# A PROFITABLE STRATEGY IN THE PELOTA BETTING MARKET 

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D.T.2006/06

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This version: August 9, 2006
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#### Abstract

In Pelota matches, games with two mutually exclusive and exhaustive outcomes, bets on the winner are made between viewers through a middleman who receives $16 \%$ of the finally paid amount. In this paper after the description of the way bets are made in the market we analyze what we call the general odds rule. Analyzing the way odds are fixed in the market we find that assuming equal return on bets there are biases in the market. Moreover, we find profitable betting strategies even taking commissions into account.


Keywords: Betting, Pelota betting system, sport betting, market efficiency.

[^0]
## 1 Introduction

We fill a gap in the literature by studying the very attractive betting system used in Pelota games in Navarra, the Basque Country and La Rioja. In what follows we call this the Pelota betting system. Before presenting the main work we would like to give a brief introduction about the Pelota games and the betting system.

Most authors who have studied the origins of the Basque sport of pelota believe that it is linked to the Medieval hand-ball game of jeu de paume or "real tennis". The history of this game was passed down orally, so the first written references to it do not appear until relatively late. In the 18th century the game began to die out in France, Italy and England, but it grew more popular in the Basque Country, where increased economic prosperity led to the building of more courts. ${ }^{1}$

Although there is evidence of betting on ball games in ancient times, there is no evidence of what type of wagers were made. We do not know when the betting system currently used on the pelota courts of the Basque Country first came into being, but we can say that no evidence has been found of the system outside this region. The rules of the system were traditionally not written down but passed on orally, which makes it harder to study. We have found a similar betting system that has been recently implemented on the Internet. The betting system followed in Betfair is similar to the Pelota betting system in that bettors can make as many bets as they want to provided that there is another bettor on the other side, and the market maker takes a percentage of the money as a commission. The main difference with the Pelota betting system is the odds scale.

In America a different kind of game known as Jai-Alai or Cesta-Punta can also be seen. Jai-alai originated from the version of Pelota called Cesta Punta but the sport is now different. Bets are also made in Jai-Alai but the betting system in America is totally different from the system described here.

Here we study the efficiency in this market. We take advantage of the peculiarities of the betting system to make some assumptions that allow us to analyze what we will call the general odds rule analyse. Analyzing the way odds are determined in this market we will find inefficiencies in the sense that profitable betting strategies are found. In Section 2 we describe the game and the betting system, a complete description can be found in Llorente, L. and J.M. Aizpurua (2006) where a theoretical explanation of the existence of bets in the market is shown. In Section 3 we take advantage of the peculiarities of the betting system to make some assumptions that allow us to analyze what we will call the general odds rule. Analyzing the way odds are determined in this market, we find inefficiencies in the sense that profitable betting strategies are found. Finally, we present some conclusions in Section 4.

[^1]
## 2 A description of the Pelota betting system

All Pelota ("Pelota Vasca") matches are played by two teams: reds $(R)$ and blues (B) play against each other by hitting a ball in turn against a wall on a court called a frontón. The team that serves first is chosen by throwing a coin. When a team makes an error the opponent scores one point and serves to start the next point. The team that reaches a pre-set number of points wins the match.
On these matches a bet is described by two quantities that inform on the odds: a quantity of money the bettor loses when he fails to predict the winning team and an amount of money the bettor wins if he guesses right. Bets can be made during the whole match, so while points are scored, odds change. A bettor can place as many bets as he wants to, provided someone can be found to accept those bets .
This betting system is popular in the Basque Country, Navarra and La Rioja, where several types of Pelota game are played. There are slight differences between the different types. In what follows we will study the betting system in a particular game called remonte. The rules of the betting system are not written, thus all our explanations are based on information obtained at the frontón. More specifically, our study is based in the information collected by at the Euskal Jai Berri, a frontón where the Pelota game played is remonte. This frontón was chosen because it has screens on which one can see the odds at which one can bet at all times during the game, a peculiarity that is very helpful for obtaining field data.

