STICKY PRICES, STICKY WAGES, AND ALSO UNEMPLOYMENT

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Abstract

This paper shows a New Keynesian model where wages are set at the value that matches household’s labor supply with firm’s labor demand. Subsequently, wage stickiness brings industry-level unemployment fluctuations. After aggregation, the rate of wage inflation is negatively related to unemployment, as in the original Phillips (1958) curve, with an additional term that provides forward-looking dynamics. The supply-side of the model can be captured with dynamic expressions equivalent to those obtained in Erceg, Henderson, and Levin (2000), though with different slope coefficients. Impulse-response functions from a technology shock illustrate the interactions between sticky prices, sticky wages and unemployment.

Keywords: New Keynesian model, sticky wages, unemployment.

JEL codes: E12, E24, E32, J30.

1 Introduction

The introduction of nominal rigidities in microfounded models (so-called New Keynesian models) brought enormous consequences for Macroeconomics, in general, and Monetary Economics, in particular. At first, nominal frictions lead to short-run real effects from demand-side shocks breaking down the classical dichotomy between nominal and real variables that was present in Neoclassical models (Fischer, 1977; Taylor, 1979; and Blanchard

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and Fischer, 1989, chapter 8). A second wave of papers (Hairault and Portier, 1993; Yun, 1996) showed how incorporating price stickiness is key to replicate, in a realistic fashion, the business-cycle responses of inflation and output to technology and monetary shocks. Moreover, New Keynesian models predict that the level of employment (total hours) falls after an expansionary technology shock as empirically supported by Galí (1999) and Francis and Ramey (2005).¹ Last but not least, New Keynesian models have become the working instrument for much of the latest monetary policy analysis due to both its theoretical appeal, as they overcome the Lucas (1976) critique, and its empirical plausibility. Two widely-used books on Monetary Economics, recently published, that rely the analysis upon the New Keynesian framework are Walsh (2003), and Woodford (2003).

However, today’s New Keynesian framework is little Keynesian in one particular sense. It is commonly presented as a General Equilibrium model that ignores the presence of unemployment in the labor market.² This fails to comply with both the original Keynesian analysis of the labor market (Keynes, 1936, chapters 18-20), and also deviates from the actual functioning of labor market in developed economies where we observe unemployment fluctuations.³

The influential paper by Erceg, Henderson and Levin (2000), henceforth EHL, brought a follow-up of New Keynesian papers with sticky wages in addition to sticky prices. Representative examples among these papers are Amato and Laubach (2003), Smets and Wouters (2003, 2007), and Christiano et al. (2005). With a somehow different labor market structure, this paper describes a New Keynesian model with sticky prices, sticky wages, and also unemployment. I voluntarily stress the word "also" because the New Keynesian literature that I just cited incorporate wage setting rigidities that, somehow surprisingly, do not deliver unemployment situations. By contrast, this paper shows how sticky wages can explain unemployment fluctuations.

Following Casares (2007b), labor contracts are set at a nominal wage that matches the amounts of heterogeneous ex ante labor supply and labor demand, expected throughout

¹In a state-of-the-art New Keynesian model, estimated for the US economy, Smets and Wouters (2007) also find a decline in total hours after a positive productivity shock.

²With the notable exception of recent papers that incorporate Mortensen-Pissarides search-and-matching frictions in the labor market (e.g., Trigari, 2004; Christoffel and Linzert, 2005; or Walsh, 2005), which provide unemployment variations imported from the separation rate and the job creation-destruction processes.

³Obviously, unemployment is not a brand new economic phenomenon. Quoting J. M. Keynes in chapter 18 of the General Theory: "the evidence indicates that full, or even approximately full, employment is of rare and short-lived occurrence."
the length of the contract. Then, unemployment arrives when there are a fraction of wage contracts that cannot be renegotiated every period, allowing possible mismatches between labor supply and labor demand. Such an interpretation of unemployment is inspired in Milton Friedman’s view of short-run unemployment variations, which hinges on the Wicksellian tradition. As described in Friedman (1968, pages 7-11), there can be a discrepancy between the observed unemployment rate and the so-called "natural rate of unemployment" that would be reached in a Walrasian competitive labor market. This flexible-wage "natural rate of unemployment" can be set as a reference value in the labor market; an actual rate of unemployment above the natural rate indicates that there is an excess supply of labor whereas a lower rate of unemployment corresponds to an excess demand for labor. Abstracting from variations in the "natural rate of unemployment" (normalizing it at zero), the model of this paper explains short-run fluctuations of unemployment by the gaps between labor supply and labor demand.

Interestingly, the analytical expressions for fluctuations on both price inflation and wage inflation happen to be equivalent for our model with unemployment and the EHL model without unemployment. Nevertheless, their slope coefficients are different reflecting the particular labor market assumptions.

The rest of the paper is organized as follows. Section 2 describes the functioning of the labor market with heterogeneous labor, sticky wages, and labor-clearing contracts. Section 3 discusses the connections and complementarities that arise between price setting and labor-clearing wage setting with nominal rigidities à la Calvo (1983). Next, the aggregation procedures provide the economy-wide price inflation and wage inflation equations that are presented in Section 4 and then compared to others belonging to the New Keynesian literature. As one applied exercise, Section 5 examines the responses to a technology shock in the baseline model and other variants having either only sticky prices or only sticky wages. The comparison is extended to the responses provided by the EHL model. Finally, Section 6 concludes the paper with a review of the most relevant findings.

\[\text{footnote}{4}\text{However, the relationship between price setting and wage setting differs here from Casares (2007b) where wage setting is subordinated to the case for optimal pricing at the firm level. Pricing and wage setting are independent in this paper, i.e., the possibility for resetting a wage contract in one particular industry is not linked to the pricing decision on that industry. This separation will result in a dynamic behavior for the rates of price inflation and wage inflation clearly distinguishable from the patterns obtained in Casares (2007b).}

\[\text{footnote}{5}\text{The new branch of models that incorporate search and matching frictions à la Mortensen-Pissarides (mentioned in footnote 2) provide theoretical justifications for the existence of Friedman's "natural rate of unemployment" and for its business cycle fluctuations.}\]
2 Heterogeneous labor market with nominal rigidities

This section describes a labor market structure that provides unemployment fluctuations due to wage setting rigidities. To start with, let us characterize a labor market by the following two main assumptions:

i) Heterogeneous labor. There is a continuum of differentiated labor services; each firm employs a specialized type of labor for the production of her differentiated good whereas the representative household supplies all the types of labor services.\(^6\)

ii) Sticky wages. Wage contracts may not be reset every period and the nominal wage remains unchanged if that is the case.\(^7\) Let us further develop this point.

Following Bénassy (1995), and more recently Casares (2007b), wage contracts are signed when firms and households get together to agree on an industry-clearing nominal wage.\(^8\) Introducing wage stickiness à la Calvo (1983), the industry-clearing nominal wage, \(W_t(i)\), is the one that satisfies

\[
E_t^{n_w} \sum_{j=0}^{\infty} \beta^{j} \eta_w \left[ n_{t+j}^d (i) - n_{t+j}^s (i) \right] = 0, \tag{1}
\]

where the demand and supply of the \(i\)-type labor service are denoted by \(n_{t+j}^d (i)\) and \(n_{t+j}^s (i)\) for period \(t + j\), \(E_t^{n_w}\) is the rational expectation operator conditional to the lack of wage contract revisions, \(\beta\) is the rate of discount per period, and \(\eta_w\) is the Calvo constant probability of not having a wage resetting. The sticky-wage formulation (1) differs from the one used in Casares (2007b) because the arrival of the market signal for wage setting is independent now from the pricing decision of the firm. More obvious are the differences with the sticky-wage specification proposed by Erceg \(et al.\) (2000), where the nominal wages are decided by heterogeneous households that bear market power to set their specific optimal wage since each household is the unique supplier of one type of labor service.

According to (1), the industry-clearing nominal wage gives a perfect matching between intertemporal labor demand and labor supply in the \(i\)-th industry that will employ the \(i\)-th type of labor to produce the \(i\)-th type of good. Future values of labor demand or supply in period \(t + j\) enter the matching condition (1) with a relative weight that corresponds to

\(^6\)Woodford (2003, chapter 3) uses this labor market scenario claiming that the existence of heterogeneous labor services is more adequate for sticky-price models than the common assumption of homogeneous labor market.

\(^7\)Wage indexation on the steady-state rate of inflation may also be considered without any effect on wage setting dynamics.

\(^8\)Blanchard and Fischer (1989, pages 518-519), also present a model where "the nominal wage is set so as to equalize expected labor demand and expected labor supply".
their discounted probability of occurrence, \( \beta^j \eta^j_k \). A compromised value of \( W_t(i) \) resulting from (1) can be obtained when inserting intertemporal labor demand curves \( n^d_{t+j}(i) \), decreasing on \( W_t(i) \); and intertemporal labor supply curves \( n^s_{t+j}(i) \), increasing on \( W_t(i) \). In a standard monopolistically competitive economy (Dixit and Stiglitz, 1977), labor demand is the amount of work hours required to produce the level of output determined by the demand curve at the firm-specific price. This can easily obtained when considering the Dixit-Stiglitz demand curve and providing the firm with a production technology. The supply of the specific \( i \)-type labor service is driven by the households’ optimal allocation between consumption and work hours.

