_p\right) E_t^{\xi_w} \widetilde{p}_{t+2}^*(i),$$

where using (A28) for $E_t^{\xi_w} \widetilde{p}_{t+1}(i)$ leads to

$$E_t^{\xi_w} \widetilde{p}_{t+2}(i) = \xi_p \left(\xi_p \widetilde{p}_t(i) + \tau_1 \left(1-\xi_p\right) \widetilde{W}_t^*(i) + \iota_p E_t \pi_{t+1} - E_t \pi_{t+2}\right) + \left(1-\xi_p\right) E_t^{\xi_w} \widetilde{p}_{t+2}^*(i). \tag{A5}$$

Recalling (14a) in period $t+2$ conditional on the lack of wage resetting, $E_t^{\xi_w}\widetilde{p}^*_{t+2}(i) = E_t\widetilde{p}^*_{t+2} + \tau_1 E_t^{\xi_w}\widetilde{W}^*_{t+1}(i) = E_t\widetilde{p}^*_{t+2} + \tau_1\left(\widetilde{W}^*_t(i) + \iota_w\pi_t + (1-\iota_w)E_t\varepsilon^w_{t+1} - E_t\pi^w_{t+1}\right)$, (A5) becomes

$$E_t^{\xi_w}\widetilde{p}_{t+2}(i) = \xi_p\left(\xi_p\widetilde{p}_t(i) + \tau_1\left(1-\xi_p\right)\widetilde{W}^*_t(i) + \iota_p E_t\pi_{t+1} - E_t\pi_{t+2}\right)$$
$$+\left(1-\xi_p\right)\left(E_t\widetilde{p}^*_{t+2} + \tau_1\left(\widetilde{W}^*_t(i) + \iota_w\pi_t + (1-\iota_w)E_t\varepsilon^w_{t+1} - E_t\pi^w_{t+1}\right)\right),$$

where using $\widetilde{p}^*_{t+2} = \frac{\xi_p}{1-\xi_p}\left(\pi_{t+2} - \iota_p\pi_{t+1}\right)$ simplifies to

$$E_t^{\xi_w}\widetilde{p}_{t+2}(i) = \xi_p^2\widetilde{p}_t(i) + \tau_1\left(1-\xi_p^2\right)\widetilde{W}^*_t(i) - \tau_1\left(1-\xi_p\right)\left(E_t\pi^w_{t+1} - \iota_w\pi_t - (1-\iota_w)E_t\varepsilon^w_{t+1}\right). \tag{A6}$$

A generalization of (A4) and (A6) results in the following rule:

$$E_t^{\xi_w}\widetilde{p}_{t+j}(i) = \xi_p^j\widetilde{p}_t(i) + \tau_1\left(1-\xi_p^j\right)\widetilde{W}^*_t(i) - \tau_1 E_t\sum_{k=1}^{j-1}\left(1-\xi_p^{j-k}\right)\left(\pi^w_{t+k} - \iota_w\pi_{t+k-1} - (1-\iota_w)E_t\varepsilon^w_{t+k}\right),$$

implying the following expected sum of discounted relative prices:

$$E_t^{\xi_w}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j\widetilde{p}_{t+j}(i) = \frac{1}{1-\overline{\beta}\xi_w\xi_p}\widetilde{p}_t(i)$$
$$+\tau_1\left(\frac{\overline{\beta}\xi_w}{1-\overline{\beta}\xi_w} - \frac{\overline{\beta}\xi_w\xi_p}{1-\overline{\beta}\xi_w\xi_p}\right)\left(\widetilde{W}^*_t(i) - E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_w^j\left(\pi^w_{t+j} - \iota_w\pi_{t+j-1} - (1-\iota_w)E_t\varepsilon^w_{t+j}\right)\right). \tag{A7}$$