### 2.1 A brief description of REMONTE

Remonte is a type of Pelota. It is played on a frontón which consists of a playing court limited by three walls at the front, the left and the back. The frontón is about 54 meters long, 12 meters wide and 11 meters high.
In this game reds and blues hit a ball with a wicker scoop attached to the players' hands called the cesta. The teams usually have two players each but may occasionally have one or three. Each team has to hit the ball in turn, starting with the team chosen by the field judge tossing a coin. When one player fails to hit correctly the opponents score one point and serve the ball in order to start the next point. The first team to reach 40 points wins the match.
To play a point, each team has two possibilities. The first and most common one is to hit the ball against the front wall so that the ball bounces on the floor inside the limits marked. The second one is to hit the ball against the left wall so that it rebounds against the front wall and then bounces on the floor inside the court. Once the ball hits the front wall, the other team is allowed to hit the ball either before or after it touches the floor (only once) or even after it bounces off the back wall.
Each game usually lasts around one hour.

Picture 1: The reds and the blues playing a Pelota game on a frontón.


### 2.2 A brief description of the betting market in REMONTE

Throughout the game, you are allowed to place as many bets as you want. In each bet you chose either the red team or the blue and wager an amount of money against another spectator that chooses the other team and wagers another amount of money (these amounts will be called the odds). If you guess right you win the money that your opponent loses.

For example, if you bet 100 euros on the reds against an opponent who bets 100 on the blues and the blues win the match, you pay your opponent 100 euros.
But all bets are placed through a middleman who works for the organisers and takes $16 \%$ of the winnings of the successful bettor, so in the previous example you pay your opponent 100 euros but he only receives 84 , because the middleman takes 16.

Throughout the Remonte match a screen (see Table 1) shows the effective odds in the market and the current score .

Table 1.The Screen

$O_{R}$ is the amount a bettor risks by betting on reds.
$O_{B}$ is the amount a bettor risks by betting on blues.
$\mathrm{s}_{\mathrm{r}}\left(\mathrm{s}_{\mathrm{b}}\right)$ is the red (blue) team's score.

The odds consist of two numbers, with the bigger one always being 100 euros and the smaller one varying between 2 and 100 as points are played. Generally the smaller odd is one of the set $\{2,3,4,5,6,8,10,12,15,20,25,30,40,50,60,70,80,90,100\}$.

Table 2. An example

|  | odds |  |
| :---: | :---: | :---: |
| (reds) | score |  |
| (blues) | 100 | 5 |
|  | 80 | 2 |
|  |  |  |

In the example in Table 2 red team has scored 5 and blue team 2. The odds are 100 to 80 , denoted by $(100,80)$. The bettor who bets on reds risks 100 euro to win 80 and the one who bets on blues risks 80 euro to win 100. From now on we will follow this convention in describing the various bets. Bets are always between spectators, so if one spectator places a bet on red there must be another who bets on blue. What happens when the game is over?
bet on reds A bettor on reds will loose 100 euros if blues win the match. Otherwise, if reds win the match the bettor will win 80 euros minus the 16 percent commission, i.e. 67.2 euros.
bet on blues A bettor on blues will loose 80 euros if reds win the match. Otherwise, if blue wins the match the bettor will win 100 euros minus 16 per cent, i.e. 84 euros.

This describes a single bet. A bettor can make as many bets as he wants to, provided that there is someone on the other side who will take his bets. For example, with the same screen as in Table 2, if a bettor places ten bets on the reds he will loose $100 * 10=1000$ euros if blues win the match. Otherwise, if reds win the match the bettor will win $67.2^{*} 10=670$ euros.

Moreover, a bettor can bet at different times during the game, choosing one team in one period and the other team in another. Therefore the result above is concerning the particular bet analyzed here.