A representative household maximizes intertemporal utility that depends positively on Dixit-Stiglitz bundles of consumption goods and negatively on all differentiated labor services supplied at the firms (indexed over the unit interval). Specifically, utility in period \( t \) amounts to

\[
U_t = \frac{c_t^{1-\sigma}}{1 - \sigma} - \int_0^1 \frac{n^*_t(i)^{1+\gamma}}{1 + \gamma} di,
\]

which conveys constant elasticities of both the consumption marginal utility, \( \frac{U_{cc}}{U_{cc}} = -\sigma \), and the marginal disutility of work hours, \( \frac{U_{nn(i)n(i)n(i)}}{U_{nn(i)}} = \gamma \). With a standard budget constraint (as in Casares, 2007b, for example), the supply of the \( i \)-th type of labor service is

\[
n^s_t(i) = \left( \frac{W_t(i)}{P_t} c_t^{-\sigma} \right)^{\frac{1}{\gamma}},
\]

where \( W_t(i) \) is the nominal wage associated to type \( i \) of labor, and \( P_t \) is the aggregate price level. Loglinearizing (2), it yields

\[
\tilde{n}^s_t(i) = \frac{1}{\gamma} \left( \log W_t(i) - \log P_t - \sigma \tilde{c}_t \right),
\]

where variables topped with a hat denote (standard) log deviations from steady state, e.g. \( \tilde{n}^s_t(i) = \log \left( \frac{n^s_t(i)}{n^s_t(i)} \right) \). Thus, fluctuations on the supply of labor \( i \), \( \tilde{n}^s_t(i) \), depend positively on the log of its specific nominal wage, \( \log W_t(i) \), at a (Frisch) labor supply elasticity given by the inverse of the elasticity on disutility of hours, \( \frac{1}{\gamma} \). Aggregating over all the industries builds up to this log deviations of total supply of labor

\[
\tilde{n}^s_t = \int_0^1 \tilde{n}^s_t(i) di = \frac{1}{\gamma} \left( \log W_t - \log P_t - \sigma \tilde{c}_t \right),
\]

where \( \log W_t = \int_0^1 \log W_t(i) di \) is the log of the aggregate nominal wage. Subtracting (4) from (3) results in this upward-sloped curve for the supply of labor \( i \)

\[
\tilde{n}^s_t(i) = \frac{1}{\gamma} \left( \log W_t(i) - \log W_t \right) + \tilde{n}^s_t.
\]
Next, let us briefly describe the behavior of firms and thus derive their labor demand equation. Firms are Calvo-style price setters in a monopolistically competitive market, as typically modelled within the New Keynesian framework. Therefore, with \( \eta_p \) is a constant probability, firms are not able to set the optimal price. The fraction of firms that are allowed to charge the optimal price will determine it by maximizing intertemporal profit conditional to situations of non-optimal price resetting for future periods and Dixit-Stiglitz demand constrained. As shown in Casares (2007b) and elsewhere, optimal pricing requires the following first order condition (for the representative \( i \)-th firm)

\[
P_t(i) = \frac{\theta}{\theta - 1} \frac{E_t^{\eta_p} \sum_{j=0}^{\infty} \Delta_{t,t+j} n_p \psi_{t+j}(i) (P_{t+j})^{\theta} y_{t+j}}{E_t^{\eta_p} \sum_{j=0}^{\infty} \Delta_{t,t+j} n_p (P_{t+j})^{\theta-1} y_{t+j}},
\]

where \( \theta \) is the Dixit-Stiglitz elasticity of substitution, \( E_t^{\eta_p} \) is the rational expectation operator conditional to the lack of future price resetting, \( \Delta_{t,t+j} \) is the stochastic discount factor, and \( \psi_{t+j}(i) \) is the real marginal cost of the \( i \)-th firm in period \( t+j \). Ignoring capital accumulation, firms have access to a production technology with decreasing labor returns that, for the \( i \)-th firm, takes this expression

\[
y_t(i) = \left( \exp(z_t) n_t^d(i) \right)^{1-\alpha}, \text{ with } 0 < \alpha < 1,
\]

where \( y_t(i) \) is the amount of output produced by firm \( i \), and \( n_t^d(i) \) is its labor demand. In addition, (7) includes the economy-wide AR(1) technology shock, \( z_t = \rho z_{t-1} + \varepsilon_t \) with \( \varepsilon_t \sim N(0, \sigma_\varepsilon) \), as an stochastic source of variability. Using the standard Dixit-Stiglitz demand curve

\[
y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} y_t,
\]

the firm-specific production function (7) can be inserted in (8) and, after loglinearizing, the following expression can be obtained for the dynamics of the demand for labor \( i^9 \)

\[
\dot{n}_t^d(i) = -\frac{\theta}{1-\alpha} (\log P_t(i) - \log P_t) + \hat{n}_t.
\]

Firm-specific labor demand inversely depends upon the relative price and positively upon the measure of demand-determined aggregate labor, \( \hat{n}_t = \int_0^1 \hat{n}_t^d(i) di \). Since prices change with a same-sign reaction to marginal costs and the latter increase with nominal wages, equation (9) provides industry-specific labor demand variations that are negatively related to the relative nominal wage. Therefore, we can loglinearize the labor-clearing condition

\(^9\text{See Casares (2007b) for details.}\)
(1), and then use equations (5) and (9) for \( \hat{n}_{t+j}^d(i) \) and \( \hat{n}_{t+j}^s(i) \) in order to determine the log of the labor-clearing wage, \( \log W_t^*(i) \). As shown in the Appendix,\(^{10}\) it is obtained

\[
\log W_t^*(i) = \log W_t - \gamma (1 - \beta \eta_w) E_t^{\eta_w} \sum_{j=0}^{\infty} \beta^j \eta_w^j \left( u_{t+j} + \frac{\theta}{1 - \alpha} (\log P_{t+j}(i) - \log P_{t+j}) \right) + E_t \sum_{j=1}^{\infty} \beta^j \eta_w^j \pi_{t+j}^w, \tag{10}
\]

where

\[
u_{t+j} = \hat{n}_{t+j}^s - \hat{n}_{t+j}^t
\]
is the rate of unemployment in period \( t+j \) defined by the log difference between labor supply and labor demand.\(^{11}\) This definition of unemployment has been recently used in Blanchard and Galí (2007) or Casares (2007b), in a way of recuperating the Wicksellian vision of business cycle unemployment due to labor gaps.\(^{12}\) As a result, there is a direct link between unemployment and sticky wages. Unemployment fluctuates because there is a positive fraction, \( \eta_w \), of total labor contracts that cannot be revised every period, which, consequently, render some mismatches between industry-specific labor supply and labor demand. When aggregating over all labor contracts, the endogenous measure of unemployment is obtained as the log difference between aggregate labor supply and the (effective) aggregate labor demand. Back to (10), the labor-clearing relative wage depends negatively on terms such as the rate of unemployment and relative prices (both at current and expected future values), and positively on the expected future rates of wage inflation. If the economy had no frictions on wage setting (\( \eta_w = 0.0 \)), which would convey that all the contracts are renegotiated every period, (10) would reduce to

\[
\log W_t^*(i) = \log W_t - \frac{\gamma \theta}{1 - \alpha} (\log P_t(i) - \log P_t),
\]

and the unemployment rate would always be at zero because all industries would have a perfect match between labor supply and labor demand.\(^{13}\)

\(^{10}\)See steps leading to equation (A15) in Part 2 of the Appendix.

\(^{11}\)A very similar way of defining the rate of unemployment is \( u_t = 1 - \frac{n_t}{n_t} \). Taking logs on both sides of the equivalent expression \( 1 - u_t = \frac{n_t}{n_t} \), and then assuming that \( \log(1 - u_t) \simeq -u_t \) because \( u_t \) is a sufficiently small number, leads to \( u_t = \hat{n}_t^s - \hat{n}_t^t \).

\(^{12}\)As mentioned above in the introductory Section 1, our measure of unemployment explains cyclical fluctuations around a zero steady-state value. The inclusion of an extensive margin would provide a positive unemployment rate in steady-state and long-run determinants of unemployment as in the Mortensen-Pissarides literature.

