

**RANK DEPENDENT EXPECTED UTILITY IN THE PELOTA  
BETTING SYSTEM:  
AN EXPERIMENT**

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# ***Rank Dependent Expected Utility in the Pelota Betting System: an Experiment***

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*Comments are welcome!*

## **Abstract**

We theoretically and experimentally study a zero sum betting market: the *Pelota* betting system, but with commonly known objective probabilities and without commissions. We know that risk-averse expected utility maximizers with identical objective probabilities cannot agree on a bet. Nevertheless, the rank dependent expected utility model allows us to explain the existence of such betting markets even assuming individuals are all identical even in utilities. We focus on behaviour in a given period in a *Pelota* betting market and we aim to explain the volume of bets assuming that all individuals are equal and their marginal utility on wealth is decreasing. We do this in two stages. First, subjects are asked to take betting decisions and the power utility function and probability weighting function are estimated. Once the underlying utility and probability weighting function are known, in a second stage subjects interact in a betting market and we test whether the volume of bets differs from proposed theoretical predictions.

**Keywords:** *Pelota* betting market, experiment, rank dependent expected utility, sport betting market.

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## 1. Introduction

Betting markets offer major opportunities for economic analysis in that they are especially simple financial markets in which the scope of the pricing problem is reduced. In “The Economics of Wagering Markets” Raymond D. Sauer analyzes the economics of betting markets in which participants take a financial position on the outcome of a sporting event such as a horse race or a football game. But there is a betting market that he does not mention at all: the one which goes with *Pelota (Jai Alai)* matches, a sport of two teams. For more details see the description of the sport and its betting system in Llorente, L. and J.M. Aizpurua (2006). We emphasize that although the *Pelota* betting system has operated for centuries in the Basque Country, *Navarra* and *La Rioja* rule are not written rules but passed on verbally. This has hindered its spread to other regions and thus it has not been studied, even though its importance in the area is remarkable. As detailed below, both its peculiarities and its theoretical simplicity makes analyzing the *Pelota* betting market an interesting exercise.<sup>1</sup>

It follows two peculiarities that differentiate it from other well-known betting systems. Unlike pari-mutuel betting systems the odds in a *Pelota* market are definitively fixed when the bet takes place. Bets are arranged by means of middlemen but unlike what happens in bookmaking, for a bet to be placed, one bettor bets on one team and another bettor bets on the other team, thus the middleman does not bet at all.

Under our point of view the *Pelota* betting system is a very attractive system from a theoretical analysis perspective, i.e., an individual decision at a given moment is simple and clear. Although there is uncertainty about who will win the match, theoretically once the person has attached a subjective probability to the chances of an event, say “*the reds will win*”, the problem he faces is simple. He decides which prospect to select from a set of only two different kinds of prospect: betting on the reds or betting on the blues. Within a set of prospects his decision is very simple: how many identical bets to place. Furthermore, each prospect entails only two possible outcomes: either as many wins as there are bets placed (a given amount of money) or the same number of losses (another given amount of money).

Here we will take advantage of the simplicity of this system’s decisions while avoiding the main drawback when analyzing this market’s data, which is the unknown objective probability. Here we implement the betting system but in a world of decision under risk and not uncertainty, i.e. a world of two possible states of nature with given objective probabilities.

Thus we implement experimentally decisions similar to those faced by individuals in a *Pelota* betting market but with commonly known objective probabilities. We study behaviour assuming the rank dependent expected utility (RDEU) model. We show that the expected utility model is a particular case of the RDEU model when the probability weighting function of the worse

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<sup>1</sup> Another important characteristic of the *Pelota* betting system is that bets are allowed throughout the match, so it can be studied how new information affects people’s decisions. An interesting avenue for future work is to analyze decisions not in a given period but as a strategy of behaviour throughout the match.

outcome is the identity  $q(p) = p$ . The attractiveness of the rank dependent expected utility model as the theoretical explanation for individual behaviour in this betting market is that it can explain the existence of the betting market even among identical bettors.

In the following Section we introduce and formalize the experimental betting system; First we study the optimal bets of rank-dependent expected utility (RDEU) maximizers. Second, we study the market; we show a necessary condition for the existence of a bet in the market and we propose predictions for the market volume of transactions. Finally we select the utility and probability weighting function to be estimated. Section 3 explains the details of the experimental protocol and the estimation procedure for each of the 2 stages. The results are discussed in Section 4, and some general conclusions are presented in Section 5.

## 2. The Theoretical Model

In this section we formalize the betting mechanism that underlies the *Pelota* betting market. First we obtain individual *demand* and *offer* functions of bets assuming bettors are rank dependent expected utility maximizers. In a second subsection we study the betting market and, after showing that bettors have to be optimistic in order to place a bet, we propose different predictions of numbers of transactions in the specific market implemented in the lab.

### 1.1. Individual optimal decision

We analyze *pelota* bets under Quiggin's RDEU model.

The theoretical framework of choice under uncertainty is the one followed in Quiggin, J. (1993). We deal with individual preferences over a set  $\mathcal{X}$  of outcomes and an associated set,  $\mathcal{Y}$ , of random variables, or *prospects*, taking values in  $\mathcal{X}$ . Elements of  $\mathcal{X}$  are denoted by  $x$ , and elements of  $\mathcal{Y}$  are denoted by  $y$ . In what follows,  $\mathcal{X}$  is assumed to be an interval in  $\mathbb{R}_+$  interpreted in terms of income or wealth levels. The outcome space is totally ordered by a preference relation, denoted by  $\succeq$ . The associated indifference relation is denoted by  $\sim$ . Prospects will be represented in the form

$$\{\mathbb{x}, \mathbb{p}\} = \{(x_1, x_2, \dots, x_n); (p_1, p_2, \dots, p_n)\}$$

where  $p_i$  is the probability of outcome  $x_i$ ,  $\sum_{i=1}^n p_i = 1$ , and the  $x_i$  are weakly ordered from worst to best, so that  $x_1 \preceq \dots \preceq x_n$ .

Formally, the *cumulative probability* of  $x$ , denoted  $F(x)$ , is given by  $P_r\{y < x\} = \sum_{x_i \leq x} p_i$ . RDEU is based on a probability weighting function  $q: [0; 1] \rightarrow [0; 1]$  applied, not to the probabilities of individual events, but to the cumulative distribution function. The RDEU functional is given by

$$V(\{\mathbb{x}; \mathbb{p}\}) = \sum_{i=1}^n U(x_i) h_i(\mathbb{p}),$$

where

$$h_i(\mathbb{p}) = q\left(\sum_{j=1}^i p_j\right) - q\left(\sum_{j=1}^{i-1} p_j\right) = q(F(x_i)) - q(F(x_{i-1}))$$

In our specific market, similarly to the betting market in *Pelota*, we consider two states of nature “reds win”,  $r$ , and “blues win”,  $b$ . Each of the  $N$  bettors has an endowment of  $W \in \mathfrak{R}_+$ .

Denote by  $p_r$  ( $p_b$ ) the objective probability for event  $r$  ( $b$ ); as ties are not allowed  $p_b = 1 - p_r$ . The odds will be represented by  $(O_R, O_B)$ , where  $O_R$  ( $O_B$ ) is the money a bettor risks if he stakes one unit on event  $r$  ( $b$ ), i.e., the money he loses if event  $r$  ( $b$ ) does not occur. It may be helpful to think of these odds as a price relation at which one can trade “money if state of nature  $b$  occurs” for “money if the state of nature  $r$  occurs”.

Therefore to stake one single bet on  $r$  means to select the prospect

$$\{(W - O_R, W + O_B), (1 - p_r, p_r)\},$$

and to stake one single bet on  $b$  means to select the prospect

$$\{(W - O_B, W + O_R), (p_r, 1 - p_r)\}.$$

As more than one bet can be staked on each of the two colors we consider two special subsets of prospects among which bettor  $i$  can choose:  $\mathcal{R}, \mathcal{B} \subseteq \mathcal{Y}$ . We denote by  $SR_i \in \mathcal{R}$  the prospect selected by bettor  $i$  when he stakes  $SR$  bets on the reds, which is the prospect

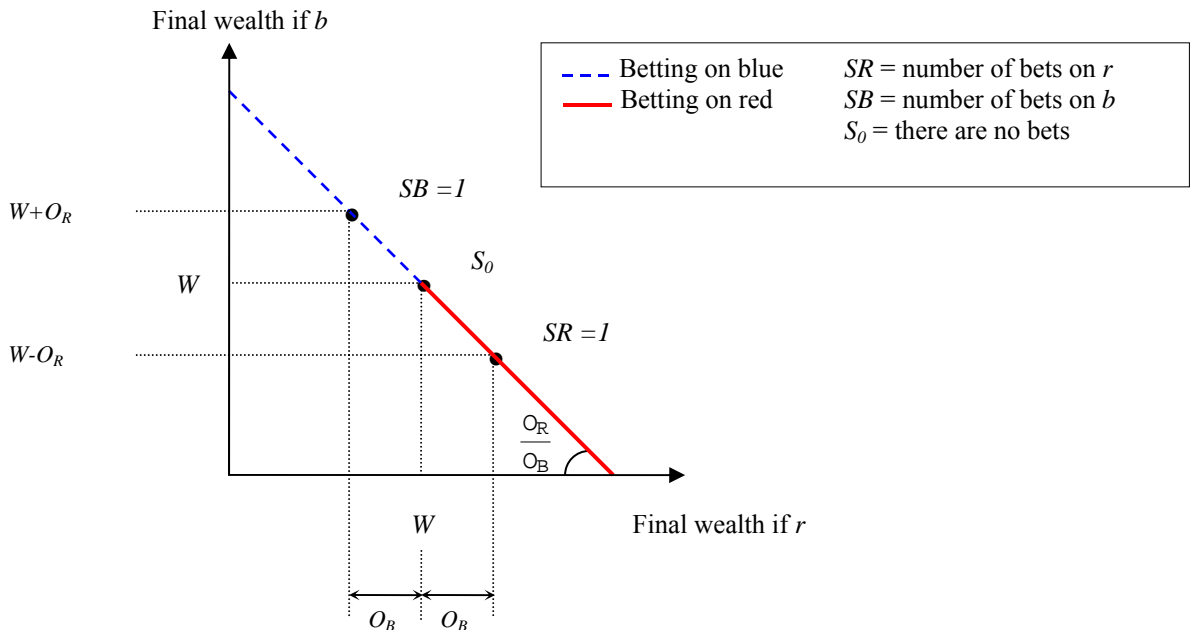
$$\{(W - O_R SR_i, W + O_B SR_i), (1 - p_r, p_r)\}, \text{ and}$$

$SB_i \in \mathcal{B}$  the prospect selected when staking  $SB$  bets on the blues, which is the prospect

$$\{(W - O_B SB_i, W + O_R SB_i), (p_r, 1 - p_r)\}.$$

These prospects can be represented in the following graph where the horizontal axis shows consumption if  $r$  occurs and the vertical axis shows consumption if  $b$  occurs.