### 2.3 The way the odds are fixed in the market

In remonte there is an "auctioneer" who posts the odds that appear on the screen. We call him the coordinator. He is usually someone who has been a player and a middleman for many years. This man is an expert on remonte, possibly the man who knows most about the game at the frontón. He chooses the hand-made balls used by players to play the match, and he posts the odds at which people bet. He sits in front of a computer, in a privileged place behind the spectators, where he can follow the match and see the spectators and all the middlemen. He posts the odds that appear on the screens. When a team scores a point it is closer to winning, and is thus more likely to win the match than before scoring. Thus the money a bettor risks betting on that team should be higher to maintain the expected value of the bet constant.

There are some general rules that the coordinator follows to set and change the odds:

If there is no reason to think of either team as the favourite just before the match starts, the score is zero-zero and the odds $\left(O_{R}, O_{B}\right)$ are $(100,100)$. If there is a favourite team, the odds may be different. For example, if red is favourite, with the same score we could have odds ( $O_{R}, O_{B}$ ) of (100, 80).

Once it is clear what the initial odds are, the match goes on and points are scored. The general rule is that the difference between the amounts of the odds increases by 10 euros on the team that has just scored a point, keeping in mind that the larger amount in the odds is always 100.

If the odds differ by more than $70(100,30)$, they change by only 5 euros for each point. If they differ by more than 90 , the odds change by only 2 euros for each point. When one team has accumulated approximately 30 points, the change in the odds doubles for each point then scored, and trebles or quadruples when the end of the match is very near. For example if the score is $(38,39)$ the odds would be $(40,100)$.

Of course, sometimes these rules are modified because of changes in supply and demand among spectators. When a middleman finds two people who want to bet at odds different from those on the screen, the middleman has to ask the coordinator to change the odds on the screen so that he can print the receipts for the bettors. There are no bets on the frontón at odds different from those on the screens. In general the odds vary mainly as the above rules indicate.
It is important to realize that the coordinator works for the firm that organizes Pelota matches, so his goal could be described as making people bet as much as possible. So we can confidently assume that the odds he posts are those at which people are willing to bet the most, i.e. the equilibrium odds where there is no excess of demand of bets .

## 3 The general odds rule

In this section we take advantage of the peculiarities of the betting system to make some assumptions that allow us to analyze what we will call the general odds rule.

We were told by the coordinator in the market that games are arranged in such a way that the chances of winning for each team are as similar as possible. From this peculiarity we assume that, in general, the probability of scoring the following point by each team is 50:50. As we will see, given the score this assumption allows us to obtain each team's theoretical probability of winning; see the following subsection 3.1.
The coordinator also told us that he follows a general rule to set odds in the market so long as nothing atypical happens (where "atypical" means that there is an excess of people willing to bet on one color). Odds differ from those of the general rule if for example it can be seen that a player has lost his ability or something happens that causes more bettors to be willing to bet on one of the teams. From that we could infer that odds in the general rule are the equilibrium odds
(there is no excess of willingness to bet on one of the two teams) when players have the same probability to score the following point. Therefore we will study the general odds rule as the equilibrium price in the market when the probability of scoreing the following point by each team is $50: 50$. From these market odds, and under some assumptions that will be explained in the subsection "Teams' probabilities of winning the match inferred from market odds", we can obtain the probabilities inferred from market odds.

We will compare the theoretical probability with the probability inferred from market odds to check if there are biases in the market.

### 3.1 Teams' theoretical probabilities of winning the match

Assuming that the likelihood of a team scoring the following point is the same throughout the match, the theoretical probability of the team's winning the match can be derived at any time. With no loss of generality we perform such a calculation for the reds. Given the reds' score, $s_{r}$, the blues' score, $s_{b}$, and the reds' probability of scoring a point at any moment during the match, $p$, the reds' probability of winning the match, $p_{r}$, is given by

$$
\begin{equation*}
p_{r}\left(s_{r}, s_{b}, p\right)=\sum_{i=0}^{40-s_{b}-1}\binom{i+40-s_{r}-1}{i} p^{40-s_{r}}(1-p)^{i} . \tag{1}
\end{equation*}
$$

This equation is obtained by the addition of as many amounts as there are different possible final scores where the reds are the winners ( $i$ is the number of points the blues could score, from 0 to 39$s_{b}$ ). For each of these possible final scores, the amount added is obtained by multiplying the probability of this final score by the number of different ways in which it can be reached.