\(^{13}\)As discussed later in Section 4, the case with \( \eta_w = 0.0 \) represents an heterogeneous labor market structure with fully-flexible wages identical to the one described in Woodford (2003, chapter 3) for his baseline New Keynesian model.
3 Price-wage complementarities at industry level

The combination of an heterogeneous labor market with Calvo-style nominal rigidities on both price and wage setting brings along a two-side connection between firm-specific prices and firm-specific wages. On the one hand, optimal prices are set taking into account the current firm-specific nominal wage since it affects both current and future marginal costs. The value of the nominal wage varies across firms due to the particular Calvo lotteries that may have occurred in the past (a firm may have renegotiated the nominal wage for the last time one period ago, or two periods ago, or three, etc.). On the other hand, the nominal wage depends on the firm-specific price of the consumption goods produced with labor employed via the labor demand schedule entering the matching condition (1). Subsequently, we can guess that the relative optimal price and the relative labor-clearing wage would evolve as indicated by these log-linear relationships

\[
\tilde{P}_t^*(i) = \tilde{P}_t^* + \tau_1 \tilde{W}_{t-1}(i) \quad \text{(11a)}
\]

\[
\tilde{W}_t^*(i) = \tilde{W}_t^* - \tau_2 \tilde{P}_t(i) \quad \text{(11b)}
\]

where \(\tau_1\) and \(\tau_2\) are undetermined coefficients to be found below. Several considerations need to be made here. First, notation must be carefully explained. Thus, \(\tilde{P}_t^*(i)\) denotes the relative optimal price as the log difference between the optimal firm-specific price and the aggregate price level, \(\tilde{P}_t^* = \log P_t^*(i) - \log P_t\), whereas \(\tilde{P}_t^*\) is the average of those across all industries, \(\tilde{P}_t^* = \log P_t^* - \log P_t\) with \(\log P_t^* = \int_0^1 \log P_t^*(i) di\). Similarly, the labor-clearing relative wage is \(\tilde{W}_t^*(i) = \log W_t^*(i) - \log W_t\), and its average value is \(\tilde{W}_t^* = \log W_t^* - \log W_t\).

There should be noticed the difference between the industry-specific optimal price, \(P_t^*(i)\), and the average of optimal prices across all the industries that receive the "adequate" Calvo signal, \(P_t^*\). Their differentiation is based on the distinctive nominal wage contracts that they have in place. Secondly, the timing of setting prices and wages is not identical as reflected in (11a) and (11b). Prices are set before wages.\(^{14}\) Therefore, it is assumed that when firms are allowed to set the optimal price they have not received the Calvo signal for wage setting yet and they cannot know whether their wage contracts will be reset or not in that period. They take the nominal wage from previous period as the reference for the case of not having wage resetting in the current period (see 11a). Meanwhile, the wage negotiation takes place (when possible) already incorporating the information on

\(^{14}\)\text{This assumption turned out to be necessary in order to derive a price inflation equation with a term involving expected next-period inflation premultiplied by } \beta \text{ as in the standard representation of the New Keynesian Phillips curve. Therefore, it was taken for technical convenience.}
the current selling price; (11b) relates the relative nominal wage to its contemporaneous relative price. Our last preliminary consideration is that the undetermined coefficients \( \tau_1 \) and \( \tau_2 \) enter (11a)-(11b) with opposite signs. The relative nominal wage affects positively the relative price because of its increasing effect on the real marginal costs faced by the price-setting firm. By contrast, an increase in the relative price lowers the relative nominal wage because labor demand falls at a higher price which pushes down the nominal wage required for matching labor demand with labor supply.

For the labor market structure described above and borrowing the undetermined-coefficients technique used in Woodford (2005), one can obtain the following solution for the values of \( \tau_1 \) and \( \tau_2 \)

\[
\tau_1 = \frac{(1 - \beta \eta_p) \eta_w}{(1 - \beta \eta_p \eta_w) \left( 1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 \left( 1 - \frac{(1 - \beta \eta_p \eta_w)}{1 - \beta \eta_p \eta_w} \right) \right)}; \quad \text{and} \quad (12a)
\]

\[
\tau_2 = \frac{\gamma \theta (1 - \beta \eta_w)}{(1 - \alpha) (1 - \beta \eta_w \eta_p) \left( 1 + \frac{\tau_1 \beta \eta_w \gamma \theta}{1 - \alpha} \left( 1 - \frac{\eta_p (1 - \beta \eta_p)}{1 - \beta \eta_p \eta_p} \right) \right)}; \quad (12b)
\]

The proof is available in the technical Appendix. As shown by the analytical solution (12a)-(12b), numerical values of \( \tau_1 \) and \( \tau_2 \) can be found by solving a non-linear pair of equations when inserting the numbers assigned to the structural parameters \( \beta, \eta_p, \eta_w, \theta, \gamma, \) and \( \alpha. \) By looking at the solution pair (12a)-(12b), it can be observed that \( \tau_1 \) and \( \tau_2 \) cannot be solved separately, which confirms the existence of interactions between price setting and wage setting at industry level.

4 Aggregation. Price inflation and wage inflation

The dynamic equation for economy-wide price inflation can be obtained from two log-linear equations: the loglinearized aggregate price level definition and the loglinearized first order condition on the optimal price. Concerning the latter, equation (6) can be loglinearized as follows

\[
\log P_t^s(i) = \left( 1 - \beta \eta_p \right) E_t^{\eta_p} \sum_{j=0}^{\infty} \beta^j \eta_p^j \left( \hat{\psi}_{t+j}(i) + \log P_{t+j} \right),
\]

which, using other relationships involved in the model, can be transformed in an expression that depends exclusively on aggregate variables

\[
\tilde{P}_t^s = \frac{1 - \beta \eta_p}{1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 (1 - \frac{(1 - \beta \eta_p \eta_w)}{1 - \beta \eta_p \eta_w})} E_t^{\eta_p} \sum_{j=0}^{\infty} \beta^j \eta_p^j \hat{\psi}_{t+j}^p + E_t^{\eta_p} \sum_{j=1}^{\infty} \beta^j \eta_p^j \eta_{t+j}, \quad (13)
\]

\[15\] See Part 1 of the technical Appendix for the details on the derivation.
where $P_t^*$ is the average relative price defined above, $\hat{\psi}_{t+j}$ is the log of the aggregate real marginal cost in $t+j$, and $\pi^p_{t+j} = \log P_{t+j} - \log P_{t+j-1}$ denotes price inflation in $t+j$. The second equation required to determine price inflation dynamics is the one reached when combining the log-linearized aggregate price level definition from Calvo-style price stickiness, 

$$\log P_t = \eta_p \log P_{t-1} + (1 - \eta_p) \log P^*_t,$$

with the price inflation definition, 

$$\pi^p_{t} = \log P_{t} - \log P_{t-1}. \text{ It leads to }$$

$$\tilde{P}_t^* = \frac{\eta_p}{1 - \eta_p} \pi^p_{t}. \quad (14)$$

Now, (14) can be substituted in the left-hand side of (13) to yield 

$$\pi^p_{t} = \frac{(1 - \beta \eta_p)(1 - \eta_p)}{\eta_p \left[ 1 + \frac{\alpha \beta}{1 - \alpha} + \tau_2 \left(1 - \frac{1 - \beta \eta_p}{1 - \beta \eta_p \eta_w}\right) \right]} E_t \sum_{j=0}^{\infty} \beta^j \eta_p \hat{\psi}_{t+j} + \frac{1 - \eta_p}{\eta_p} E_t \sum_{j=1}^{\infty} \beta^j \eta_p \pi^p_{t+j}. \quad (15)$$

Interestingly, the analytical expression (15) governing the dynamics of price inflation coincides with the standard representation of the so-called New Keynesian Phillips Curve (NKPC) derived with either fully-flexible competitive wages or with sticky wages set by households.\footnote{See Yun (1996) for a NKPC with flexible wages and homogeneous labor, Woodford (2003, chapter 3) for the NKPC with flexible wages and heterogeneous labor, and Erceg et al. (2000) for the NKPC with sticky wages set by households.} Price inflation is forward-looking and changes in response to fluctuations on current and expected future real marginal costs. Even though the general expression is not altered, the slope coefficient in (15) is affected by the presence of nominal rigidities on labor-clearing contracts because it depends on the value of the sticky-wage probability, $\eta_w$. Therefore, our proposal for a sticky-wage specification with labor-clearing contracts keeps the standard NKPC expression with a different slope coefficient, reflecting the existence of price-wage complementarities discussed above.