Graph 1: “Bettor’s consumption set”



Then a bettor's utilities for each of the possible prospects are

$$V(SR) = q(1 - p_r)U(W - O_R SR) + (1 - q(1 - p_r))U(W + O_B(1 - t)SR), \text{ and} \\ V(SB) = q(p_r)U(W - O_B SB) + (1 - q(p_r))U(W + O_R(1 - t)SB),$$

From these expressions we obtain the graphical representation of indifference curves (IC) in Llorente, L. and J.M. Aizpurua (2006).

A bettor's optimal decision is given by

$$S^{**}(p_r, O_R, O_B, W) = \begin{cases} SR^* & \text{if } V(SR^*) \geq V(SB^*) \\ or \\ SB^* & \text{if } V(SB^*) \geq V(SR^*), \end{cases}$$

where  $SR^*$  is obtained by solving the problem

$$\max_{SR_r \in \left[0, \frac{W}{O_R}\right]} V(SR),$$

and  $SB^*$  is obtained by solving the problem

$$\max_{SB_b \in \left[0, \frac{W}{O_B}\right]} V(SB).$$

Thus a bettor decides either to stake on  $r$ ,  $SR \in \left[0, \frac{W}{O_R}\right]$ , or to stake on  $b$ ,  $SB \in \left[0, \frac{W}{O_B}\right]$ , it is not possible to bet a negative amount or to bet an amount unaffordable under a bettor's endowment. We avoid the possibility of betting on both colors. The prospect selected when betting  $SR = SB$  is the same as when not betting at all. In the experiment we do not allow this to happen and in the real *Pelota* market there are commissions, which makes this prospect a sure loss of commission.

The first order conditions for an interior solution are

$SR^*(p_r, O_R, O_B, W)$  such that

$$\frac{(1 - q(1 - p_r))}{q(1 - p_r)} \frac{U'(W + O_B SR)}{U'(W - O_R SR)} = \frac{O_R}{O_B}, \quad (1)$$

*IC's slope*<sup>2</sup> *CS's slope*

and  $SB^*(p_r, O_R, O_B, W)$  such that

$$\frac{q(p_r)}{1 - q(p_r)} \frac{U'(W - O_B SB)}{U'(W + O_R SB)} = \frac{O_R}{O_B}. \quad (2)$$

*IC's slope* *CS's slope*

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<sup>2</sup> See Llorente, L. and J.M. Aizpurua (2006), the section "RDEU maximizers' indifference curves" to see the shape of the IC's under this model.

The second order condition tells us that  $U''(x) < 0$  is a sufficient condition for maximization. See proof in “Appendix 1: The general optimization problem”.

We refer to  $SR^*(O_R, O_B)$  as either the offer function on  $r$  or the demand function on  $b$ , and to  $SB^*(O_R, O_B)$  as either the demand function on  $r$  or the offer function on  $b$ .

## 1.2. The market

In this subsection we first show the necessary conditions for the existence of bets in the market, then we propose predictions for the number of transactions in each period and finally we show the maximum possible bets in the market.

### 1.2.1. Necessary condition for the existence of bets in the market

**Proposition 1** Let a *Pelota* betting market be composed of identical RDEU maximizers with a concave utility function on wealth. Then a necessary condition for a bet to take place is that bettors should be optimistic as defined in Quiggin, J. (1982), i.e.,  $q(p) + q(1-p) < 1$

Proof:

A necessary condition for the existence of a bet is one bettor betting on reds and another on blues but as bettors are equal we need to obtain  $SR_i^*$  and  $SB_i^*$  greater than 0 “for the same bettor”. Therefore either (i) or (ii) must happen;

(i) equations (1) and (2) have to be fulfilled, which implies that left-hand sides of the two equations have to be equal,

$$\frac{(1-q(1-p_r))}{q(1-p_r)} \frac{U'(W+O_B SR)}{U'(W-O_R SR)} = \frac{q(p_r)}{1-q(p_r)} \frac{U'(W-O_B SB)}{U'(W+O_R SB)} \quad (3)$$

In addition we assume decreasing marginal utility on wealth, therefore

$$\frac{U'(W+O_B SR)}{U'(W-O_R SR)} < \frac{U'(W-O_B SB)}{U'(W+O_R SB)}$$

thus, in order for (3) to be true,

$$\frac{1-q(1-p_r)}{q(1-p_r)} > \frac{q(p_r)}{1-q(p_r)}, \text{ operating } 1-q(1-p_r) > q(p_r) \text{ which is Quiggin's definition of optimistic.}$$

(ii)  $SR^*$  or  $SB^*$  is a corner solution. If  $SR^*$  is a corner solution where  $SR^* = W/O_R$  then

$$SR^*(p_r, O_R, O_B, W) \text{ such that } \frac{(1-q(1-p_r))}{q(1-p_r)} \frac{U'(W+O_B SR)}{U'(W-O_R SR)} > \frac{O_R}{O_B}, \quad (1') \text{ which implies that}$$

$$\frac{(1-q(1-p_r))}{q(1-p_r)} \frac{U'(W+O_B SR)}{U'(W-O_R SR)} > \frac{q(p_r)}{1-q(p_r)} \frac{U'(W-O_B SB)}{U'(W+O_R SB)} \quad (3')$$

and the proof is similar.

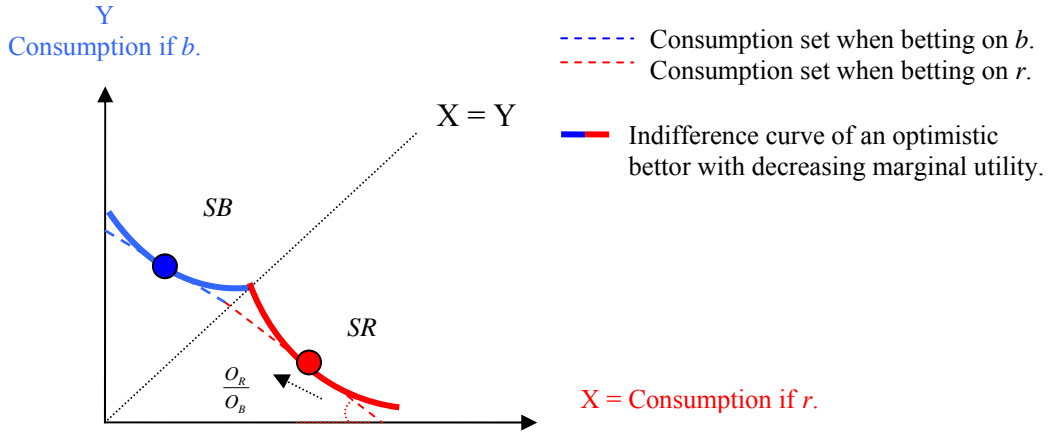
and if  $SB^*$  is a corner solution then

$$SB^*(p_r, O_R, O_B, W) \text{ such that } \frac{q(p_r)}{1-q(p_r)} \frac{U'(W-O_B SB)}{U'(W+O_R SB)} < \frac{O_R}{O_B}. \quad (2') \text{ which implies that}$$

$$\frac{(1-q(1-p_r))}{q(1-p_r)} \frac{U'(W+O_B SR)}{U'(W-O_R SR)} > \frac{q(p_r)}{1-q(p_r)} \frac{U'(W-O_B SB)}{U'(W+O_R SB)} \quad (3')$$

and the proof is similar.

The presence of these optimistic RDEU maximizers provides us with a model where an interior solution makes it possible for bets to exist.



In the graph above we see that the bettor is indifferent between betting on *SB* or *SR*. Therefore one bettor is willing to bet on the reds, another bettor on the blues and therefore bets can be explained in the market.

### 1.2.2. Market volume of bets

In *Pelota* betting markets, given the probabilities  $(p_r, 1 - p_r)$  and the odds  $(O_R, O_B)$ , a bettor decides how much to stake on *r*, *SR*, or on *b*, *SB*. The realization of this decision depends on whether a middleman can find a bettor willing to bet on the other team. The equilibrium price in a *Pelota* betting market is such that the bettor's utility betting his optimum on the reds must be equal to his utility betting the optimum on the blues. Therefore in a "big" market of  $N$  identical bettors whose utility and probability weighting functions are known, the individual optimal number of bets is obtained by applying equations (1) and (2) and the number of transactions and the number of bettors betting on each of the two colors can be easily calculated as explained in "Appendix 2: Equilibrium in the *frontón* betting market".

But in this experiment the market is slightly different. First of all we are not able to run an experiment with a large number of bettors, so bettors could realize that they can influence market odds. In addition, the underlying probability weighting function is unknown in advance; so we are not able to obtain the equilibrium odds. We decided to simplify the market. In a particular period the odds (prices) are given but they may or may not be the equilibrium odds. In a particular period, given the commonly known probability and the odds, bettors post their offers to bet on one color or the other, or on both. They can also accept offers already posted on the market by other bettors. Once they decide to bet on one color they cannot bet on the other in the same period. In such a market we propose various predictions concerning the number of transactions.



In a period,  $(p_r, O_R, O_B, W)$  are given. From equations (1) and (2) we can calculate  $SR^*(p_r, O_R, O_B, W)$  and  $SB^*(p_r, O_R, O_B, W)$ . In the market only an integer number of bets can be placed, so from now on  $SR^*$  and  $SB^*$  are rounded.

**Prediction A:** We predict that in a market comprising  $N$  identical bettors the number of bets realized is  $S_A^* = \text{Min}\{SR^*, SB^*\} * \left\lceil \frac{N}{2} \right\rceil$ , where the subindex A refers to “prediction A” and  $\lceil \cdot \rceil$  denotes the integer part of  $\cdot$ .

We can interpret this prediction as  $\left\lceil \frac{N}{2} \right\rceil$  pairs of bettors in the market betting the minimum between the optimal when betting on the reds and the optimal when betting on the blues. This adds up to  $S_A^*$  bets in the market.

**Prediction B:** Denote  $x = \text{Min}\{SR^*, SB^*\}$ ,  $y = \text{Max}\{SR^*, SB^*\}$  and  $z = \frac{y}{x}$ . We interpret  $\lceil z \rceil$  as the number of bettors on one color that bet against a bettor on the other color. These bettors all together form what we call a betting group. The number of bettors in a betting group is then defined as  $\lceil nbg \rceil = \text{Min}\{1 + z, N\}$  where betting groups comprising more than the number of bettors in the entire market,  $N$ , are not permitted and bettors are not divisible. The number of possible groups in a market is given by  $\lceil ng \rceil = \frac{N}{\lceil nbg \rceil}$ . Therefore we predict that the number of transactions in a market is  $S_B^* = x * \lceil z \rceil * \lceil ng \rceil$ . Bettors post their optimal decisions and accept the optimal decisions of others.

**Prediction C:** Given the notation in “prediction B”, we define  $r = N - \lceil ng \rceil * \lceil nbg \rceil$  as the number of bettors in prediction B that do not belong to a group. We predict that the total number of transactions in the betting market will be  $S_C^* = x * \lceil z \rceil * \lceil ng \rceil + \left( x * \left\lceil \frac{r}{2} \right\rceil \right)$ .