In Pelota games the probability of each team scoring the following point and the probability of wining the match are unknown but we know that matches are arranged so that the chances of winning for each team are as similar as possible according to the subjective perception of the organizer of the match: when one team is superior the odds are shortened by using match-balls that favour the worse team and so on. When one team is superior the other may even be allowed to use one more player. Thus, for the sake of simplicity, we set the reds’ probability of scoring the following point at $p=0.5$. This in conjunction with the equation above, (1), allows us to calculate, for a given score, the reds' probability of wining the match, $p_{r}$,

$$
p_{r}\left(s_{r}, s_{b}\right)=\sum_{i=0}^{39-s_{b}}\binom{i+39-s_{r}}{i} 0.5^{40-s_{r}} 0.5^{i} .
$$

We call this the reds' theoretical probability of winning the match. These theoretical probabilities for some possible scores are shown below.

Table 1: "The reds' theoretical probabilities of winning the match given the current score."

| $\mathbf{( s r , s b})$ | Theoretical $\boldsymbol{p}_{\boldsymbol{r}}$ |
| :---: | :---: |
| $(1,0)$ | 0,545 |
| $(2,0)$ | 0,5901 |
| $(3,0)$ | 0,6345 |
| $(4,0)$ | 0,6778 |
| $(5,0)$ | 0,7193 |
| $(6,0)$ | 0,7586 |
| $(7,0)$ | 0,7952 |
| $(8,0)$ | 0,8288 |
| $(9,0)$ | 0,859 |
| $(10,0)$ | 0,8858 |
| $(11,0)$ | 0,9091 |
| $(12,0)$ | 0,929 |
| $(13,0)$ | 0,9456 |
| $(14,0)$ | 0,9592 |
| $(15,0)$ | 0,97 |
| $(16,0)$ | 0,9785 |
| $(17,0)$ | 0,985 |

### 3.2 Teams' probabilities of winning the match inferred from market odds

In this subsection we try to analyze the market with orthodox methods, here in particular we analyze the general odds rule followed in the Pelota betting system under certain assumptions that may be somewhat strong, but are made in studies of other wagering markets. Thus we start by assuming that bettors are expected value maximizers. A condition for equilibrium is that the expected value of a bet on the reds should be equal to the expected value of a bet on the blues, because if not all bettors prefer to bet on the colour with the higher return. In the section "Efficiency of the general odds rule assuming equal return on bets" we study the probability inferred from the market assuming equal returns on each bet and we obtain that there is a difference between the probability inferred from market odds and the theoretical probability of a team winning the match. Low probabilities are overestimated while high probabilities are underestimated.
In these markets there are commissions, so the equilibrium condition of equal return of bets implies that each bet has a negative expected return. Therefore it seems more convenient to introduce the less restrictive restriction of not allowing profitable bets in the market. This is done in "Efficiency of the general odds rule assuming no profitable bets" where we check what the probabilities inferred from market odds must be to satisfy this less restrictive restriction of no profitable bets, i.e. to satisfy the condition of expected value of a bet lower than or at most equal to zero. Comparing this probability inferred from market odds with the actual probabilities we find that in these markets there are profitable betting strategies.

### 3.2.1 The general odds rule

As already mentioned, for each score we can derive the odds in the market by applying the general odds rule. The table below shows some scores and their corresponding market odds .