Now, let us derive the equation that governs the dynamic behavior of aggregate wage inflation. Like for the case of price inflation, two log-linear relations implied by the model are required: one obtained by log-linearizing the intertemporal labor-matching condition...
(1), and the other one that describes the aggregation of nominal wages. As shown in Section 2, the log of the nominal wage consistent with (1) is given by equation (10), which after some algebra leads to

\[ \tilde{W}_t^* = - \frac{\gamma(1-\beta \eta_w)(1-\eta_w)}{\eta_w(1+\tau_1 \beta \eta_w \eta_p(1-\eta_w))} E_t \sum_{j=0}^\infty \beta^j \eta_w^j u_{t+j} + E_t \sum_{j=1}^\infty \beta^j \eta_w^j \pi_{t+j}^w, \]

(16)

where \( \tilde{W}_t^* \) is the average relative wage defined above, and \( \pi_{t+j}^w = \log W_{t+j} - \log W_{t+j-1} \) is the rate of wage inflation in \( t+j \). Loglinearizing the definition of the aggregate nominal wage with Calvo staggered wages results in a log of the aggregate nominal wage obtained as a weighted average of its previous observation and the average value of current labor-matching contracts, i.e., \( \log W_t = \eta_w \log W_{t-1} + (1-\eta_w) \log W_t^* \), where using the wage inflation definition for period \( t \), it is obtained

\[ \tilde{W}_t^* = \frac{\eta_w}{1-\eta_w} \pi_t^w. \]

(17)

Combining (16) and (17), it yields

\[ \pi_t^w = - \frac{\gamma(1-\beta \eta_w)(1-\eta_w)}{\eta_w(1+\tau_1 \beta \eta_w \eta_p(1-\eta_w))} E_t \sum_{j=0}^\infty \beta^j \eta_w^j u_{t+j} + \frac{1-\eta_w}{\eta_w} E_t \sum_{j=1}^\infty \beta^j \eta_w^j \pi_{t+j}^w. \]

Analogously to the procedure used above for price inflation, we can rewrite the last expression in period \( t+1 \) and then compute \( \pi_t^w - \beta \eta_w E_t \pi_{t+1}^w \) to find

\[ \pi_t^w - \beta \eta_w E_t \pi_{t+1}^w = - \frac{\gamma(1-\beta \eta_w)(1-\eta_w)}{\eta_w(1+\tau_1 \beta \eta_w \eta_p(1-\eta_w))} u_t + \frac{1-\eta_w}{\eta_w} \beta \eta_w E_t \pi_{t+1}^w, \]

which collapses to the wage inflation equation

\[ \pi_t^w = \beta E_t \pi_{t+1}^w - \frac{\gamma(1-\beta \eta_w)(1-\eta_w)}{\eta_w(1+\tau_1 \beta \eta_w \eta_p(1-\eta_w))} u_t. \]

(18)

Notably, wage dynamics are forward-looking and inversely driven by the rate of unemployment, \( u_t = \tilde{n}_t^s - \tilde{n}_t \). The first aspect (forward-lookingness) is typical from a New Keynesian model and the second aspect (negative relationship between nominal wage changes and unemployment) belongs to the Old Keynesian tradition that relates money wages to employment developments (Keynes, 1936, chapter 19; Modigliani, 1947). Actually, (18) is a forward-looking representation of the empirical relationship between nominal wage inflation and unemployment estimated by Phillips (1958) and microfounded by Lipsey (1960) and Phelps (1968). The slope coefficient attached to unemployment in (18) is determined by

\[ ^{17} \text{The steps and algebra involved are included in Part 2 of the technical Appendix.} \]
parameters related to the wage setting procedure \((\eta_w, \gamma)\) as well as other related to price setting \((\eta_p, \tau_1, \theta)\). The latter is another example of the connections between price setting and wage setting, embedded in the model setup discussed above.

One comparison with the sticky-price, sticky-wage model by Erceg et al. (2000)

The common practice for a sticky-wage specification in the New Keynesian framework is to let households decide on the nominal wage contract as first assumed by EHL (2000). Their labor market structure also incorporates heterogeneous types of labor services, although each of them is supplied by one differentiated household and demanded by all firms. Thus, there are household-specific nominal wages that become staggered as subject to the arrival of the right probability à la Calvo (1983). Each household may be able to set the nominal wage whereas the amount of work hours supplied is given by the labor-demand constraint. Firms demand bundles of labor obtained using a Dixit-Stiglitz aggregator that combines all types of labor services, which allows substitutions between differentiated labor services with a constant elasticity. The EHL model is a general equilibrium model where the labor market clears in every period because all industries present a perfect match between labor supply and labor demand. For a separable utility function introduced in Section 2, the EHL model explains fluctuations of wage inflation as given by the following forward-looking equation

\[
\pi_t^w = \beta E_t \pi_{t+1}^w + \frac{(1-\eta_w)(1-\beta \eta_w)}{\eta_w (1+\gamma \theta_f)} \left( \bar{m} \bar{r} s_t - \hat{w}_t \right),
\]

where \(\bar{m} \bar{r} s_t - \hat{w}_t\) is the log difference between the aggregate marginal rate of consumption/hours substitution and the real wage, and \(\theta_f\) is the firms’ labor demand elasticity of substitution. Recalling the specification of the utility function introduced above and using the equilibrium condition \(\hat{c}_t = \hat{y}_t\), we have

\[
\bar{m} \bar{r} s_t - \hat{w}_t = \gamma \hat{n}_t + \sigma \hat{y}_t - \hat{w}_t.
\]

Meanwhile, total supply of labor \((4)\) and the definition of the log real wage, \(\hat{w}_t = \hat{W}_t - \hat{P}_t\), can be inserted in the equation that determines the rate of unemployment of our model to yield

\[
u_t = \hat{n}_t^* - \hat{n}_t = \frac{1}{\gamma} (\hat{w}_t - \sigma \hat{y}_t) - \hat{n}_t.
\]

\(^{18}\)Other recent papers with household-specific sticky wages are Amato and Laubach (2003), Smets and Wouters (2003, 2007), Christiano et al. (2005), and Casares (2007a).
Noteworthily, there is a semi-loglinear relationship between the rate of unemployment used in our model, equation (21), and the gap that drives wage inflation fluctuations in the EHL model, \( \hat{m}r \hat{s}_t - \hat{w}_t \), defined in (20).\(^{19}\) Comparing them, it is straightforward to see that

\[
\hat{m}r \hat{s}_t - \hat{w}_t = -\gamma u_t, \tag{22}
\]

which obviously is only a valid statement for our model with endogenous unemployment because in the EHL model the unemployment rate is always at zero. The last result allows us to rewrite the wage inflation equation (18) as follows

\[
\pi^w_t = \beta E_t \pi^w_{t+1} + \frac{(1-\beta \eta_p)(1-\eta_w)}{\eta_w(1+\tau_1)(1-\alpha)(1-\eta_p(1+\beta \eta_w))} (\hat{m}r \hat{s}_t - \hat{w}_t), \tag{23}
\]

whose analytical form is identical to the wage inflation equation derived in EHL (2000). Hence, the dynamic behavior of wage inflation is governed by an equation equivalent to that of the EHL model, with the only difference in the numerical value of the slope coefficient.\(^{20}\)

Moreover, the price inflation equation of the EHL model has also the same forward-looking expression that we obtained above as the NKPC of our model (see equation 15). The slope coefficient that gives the real marginal cost semi-elasticity is \( \frac{(1-\beta \eta_p)(1-\eta_p)}{\eta_p(1+\alpha \tau_0 / \tau_0^2)} \) in EHL (2000) with happens to be higher than the number taken from equation (15) of our model. Therefore, the comparison of our sticky-wage model with EHL (2000) shows that both models share the same analytical (forward-looking) expressions for dynamic changes on price inflation and wage inflation, although the slope coefficients on those expressions are built up differently from the respective structural parameters.\(^{21}\)

**Two limit cases**

The model presented in this paper provides a general structure for both sticky prices and sticky wages, ranging from perfectly flexible prices (or wages) to completely rigid prices (or wages). In other words, our setup with two different Calvo probabilities allows particular specifications that can be of interest as limiting the sources of nominal rigidities exclusively on either price or wage setting. Hence, there are two limit cases that deserve special attention:

\(^{19}\)This point has already been shown in Casares (2007b).

\(^{20}\)By contrast, Casares (2007b) shows that when sticky wages are subordinated to firm-specific sticky pricing, the wage inflation equation includes another term involving the gap between labor productivity and the real wage.

\(^{21}\)Nevertheless, these structural parameters could be calibrated in a particular way that yield a perfect matching on the slope coefficients of both models, which would make them equivalent from a business cycle perspective.
i) the case for sticky prices ($\eta_p > 0.0$) and fully-flexible wages ($\eta_w = 0.0$).

ii) the case for sticky wages ($\eta_w > 0.0$) and fully-flexible prices ($\eta_p = 0.0$).

In the first specification, the model collapses to the baseline New Keynesian setup introduced by Woodford (2003, chapter 3). All industries have labor clearing in the sense that the nominal wage is adjusted to match the labor demand of the firm with the labor supply of the household. In turn, the unemployment rate is always maintained at zero. Nominal (and real) wages are distinctive across industries because pricing and labor demand is firm specific (driven by Calvo lotteries). After aggregation, the log of the real wage coincides with the log of households’ marginal rate of substitution as required to guarantee the absence of unemployment.\footnote{This result can be obtained from the labor supply curve of the model (equation 4) when considering that total labor supply is equal to total labor demand (zero unemployment).}

The slope coefficient in the wage inflation equation (18) –or the equivalent (23)– soars to infinity as $\eta_w$ approaches to zero, in practice implying a zero unemployment or a zero gap between the marginal rate of substitution and the real wage. In the price inflation equation (15), the slope coefficient of the real marginal cost simplifies to $\frac{(1-\beta\eta_p)(1-\eta_p)}{\eta_p(1+\lambda\alpha\delta)}$ when $\eta_w = 0.0$, which coincides with that derived in the baseline model of Woodford (2003, chapter 3).