**Prediction D:** Denote  $x = \text{Min}\{SR^*, SB^*\}$ ,  $y = \text{Max}\{SR^*, SB^*\}$  and  $z = \frac{y}{x}$ . We interpret  $z$  as the number of bettors on one color that bet against a bettor on the other color. This is what we call a betting group. The number of bettors in a betting group is defined as  $nbg = \text{Min}\{1 + z, N\}$ , and here we could find a non-integer number of bettors. The number of possible groups in a market is given by  $ng = \frac{N}{nbg}$ . We predict that the number of transactions in a market is  $S_C^* = y * ng$ .

In this prediction there can be bettors who post a number of bets lower than their optimal.

### 1.2.3. Maximum number of transactions

Denote by  $\overline{SR} = \frac{W}{O_R}$  the maximum number of bets that an individual can place on  $r$  and by

$\overline{SB} = \frac{W}{O_B}$  the maximum a bettor can place on  $b$ . Call  $\overline{x} = \min\{\overline{SR}, \overline{SB}\}$ ,  $\overline{y} = \max\{\overline{SR}, \overline{SB}\}$  and

$\overline{z} = \frac{\overline{y}}{\overline{x}}$ . We interpret  $\overline{z}$  as the number of bettors betting the maximum on one color who bet

against a bettor who bets the maximum on the other color. This is what we call a betting group where bettors bet the maximum. The number of bettors in this betting group is defined as  $\overline{nbg} = \min\{1 + \overline{z}, N\}$ , and here we could find a non-integer number of bettors: half a bettor can be

interpreted as a bettor who bets half the amount he can afford from his endowment. The number of possible groups in a market is given by  $\overline{ng} = \frac{N}{\overline{nbg}}$ . Then the maximum number of

transactions in a market composed by  $N$  bettors is  $\overline{S} = \lceil \overline{y} * \overline{ng} \rceil$ .

## 1.3. The utility and probability weighting function

In this experiment we try to explain the volume of bets by applying the model of equal RDEU bettors. In a given period of a betting market the variables  $(p_r, O_R, O_B)$  are commonly known. In order to predict the volume of bets we would like to know the optimal decision of each bettor. Assuming bettors are RDEU maximizers the optimum amount bet is the amount that fulfils equation (1) or (2). We see in these equations that the optimal amount of bets depends on both the utility and the probability weighting functions. Therefore, if we know the utility function and the probability weighting function we will be able to predict the volume of bets. At this point we decided to divide the experiment into two stages. The data from the first stage were used to estimate the utility and probability weighting functions and in the second stage the betting market was implemented to check whether we could predict volume of bets with the RDEU model.

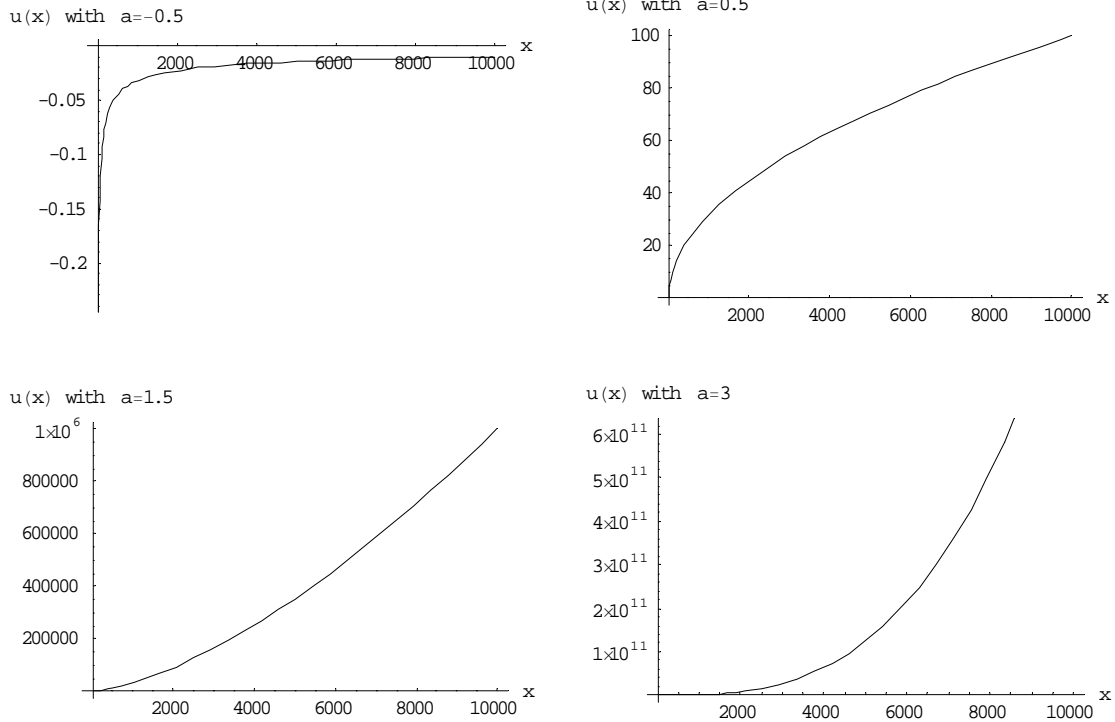
Therefore we ran a first stage in the experiment where, given  $(p_r, O_R, O_B)$ , subjects were asked how much they wanted to bet and we were forced to accept the bet on the other side. We estimated the optimal decision of bettors assuming both a utility and a probability weighting functional form. Once we had the parameters that best fit the data in the first stage, in the second stage we used the utility and probability weighting function obtained to predict the number of transactions in the market.

### 1.3.1. The utility function

We estimated the parameter of the power utility function  $u(x) = \begin{cases} x^a & \text{if } a \geq 0 \\ -x^a & \text{if } a < 0 \end{cases}$  that allows for

decreasing marginal utility whenever  $a < 1$ , see proof in Appendix 3: The estimated utility function. The attractiveness of this utility function is that it has only one parameter but it allows

for plenty of different attitudes towards risk. We want to allow for the greatest possible variability in risk aversion but with as few parameters as possible. That is why we estimate this utility function. The utility function for different  $a$  values follows.



We calculate the Arrow-Pratt coefficient of risk aversion  $r(x) = -\frac{u''(x)}{u'(x)} = \frac{1-a}{x}$  or, rewriting,

$$r(x) = \begin{cases} \frac{1-a}{x} & \text{if } a \geq 0 \\ \frac{1+|a|}{x} & \text{if } a < 0 \end{cases}. \text{ Therefore negative } a \text{ allows for more risk-averse attitudes than allowing}$$

only positive values of the parameter  $a$ .

### 1.3.2. The probability weighting function

We estimate the probability weighting function  $q(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$  studied in Lattimore, P.K., J. K. Baker, and A. D. Witte. (1992). This has been estimated in empirical papers such as Gonzalez, R. and G. Wu (1999). For the median data they obtain  $\delta = 0.77$  and  $\gamma = 0.44$ .<sup>3</sup> We chose this functional form because it allows us to study optimism (whenever  $\delta < 1$ ) separately from the widespread behaviour of overestimating low probabilities while underestimating high ones ( $\gamma < 1$ ). See Gonzalez, R. and G. Wu (1999) page 139 for an explanation of these two parameters.

<sup>3</sup> These data are obtained for the power utility function on wealth,  $u(x) = x^a$ , with  $a = 0.49$ .

### 3. The design of the experiment and its protocol

In general economic theory relies on the assumption that individuals are risk averse, i.e., the utility function on wealth has decreasing marginal utility. When trying to explain betting games of zero sum between individuals with decreasing marginal utility on wealth the expected utility theory has little success.<sup>4</sup> We have shown above that theoretically we can explain the existence of transactions in *pelota* betting markets when there are optimistic RDEU maximizer bettors. Moreover, if we know  $u(x)$  and  $q(p)$  we can predict, for each vector  $(p_r, O_R, O_B, W)$ , the amount transacted in the market. We have two main objectives: first of all to test whether the RDEU model with decreasing marginal utility on wealth fits the experimental data better than the expected utility model (EU). Secondly we want to study the equilibrium amount of transactions and to check whether the theoretical prediction is close enough to the real number of transactions. In order to achieve our objectives we divided our experiment into two stages. In the first stage we assumed subjects were RDEU maximizers with a utility function of the form

$$u(x) = \begin{cases} x^a & \text{if } a \geq 0 \\ -x^a & \text{if } a < 0 \end{cases} \quad \text{and a probability weighting function } q(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}. \quad \text{Our objective}$$

was to estimate the parameters  $a$ ,  $\delta$  and  $\gamma$ , in order to obtain the demand and offer functions on a given event, which determine the market equilibrium. In the second stage we implemented a betting market where, given the vector  $(p_r, O_R, O_B, W)$ , subjects either demanded or offered bets on an event. We study whether the amount of bets fits the theoretical prediction. Below we explain in detail each the design of the experiment for each stage.

#### 1.1. Stage 1: Estimating bettor's demand and offer function on event $r$

80 students at the Universidad Pública de Navarra participated in the computerized experiment, using the z-Tree software package (Fischbacher 1999, Zurich Toolbox for Ready-made Economic Experiments). In the first stage subjects took 36 independent decisions on the amount to bet on a given color  $r$ . In each period all subjects were given the same endowment  $W = 10.000$  points that could be used to bet on  $r$ . The commonly known probability for event  $r$  was shown as the chances of extracting a ball of color  $r$  from a box containing eight balls of two different colors  $r$  and  $b$ . The odds were shown as the money won on a bet if the ball randomly selected was  $r$  and the money lost on a bet if the ball was not  $r$ . In order to be sure that subjects knew exactly how much money they could obtain with their decision, they were able to check, for each possible decision, the total amount of points they would obtain depending on the color of the ball extracted. Unlike stage 2, here we were forced to accept the amount bet on the other color. Thus, in each period of this stage, given the vector  $(p_r, O_R, O_B, W)$ , subjects were asked

$$\text{to choose the amount demanded on event } r, \quad SR_i \in \left[ 0, \frac{10.000}{O_R} \right].$$

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<sup>4</sup> One bet in our betting market is a zero sum game. In real life there are *Pelota* betting markets even more difficult to explain with expected utility theory because if there is a bet the sum of the expected value of both bettors is less than zero. In real *Pelota* betting markets there is a middleman who takes 16% of the amount the winner obtains in a bet as commission, so the sum of bettors' expected value is  $-0.16(O_B p_r + O_R(1-p_r))$ .

The parametrization established was  $p_r \in \{1/8, 3/8, 4/8, 5/8, 7/8\}$ ,<sup>5</sup> and either  $O_R = 100$  and  $O_B \in \{5, 7, 10, 12, 15, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$  or vice versa.