Table 2:"General odds rule."

| Score <br> $(\mathbf{s r}, \mathbf{s b})$ | Odds <br> $(\mathbf{O R}, \mathbf{O B})$ |
| :---: | :---: |
| $(1,0)$ | $(100,90)$ |
| $(2,0)$ | $(100,80)$ |
| $(3,0)$ | $(100,70)$ |
| $(4,0)$ | $(100,60)$ |
| $(5,0)$ | $(100,50)$ |
| $(6,0)$ | $(100,40)$ |
| $(7,0)$ | $(100,30)$ |
| $(8,0)$ | $(100,25)$ |
| $(9,0)$ | $(100,20)$ |
| $(10,0)$ | $(100,15)$ |
| $(11,0)$ | $(100,10)$ |
| $(12,0)$ | $(100,8)$ |
| $(13,0)$ | $(100,6)$ |
| $(14,0)$ | $(100,4)$ |
| $(15,0)$ | $(100,2)$ |
| $(16,0)$ | $(100,2)$ |
| $(17,0)$ | $(100,2)$ |

### 3.2.2 Efficiency of the general odds rule assuming equal return on bets

Now that we know the odds in the market for each score, we can derive the probabilities inferred from these odds as follows. If the market is efficient, where "efficiency" means that the expected returns are equal on the various bets (see the different meanings of "efficiency" in Sauer, R.D. (1998), p. 2024), then equation (2) should happen.

Proposition 1 A Pelota betting market has equal returns on bets if equation (2) is satisfied.

$$
\begin{equation*}
\frac{p_{r}}{\left(1-p_{r}\right)}=\frac{O_{R}}{O_{B}} \tag{2}
\end{equation*}
$$

where $p_{r}$ is the likelihood of the reds winning the match, $O_{R}$ is the money risked in a bet on the reds and $O_{B}$ is the money risked in a bet on the blues.

Proof. Denote by $V E_{R}$ the expected value of a bet on the reds, and $V E_{B}$ the expected value of a bet on the blues. We know that $V E_{R}=p_{r} O_{B}(1-t)-\left(1-p_{r}\right) O_{R}$, and $V E_{B}=\left(1-p_{r}\right) O_{R}(1-t)-p_{r}$ $O_{B}$ where $t$ is the middlemen's commission. If the market is efficient the two expected values should be equal, thus $p_{r} O_{B}(1-t)-\left(1-p_{r}\right) O_{R}=\left(1-p_{r}\right) O_{R}(1-t)-p_{r} O_{B}$, and by operating we obtain $\frac{\boldsymbol{p}_{\boldsymbol{r}}}{\left(\mathbf{1}-\boldsymbol{p}_{\mathbf{r}}\right)}=\frac{\left(\mathbf{2 - t )} \boldsymbol{O}_{\boldsymbol{R}}\right.}{(2-\boldsymbol{t}) \boldsymbol{O}_{\boldsymbol{B}}}$, which proves that condition (2) is true no matter what the commission is.

Rearranging equation (2); $p_{r}=\frac{O_{R}}{O_{B}}\left(1-p_{r}\right) ; p_{r}\left(1+\frac{O_{R}}{O_{B}}\right)=\frac{O_{R}}{O_{B}} ; p_{r}=\frac{O_{R} / O_{B}}{\left(\frac{O_{B}+O_{R}}{O_{B}}\right)}$; therefore

$$
p_{r}=\frac{O_{R}}{O_{B}+O_{R}} \cdot \text { (3) }
$$

The probability inferred from the odds is obtained by equation (3). These probabilities are shown in Table 3 below.

Table 3: "The probability of the reds winning the match inferred from market odds assuming equal return of bets. It is obtained by equation (3)."

| Odds <br> (OR,OB) | $\boldsymbol{p}_{\mathbf{r}}$ derived from <br> market odds $=\boldsymbol{\pi}_{\mathbf{r}}$ |
| :---: | :---: |
| $(100,90)$ | 0,5263 |
| $(100,80)$ | 0,5556 |
| $(100,70)$ | 0,5882 |
| $(100,60)$ | 0,625 |
| $(100,50)$ | 0,6667 |
| $(100,40)$ | 0,7143 |
| $(100,30)$ | 0,7692 |
| $(100,25)$ | 0,8 |
| $(100,20)$ | 0,8333 |
| $(100,15)$ | 0,8696 |
| $(100,10)$ | 0,9091 |
| $(100,8)$ | 0,9259 |
| $(100,6)$ | 0,9434 |
| $(100,4)$ | 0,9615 |
| $(100,2)$ | 0,9804 |
| $(100,2)$ | 0,9804 |
| $(100,2)$ | 0,9804 |

In Table 4 below we put together four columns: first of all the score, secondly the market odds for each score applying the general odds rule, thirdly the reds' theoretical probability of winning the match obtained for each score, and finally the reds’ probability of winning the match obtained from the market odds by applying equation (3).