Substituting (A7) in the relative wage equation (A1), we obtain:

$$\left(1 + \frac{\tau_1\overline{\beta}\xi_w\theta}{\sigma_l^{-1}+\alpha}\left(1 - \frac{\xi_p\left(1-\overline{\beta}\xi_w\right)}{1-\overline{\beta}\xi_w\xi_p}\right)\right)\widetilde{W}^*_t(i) = -\frac{\theta\left(1-\overline{\beta}\xi_w\right)}{\left(\sigma_l^{-1}+\alpha\right)\left(1-\overline{\beta}\xi_w\xi_p\right)}\widetilde{p}_t(i) - \frac{1-\overline{\beta}\xi_w}{\sigma_l^{-1}+\alpha}E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_w^j u_{t+j}$$
$$+\left(1 + \frac{\tau_1\overline{\beta}\xi_w\theta}{\sigma_l^{-1}+\alpha}\left(1 - \frac{\xi_p\left(1-\overline{\beta}\xi_w\right)}{1-\overline{\beta}\xi_w\xi_p}\right)\right)E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_w^j\left(\pi^w_{t+j} - \iota_w\pi_{t+j-1} - (1-\iota_w)E_t\varepsilon^w_{t+j}\right),$$

that corresponds to equation (15) displayed in the main text.

3. Derivation of equation (26) on the dynamics of relative optimal prices.

Equation (25) from section 1 in the main text indicates that the loglinearized relative price depends upon terms on relative wages as follows

$$\widetilde{p}_t^*(i) = A\left(1-\overline{\beta}\xi_p\right) E_t^{\xi_p} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_p^j \left(mc_{t+j} + (1-\alpha)\widetilde{W}_{t+j}(i) + \lambda_{t+j}^p\right) + E_t \sum_{j=1}^{\infty} \overline{\beta}^j \xi_p^j \left(\pi_{t+j} - \iota_p \pi_{t+j-1}^p\right). \tag{A8}$$

To be consistent with the value of the undetermined coefficient τ_1 implied by the linear relationship (14a) from section 2, we must relate $E_t^{\xi_p} \sum_{j=0}^{\infty} \overline{\beta}^j \xi_p^j \widetilde{W}_{t+j}(i)$ to $\widetilde{W}_{t-1}(i)$. The Calvo scheme applied for wage setting in period t results in

$$\widetilde{W}_t(i) = \xi_w\left(\widetilde{W}_{t-1}(i) + \iota_w \pi_{t-1} + (1-\iota_w)\varepsilon_t^w - \pi_t^w\right) + (1-\xi_w)\widetilde{W}_t^*(i).$$

Using the proposed conjecture (14b) conditional on optimal pricing in period t allows us to write $\widetilde{W}_t^*(i)$ depending upon the average relative value of new contracts, $\widetilde{W}_t^*$, and also upon the relative optimal price, $\widetilde{p}_t^*(i)$:

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \widetilde{p}_t^*(i),$$

which can be inserted in the previous expression to reach

$$\widetilde{W}_t(i) = \xi_w\left(\widetilde{W}_{t-1}(i) + \iota_w \pi_{t-1} + (1-\iota_w)\varepsilon_t^w - \pi_t^w\right) + (1-\xi_w)\left(\widetilde{W}_t^* - \tau_2 \widetilde{p}_t^*(i)\right). \tag{A9}$$

Recalling that $\widetilde{W}_t^* = \frac{\xi_w}{1-\xi_w}\left(\pi_t^w - \iota_w \pi_{t-1} - (1-\iota_w)\varepsilon_t^w\right)$ from Calvo-type sticky wages, and cancelling terms in (A9), we obtain

$$\widetilde{W}_t(i) = \xi_w \widetilde{W}_{t-1}(i) - \tau_2 (1-\xi_w)\widetilde{p}_t^*(i). \tag{A10}$$

Repeating the procedure one period ahead for $E_t^{\xi_p}\widetilde{W}_{t+1}(i)$, we have

$$E_t^{\xi_p}\widetilde{W}_{t+1}(i) = \xi_w\left(\widetilde{W}_t(i) + \iota_w \pi_t + (1-\iota_w)E_t\varepsilon_{t+1}^w - E_t\pi_{t+1}^w\right) + (1-\xi_w)E_t^{\xi_p}\widetilde{W}_{t+1}^*(i). \tag{A11}$$

Using (14b) conditional on no-optimal pricing in $t+1$ yields

$$E_t^{\xi_p}\widetilde{W}_{t+1}^*(i) = \widetilde{W}_{t+1}^* - \tau_2\left(\widetilde{p}_t^*(i) + \iota_p \pi_t - E_t \pi_{t+1}\right),$$

which can be inserted in (A11) together with (A10) and also $\widetilde{W}_{t+1}^{*}=\frac{\xi_w}{1-\xi_w}\left(\pi_{t+1}^{w}-\iota_w\pi_t-(1-\iota_w)\,\varepsilon_{t+1}^{w}\right)$ to obtain (after dropping terms that cancel out)

$$E_t^{\xi_p}\widetilde{W}_{t+1}(i)=\xi_w^2\widetilde{W}_t(i)-\tau_2\left(1-\xi_w^2\right)\widetilde{p}_t^{*}(i)+\tau_2\left(1-\xi_w\right)\left(E_t\pi_{t+1}-\iota_p\pi_t\right). \tag{A12}$$