The second notable specification is the one with flexible prices and sticky wages, obtained when setting $\eta_p = 0.0$ in our model. Firms can optimize on their prices all the periods. They will decide a different optimal price depending on the specific nominal wage that they face given the Calvo lotteries on wage setting. As typical from a flexible-price scenario, the aggregate real marginal cost is constant as all firms keep a constant mark up of prices over marginal costs.\footnote{The optimality condition on the price (equation 6) with no rigidities becomes $P_t(i) = \frac{\vartheta}{\beta-1} P_t \psi_t(i)$, that can be loglinearized to $\log P_t(i) = \log P_t + \hat{\psi}_t(i)$. Aggregating over the industries forming the economy, it yields}

\[
\int_0^1 \log P_t(i) di = \log P_t + \int_0^1 \hat{\psi}_t(i) di,
\]

where using $\int_0^1 \log P_t(i) di = \log P_t$, we obtain zero fluctuations on the aggregate real marginal cost

\[
\int_0^1 \hat{\psi}_t(i) di = \hat{\psi}_t = 0.
\]
marginal cost, \( \hat{\psi}_t = 0.0 \), that becomes the equation that explains price inflation dynamics. Wage inflation dynamics are still provided by equation (18), or equation (23), since wage rigidities give rise to the presence of unemployment in the labor market and also existing gaps between the marginal rate of substitution and the real wage. However, flexible prices reduce the slope coefficient on wage inflation dynamics. Thus, the slope in (23) gets down to \( \frac{(1-\beta_{\eta})}{\eta_{\pi}(1+r_{\pi}(1-w_{\pi}))}. \)

As a summary, Table 1 collects the slope coefficients for price and wage inflation dynamics at various levels of price-wage rigidities, using expression (23) for wage dynamics to make it comparable to the one obtained in the EHL model.

5 Nominal rigidities and technology shocks

This section examines the responses to a positive innovation in technology under different price/wage settings and also compares the effects to those obtained in the EHL model. Let us recall that a technology shock enters the firm’s production function as assumed in (7). Taking logs in (7) and aggregating across firms yield

\[ \hat{y}_t = (1 - \alpha) \hat{n}_t + (1 - \alpha) z_t, \]

that relates output fluctuations to those of total labor employed and to the realization of the AR(1) technology shock.

The model has to be completed with the demand sector, and also a numerical calibration for its structural parameters needs to be provided. Regarding the former, the forward-looking IS curve explains output fluctuations in response to the real interest rate

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} (R_t - E_t \hat{\pi}_{t+1}^p), \]

which is consistent with the optimizing behavior of the representative household depicted above (shown in Casares, 2007b). The second required equation for the demand sector is a monetary policy rule. On a standard fashion, it is assumed that the nominal interest rate is set as indicated by the following Taylor (1993)-type rule with an smoothing component

\[ R_t = \mu_R R_{t-1} + (1 - \mu_R) \left[ \mu_{\pi_p} E_t \hat{\pi}_{t+1}^p + \mu_y \left( \hat{y}_t - \hat{\gamma}_t \right) \right], \]

with \( \mu_{\pi_p} > 1.0, \mu_y \geq 0.0, 0.0 < \mu_R < 1.0 \). The output gap term, \( \hat{y}_t - \hat{\gamma}_t \), defines log deviations between current and potential output, being the latter the amount of output
produced if the economy would be released from nominal rigidities. Setting $\eta_p = \eta_w = 0.0$, it can be found

$$\frac{\alpha}{1-\alpha} \gamma + \sigma \widehat{y}_t = (1 + \gamma)z_t.$$  \hfill (27)

Hence, the complete model consists of the price inflation equation (15), the wage inflation equation (18), the endogenous unemployment definition given in (21), the production technology (24), the IS curve (25), the Taylor rule (26) where potential output is given by (27), and the following definitions for log deviations of the the real marginal cost and the real wage

$$\widehat{\psi}_t = \widehat{w}_t - (\widehat{y}_t - \widehat{n}_t).$$  \hfill (28)

$$\widehat{w}_t = \widehat{w}_{t-1} + \pi^w_t - \pi^p_t.$$  \hfill (29)

In total, there are nine equations that may give solution paths for the following nine endogenous variables $\pi^w_t, \pi^p_t, u_t, \hat{y}_t, \hat{n}_t, R_t, \widehat{y}_t, \widehat{\psi}_t,$ and $\widehat{w}_t$. Alternatively, wage inflation dynamics can be expressed by (23) and the model can be solved when adding one extra equation for fluctuations of the marginal rate of substitution (equation 20).

For a quarterly calibration of the structural parameters, the intertemporal discount factor is set at $\beta = 0.99$ which implies a 1% quarterly real rate of interest in steady state, 4% in annualized terms. The production technology is set with decreasing marginal returns at $\sigma = 0.36$, and the coefficient of serial correlation on the technology shock is equal to 0.95. Households’ utility function comes with the same elasticities $\sigma = \gamma = 2.0$. The value assigned to $\gamma$ implies a low Frisch elasticity of labor supply, $\gamma^{-1} = 0.5$, consistent with most micro evidence.\textsuperscript{24} With respect to nominal rigidities, it is assumed that prices and wages are reset one time per year on average as suggested by Taylor (1999) and also used in EHL (2000). Accordingly, the Calvo probabilities are set at $\eta_p = \eta_w = 0.75$. The Dixit-Stiglitz elasticity of demand is $\theta = 6.0$ in order to have a 20% mark-up of prices over marginal costs in steady state. Finally, the coefficients of the monetary policy rule (26) are the ones originally advocated by Taylor (1993), $\mu_{\pi^p} = 1.5$ and $\mu_y = \frac{0.5}{4}$, with a significant degree of interest-rate smoothing, $\mu_R = 0.8$.

For the calibration of the EHL model, all the parameters take the same values as the ones set in our model, with the additional parameter for the labor demand elasticity set at $\theta_f = 4.0$ as in EHL (2000).

Now we are ready to examine the responses of the model to a 1% expansionary technology shock entering (24). Figure 1 shows a graphical display with the impulse-response

\textsuperscript{24}The empirical evidence supports values for the Frisch labor supply elasticity ranging between 0 and 0.5 (see Pencavel, 1986; Altonji, 1986, and Domeij and Flodén, 2006).
functions and Table 2 provides the numerical values of the peak responses relative to those of output. Both Figure 1 and Table 2 compare results obtained in our model with sticky prices and sticky wages ($\eta_p = \eta_w = 0.75$), with only sticky prices ($\eta_p = 0.75$ and $\eta_w = 0.0$) and with only sticky wages ($\eta_p = 0.0$ and $\eta_w = 0.75$). Furthermore, the reactions observed are compared with the ones obtained in the EHL model that features sticky prices set by firms ($\eta_p = 0.75$) and sticky wages set by households ($\eta_w = 0.75$).

The reaction of output is very similar in three of the four cases displayed in Figure 1. Thus, the three variants with sticky prices give a moderate output response at the quarter of the shock, which is yet slightly further increased for the next few quarters and then slowly returns to the steady-state level. The model with flexible prices is the only case where output separates from this hump-shaped pattern. Output has an immediate sharper response (about twice the size observed in the other cases) with no delayed peak. The deeper reductions in nominal and real interest rates (not reported in Figure 1) explain, via the IS curve, why output rises more with no pricing frictions.

As shown in Figure 1, price inflation also reports similar drops for all the cases except for the model with flexible prices. When all prices can optimally reset, the fall of price inflation relative to the change in output is several times greater than the ones observed with price stickiness (see Table 2). This result is easily understandable as price stickiness makes 75% of prices remain unchanged. Comparing between sticky-wage models, the response of inflation to the technology shock is of larger magnitude in the EHL model than in our baseline model as also documented in Table 2. The reason for the difference is that the slope coefficient governing the reaction of price inflation to the real marginal cost is higher in the EHL model compared to our model (see Table 1).

Wage inflation responds to the technology shock with a minor drop (less than one sixth of the output percent change) except in the case when wage contracts are fully flexible ($\eta_w = 0.0$) that shows a downwards reaction even larger in size than the percent increase observed in output (see Table 2).