The stage lasted approximately thirteen minutes. The ending time for each period was 30 seconds but the period did not finish until all subjects had entered their decision. Subjects were informed that three of the 36 periods would be randomly selected at the end of the experiment. Then the uncertainty for each period was solved independently, event  $r$  was selected with the previously fixed probability in the period, and subjects received the points related to their decision in all the periods selected.

The estimating procedure follows. Remember we assume subjects are RDEU maximizers with

$$u(x) = \begin{cases} x^a & \text{if } a \geq 0 \\ -x^a & \text{if } a < 0 \end{cases} \quad \text{and} \quad q(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}. \quad \text{In order to estimate the parameters } a, \delta \text{ and } \gamma,$$

we run a non-linear regression using the Levenberg-Marquardt method with the software SPSS v.11.5.1 for Windows. We specified a non-linear model where each subject's decisions on the amount demanded on  $r$  were predicted by the theoretical optimal decision

$$SR_i^* = \begin{cases} \frac{W}{O_R} \frac{\left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}} - 1}{\frac{O_B}{O_R} + \left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}}} & \text{if } \frac{W}{O_R} \frac{\left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}} - 1}{\frac{O_B}{O_R} + \left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}}} \in \left[ 0, \frac{W}{O_R} \right] \\ 0 & \text{if } \frac{W}{O_R} \frac{\left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}} - 1}{\frac{O_B}{O_R} + \left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}}} < 0 \\ \frac{W}{O_R} & \text{if } \frac{W}{O_R} \frac{\left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}} - 1}{\frac{O_B}{O_R} + \left( \frac{1-q(1-p_r)}{q(1-p_r)} \frac{O_B}{O_R} \right)^{\frac{1}{1-a}}} > \frac{W}{O_R} \end{cases} \quad (4a)$$

when  $a < 1$ , and

$$SR_i^* = \begin{cases} \frac{W}{O_R} & \text{if } (1-q(1-p_r))^{\frac{1}{a}} > \frac{O_R}{O_B + O_R} \\ 0 & \text{if } (1-q(1-p_r))^{\frac{1}{a}} \leq \frac{O_R}{O_B + O_R} \end{cases} \quad (4b)$$

when  $a \geq 1$  (we allow for risk-taking behaviour).

The first arrow in equation (4a) is the interior solution obtained in equation (1) on page 5, the second arrow is the corner solution where a negative amount cannot be bet on a color and the third arrow is the corner solution where the full amount affordable under the endowment

<sup>5</sup> All the probabilities were multiplies of one-eighth to avoid any misperception of the probabilities as in Hey, J.D. and C. Orme (1994).

$W/O_R$  is bet. Now we explain the first row in equation (4 b); when  $a > 1$  the IC's are concave, therefore the optimal solution is either not to bet at all or to bet all one's wealth,  $W/O_R$ . We compare the utility of not betting with the utility of betting all one's wealth and the latter is lower than the former whenever  $(1 - q(1 - p_r))^{1/a} > \frac{O_R}{O_R + O_B}$ .

**Proof.** We have assumed  $u(x) = x^a$  if  $a \geq 0$ , therefore when  $a \geq 1$  the utility function is assumed to be  $u(x) = x^a$ . The utility of not betting is the utility of obtaining  $W$  with certainty, which is  $W^a$ . The utility of betting the maximum possible on  $r$ ,  $SR = \frac{W}{O_R}$ , is

$$q(1 - p_r)u(0) + (1 - q(1 - p_r))u\left(W + O_B \frac{W}{O_R}\right); \quad (1 - q(1 - p_r))\left(W + O_B \frac{W}{O_R}\right)^a.$$

Therefore not betting is preferred to betting the maximum possible when

$$W^a \geq (1 - q(1 - p_r))\left(W + O_B \frac{W}{O_R}\right)^a, \text{ rearranging } (1 - q(1 - p_r))^{1/a} \geq \frac{O_R}{O_R + O_B}.$$

The proof is similar for the inequality in the second row of equation (4 b). In the case  $a = 1$  bettors' IC's are linear and therefore either they decide to bet all that is allowed with their wealth or they decide to not bet at all.

In the especial case of equal utility of betting all or nothing, we take the convention of assuming zero bets.

## 1.2. Stage 2: The betting market

Once the 80 subjects had made their decisions in stage 1, they participated in stage 2, in partner groups of 10 subjects each, where we replicated the betting system found in *Pelota* games. The experiment was computerized using z-Tree software. In each period the mechanism followed was similar to a double auction where subjects post their bids and offers, so there were no middlemen. The biggest difference from a double auction is that in our market subjects bid and offer not the price but the amount to be transacted. Once a subject has posted a bid or an offer, any other can accept it and it is precisely then that a transaction is realized. Subjects in this stage bet against one another. In each period subjects were allowed to either demand or offer any amount on  $r$  (or  $b$ ), within their endowments. They were not allowed to buy and sell in the same period, i.e. given a period they could either demand any amount on  $r$  (or  $b$ ) or offer any amount on  $r$  (or  $b$ ) but once they had bought (sold) a bet on  $r$  (or  $b$ ) they could only buy (sell) in that period. Three markets were run with different parametrizations. The probabilities for event  $r$  were  $\left\{\frac{4}{8}, \frac{5}{8}, \frac{7}{8}\right\}$  for markets  $\{1, 2, 3\}$  respectively. In each market subjects played different periods<sup>6</sup>, in which their endowments and the odds for event  $r$  were fixed ( $W_i = 10.000$  points,  $O_R = 100$  points). The odds on  $O_B$  in the first period of a market were approximately 30 per cent

<sup>6</sup> 4 in market 1, 8 in market 2 and 6 in market 3.

above the odds under which the expected value of a bet on  $r$  is zero. The odds decreased with each period to approximately 30 per cent below the odds of zero expected value.

At the end of the experiment three periods were randomly selected from the 18 played in all the markets (one period per market). The random device correspondent for each period was realized and subjects obtained, for each period, the endowment plus (minus) the amount earned (lost) due to their decisions.

Each period lasted an average of one and a half minutes, then stage two lasted 27 minutes.

### **1.3. Payment**

The total amount obtained in the experiment was the amount obtained in stage 1 plus that obtained in stage 2. Both payments were decided after all subjects' decision periods had finished. The participants were all students on a Microeconomics course and instead of money they were paid with extra points in their grades. The ECU<sup>7</sup> exchange rate was 60,000 points = 1 point added to their grade in a subject. Therefore a student that decided not to bet at all would have 10,000 ECU multiplied by 6 periods which adds up to 60,000 points, i.e., 1 point to be added to their final grade in Microeconomics, where 10 is the best grade.

We decided to pay via final grades in Microeconomics to induce risk aversion. In experiments with payoffs, low payments seem to induce risk neutrality. Nevertheless we aim to analyze the experimental data assuming that the endowment is the wealth level. This makes sense when paying with final grade points because the point we give subjects to bet is the only point they have (they have not yet sat their final exam). In order to check if we were right we ran exactly the same experiment but paying with cash. We found that when subjects are paid with money instead of final grade points they are less averse: more bets are placed. In the section "Identical RDEU bettors: estimated parameters", (p. 15), we show the estimated parameters. We obtain a significantly lower  $a$  value in the experiment paid with grade points than the one with money, which indicates a higher risk aversion (higher Arrow-Pratt aversion coefficient).

### **1.4. Questionnaire**

Subjects were asked about their comprehension of the experiment's rules and about the strategy they followed to decide how much to bet in a period. In general they had no problems in understanding the game and expressed willingness to participate in other experiments.

### **1.5. Instructions**

Two different instruction forms were given to subjects, one at the beginning of each stage. At the end of stage 1's instructions, they had to complete a test to check their understanding. They had plenty of time for this. They also had four practice periods in stage 1 and two in stage 2. The practice periods could not be selected as paid periods.

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<sup>7</sup> Experimental currency unit.

## 4. Results

### 1.1. EU does not explain individual behaviour when selecting prospects

Although we have shown theoretically that bets between **equal** individuals -when they are EU maximizers with decreasing marginal utility on wealth- are not possible, we want to give the EU model a chance, so we study behaviour individually in stage 1. Remember that in this first stage individuals decide how much to bet on a colour and we are forced to bet on the other color. Therefore we can study each individual's optimal betting decision and analyze attitudes toward risk individually. Allowing risk-taking behaviour it might happen that a number of risk-takers allow for the existence of bets when different individuals interact in a market. Therefore we studied individual attitudes towards risk of the bettors assuming that they were EU maximizers. The expected utility model is the special case of the RDEU model where  $\delta = \gamma = 1$ , if this is the case the probability weighting function is  $q(p) = p$ . Therefore we estimated parameter  $a$  independently for each of the 80 subjects making  $\delta = \gamma = 1$  and we found that  $a$  estimated was lower than 1 in 77 out of 80 people (there was only one risk neutral bettor, individual 34 with  $a = 1$ , and 2 risk takers, individual 4 with  $a = 1.5$  and individual 21 with  $a = 1.8$ ).

Therefore the data show us that if subjects are assumed to be expected utility maximizers, they are risk averse. Thus we can conclude that the expected utility theory is unsuccessful in explaining the existence of bets in *Pelota* markets even allowing for individually different risk attitudes.

### 1.2. Identical RDEU bettors: estimated parameters

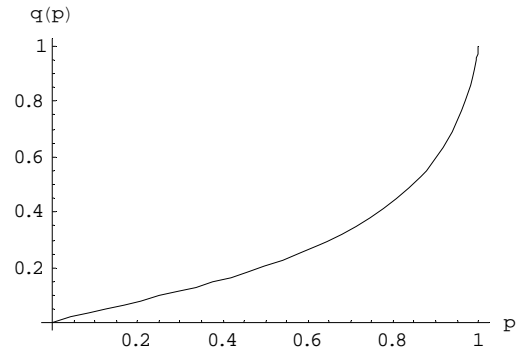
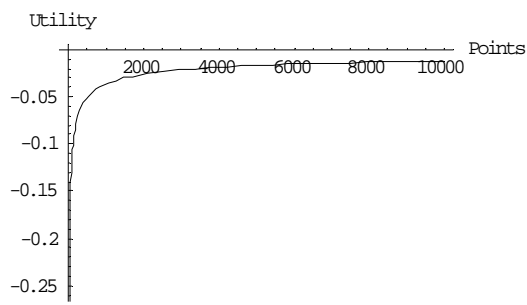
Having shown that there is no way for EU to explain the betting market, even allowing bettors to be different and with different risk-attitudes, we go on to study bettors' behavior under the RDEU model. Assuming all individuals are equal we estimate the utility function

$$u(x) = \begin{cases} x^a & \text{if } a \geq 0 \\ -x^a & \text{if } a < 0 \end{cases} \quad \text{and the probability weighting function } q(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$$

and we obtain, with an R square = 0.262, that  $a = -0.48$ ;  $\delta = 0.26$  and  $\gamma = 0.79$ . Each of the parameters obtained is lower than 1 with a 95% degree of confidence. Therefore we have found that the marginal utility of wealth is decreasing ( $a < 1$ ), bettors are optimistic ( $\delta < 1$ ) and bettors overestimate the worst outcome chances to win when the probability is low and underestimate it when it is high ( $\gamma < 1$ ).