Table 4: "The probability of the reds winning; theoretical and derived from market odds."

| Score <br> (sr,sb) | Odds <br> $($ OR,OB $)$ | Theoretical <br> $\boldsymbol{p}_{\boldsymbol{r}}$ | $\boldsymbol{p}_{\boldsymbol{r}}$ derived from <br> market odds |
| :---: | :---: | :---: | :---: |
| $(1,0)$ | $(100,90)$ | 0,545 | 0,5263 |
| $(2,0)$ | $(100,80)$ | 0,5901 | 0,5556 |
| $(3,0)$ | $(100,70)$ | 0,6345 | 0,5882 |
| $(4,0)$ | $(100,60)$ | 0,6778 | 0,625 |
| $(5,0)$ | $(100,50)$ | 0,7193 | 0,6667 |
| $(6,0)$ | $(100,40)$ | 0,7586 | 0,7143 |
| $(7,0)$ | $(100,30)$ | 0,7952 | 0,7692 |
| $(8,0)$ | $(100,25)$ | 0,8288 | 0,8 |
| $(9,0)$ | $(100,20)$ | 0,859 | 0,8333 |
| $(10,0)$ | $(100,15)$ | 0,8858 | 0,8696 |
| $(11,0)$ | $(100,10)$ | 0,9091 | 0,9091 |
| $(12,0)$ | $(100,8)$ | 0,929 | 0,9259 |
| $(13,0)$ | $(100,6)$ | 0,9456 | 0,9434 |
| $(14,0)$ | $(100,4)$ | 0,9592 | 0,9615 |
| $(15,0)$ | $(100,2)$ | 0,97 | 0,9804 |
| $(16,0)$ | $(100,2)$ | 0,9785 | 0,9804 |
| $(17,0)$ | $(100,2)$ | 0,985 | 0,9804 |
| $(0,1)$ | $(90,100)$ | 0,455 | 0,4737 |
| $(0,2)$ | $(80,100)$ | 0,4099 | 0,4444 |
| $(0,3)$ | $(70,100)$ | 0,3655 | 0,4118 |
| $(0,4)$ | $(60,100)$ | 0,3222 | 0,375 |
| $(0,5)$ | $(50,100)$ | 0,2807 | 0,3333 |
| $(0,6)$ | $(40,100)$ | 0,2414 | 0,2857 |
| $(0,7)$ | $(30,100)$ | 0,2048 | 0,2308 |
| $(0,8)$ | $(25,100)$ | 0,1712 | 0,2 |
| $(0,9)$ | $(20,100)$ | 0,141 | 0,1667 |
| $(0,10)$ | $(15,100)$ | 0,1142 | 0,1304 |
| $(0,11)$ | $(10,100)$ | 0,0909 | 0,0909 |
| $(0,12)$ | $(8,100)$ | 0,071 | 0,0741 |
| $(0,13)$ | $(6,100)$ | 0,0544 | 0,0566 |
| $(0,14)$ | $(4,100)$ | 0,0408 | 0,0385 |
| $(0,15)$ | $(2,100)$ | 0,03 | 0,0196 |
| $(0,16)$ | $(2,100)$ | 0,0215 | 0,0196 |
| $(0,17)$ | $(2,100)$ | 0,015 | 0,0196 |
|  |  |  |  |

In Figure 1 a scatterplot is shown of the probabilities inferred from markets odds, $\pi_{r}$, against the corresponding theoretical probability, $p_{r}$.