The response of the real wage is the result of combining the reactions of wage inflation and price inflation. Therefore, such real wage responses are very helpful to understand the implications of the presence or absence of price/wage rigidities. The model with flexible prices ($\eta_p = 0.0$) requires that the real wage and productivity move along together because the real marginal cost never fluctuates (due to the constant mark-up in monopolistic competition). In turn, the real wage immediately responds to the shock in order to replicate the labor productivity peak and its gradual return to steady state. By contrast, the model with sticky prices and flexible wages (equivalent to the model used by Woodford, 2003,
requires that the real wage fluctuates in the same way as the households’ marginal rate of substitution. Under our selected calibration of parameters, the marginal rate of substitution falls and so equally does the real wage under fully-flexible nominal wages.\footnote{If the labor elasticity in the utility function were set at a lower value (for example $\gamma = 1.0$), the marginal rate of substitution and the real wage would both respond with increases to the technology shock. However, that calibration would imply a Frisch elasticity higher than the numbers provided by the microeconomic empirical evidence.} A countercyclical real wage in the presence of technology shocks is not supported by the empirical evidence (e.g., Francis and Ramey, 2005). Finally, both our baseline model with sticky prices and sticky wages and the EHL model show a procyclical and gradual real wage response with a late peak reached approximately ten quarters after the shock. As this paper shows, both models have identical expressions for price and wage inflation dynamics with different slope coefficients that may explain the quantitative difference in the reaction to the technology shock. In both settings, the real wage changes as a combined reaction to changes in productivity (that affects price inflation) and to changes in the marginal rate of substitution (that affects wage inflation).

Total hours fall markedly after the technology shock in all the cases with sticky prices. This is a typical finding on the New Keynesian framework that is supported by a significant portion of the empirical evidence.\footnote{See Galí (1999) and, more recently, Francis and Ramey (2005) and Smets and Wouters (2007).} The model with flexible prices shows an initial positive reaction of hours followed by a severe decline that ends up converging towards the other cases.

Finally, the impact of a technology shock on the rate of unemployment also provides insightful differences across the model variants at hand. Our baseline model with sticky prices and sticky wages suggests that the rate of unemployment temporarily would rise with the productivity shock because labor demand (i.e., total hours) falls further below the drop of labor supply. If frictions on wage contracts are lifted ($\eta_w = 0.0$), the model reports no reaction in unemployment because total labor demand and labor supply perfectly match at the current wage rate. As discussed above, the EHL model also delivers no response on the unemployment rate in spite of featuring sticky wages.\footnote{The sticky-wage EHL model is built upon the assumption that all pairs of differentiated labor demand and labor supplied are well matched at their current nominal wage rates. Even though there are no labor gaps, the aggregation delivers gaps between the real wage and the effective marginal rate of substitution.} Lastly, the case with flexible prices and sticky wages brings a reduction in the unemployment rate because labor demand barely changes and the fall of labor supply dominates.
6 Conclusions

This paper presents a model that features two sources of nominal rigidities: Calvo-style sticky prices for monopolistically competitive firms and Calvo-style sticky wages on heterogeneous labor contracts. Wage contracts are jointly agreed by households and firms because they are set at the nominal wage that matches labor supply with labor demand. Consequently, the existence of wage rigidities is crucial to explain the presence of endogenous unemployment in the model. After doing the aggregation algebra, the dynamic fluctuations on the rate of wage inflation depend negatively on the rate of unemployment—which recalls the relationship found in the old empirical Phillips (1959) curve—, and also on next period’s expected wage inflation.

Comparing the model described in this paper with the popular sticky-price sticky-wage model by Erceg et al. (2000), we found that price inflation and wage inflation dynamics are governed by the same analytical expressions. However, the slope coefficients on those expressions are different, reflecting the disparities in the labor market structures of these two models. In addition, the unemployment rate is absent in the model by Erceg et al. (2000) and present in the model shown here.

Finally, the responses to a technology shock were analyzed under different levels of nominal rigidities. In the baseline variant with both sticky prices and sticky wages, output increases describing a hump-shaped pattern, price inflation and wage inflation slightly fall, the real wage shows a slow procyclical reaction and the unemployment rate temporarily rises. When wages turn fully flexible (as in the baseline New Keynesian model described in Woodford, 2003), output and inflation react similarly but the real wage responds with an countercyclical fall and there is no unemployment. If prices are flexible and wages remain sticky, price inflation falls much more sharply, output and the real wage increase more than with sticky prices, and the unemployment rate drops.
Technical Appendix. How to find the analytical solution for the undetermined coefficients, \( \tau_1 \) and \( \tau_2 \), and how to obtain the loglinear expressions used for the average values of the relative price, \( \bar{P}_t^* \), and the relative wage, \( \bar{W}_t^* \), set in period \( t \).

Part 1. Finding the solution for the undetermined coefficient \( \tau_1 \) and the analytical expression for the average relative prices set in period \( t \), \( \bar{P}_t^* \).

Loglinearizing (6), the (log-linear) optimality condition on the price setting decision of the \( i \)-th firm with Calvo-type stickiness is

\[
\log P_t(i) = (1 - \beta \eta_p) E_t^{\eta_p} \sum_{j=0}^{\infty} \beta^j \eta_p^j \tilde{\psi}_{t+j}(i) + \log P_{t+j}. \tag{A1}
\]

Using \( \log P_{t+j} = \log P_t + \sum_{k=1}^{j} \pi_{t+k}^p \) in (A1), it is obtained

\[
\tilde{P}_t^*(i) = \log P_t^*(i) - \log P_t = (1 - \beta \eta_p) E_t^{\eta_p} \sum_{j=0}^{\infty} \beta^j \eta_p^j \tilde{\psi}_{t+j}(i) + E_t \sum_{j=1}^{\infty} \beta^j \eta_p^j \pi_{t+j}^p. \tag{A2}
\]

The real marginal cost is the ratio between the real wage and labor productivity. Thus, firm-specific real marginal costs are influenced by both the firm’s nominal wage (affecting real wage) and the selling price (affecting labor productivity via output demand). In formal terms

\[
E_t^{\eta_p} \tilde{\psi}_{t+j}(i) = E_t^{\eta_p} \log W_{t+j}(i) - E_t \log P_{t+j} - E_t^{\eta_p} \text{mpl}_{t+j}(i) =
E_t^{\eta_p} \log W_{t+j}(i) - E_t \log P_{t+j} + \frac{\alpha}{1 - \alpha} E_t^{\eta_p} \tilde{y}_{t+j}(i), \tag{A3}
\]

where \( \text{mpl}_{t+j}(i) = -\frac{\alpha}{1 - \alpha} \tilde{y}_{t+j}(i) \) is the log of the marginal product of labor given the decreasing-marginal returns production technology introduced in the main text. The Dixit-Stiglitz log-linear demand curve with no optimal price adjustment since period \( t \) links the log of firm-specific output, \( \tilde{y}_{t+j}(i) \), with the relative price

\[
E_t^{\eta_p} \tilde{y}_{t+j}(i) = -\theta (\log P_t^*(i) - E_t \log P_{t+j}) + E_t \tilde{y}_{t+j},
\]

which, using \( \log P_{t+j} = \log P_t + \sum_{k=1}^{j} \pi_{t+k}^p \), implies

\[
E_t^{\eta_p} \tilde{y}_{t+j}(i) = -\theta \left( \tilde{P}_t^* - \sum_{k=1}^{j} E_t \pi_{t+k}^p \right) + E_t \tilde{y}_{t+j}. \tag{A4}
\]

Inserting (A4) onto (A3), it is obtained

\[
E_t^{\eta_p} \tilde{\psi}_{t+j}(i) = E_t^{\eta_p} \log W_{t+j}(i) - E_t \log P_{t+j} - \frac{\alpha \theta}{1 - \alpha} \left( \tilde{P}_t^* - \sum_{k=1}^{j} E_t \pi_{t+k}^p \right) - E_t \text{mpl}_{t+j},
\]

20
with $\widetilde{mpl}_{t+j}$ denoting the log of the aggregate marginal product of labor. Summing and subtracting $\log W_{t+j}$, i.e. the log of the aggregate nominal wage in $t+j$, and defining the log of the real marginal cost as $\widetilde{\psi}_{t+j} = \log W_{t+j} - \log P_{t+j} - \widetilde{mpl}_{t+j}$, we can easily transform the last expression into the following one

$$E_t^{p,\eta} \widetilde{\psi}_{t+j}(i) = E_t \widetilde{\psi}_{t+j} + E_t^{p,\eta} \widetilde{W}_{t+j}(i) - \frac{\alpha \theta}{1 - \alpha} \left( \tilde{P}_t^* - \sum_{k=1}^{j} E_t \pi_{t+k}^p \right),$$  \hfill (A5)

where $\widetilde{W}_{t+j}(i) = \log W_{t+j}(i) - \log W_t$ is the relative marginal wage in $t+j$. Next, (A5) is plugged into the first order condition (A2) and terms are rearranged to yield

$$\left( 1 + \frac{\alpha \theta}{1 - \alpha} \right) \tilde{P}_t^*(i) = \left( 1 - \beta \eta_p \right) E_t^{p,\eta} \sum_{j=0}^{\infty} \beta^j \eta_p^j \left( \widetilde{\psi}_{t+j} + \widetilde{W}_{t+j}(i) \right) + \left( 1 + \frac{\alpha \theta}{1 - \alpha} \right) E_t \sum_{j=1}^{\infty} \beta^j \eta_p^j \pi_{t+j}^p. \hfill (A6)$$