Here we show the utility and probability weighting functions obtained.

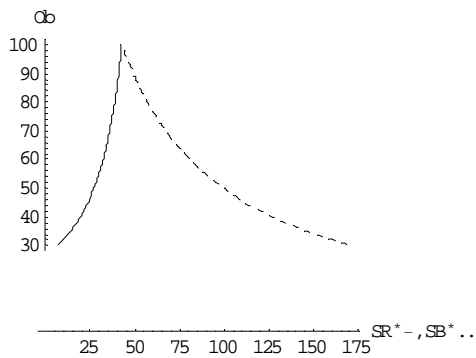




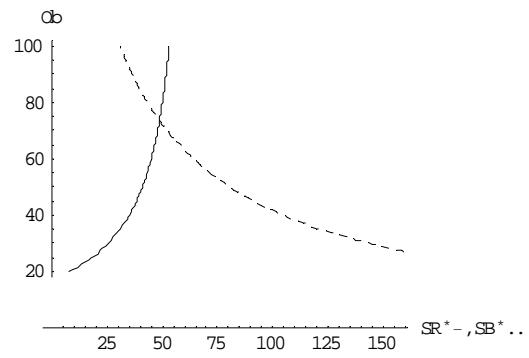
Once parameters are estimated we can obtain the demand function on  $b$ ,  $SB^*(O_B)$ , and the offer function on  $b$ ,  $SR^*(O_B)$  for the different markets as shown in the graphs below.

*Graphs: Demand and offer function on  $b$*

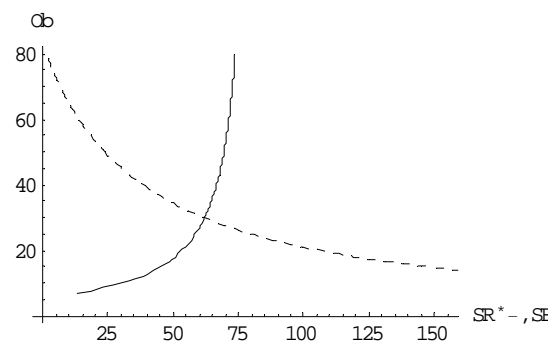
Pr = 0.5 (Market 1)



Pr = 0.625 (Market 2)



Pr = 0.875 (Market 3)



### 1.3. Paying with grade points induces risk aversion

When we pay subjects with money we obtain that, with an  $R^2 = 0.30248$ ,  $a = -0.12$ ;  $\delta = 0.32$  and  $\gamma = 0.84$ . The confidence intervals for the parameters in both experiments are shown below:

Table: Estimated parameters

| Monetary reward         |                 |                |               |
|-------------------------|-----------------|----------------|---------------|
|                         | $a$             | $\delta$       | $\gamma$      |
| Estimate                | -0,12           | 0,32           | 0,84          |
| 95% Confidence interval | [-0,259, 0,122] | [0,271, 0,374] | [0,782, 0,90] |

| Grade points reward     |                  |                |                |
|-------------------------|------------------|----------------|----------------|
|                         | $a$              | $\delta$       | $\gamma$       |
| Estimate                | -0,48            | 0,26           | 0,79           |
| 95% Confidence interval | [-0,702, -0,268] | [0,206, 0,321] | [0,722, 0,866] |

We conclude that parameter  $a$  differs significantly (at the 95% degree of confidence) from one treatment to the other.

We run the Lower-Tailed Mann Whitney Test for independent samples as a non-parametric test to check whether there is a significant difference in the optimum bet between an experiment with monetary reward and one with a grade point reward. For each experiment we have a sample of 36 mean decisions on betting, one for each period in stage 1. We run the Mann Whitney test to compare the distribution functions corresponding to optimal decisions on betting in each experiment. The null hypothesis is that the optimal bets are equal in the two experiments, while the alternative hypothesis is that optimal bets under grade point rewards are lower than the optimal bets under monetary rewards.

| Ranks   |             |    |           |              | Test Statistics(b)           |         |
|---------|-------------|----|-----------|--------------|------------------------------|---------|
|         | Experiment  | N  | Mean Rank | Sum of Ranks |                              | Average |
| Average | Grade point | 36 | 32,42     | 1167         | Mann-Whitney U               | 501,00  |
|         | money       | 36 | 70,58     | 1461         | Wilcoxon W                   | 1167,00 |
|         | Total       | 72 |           |              | Z                            | -1,656  |
|         |             |    |           |              | Asymp. Sig. (2-tailed)       | 1,000   |
|         |             |    |           |              | Exact Sig. [(1-tailed Sig.)] | 0,049   |

b Grouping Variable: average

We can conclude that paying with grade points induces risk aversion.

### 1.4. Predictions

First we calculate the predicted volume of bets in each period of stage 2 (Table: *Predicted volume of bets*) and then we compare them with the actual number of transactions.

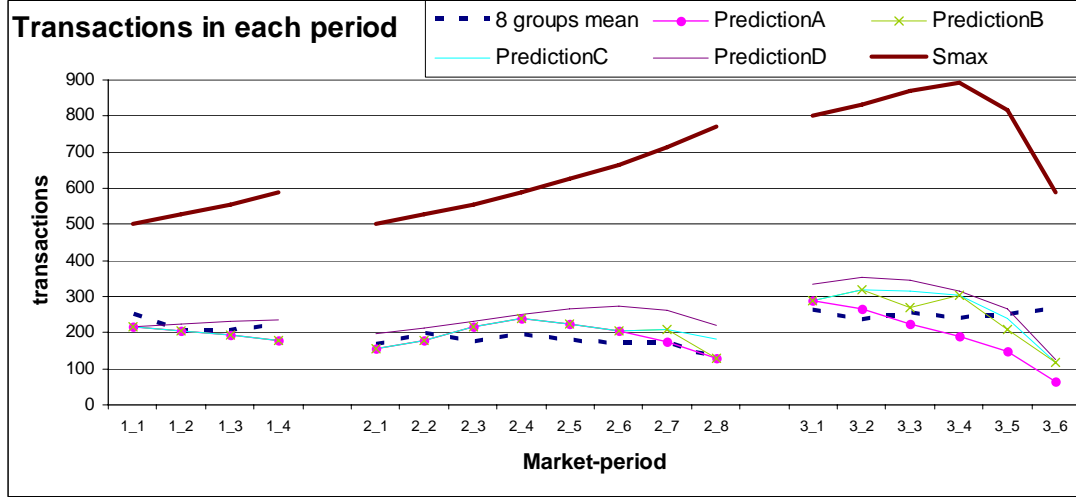
The following table shows the different predictions of volume of transactions in each period of stage 2 assuming that all bettors are identical, with the utility and probability weighting function obtained in stage 1.

Table: Predicted volume of bets.

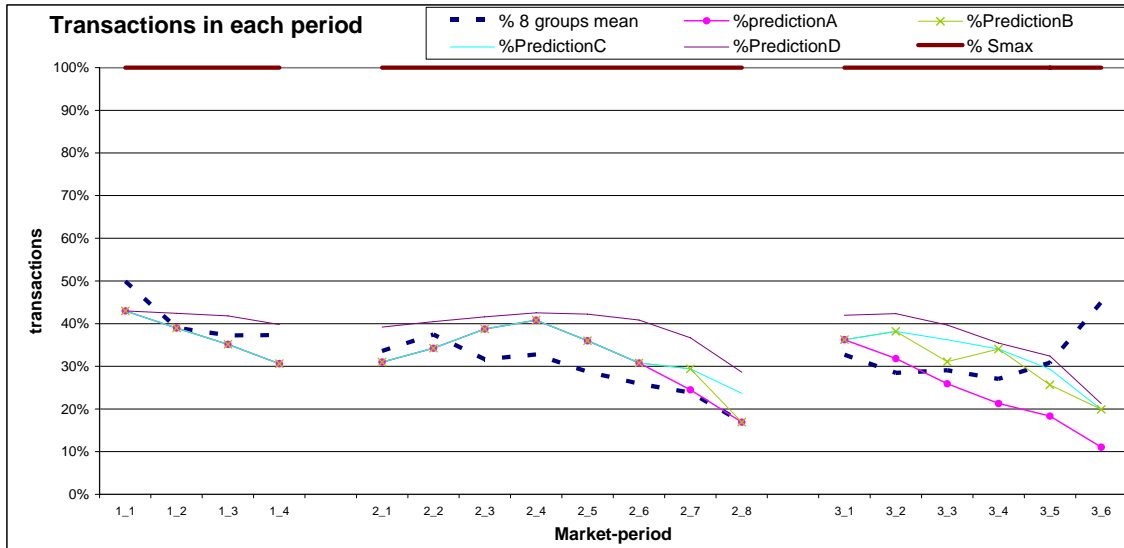
| Market 1: $p_r = 0,5$   |       |       |        |        |              |              |              |              |
|-------------------------|-------|-------|--------|--------|--------------|--------------|--------------|--------------|
|                         | $O_R$ | $O_B$ | $SR^*$ | $SB^*$ | Prediction A | Prediction B | Prediction C | Prediction D |
| Period 1                | 100   | 100   | 43     | 43     | 215          | 215          | 215          | 215          |
| Period 2                |       | 90    | 41     | 49     | 205          | 205          | 205          | 223          |
| Period 3                |       | 80    | 39     | 57     | 195          | 195          | 195          | 232          |
| Period 4                |       | 70    | 36     | 67     | 180          | 180          | 180          | 234          |
| Market 2: $p_r = 0,625$ |       |       |        |        |              |              |              |              |
|                         | $O_R$ | $O_B$ | $SR^*$ | $SB^*$ | Prediction A | Prediction B | Prediction C | Prediction D |
| Period 1                | 100   | 100   | 53     | 31     | 155          | 155          | 155          | 196          |
| Period 2                |       | 90    | 52     | 36     | 180          | 180          | 180          | 213          |
| Period 3                |       | 80    | 50     | 43     | 215          | 215          | 215          | 231          |
| Period 4                |       | 70    | 48     | 52     | 240          | 240          | 240          | 250          |
| Period 5                |       | 60    | 45     | 64     | 225          | 225          | 225          | 264          |
| Period 6                |       | 50    | 41     | 80     | 205          | 205          | 205          | 272          |
| Period 7                |       | 40    | 35     | 104    | 175          | 210          | 210          | 262          |
| Period 8                |       | 30    | 26     | 143    | 130          | 130          | 182          | 220          |
| Market 3: $p_r = 0,875$ |       |       |        |        |              |              |              |              |
|                         | $O_R$ | $O_B$ | $SR^*$ | $SB^*$ | Prediction A | Prediction B | Prediction C | Prediction D |
| Period 1                | 100   | 25    | 58     | 80     | 290          | 290          | 290          | 336          |
| Period 2                |       | 20    | 53     | 106    | 265          | 318          | 318          | 353          |
| Period 3                |       | 15    | 45     | 147    | 225          | 270          | 315          | 345          |
| Period 4                |       | 12    | 38     | 186    | 190          | 304          | 304          | 316          |
| Period 5                |       | 10    | 30     | 224    | 150          | 210          | 240          | 265          |
| Period 6                |       | 7     | 13     | 314    | 65           | 117          | 117          | 125          |

In all periods  $O_R = 100$  and  $W = 10.000$ , so given  $(p_r, O_B)$  and applying equations (1) and (2), (pp. 5-5), we calculate  $SR^*$  and  $SB^*$ , which are shown in columns 4 and 5. Once these individuals' optimal bet amounts on  $r$  and  $b$  are obtained, predictions of transactions can be obtained with the equations in "Market volume of bets" (p. 7).