## Figure 1.



It can be seen in the graph above that for low theoretical probabilities the probability inferred from market odds is higher, and for high probabilities the probability inferred from market odds is lower than is actually the case. Thus this analysis, as well as other studies, supports the longshot bias. Empirical evidence on horseracing is found in Dowie, J. (1976), Henery, R.J. (1985), Thaler, R.H. and W.T. Ziemba (1988), and Vaughan, L. and D. Paton (1997). And empirical evidence of the long-shot bias is found in greyhound in Cain, M., D. Law, and D.A. Peel (1992).

### 3.2.3 Efficiency of the general odds rule assuming no profitable bets

Nevertheless, as there are commissions, $t=0.16$, when a bet takes place both bettors' expected values add up to a negative amount. Thus when analysing efficiency it is more convenient to ask for no possible profitable bets in the market. This implies that the expected value of a bet, both on the reds and on the blues, has to be lower than or equal to zero and, as shown in the following proposition, this implies that both equations (4) and (5) have to be fulfilled.

Proposition 3 In a Pelota betting market there are no profitable bets iff

$$
\begin{equation*}
p_{r} \leq \frac{o_{R}}{o_{B}(1-t)+O_{R}} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{O_{R}(1-t)}{O_{B}+O_{R}(1-t)} \leq p_{r} \tag{5}
\end{equation*}
$$

Proof. The expected value of a bet on the reds lower than zero and the expected value of a bet o on the blues lower than zero implies

$$
\begin{equation*}
p_{r} O_{B}(1-t)-\left(1-p_{r}\right) O_{R} \leq 0 \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(1-p_{r}\right) O_{R}(1-t)-p_{r} O_{B} \leq 0 \tag{7}
\end{equation*}
$$

Respectively. Rearranging (6) and (7) we obtain equations (4) and (5).

We represent equations (4) and (5) first in two separate graphs, Figure 2 and Figure 3, and then together in Figure 4.

Figure 2. $p_{r}$ assuming that the expected value of a bet on the reds is at most zero (4).


Given a score, we know both the theoretical probability of winning for the reds (applying equation (1)), and the market odds (applying the general odds rule). Therefore in Figure 2 the horizontal axis shows the theoretical probability and the vertical axis shows the upperbound probability inferred from market odds (applying equation (4)). The marked area corresponds to the probabilities inferred from markets odds assuming that the expected value of a bet on the reds is lower than zero.

Figure 3. $p_{r}$ assuming expected value of a bet on the blues is at most zero (5).


Given a score, we know both the theoretical probability of winning for the reds (applying equation (1)), and the market odds (applying the general odds rule). Therefore in Figure 3 the horizontal axis shows the theoretical probability of winning for the reds. The vertical axis shows the lower bound probability inferred from market odds (applying equation (5)). The marked area corresponds to the probabilities inferred from markets odds assuming that the expected value of a bet on the blues is lower than zero.

As both conditions (4) and (5) are necessary for the market to have no possible profitable bets, the probability inferred from market odds should be in the area between the two lines in the following graph.

Figure 4


From Figure 4 above it can be seen that there are odds in the market at which profitable bets could be made. It can be seen that when the theoretical probability of the reds winning is $\{.68$, $.72, .75\}$ it is profitable to bet on the reds: the odds are respectively $\{(100,60),(100,50),(100$, $40)\}$. Symmetrically, when the theoretical probability of the reds winning is $\{.24, .28, .32\}$ the odds are respectively $\{(40,100),(50,100),(60,100)\}$, and it is profitable to bet on the blues whose probabilities of winning are complementary. Overall, when the odds differ by 40,50 or 60 euros, it is profitable to bet on the favourite.

Woodland, L. M., and B. M. Woodland (1994) find deviation from efficiency in the baseball betting market but insufficient to allow for profitable betting strategies when commissions are considered. Here we find profitable betting strategies taking commissions into account.

If the expected value of a bet on the reds must be equal to the expected value of a bet on the blues, equation (2), the probability inferred from market odds is just in the middle of the area between the two lines, a probability for which the expected value of a bet is negative. This is shown in Figure 5.