To be consistent with the value of the undetermined coefficient $\tau_1$ implied by the linear relationship guessed in (11a), we must find some expression for $E_t^{p,\eta} \sum_{j=0}^{\infty} \beta^j \eta_p^j \widetilde{W}_{t+j}(i)$ depending upon the observed value of $\widetilde{W}_{t-1}(i)$. The Calvo scheme applied for wage setting in period $t$ results in

$$\widetilde{W}_t(i) = \eta_w \left( \widetilde{W}_{t-1}(i) - \pi_t^w \right) + (1 - \eta_w) \widetilde{W}_t^*(i),$$

where $\widetilde{W}_t^*(i) = \log W_t^*(i) - \log W_t$ is the optimal relative wage that may be set by the $i$-th firm in period $t$. Using the proposed conjecture (11b) conditional to optimal pricing in period $t$ allows us to write $\widetilde{W}_t^*(i)$ depending upon the average (relative) value of new contracts and also upon the relative optimal price: $\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \tilde{P}_t^*(i)$, which can be inserted in the previous expression to reach

$$\widetilde{W}_t(i) = \eta_w \left( \widetilde{W}_{t-1}(i) - \pi_t^w \right) + (1 - \eta_w) \left( \widetilde{W}_t^* - \tau_2 \tilde{P}_t^*(i) \right).$$  \hfill (A7)

Recalling $\widetilde{W}_t^* = \frac{\eta_w}{1 - \eta_w} \pi_t^w$ from equation (17) of the text, and canceling terms in (A7), we have

$$\widetilde{W}_t(i) = \eta_w \widetilde{W}_{t-1}(i) - \tau_2 (1 - \eta_w) \tilde{P}_t^*(i).$$  \hfill (A8)

Repeating the procedure one period ahead for $E_t^{p,\eta} \widetilde{W}_{t+1}(i)$, replacing $\widetilde{W}_t(i)$ for its value obtained in (A8), and using (11b) conditional to the lack of optimal price setting in $t+1$, $E_t^{p,\eta} \widetilde{W}_{t+1}(i) = E_t \widetilde{W}_{t+1} - \tau_2 (\tilde{P}_t^*(i) - E_t \pi_t^p)$, result in

$$E_t^{p,\eta} \widetilde{W}_{t+1}(i) = \eta_w^2 \widetilde{W}_{t-1}(i) - \tau_2 (1 - \eta_w^2) \tilde{P}_t^*(i) + \tau_2 (1 - \eta_w) E_t \pi_t^p.$$  \hfill (A9)
A generalization of (A8) and (A9) for a \( t + j \) period gives the following expression

\[
E_t^{\eta_w} \tilde{W}_{t+j}(i) = \eta_w^{j+1} \tilde{W}_{t-1}(i) - \tau_2 \left( 1 - \eta_w^{j+1} \right) \tilde{P}^*_t(i) + \tau_2 E_t \sum_{k=1}^j \left( 1 - \eta_w^{-k+1} \right) \pi^p_{t+k}. \tag{A10}
\]

Using (A10), the stream of conditional relative wages becomes

\[
E_t^{\eta_w} \sum_{j=0}^\infty \beta^j \eta_p^j \tilde{W}_{t+j}(i) = \frac{\eta_w}{1 - \beta \eta_p} \tilde{W}_{t-1}(i) - \tau_2 \left( \frac{1}{1 - \beta \eta_p} \frac{\eta_w}{1 - \beta \eta_p \eta_p} \right) \tilde{P}^*_t(i) \\
+ \tau_2 \left( \frac{1}{1 - \beta \eta_p} \frac{\eta_w}{1 - \beta \eta_p \eta_p} \right) E_t \sum_{j=1}^\infty \beta^j \eta_p^j \pi^p_{t+j}. \tag{A11}
\]

Substituting (A11) in (A6) yields

\[
\left( 1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 \left( 1 - \frac{1 - \beta \eta_p}{1 - \beta \eta_p \eta_p} \right) \right) \tilde{P}^*_t(i) = \frac{1 - \beta \eta_p}{1 - \beta \eta_p \eta_p} \tilde{W}_{t-1}(i) + \left( 1 - \beta \eta_p \right) E_t \sum_{j=0}^\infty \beta^j \eta_p^j \tilde{W}_{t+j} \\
+ \left( 1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 \left( 1 - \frac{1 - \beta \eta_p}{1 - \beta \eta_p \eta_p} \right) \right) E_t \sum_{j=1}^\infty \beta^j \eta_p^j \pi^p_{t+j},
\]

which confirms the validity of the guess (11a), \( \tilde{P}^*_t(i) = \tilde{P}^*_t + \tau_1 \tilde{W}_{t-1}(i) \), with the following implied value for the undetermined coefficient

\[
\tau_1 = \frac{(1 - \beta \eta_p) \eta_w}{(1 - \beta \eta_p \eta_p) \left( 1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 \left( 1 - \frac{1 - \beta \eta_p}{1 - \beta \eta_p \eta_p} \right) \right)}, \tag{A12}
\]

and the following expression to determine the average relative price set in period \( t \)

\[
\tilde{P}^*_t = \frac{1 - \beta \eta_p}{1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 \left( 1 - \frac{1 - \beta \eta_p \eta_p}{1 - \beta \eta_p \eta_p} \right)} E_t \sum_{j=0}^\infty \beta^j \eta_p^j \tilde{W}_{t+j} + E_t \sum_{j=1}^\infty \beta^j \eta_p^j \pi^p_{t+j}. \tag{A13}
\]

In the text, (A12) corresponds to equation (12a) and (A13) to equation (13).

Part 2. Finding the solution for the undetermined coefficient \( \tau_2 \) and the analytical expression for the average relative wages set in period \( t \), \( \tilde{W}^*_t \).

The wage setting behavior described in the text can be used to identify the second undetermined coefficient, \( \tau_2 \), as well as the average relative wage, \( \tilde{W}^*_t \). If the contract on the \( i \)-th labor service is negotiated in period \( t \), the (log of) the labor-matching nominal wage is the one that satisfies

\[
E_t^{\eta_w} \sum_{j=0}^\infty \beta^j \eta_p^j \left( \tilde{n}_{t+j}^d(i) - \tilde{n}_{t+j}^s(i) \right) = 0,
\]

which conﬁrms the validity of the guess (11a), \( \tilde{P}^*_t(i) = \tilde{P}^*_t + \tau_1 \tilde{W}_{t-1}(i) \), with the following implied value for the undetermined coefficient

\[
\tau_1 = \frac{(1 - \beta \eta_p) \eta_w}{(1 - \beta \eta_p \eta_p) \left( 1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 \left( 1 - \frac{1 - \beta \eta_p}{1 - \beta \eta_p \eta_p} \right) \right)}, \tag{A12}
\]

and the following expression to determine the average relative price set in period \( t \)

\[
\tilde{P}^*_t = \frac{1 - \beta \eta_p}{1 + \frac{\alpha \theta}{1 - \alpha} + \tau_2 \left( 1 - \frac{1 - \beta \eta_p \eta_p}{1 - \beta \eta_p \eta_p} \right)} E_t \sum_{j=0}^\infty \beta^j \eta_p^j \tilde{W}_{t+j} + E_t \sum_{j=1}^\infty \beta^j \eta_p^j \pi^p_{t+j}. \tag{A13}
\]
Taking equation (14) from the text in period \( t \), having a new wage contract set in the main text. Our next task is to express the stream of relative prices, \( E_t \quad \frac{\partial}{\partial t} E_t \), that shows a negative impact of relative prices on relative wages, equation (10) of the main text. Our next task is to express the stream of relative prices, \( E_t \quad \frac{\partial}{\partial t} E_t \), depending upon the current value of the relative price in order to have an expression for \( \tilde{W}_t(i) \) consistent with (11b). Beginning with \( \tilde{P}_{t+1}(i) \), the Calvo aggregation scheme implies

\[
E_t^{\eta_u} \tilde{P}_{t+1}(i) = \eta_p \left( \tilde{P}_t(i) - E_t \pi_{t+1}^p \right) + (1 - \eta_p) E_t^{\eta_u} \tilde{P}_{t+1}(i),
\]

where the second term is \( E_t^{\eta_u} \tilde{P}_{t+1}^*(i) = E_t \tilde{P}_{t+1}^* + \tau_1 \tilde{W}_t^*(i) \) using (11a) in \( t + 1 \) conditional to having a new wage contract set in \( t \). Making such replacement in (A16), it yields

\[
E_t^{\eta_u} \tilde{P}_{t+1}(i) = \eta_p \left( \tilde{P}_t(i) - E_t \pi_{t+1}^p \right) + (1 - \eta_p) \left( E_t \tilde{P}_{t+1}^* + \tau_1 \tilde{W}_t^*(i) \right).
\]

Taking equation (14) from the text in period \( t + 1 \) says \( \tilde{P}_{t+1} = \eta_p E_t^{\eta_u} \pi_{t+1}^p \), which can be used to simplify (A17) to

\[
E_t^{\eta_u} \tilde{P}_{t+1}(i) = \eta_p \tilde{P}_t(i) + \tau_1 (1 - \eta_p) \tilde{W}_t^*(i).
\]