In the following graph we compare predictions with the actual number of transactions. on the horizontal axis we have each period, we denote by  $i\_j$  period  $j$  in market  $i$ , and on the vertical axis we have the number of transactions. There are different series plotted on the graph: the dotted line is the mean number of transactions for the eight independent groups. The thicker line is the maximum number of transactions in the market. It is obtained as explained in "Maximum number of transactions" (p. 9). The other four series correspond to the different predictions defined in "Market volume of bets" (p.7). We see that in markets 1 and 2 the predictions are close to the actual transactions while in market 3 a decrease in volume of bets is predicted as  $O_B$  is lower, but in the experiment the number of bets is not so sensitive to changes in odds. Notice that here the  $O_B$  is very low and the variation from one period to another is also low.



To show at first sight the accuracy of the predictions compared to the actual number of transactions, the following graph plots the same data, this time expressed as a percentage of the maximum possible number of bets in the period.



We emphasize that prediction A is accurate in markets 1 and 2. In these markets it differs from actual decisions by at most 10% of the maximum possible bets in the period. But in market 3 as  $O_B$  is lower, the prediction shifts away from the actual volume of bets. It seems that when the odds are very low,  $O_B \in \{5, 7\}$ , the model loses prediction power. Prediction A is not a good prediction when the odds are so extreme because  $SR^* \ll SB^*$ , so predicting that a bettor on  $b$  will accept bets from one, and only one, bettor on  $r$ , is not accurate. Predictions B and C seem to predict better than A. But still in the last period, where  $O_B = 5$ , there is a gap between predictions and actual volume of bets. Prediction D is in general above the actual volume of transactions. This could be due to the assumption of more rationality in the sense that bettors post offers or bids different from their optimum in order to obtain more utility.

In what follows we analyze the difference between the actual and predicted volumes of bets. First of all, in the following table, we describe this variable showing its mean and standard deviation. Explicitly, this variable is obtained as the mean volume of bets of all 8 groups (as a percentage of the maximum possible volume of bets) minus the predicted volume of bets (also as a percentage of the maximum possible bets).

*Table: Actual volume of bets minus prediction.*

|                                      | Mean<br>(18 periods) | Standard<br>deviation |
|--------------------------------------|----------------------|-----------------------|
| Mean volume of bets* - Prediction A* | 2,34%                | 9,66%                 |
| Mean volume of bets* - Prediction B* | -0,18%               | 8,24%                 |
| Mean volume of bets* - Prediction C* | -1,05%               | 8,42%                 |
| Mean volume of bets* - Prediction D* | -5,83%               | 9,23%                 |

\*Variables as percentages of the maximum volume of bets

We emphasize that prediction B is the closest to the actual volume of bets, with a mean difference of 0.18% and the lowest standard deviation at 8.24%. It seems to be the best prediction. But to check whether there was a significant difference between prediction and actual volume of bets we ran four non-parametric tests (Wilcoxon Matched-Pairs Signed Rank tests), one for each prediction. The null hypothesis is that the median difference between actual and predicted volume of bets is zero. We find that we cannot reject the null hypothesis of equal median of bets for predictions A, B and C. However we find that the median volume of bets predicted with Prediction D is higher than the actual volume of bets. The test results follow.

#### Test Statistics(c)

|                        | Prediction A –<br>8 groups' mean | PPredictionB -<br>P8 groups<br>mean | PPredictionC -<br>P8 groups<br>mean | PPredictionD -<br>P8 groups<br>mean |
|------------------------|----------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| Z                      | -,457(a)                         | -,762(b)                            | -1,415(b)                           | -2,635(b)                           |
| Asymp. Sig. (2-tailed) | ,647                             | ,446                                | ,157                                | ,008                                |

a Based on positive ranks.

b Based on negative ranks.

c Wilcoxon Signed Ranks Test

We conclude that prediction B based on equal RDEU maximizers is a good predictor of the mean volume of bets in a *Pelota* betting system.

## 5. Summary and conclusion

We conclude that the expected utility theory is unsuccessful in explaining the existence of bets in *Pelota* markets even allowing for different individuals with different risk attitudes. Moreover, under equal RDEU maximizers we obtain accurate prediction of the volume of bets transacted

in the market. The estimated utility and probability weighting function are  $u(x) = -\frac{1}{x^{0.48}}$  and

$$q(p) = \frac{0.26p^{0.79}}{0.26p^{0.79} + (1-p)^{0.79}} .$$

We also find that paying with grade points in an experiment induces risk aversion.

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## Appendix

### Appendix 1: The general optimization problem

Bettor  $i$ 's optimal decision is given by

$$S^{**}(p_r, O_R, O_B, W) = \begin{cases} SR_i^* & \text{if } V(SR_i^*) \geq V(SB_i^*) \\ \text{or} \\ SB_i^* & \text{if } V(SB_i^*) \geq V(SR_i^*) \end{cases}$$

where  $SR_i^*$  that is obtained solving the problem

$$\max_{SR_i \in \left[0, \frac{W}{O_R}\right]} V(SR_i),$$

and  $SB_i^*$  is such that

$$\max_{SB_i \in \left[0, \frac{W}{O_B}\right]} V(SB_i).$$

Thus bettor  $i$  decides either to stake on  $r$ ,  $SR_i \in \left[0, \frac{W}{O_R}\right]$ , or to stake on  $b$ ,  $SB_i \in \left[0, \frac{W}{O_B}\right]$ . The

RDEU for each of the considered prospects is

$$V(SR_i) = q(1-p_r)U(W - O_R SR_i) + (1-q(1-p_r))U(W + O_B SR_i), \text{ and} \\ V(SB_i) = q(p_r)U(W - O_B SB_i) + (1-q(p_r))U(W + O_R SB_i).$$

Solving first order condition for the optimization,

$$\frac{\delta V(SR_i)}{\delta SR_i} = 0 \Rightarrow q(1-p_r)U'(W - O_R SR_i)(-O_R) + (1-q(1-p_r))U'(W + O_B SR_i)O_B = 0$$

and

$$\frac{\delta V(SB_i)}{\delta SB_i} = 0 \Rightarrow q(p_r)U'(W - O_B SB_i)(-O_B) + (1-q(p_r))U'(W + O_R SB_i)O_R = 0$$

We obtain that

$$SR_i^*(p_r, O_R, O_B, W) \text{ such that } \frac{(1-q(1-p_r))U'(W + O_B SR_i)}{q(1-p_r)U'(W - O_R SR_i)} = \frac{O_R}{O_B}, \text{ and} \\ SB_i^*(p_r, O_R, O_B, W) \text{ such that } \frac{q(p_r)U'(W - O_B SB_i)}{1-q(p_r)U'(W + O_R SB_i)} = \frac{O_R}{O_B}.$$

Second order condition for a maximization establishes that the following is true

$$\frac{\delta^2 V(SR_i)}{\delta SR_i^2} < 0 \Rightarrow q(1-p_r)U''(W - O_R SR_i)(-O_R)^2 + (1-q(1-p_r))U''(W + O_B SR_i)O_B^2 < 0 \text{ and} \\ \frac{\delta^2 V(SB_i)}{\delta SB_i^2} < 0 \Rightarrow q(p_r)U''(W - O_B SB_i)(-O_B)^2 + (1-q(p_r))U''(W + O_R SB_i)O_R^2 < 0. \text{ These}$$

conditions are satisfied when  $U''(x) < 0$ , i.e., when bettors' utility function is concave.



## Appendix 2: Equilibrium in the *frontón* betting market

We have already found that it is possible to find bets between identical RDEU maximizers when they are optimistic bettors with decreasing marginal utility on wealth. Now, we want to study which the behaviour will be in the market when there are  $N$  of such bettors each one with a wealth  $W$ . We will assume there are a large enough number of bettors ( $N \rightarrow \infty$ ). Thus in a period, where  $p_r$  is constant, each individual is informed about the odds fixed by the market and decides the amount to bet that maximizes his RDEU functional.

In a “competitive” market with given prices like this one, a necessary condition for the market to be in equilibrium is that bettor’s utility betting his optimum on the reds must be equal to his utility betting the optimum on the blues.

If this is not the case, without loss of generality imagine the RDEU of betting  $S_r^*$  is greater than the RDEU of betting  $S_b^*$ , then all bettors prefer to bet on  $r$ , thus there is a pressure for increase  $O_R$ , i.e., the money risked by the bettor who bets on  $r$ .

When utility obtained by bettors betting on the reds equals the utility of bettors betting on the blues, the number of bettors betting on reds and the number of bettor betting on blues will depend on which one is the optimal amount betted when betting on blues,  $S_b^*$ , and the optimal amount betted when betting on reds,  $S_r^*$ .

Call  $n_r$  ( $n_b$ ) the number of bettors betting on  $r$  ( $b$ ). Notice that the number of bettors betting on reds plus the ones betting on blues must be equal to the total number of bettors

$$n_r + n_b = N, \quad (1)$$

It is worthy to remember that in this betting system, in order to arrange a bet, there must be one bettor betting on one color and another betting on the other color, i.e., *there have to be the same number of bets on reds than on blues*. As the number of bets on one color will be the number of bettors on this color multiplied by each bettor’s optimal number of bets on this color, we automatically have that the following must be fulfilled,

$$n_r S_r^* = n_b S_b^*. \quad (2)$$

From equations (1) and (2) we obtain

$$n_i = \frac{S_j^*}{S_r^* + S_b^*} N, \quad \text{where } i, j \in \{r, b\} \text{ and } i \neq j. \quad ^8$$

---

<sup>8</sup> If (1) and (2) are true; from (1)  $n_r = N - n_b$ , substituting in (2)  $(N - n_b) S_r^* = n_b S_b^*$ ;  $N S_r^* - n_b S_r^* = n_b S_b^*$ ;  $\frac{N S_r^*}{S_b^* + S_r^*} = n_b$ . Replacing this in (1) we have  $n_r = N - \frac{N S_r^*}{(S_b^* + S_r^*)} = N \left( 1 - \frac{S_r^*}{(S_b^* + S_r^*)} \right) = N \frac{S_b^*}{(S_b^* + S_r^*)}$ .