Figure 5.


## 4 Summary and conclusion

We have described the Pelota betting system. We point out two peculiarities that differentiate it from other well-known betting systems. Unlike pari-mutuel betting systems the odds in a Pelota market are definitively set when the bet takes place. Bets are arranged by means of middlemen but, unlike what happens in bookmaking, for a bet to be placed one bettor bets on one team and another bettor bets on the other team, thus the middleman does not bet at all. We have found a similar betting system that has been recently implemented on the Internet. The betting system followed in Betfair is similar to the Pelota betting system in that bettors can make as many bets as they want to provided that there is another bettor on the other side, and the market maker takes a percentage of the money as a commission. The main difference with the Pelota betting system is the odds scale.

Here we seek to analyze the market with orthodox methods, and in particular we analyze the general odds rule followed in the Pelota betting system under certain assumptions that, though they may be somewhat strong, are made in studies of other wagering markets. Thus we start by assuming that bettors are expected value maximizers. A condition for equilibrium is that the expected value of a bet on the reds must be equal to the expected value of a bet on the blues, because if not all bettors prefer to bet on the colour with the higher return. We study the probability inferred from the market odds assuming equal returns on each bet and we obtain that there is a difference between the probability inferred from market odds and the theoretical probability of a team winning the match. Low probabilities are overestimated while high probabilities are underestimated. Therefore in this the Pelota betting market we find evidence of the long-shot bias.

In these markets there are commissions, so the equilibrium condition of equal returns on bets implies that each bet has a negative expected return. Thus it seems more convenient to introduce the less restrictive restriction of not allowing profitable bets in the market. We check what the probability inferred from market odds is to satisfy this less restrictive restriction of no profitable bets, i.e. to satisfy the condition of expected value of a bet lower than or at most equal to zero. We obtain that the probability inferred from market odds must be in an area satisfying inequalities (4) and (5) on pages 12 and 13 . We find that under the assumption of no profitable bets there are odds at which subjective probabilities differ from real ones. Thus we have found that there are odds in the market at which profitable bets could be made; when the odds differ by 40,50 or 60 euros, it is profitable to bet on the favourite.

## References

Cain, M., D. Law, and D.A. Peel (1992): "Greyhound racing: further empirical evidence on market efficiency in a wagering market," Aberystwyth Economic Research Papers, March.

Dowie, J. (1976): "On the efficiency and equity of betting markets," Economica, 43, 139-150.

Henery, R.J. (1985): "On the average probability of losing bets on horses with given starting price odds," Journal of the Royal Statistical Society, 148 (4), 342-349.

Llorente, L. and J.M. Aizpurua (2006): "A BETTING MARKET: Description and a theoretical explanation of bets in Pelota Matches". Working paper D.T.0603, Universidad Pública de Navarra (Downloaded at http://www.econ.unavarra.es/investig/papers/index.html).

Sauer, R.D. (1998): "The Economics of Wagering Market," Journal of Economic Literature, XXXVI, 2021-2064.

Smith, M.A., D. Paton and L.V. Williams (2006): "Market Efficiency in PersontoPerson Betting," Economica.

Thaler, R.H. and W.T. Ziemba (1988): "Parimutual Betting Markets: Racetracks and Lotteries." Journal of Economic Perspectives 2(2), (1988):161-174.

Vaughan, L. and D. Paton (1997): "Why is There a Favourite-Longshot Bias in British Racetrack Betting Markets)," The Economic Journal, 107, 150-158.

Woodland, L. M., and B. M. Woodland (1994): "Market Efficiency and the Favorite Long Shot Bias: The Baseball Betting Market," Journal of Finance, 49:1, 269-80.


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[^1]:    ${ }^{1}$ Data obtained on-line from the paper "Origen y desarrollo de las distintas modalidades del juego de la pelota vasca" on the website of the conference Confederación Internacional del Juego de la Pelota; http://www.cijb.org.