Similarly to (A16), the value of \( E_t^{\eta_u} \tilde{P}_{t+2}(i) \) in the stream of relative prices is a Calvo-type linear combination of non-adjusted prices and optimal prices

\[
E_t^{\eta_u} \tilde{P}_{t+2}(i) = \eta_p \left( E_t^{\eta_u} \tilde{P}_{t+1}(i) - E_t \pi_{t+2}^p \right) + (1 - \eta_p) E_t^{\eta_u} \tilde{P}_{t+2}^*(i),
\]

where using (A18) for the first term leads to

\[
E_t^{\eta_u} \tilde{P}_{t+2}(i) = \eta_p \left( \eta_p \tilde{P}_t(i) + \tau_1 (1 - \eta_p) \tilde{W}_t^*(i) - E_t \pi_{t+2}^p \right) + (1 - \eta_p) E_t^{\eta_u} \tilde{P}_{t+2}^*(i).
\]
Recalling (11a) in period $t + 2$ conditional to the lack of wage resetting for substitution in the second term of (A19), $E_t^{n_w} \tilde{P}_{t+2}^*(i) = E_t \tilde{P}_{t+2}^* + \tau_1 E_t \tilde{W}_{t+1}^*(i) = E_t \tilde{P}_{t+2}^* + \tau_1 \left( \tilde{W}_{t}^*(i) - E_t \pi_t^w \right)$, it is obtained

$$E_t^{n_w} \tilde{P}_{t+2}^*(i) = \eta_p \left( \eta_p \tilde{P}_t(i) + \tau_1 (1 - \eta_p) \tilde{W}_t^*(i) - E_t \pi_t^w \right) + (1 - \eta_p) \left( E_t \tilde{P}_{t+2}^* + \tau_1 \left( \tilde{W}_t^*(i) - E_t \pi_t^w \right) \right),$$

where inserting $\tilde{P}_{t+2}^* = \frac{\eta_p}{1 - \eta_p} \pi_{t+2}^w$ simplifies to

$$E_t^{n_w} \tilde{P}_{t+2}^*(i) = \eta_p \tilde{P}_t(i) + \tau_1 (1 - \eta_p^2) \tilde{W}_t^*(i) + \tau_1 (1 - \eta_p) E_t \pi_t^w. \quad (A20)$$

A generalization of (A18) and (A20) results in the following rule

$$E_t^{n_w} \tilde{P}_{t+j}^*(i) = \eta_p^j \tilde{P}_t(i) + \tau_1 (1 - \eta_p^j) \tilde{W}_t^*(i) + \tau_1 E_t \sum_{k=1}^{j-1} (1 - \eta_p^{j-k}) \pi_t^w, \quad (A21)$$

that serves to compute the stream of relative prices as follows

$$E_t^{n_w} \sum_{j=0}^{\infty} \beta^j \eta_w^j \tilde{P}_{t+j}^*(i) = \frac{1}{1 - \beta \eta_w \eta_p} \tilde{P}_t(i) + \tau_1 \left( \frac{\beta \eta_w}{1 - \beta \eta_w} - \frac{\beta \eta_w \eta_p}{1 - \beta \eta_w} \right) \tilde{W}_t^*(i)$$

$$- \tau_1 \left( \frac{\beta \eta_w}{1 - \beta \eta_w} - \frac{\beta \eta_w \eta_p}{1 - \beta \eta_w} \right) E_t \sum_{j=1}^{\infty} \beta^j \eta_w^j \pi_t^w. \quad (A21)$$

Substituting (A21) in the relative wage equation (A15), it is obtained

$$\left( 1 + \frac{\tau_1 \beta \eta_w \theta \gamma}{(1 - \alpha)} \left( 1 - \eta_p \frac{1 - \beta \eta_w}{1 - \beta \eta_w \eta_p} \right) \right) \tilde{W}_t^*(i) = - \frac{\theta \gamma (1 - \beta \eta_w)}{(1 - \alpha) \left( 1 - \beta \eta_w \eta_p \right)} \tilde{P}_t(i)$$

$$- \gamma (1 - \beta \eta_w) E_t \sum_{j=0}^{\infty} \beta^j \eta_w^j u_{t+j} + \left( 1 + \frac{\tau_1 \beta \eta_w \theta \gamma}{(1 - \alpha)} \left( 1 - \eta_p \frac{1 - \beta \eta_w}{1 - \beta \eta_w \eta_p} \right) \right) E_t \sum_{j=1}^{\infty} \beta^j \eta_w^j \pi_t^w, \quad (A22)$$

which proves right the proposed guess (11b) with the following implied value for the undetermined coefficient $\tau_2$

$$\tau_2 = \frac{\theta \gamma (1 - \beta \eta_w)}{(1 - \alpha)(1 - \beta \eta_w \eta_p) \left( 1 + \frac{\tau_1 \beta \eta_w \theta \gamma}{(1 - \alpha)} \left( 1 - \eta_p \frac{1 - \beta \eta_w}{1 - \beta \eta_w \eta_p} \right) \right)}, \quad (A23)$$

and the following expression for the average relative wage contract set in period $t$

$$\tilde{W}_t^* = - \frac{\gamma (1 - \beta \eta_w)}{(1 + \frac{\tau_1 \beta \eta_w \theta \gamma}{(1 - \alpha) \left( 1 - \eta_p \frac{1 - \beta \eta_w}{1 - \beta \eta_w \eta_p} \right)} ) E_t \sum_{j=0}^{\infty} \beta^j \eta_w^j u_{t+j} + E_t \sum_{j=1}^{\infty} \beta^j \eta_w^j \pi_t^w, \quad (A24)$$

In the text, (A23) corresponds to equation (12b) and (A24) to equation (16).
References


Walsh, C. E. (2005). Labor market search, sticky prices, and interest rate policies,


Table 1. Analytical slope coefficients under alternative price/wage settings

<table>
<thead>
<tr>
<th></th>
<th>Price inflation, eq (15)</th>
<th>Wage inflation, eq (23)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sticky prices, sticky wages</td>
<td>( \frac{(1-\beta \eta_p)(1-\eta_w)}{\eta_p(1+\frac{\omega \theta}{1-\alpha} + \tau_2(1-\eta_w \beta \eta_p/\eta_w))} )</td>
<td>( \frac{(1-\beta \eta_w)(1-\eta_w)}{\eta_w(1+\tau_1 \frac{\eta_w \theta w}{1-\alpha} (1-\eta_w \beta \eta_p/\eta_w))} )</td>
</tr>
<tr>
<td>Sticky prices, flexible wages</td>
<td>( \frac{(1-\beta \eta_p)(1-\eta_w)}{\eta_p(1+\frac{\omega \theta}{1-\alpha} + \frac{\eta_w \theta w}{1-\alpha})} )</td>
<td>( \infty ), since ( \hat{\psi}_t = \hat{m} \hat{r} s_t ) or ( u_t = 0 )</td>
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<tr>
<td>Flexible prices, sticky wages</td>
<td>( \infty ), since ( \hat{\psi}_t = 0 )</td>
<td>( \frac{(1-\beta \eta_w)(1-\eta_w)}{\eta_w(1+\tau_1 \frac{\eta_w \theta w}{1-\alpha})} )</td>
</tr>
<tr>
<td>EHL (2000) model</td>
<td>( \frac{(1-\beta \eta_p)(1-\eta_p)}{\eta_p(1+\frac{\omega \theta}{1-\alpha})} )</td>
<td>( \frac{(1-\beta \eta_w)(1-\eta_w)}{\eta_w(1+\gamma \theta_f)} )</td>
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Table 2. (Peak) responses to a technology shock relative to output.

<table>
<thead>
<tr>
<th></th>
<th>( \pi^p/\hat{y} )</th>
<th>( \pi^w/\hat{y} )</th>
<th>( \hat{w}/\hat{y} )</th>
<th>( \hat{n}/\hat{y} )</th>
<th>( u/\hat{y} )</th>
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<tbody>
<tr>
<td>Sticky prices, sticky wages</td>
<td>-0.20</td>
<td>-0.16</td>
<td>0.62</td>
<td>-1.41</td>
<td>0.53</td>
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<td>( \eta_p = \eta_w = 0.75 )</td>
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<tr>
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<td>-1.71</td>
<td>-1.57</td>
<td>-1.72</td>
<td>0.0</td>
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<td>(Woodford, 2003) ( \eta_p = 0.75 ) and ( \eta_w = 0.0 )</td>
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<tr>
<td>Flexible prices, sticky wages</td>
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<td>-0.09</td>
<td>0.97</td>
<td>-0.54</td>
<td>-0.54</td>
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<tr>
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<tr>
<td>EHL (2000) model</td>
<td>-0.26</td>
<td>-0.08</td>
<td>0.78</td>
<td>-1.24</td>
<td>0.0</td>
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<td>( \eta_p = \eta_w = 0.75 )</td>
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Figure 1: Responses to a 1% technology shock.