A generalization of (A10) and (A12) for a $t+j$ future period gives the following expression

$$E_t^{\xi_p}\widetilde{W}_{t+j}(i)=\xi_w^{j+1}\widetilde{W}_{t-1}(i)-\tau_2\left(1-\xi_w^{j+1}\right)\widetilde{p}_t^{*}(i)+\tau_2\left(1-\xi_w^{j-k+1}\right)E_t\sum_{k=1}^{j}\left(\pi_{t+k}-\iota_p\pi_{t+k-1}\right). \tag{A13}$$

Using (A13), the expected sum of the stream of conditional relative wages becomes

$$\begin{aligned}E_t^{\xi_p}\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\widetilde{W}_{t+j}(i)=\ &\frac{\xi_w}{1-\overline{\beta}\xi_w\xi_p}\widetilde{W}_{t-1}(i)-\tau_2\left(\frac{1}{1-\overline{\beta}\xi_p}-\frac{\xi_w}{1-\overline{\beta}\xi_w\xi_p}\right)\widetilde{p}_t^{*}(i)\\ &+\tau_2\left(\frac{1}{1-\overline{\beta}\xi_p}-\frac{\xi_w}{1-\overline{\beta}\xi_w\xi_p}\right)E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j}-\iota_p\pi_{t+j-1}\right).\end{aligned} \tag{A14}$$

Substituting (A14) in (A8) yields

$$\begin{aligned}\left(1+\tau_2A(1-\alpha)\left(1-\frac{\left(1-\overline{\beta}\xi_p\right)\xi_w}{1-\overline{\beta}\xi_w\xi_p}\right)\right)\widetilde{p}_t^{*}(i)=\ &\frac{A(1-\alpha)\left(1-\overline{\beta}\xi_p\right)\xi_w}{1-\overline{\beta}\xi_w\xi_p}\widetilde{W}_{t-1}(i)\\ &+A\left(1-\overline{\beta}\xi_p\right)E_t\sum_{j=0}^{\infty}\overline{\beta}^j\xi_p^j\left(mc_{t+j}+\lambda_{t+j}^{p}\right)\\ &+\left(1+\tau_2A(1-\alpha)\left(1-\frac{\left(1-\overline{\beta}\xi_p\right)\xi_w}{1-\overline{\beta}\xi_w\xi_p}\right)\right)E_t\sum_{j=1}^{\infty}\overline{\beta}^j\xi_p^j\left(\pi_{t+j}-\iota_p\pi_{t+j-1}\right),\end{aligned}$$

which is equation (26) displayed in the main text.

4. New-Keynesian model with unemployment as excess supply of labor and variable capital.

Log-linearized set of model equations:

- Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{A15}$$

where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, and $z_y = r^k \frac{K}{Y}$ are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.

- Consumption equation:

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (R_t - E_t \pi_{t+1}) + \varepsilon_t^b, \tag{A16}$$

where $c_1 = \frac{\lambda/\gamma}{1+\lambda/\gamma}$, $c_2 = \frac{[(\sigma_c - 1) wL/C]}{\sigma_c(1+\lambda/\gamma)}$ and $c_3 = \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}$.[24]

- Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \tag{A17}$$

where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{\left(1+\overline{\beta}\right)\gamma^2\varphi}$ with $\overline{\beta} = \beta\gamma^{(1-\sigma_c)}$.

- Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \tag{A18}$$

where $q_1 = \overline{\beta}\gamma^{-1}(1-\delta) = \frac{(1-\delta)}{\left(r^k + 1 - \delta\right)}$.

[24] Notice that we introduce the risk premium shock ε_t^b as expansionary, akin to a positive preference shock, whereas Smets and Wouters (2007) introduce this same shock with the opposite sign, thus contractionary.