Number of bettors in one color will be directly proportional to the optimal bets on the other color. When the optimal amount to bet on  $r$  is higher than the optimal amount to bet on  $b$ ,  $S_r^* > S_b^*$ , we need more bettors on  $b$  in order to balance the number of bets on  $r$  with number of bets on  $b$ .

Of course, in case the optimal amount of bets on  $r$  equals the optimal amount of bets on  $b$ ,  $S_r^* = S_b^*$ , there will be the same number of bettor on each color.

### Appendix 3: The estimated utility function

We estimated the following utility function  $u(x) = \begin{cases} x^a & \text{if } a \geq 0 \\ -x^a & \text{if } a < 0 \end{cases}$

We study this family of utility functions separately. First we study the case  $a \geq 0$ :

$u(x) = x^a$ ;  $u'(x) = ax^{a-1}$ ;  $u''(x) = a(a-1)x^{a-2}$ . We see that  $u''(x) < 0$  whenever  $0 < a < 1$ .

The second part can be written as  $u(x) = -\frac{1}{x^a}$  if  $a > 0$ . Deriving

$u'(x) = \frac{ax^{a-1}}{x^{2a}} = ax^{a-1-2a} = ax^{-a-1}$ ;  $u''(x) = a(-a-1)x^{-a-2}$ . We see that  $u''(x) < 0$  always.

## Appendix 4: Instructions

### Bienvenido al experimento

Estás participando en un experimento económico. El propósito de este experimento es el estudio científico de comportamiento de los individuos en mercados.

A partir de ahora está prohibida toda comunicación con otro participante que no sea la comunicación permitida a través del ordenador. Infringir esta regla supone la invalidez científica del experimento y la obligación de abandonar el mismo. Lee atentamente las instrucciones y si tienes cualquier duda, levanta la mano y cualquiera de nosotros acudirá a tu sitio y te responderá personalmente.

Con tu actuación puedes obtener puntos extras a la calificación final de la asignatura. Durante el experimento hablaremos de puntos que se convertirán en calificación extra al examen final de la asignatura según la siguiente relación:

60000 puntos del experimento = 1 punto a añadir en la calificación final.

**IMPORTANTE:** En ningún caso evaluaremos tus conocimientos sobre la asignatura Microeconomía IV. Decide sin miedo según tus criterios, NO necesitas aplicar ningún conocimiento teórico explicado en clase.

En estas instrucciones encontrarás información sobre las decisiones que vas a tomar y sobre las consecuencias de estas decisiones.

### El experimento

El experimento consta de dos partes diferenciadas, Fase 1 y Fase 2. Cada Fase consta de un número diferente de periodos. Un periodo de decisión es independiente del resto y en cada periodo debes decidir el número de apuestas a realizar por un color. A continuación se describen las características generales de todo el experimento. Después obtendrás instrucciones concretas para cada Fase.

#### **Características comunes de UN PERIODO de decisión (en ambas fases)**

Hay una urna con 8 bolas de dos colores diferentes, rojo y azul. Al final del experimento se extrae al azar una de entre las 8 bolas.

Realizar una apuesta por un color significa que ganas una cantidad de puntos si aciertas el color de la bola extraída pero pierdes otra cantidad de puntos si fallas en tu predicción (sale

una bola del otro color). Para que tu apuesta por un color pueda llevarse a cabo tiene que haber otra persona que la acepte, es decir, otra persona que apueste por el otro color.

Al principio de cada periodo tendrás 10000 puntos que te permitirán realizar apuestas. No te permitiremos realizar un número de apuestas tan elevado que puedas perder más puntos que los que tienes de modo que en ningún caso obtendrás puntos negativos en este experimento; por tanto, en cada periodo serás informado de este número *máximo* de apuestas que puedes realizar con los 10000 puntos de dotación.

### **Diferencias entre las dos Fases del experimento**

1ª Fase. En cada periodo TU decides el número de apuestas que quieres realizar por rojo y nosotros estamos obligados a aceptar la apuesta (apostamos por azul).

2ª Fase. Participas en un mercado junto a otros 9 participantes en este aula. En el mercado podrás *lanzar* apuestas por un color o por el otro pero estas apuestas sólo se llevarán a cabo si otro compañero decide *aceptar* la apuesta (apostando él por el otro color).

### **Puntos finales obtenidos en el experimento**

Al finalizar el experimento habrás realizado las decisiones correspondientes a la Fase 1 (36 decisiones independientes de cuanto apostar por rojo), y las decisiones correspondientes a la Fase 2 en la que la decisión de cuánto apostar también es independiente de un periodo a otro.

Una vez que todos hayáis introducido vuestras decisiones, al finalizar el experimento se seleccionarán los periodos que realmente se pagan; 3 periodos al azar de la Fase 1 y otros 3 de la Fase 2.

Una vez elegidos al azar los periodos a pagar, y *para cada periodo por separado*, se extraerá una bola de la correspondiente urna y se calcularán los puntos que hayas obtenido con las apuestas que hayas realizado en ese periodo.

El total de puntos que obtendrás será la suma de los calculados para cada uno de los 6 periodos seleccionados aleatoriamente.

## Instrucciones 1ª Fase del experimento

Esta fase consta de 36 periodos independientes.

Al principio de cada periodo obtienes 10000 puntos para apostar por ROJA. La información que recibes en la pantalla de tu ordenador es la siguiente:

Tu dotación de puntos 10000

Hay 8 bolas en una urna

|              |     |
|--------------|-----|
| Bolas rojas  | ... |
| Bolas azules | ... |

Si realizas UNA apuesta por roja

|                      |     |
|----------------------|-----|
| Si sale roja ganas   | ... |
| Si sale azul pierdes | ... |

Máximo número de apuestas que puedes realizar

Número de apuestas por ROJA

ver consecuencias

Confirmar

No te permitimos apostar por encima de lo que puedes pagar con tus 10000 puntos de dotación

Para introducir tu decisión de número de apuestas por ROJA que quieres realizar, la escribirás dentro del recuadro. A continuación deberás presionar **ver consecuencias** y se te mostrarán los puntos que obtienes con esa decisión dependiendo del color de la bola extraída (10000 puntos de dotación más o menos las ganancias o pérdidas). Una vez leída esta información estarás a tiempo para cambiar tu decisión, basta con que escribas un nuevo número de apuestas en el recuadro y vuelvas a presionar en "ver consecuencias". Para introducir tu decisión definitiva, y siempre tras presionar "ver consecuencias", presionarás **Confirmar**. Empezará un nuevo periodo en el que tendrás otros 10000 puntos de dotación para apostar. Cada periodo se distingue del resto por los valores que aparecerán en vez de los puntos suspensivos (...).

### Ejemplo 1

Tu dotación de puntos 10000

Hay 8 bolas en una urna

|              |   |
|--------------|---|
| Bolas rojas  | 3 |
| Bolas azules | 5 |

Si realizas UNA apuesta por roja

|                      |     |
|----------------------|-----|
| Si sale roja ganas   | 100 |
| Si sale azul pierdes | 30  |

Máximo número de apuestas que puedes realizar 333

Número de apuestas por ROJA

Si decides realizar 50 apuestas (este número podría haber sido cualquiera entre 0 y 333, es un ejemplo para que veas como calcular las ganancias o pérdidas):

Si sale roja ganas 100 puntos por cada una de las 50 apuestas lo que hace un total de 5000 puntos que añades a tu dotación inicial de 10000 puntos. Por tanto en este periodo obtienes un total de 15.000 puntos.

Si por el contrario sale azul pierdes 30 puntos por cada una de las 50 apuestas lo que hace un total de 1500 puntos que restas de tu dotación inicial de 10000 puntos. Por tanto en este periodo obtienes un total de 8.500 puntos.

### Ejemplo 2

Tu dotación de puntos 10000

Hay 8 bolas en una urna

|              |   |
|--------------|---|
| Bolas rojas  | 5 |
| Bolas azules | 3 |

Si realizas UNA apuesta por roja

|                      |     |
|----------------------|-----|
| Si sale roja ganas   | 30  |
| Si sale azul pierdes | 100 |

Máximo número de apuestas que puedes realizar 100

Número de apuestas por ROJA

Si decides realizar 50 apuestas:

Si sale roja ganas 30 puntos por cada una de las 50 apuestas lo que hace un total de 1500 puntos que añades a tu dotación inicial de 10000 puntos. Por tanto en este periodo obtienes un total de 11.500 puntos.

Si por el contrario sale azul pierdes 100 puntos por cada una de las 50 apuestas lo que hace un total de 5000 puntos que restas de tu dotación inicial de 10000 puntos. Por tanto en este periodo obtienes un total de 5.000 puntos.

Recuerda que cada periodo es independiente del resto. Cada vez tendrás 10000 nuevos puntos de dotación para apostar independientemente de las apuestas que hayas realizado en periodos anteriores.

### **Información adicional en la pantalla**

En la esquina superior izquierda de la pantalla podemos ver el periodo en el que nos encontramos. En la parte superior derecha vemos el tiempo que nos queda para responder en el actual periodo. En la parte superior central aparece una calculadora en un icono. Si pinchas con el ratón en este icono aparece una calculadora que puedes utilizar en caso de que lo necesites.

Una vez que todos los participantes hayáis respondido a las 36 preguntas se os facilitarán las instrucciones de la segunda parte del experimento.

## Test de comprensión

Si la información que tienes en la pantalla es

|   |   |
|---|---|
| Tu dotación de puntos 10000                   |   |
| Hay 8 bolas en una urna                       |   |
| Bolas rojas                                   | 6   |
| Bolas azules                                  | 2   |
| Si realizas UNA apuesta por roja              |   |
| Si sale roja ganas                            | 90  |
| Si sale azul pierdes                          | 100   |
| Máximo número de apuestas que puedes realizar | 100   |
| Número de apuestas por ROJA                   | <input type="text"/> <input type="button" value="ver consecuencias"/> |
| <input type="button" value="Confirmar"/>      |   |

responde a las siguientes preguntas.

Si decides 100 apuestas por roja:

- Si la bola extraída al final del experimento es roja,  

Ganarías la apuesta ]
Perderías la apuesta ]
- ¿Cuántos puntos ganarías por cada apuesta en el caso de que la bola extraída fuera roja?  

80 ]
90 ]
100 ]
- ¿Cuántos puntos ganarías por las 100 apuestas realizadas en el caso de que la bola extraída fuera roja?  

90 ]
9000 ]
10000 ]
- ¿Cuántos puntos obtendrías en total en este periodo si la bola extraída fuera roja?  

0 ]
20000 ]
19000 ]
- Si la bola extraída al final del experimento es azul,  

Ganarías la apuesta ]
Perderías la apuesta ]
- ¿Cuántos puntos perderías por cada apuesta en el caso de que la bola extraída fuera azul?  

80 ]
90 ]
100 ]
- ¿Cuántos puntos perderías por las 100 apuestas realizadas en el caso de que la bola extraída fuera azul?  