- Log-linearized aggregate production function:

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha)l_t + \varepsilon_t^a\right), \tag{A19}$$

where $\phi_p = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and α is the capital-share in the production function.[25]

- Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \tag{A20}$$

- Capital utilization:

$$z_t = z_1 r_t^k, \tag{A21}$$

where $z_1 = \frac{1-\psi}{\psi}$.

- Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1-k_1)i_t + k_2 \varepsilon_t^i, \tag{A22}$$

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)\left(1+\overline{\beta}\right)\gamma^2\varphi$.

- Price mark-up (negative of the log of the real marginal cost):

$$\mu_t^p = mpl_t - w_t = \alpha\left(k_t^s - l_t\right) + \varepsilon_t^a - w_t. \tag{A23}$$

- New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p, \tag{A24}$$

[25] From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, and $\pi_3 = \frac{1}{1+\overline{\beta}\iota_p}\left[\frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(\left(\phi_p-1\right)\varepsilon_p+1\right)(1+\Theta)}\right]$ with $\Theta = \tau_2\left(1 - \frac{\left(1-\overline{\beta}\xi_p\right)\xi_w}{1-\overline{\beta}\xi_p\xi_w}\right)$. The coefficient of the curvature of the Kimball goods market aggregator is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).[26]

- Optimal demand for capital by firms

$$-\left(k_t^s - l_t\right) + w_t = \frac{1}{r^k} r_t^k. \tag{A25}$$

- Unemployment equation:

$$u_t = l_t^s - l_t = \left(\tfrac{1}{\sigma_l} w_t - \tfrac{1}{\sigma_l(1-\lambda/\gamma)}\left(c_t - \left(\lambda/\gamma\right) c_{t-1}\right)\right) - l_t, \tag{A26}$$

where l_t^s denotes log-fluctuations of the aggregate labor supply and l_t denotes log-fluctuations of demand-determined aggregate labor.

- Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 u_t + w_5\left(\varepsilon_t^w - \overline{\beta} E_t \varepsilon_{t+1}^w\right), \tag{A27}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{\left(1-\overline{\beta}\xi_w\right)\left(1-\xi_w\right)}{\xi_w\left(\sigma_l^{-1}+\alpha\right)(1+\Lambda)}\right]$ with $\Lambda = \frac{\tau_1\overline{\beta}\xi_w\theta}{\sigma_l^{-1}+\alpha}\left(1 - \frac{\xi_p\left(1-\overline{\beta}\xi_w\right)}{1-\overline{\beta}\xi_w\xi_p}\right)$, and $w_5 = \frac{1-\iota_w}{1+\overline{\beta}}$.

- Monetary policy rule, a Taylor-type rule for nominal interest rate management:

$$R_t = \rho R_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_Y\left(y_t - y_t^p\right)\right] + r_{\triangle y}\left[\left(y_t - y_t^p\right) - \left(y_{t-1} - y_{t-1}^p\right)\right] + \varepsilon_t^R. \tag{A28}$$

Potential (natural-rate) variables are obtained assuming flexible prices, flexible wages and shutting down price mark-up and wage indexation shocks. They are denoted by adding a superscript "p".

[26] Using Dixit-Stiglitz output aggregators as in Smets and Wouters (2003) or Christiano *et al.* (2005), the slope coefficient on the prime mark-up changes to $\pi_3 = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\left[\frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p(1+\Theta)}\right]$.

- Flexible-price condition (no price mark-up fluctuations, $\mu_t^p = mpl_t - w_t = 0$):

$$\alpha\left(k_t^{s,p} - l_t^p\right) + \varepsilon_t^a = w_t^p. \tag{A29}$$

- Flexible-wage condition (no wage mark-up fluctuations, $\mu_t^w = w_t - mrs_t = 0$):

$$w_t^p = \sigma_l l_t^p + \tfrac{1}{1-\lambda/\gamma}\left(c_t^p - \lambda/\gamma c_{t-1}^p\right). \tag{A30}$$

- Potential aggregate resources constraint:

$$y_t^p = c_y c_t^p + i_y i_t^p + z_y z_t^p + \varepsilon_t^g. \tag{A31}$$

- Potential consumption equation:

$$c_t^p = c_1 c_{t-1}^p + (1-c_1)\,E_t c_{t+1}^p + c_2\left(l_t^p - E_t l_{t+1}^p\right) - c_3\left(R_t^p - E_t \pi_{t+1}^p\right) + \varepsilon_t^b. \tag{A32}$$