90 ]
9000 ]
10000 ]
- ¿Cuántos puntos obtendrías en total en este periodo si la bola extraída fuera azul?  

0 ]
1000 ]
19000 ]

Tienes 4 periodos de prueba para familiarizarte con la pantalla del ordenador. En estos 4 periodos las decisiones que introduzcas NO se tendrán en cuenta.



## Instrucciones 2ª Fase del experimento

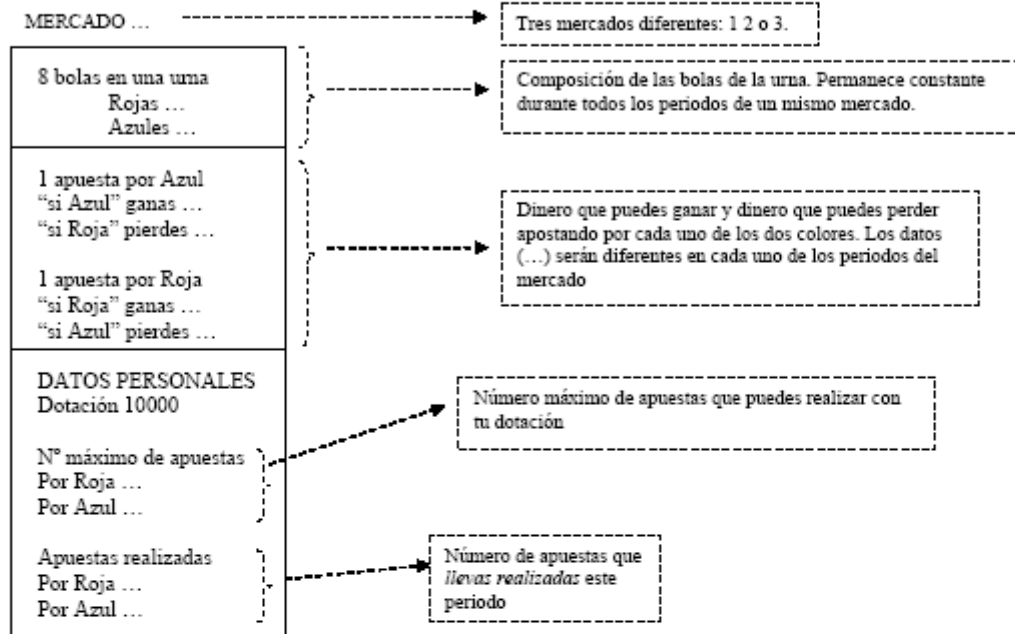
En esta Fase participarás en el mercado junto a otras 9 personas de esta sala. Tú y tus 9 compañeros vais a participar en tres mercados de apuestas.

Cada mercado consta de un número diferente de periodos independientes, 4, 8 y 6 respectivamente. En *cada periodo* de decisión verás la siguiente pantalla que explicaremos por partes a continuación:

### MERCADO ...

|  |                      |                      |  |                      |                      |
|--|----------------------|----------------------|--|----------------------|----------------------|
| 8 bolas en una urna<br>Rojas ...<br>Azules ...   |                      | Apuestas<br>por Azul |  | Apuestas<br>por Roja |                      |
| 1 apuesta por Azul<br>"si Azul" ganas ...<br>"si Roja" pierdes ...   |                      |                      |  |                      |                      |
| 1 apuesta por Roja<br>"si Roja" ganas ...<br>"si Azul" pierdes ...   | <input type="text"/> |                      |  |                      | <input type="text"/> |
| DATOS PERSONALES<br>Dotación 10000<br><br>Nº máximo de apuestas<br>Por Roja ...<br>Por Azul ...<br><br>Apuestas realizadas<br>Por Roja ...<br>Por Azul ... |                      |                      |  |                      |                      |
|  | Lanzar (Azul)        | Aceptar (Roja)       |  | Aceptar (Azul)       | Lanzar (Roja)        |

En la parte izquierda de la pantalla tienes la siguiente información:



En el resto de la pantalla aparecen los *datos del mercado* en el que tú puedes participar lanzando apuestas o aceptando apuestas que otros participantes hayan lanzado previamente al mercado. A continuación se muestra esta información:

#### Datos del mercado:

|                      | Apuestas por Azul |  | Apuestas por Roja |                      |
|----------------------|-------------------|--|-------------------|----------------------|
| Apuestas Azul        |                   |  |                   | Apuestas Roja        |
| <input type="text"/> |                   |  |                   | <input type="text"/> |
| Lanzar (Azul)        | Aceptar (Roja)    |  | Aceptar (Azul)    | Lanzar (Roja)        |

#### Como LANZAR apuestas al mercado

Si, por ejemplo, quieres lanzar al mercado 10 apuestas por azul debes pinchar con el ratón en "Apuestas Azul", escribir "10" y pinchar en el botón "Lanzar(Azul)". En paréntesis te recordamos el color por el que apuestas en caso de que te acepten la apuesta que vas a lanzar.

Si por el contrario quieres lanzar apuestas por Roja debes introducir el número de apuestas deseado en "Apuestas Roja" y apretar con el ratón en el botón debajo "Lanzar(Roja)". Entre paréntesis te recordamos el color por el que apuestas en caso de que te acepten la apuesta que vas a lanzar.

Recuerda que sólo realizarás las apuestas lanzadas si otro participante acepta la apuesta apostando él por el otro color.

### Como ACEPTAR apuestas en el mercado

Las apuestas que lances tanto tú como tus compañeros aparecerán en las columnas centrales del cuadro anterior que reproducimos a continuación:

| Apuestas por Azul |  | Apuestas por Roja |
|-------------------|--|-------------------|
| 100               |  | 90                |
| 60                |  | 80                |
| 40                |  | 50                |
| Aceptar (Roja)    |  | Aceptar (Azul)    |

En la columna "Apuestas por Azul" aparecen los números de apuestas que tanto tú como el resto de participantes hayáis lanzado por Azul. Si quieres aceptar una de estas apuestas apostando tú por el otro color (Roja), primero debes seleccionarla pinchando con el ratón encima del número de apuestas que quieres aceptar, este número quedará marcado. Seguidamente deberás apretar el botón "Aceptar(Roja)". Entre paréntesis te recordamos el color por el que tú apuestas.

De manera similar, en la columna "Apuestas por Roja" aparecen los números de apuestas que tanto tú como el resto de participantes hayáis lanzado por Roja. Si quieres aceptar una de estas apuestas apostando tú por el otro color (Azul), primero debes seleccionarla pinchando con el ratón encima del número de apuestas que quieres aceptar, este número quedará marcado. Seguidamente deberás apretar el botón "Aceptar(Azul)". Entre paréntesis te recordamos el color por el que tú apuestas.

Una vez que alguien acepte una apuesta lanzada el número de apuestas realizadas aparecerá en la columna central.

Puedes lanzar apuestas tantas veces como quieras, pero recuerda que una vez te han aceptado una apuesta lanzada, el resto de tus lanzamientos desaparecerán automáticamente del mercado con lo que, si quieres, puedes volver a lanzar apuestas por el mismo color por el que ya has apostado con los puntos que te quedan para apostar. En un mismo periodo, una vez que hayas

realizado una apuesta por un color, el resto de apuestas durante el mismo periodo (y hasta que no cambiemos de pantalla) han de ser por el mismo color.

No te permitiremos perder más puntos que los 10000 que tienes de dotación en cada periodo.

### **Información del periodo**

Cada periodo dura 1 minuto. En cada momento podrás leer en la parte superior derecha de la pantalla el tiempo restante. Si cuando se acerque el final del periodo todavía hay movimiento en el mercado el reloj se detendrá con lo que el tiempo se alargará.

Terminado un periodo, automáticamente aparecerá una pantalla donde obtendrás información sobre el periodo que ha finalizado. En esta pantalla podrás ver el total de apuestas que has realizado por cada color y el total de puntos, incluida la dotación, que, dependiendo del color de la bola extraída, obtendrás en este periodo en caso de ser seleccionado como periodo de pago.

Tienes 2 periodos de prueba para familiarizarte con la entrada de decisiones. Estas decisiones NO se tendrán en cuenta así que lanza y acepta apuestas sin miedo para aprender el funcionamiento de la entrada de datos en el mercado.

Appendix 5: Screens.

Screen: Stage 1; Period 1; Screen 1.

Periodo1 de 36Tiempo restante3

Tu dotacion de puntos10000

Hay 8 bolas en una urna

Bolas rojas5Bolas azules3

Si realizas UNA apuesta por roja

Si sale roja ganas60Si sale azul pierdes100

Máximo número de apuestas que puedes realizar100

Número de apuestas por ROJA10

ver consecuencias

Posibles puntos totales de este periodo:

Si sale bola roja ( 5 de 8):10600Si sale bola azul ( 3 de 8):9000

Tu decisión10

Confirmar

Periodo

1 de 8

Tiempo restante [seg]: 32

MERCADO 2

8 bolas en una urna:  
Rojas 5  
Azules 3

1 apuesta por Azul:  
"Si azul" ganas 100  
"Si roja" pierdes 100

1 apuesta por Roja:  
"Si roja" ganas 100  
"Si azul" pierdes 100

DATOS PERSONALES  
Dotación 10000  
Número máximo de apuestas:  
Por Roja 100  
Por Azul 100  
Apuestas realizadas:  
Por Roja 10  
Por Azul 0

Apuestas por Azul

Apuestas realizadas 10

Apuestas por Roja 50

Apuestas Rojo

Apuestas Azul

Lanzar (Azul)

Aceptar (Roja)

Aceptar (Azul)

Lanzar (Roja)

Periodo

1 de 8

continuar

RESUMEN DE TUS APUESTAS EN ESTE PERIODO

Número total de apuestas por Roja

10

Número total de apuestas por Azul

0

Total puntos si Roja

11000

Total puntos si Azul

9000



Screen: *Payment.*

PANTALLA INFORMATIVA SOBRE PUNTOS OBTENIDOS

FASE 1

| Periodos seleccionados | Tus apuestas por ROJA | Color bola extraida | Tus Ganancias |
|------------------------|-----------------------|---------------------|---------------|
| Periodo 22 :           | 12                    | <div>VER</div> AZUL | 9040          |
| Periodo 19 :           | 85                    | <div>VER</div> AZUL | 1500          |
| Periodo 29 :           | 20                    | <div>VER</div> ROJA | 11000         |

FASE 2

| Periodo seleccionado en cada mercado | Tus apuestas<br>Por ROJA      Por AZUL | Color bola extraida | Tus Ganancias |
|--------------------------------------|--|---------------------|---------------|
| Mercado 1 - Periodo 2 :              | 8      0                               | <div>VER</div> AZUL | 9200          |
| Mercado 2 - Periodo 4 :              | 50      0                              | <div>VER</div> ROJA | 13500         |
| Mercado 3 - Periodo 1 :              | 0      58                              | <div>VER</div> ROJA | 8550          |

TOTAL PUNTOS OBTENIDOS EN EL EXPERIMENTO

Sumar

52790

OK