- Potential investment equation:

$$i_t^p = i_1 i_{t-1}^p + (1-i_1)\,E_t i_{t+1}^p + i_2 q_t^p + \varepsilon_t^i. \tag{A33}$$

- Arbitrage condition (value of potential capital, q_t^p):

$$q_t^p = q_1 E_t q_{t+1}^p + (1-q_1) E_t r_{t+1}^{k,p} - \left(R_t^p - E_t \pi_{t+1}^p\right) + c_3^{-1}\varepsilon_t^b. \tag{A34}$$

- Log-linearized potential aggregate production function:

$$y_t^p = \phi_p\left(\alpha k_t^{s,p} + (1-\alpha) l_t^p + \varepsilon_t^a\right). \tag{A35}$$

- Potential capital (with one period time-to-build):

$$k_t^{s,p} = k_{t-1}^p + z_t^p. \tag{A36}$$

- Potential capital utilization:

$$z_t^p = z_1 r_t^{k,p}. \tag{A37}$$

- Potential capital accumulation equation:

$$k_t^p = k_1 k_{t-1}^p + (1-k_1) i_t^p + k_2 \varepsilon_t^i. \tag{A38}$$

- Potential demand for capital by firms ($r_t^{k,p}$ is the potential log of the rental rate of capital):

$$-\left(k_t^{s,p} - l_t^p\right) + w_t^p = \frac{1}{r^k} r_t^{k,p}. \tag{A39}$$

- Monetary policy rule (under flexible prices and flexible wages):

$$R_t^p = \rho R_{t-1}^p + (1-\rho)\left[r_\pi \pi_t^p\right] + \varepsilon_t^R. \tag{A40}$$

Equations-and-variables summary

- Set of equations:

Equations (A15)-(A40), which may determine solution paths for 26 endogenous variables. The subset (A29)-(A40) is introduced to solve the potential (natural-rate) block of the model.

- Set of variables:

Endogenous variables (26): y_t, c_t, i_t, z_t, l_t, R_t, π_t, q_t, r_t^k, k_t^s, k_t, μ_t^w, μ_t^p, w_t, y_t^p, c_t^p, i_t^p, z_t^p, l_t^p, R_t^p, π_t^p, q_t^p, $r_t^{k,p}$, $k_t^{s,p}$, k_t^p, and w_t^p.

Predetermined variables (12): c_{t-1}, i_{t-1}, k_{t-1}, π_{t-1}, w_{t-1}, R_{t-1}, y_{t-1}, y_{t-1}^p, c_{t-1}^p, i_{t-1}^p, k_{t-1}^p, and R_{t-1}^p.

Exogenous variables (7): AR(1) technology shock $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, AR(1) risk premium shock $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, AR(1) exogenous spending shock cross-correlated to technology innovations $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a$, AR(1) investment shock $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$, AR(1) monetary policy shock $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$, ARMA(1,1) price mark-up shock $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$ and ARMA(1,1) wage mark-up shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$.

5. Model comparison.

In order to recover the Smets and Wouters (2007) model, we must introduce the following modifications in equations (A24), (A26) and (A27):

i) New-Keynesian Phillips Curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p, \tag{A24'}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, and $\pi_3 = \frac{1}{1+\overline{\beta}\iota_p}\left[\frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(\left(\phi_p-1\right)\varepsilon_p+1\right)}\right]$. Notice the changes in the slope coefficient π_3.

ii) Wage mark-up (log difference between the marginal rate of substitution between working and consuming and the real wage) which replaces unemployment present in the CMV model:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1-\lambda/\gamma}\left(c_t - \lambda/\gamma c_{t-1}\right)\right) \tag{A26'}$$

iii) Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w, \tag{A27'}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, and $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$. Notice the changes in the slope coefficient w_4.

In addition, the coefficient c_2 from equations (A16) and (A32) suffers a slight change to accommodate the fact that there is a wage mark-up in the SW model. In turn, the coefficient to be used in the SW model is $c_2 = \frac{[(\sigma_c-1)wL/(\phi_w C)]}{\sigma_c(1+\lambda/\gamma)}$ with a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007).

The set of variables is identical in the two models except for the replacement of the rate of unemployment, u_t, instead of the wage mark-up μ_t^w. In turn there are 26 equations and 26 endogenous variables in both models.