MOTIVATIONAL CAPITAL AND INCENTIVES IN HEALTH CARE ORGANIZATIONS

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Abstract

This paper explores optimal incentive schemes in public health institutions when agents (doctors) are intrinsically motivated. We develop a principal-agent dynamic model with moral hazard in which agents’ intrinsic motivation could be promoted (crowding-in) by combining monetary and non-monetary rewards, but could also be discouraged (crowding-out) when the health manager uses only monetary incentives.

We discuss the conditions under which investing in doctors’ motivational capital by the use of well designed nonmonetary rewards is optimal for the health organizations manager. Our results show that such investments will be more efficient than pure monetary incentives in the long run. We will also prove that when doctors are risk-averse, it is profitable for the health manager to invest in motivational capital.

Keywords: contracts, moral hazard, intrinsic motivation, crowding effects, motivational capital.


1 Introduction

The purpose of this work is to investigate the roles played by the intrinsic motivation of doctors working in public health systems and by crowding effects, which can either undermine or enhance these inner motivations. Should health care organisations invest in motivating doctors? How should organisations’ managers design incentive schemes so that they can benefit from what Akerlof and Kranton [1] call motivational capital? Could intrinsic motivation be the key to avoiding opportunistic behaviour?

People who work in the provision of collective goods are usually intrinsically motivated agents who get satisfaction from the very act of doing their work. There are motives such as altruism, reciprocity, intrinsic pleasure in helping others and ethical commitments, that induce people to help others more than would an own-material-maximizing individual [7]. Teachers, doctors, firefighters, policemen and social workers are good examples of such intrinsically motivated workers [2, 4, 20]. We use the term “intrinsic motivation” to refer to doing something because it is inherently interesting or enjoyable [10, 11]. In health care, intrinsic motivation refers to doctors’ willingness to exert effort performing in medical activities that are of non-material interest like research, teaching, further education, health prevention activities or clinical management.

A new branch of contract theory investigates optimal contracts and incentives when agents are intrinsically motivated and when incentives beyond the money work [13, 14, 21, 28]. Dewatripont, Jewitt and Tirole [14] explore the effects of implicit incentives in the form of career concerns. Murdock [21] shows that in presence of implicit contracts, the firm can commit to implement some financially non-profitable projects with positive intrinsic value for the agent because doing that, agents will respond putting high effort to generate more projects and increasing the expected returns of the firm.

Another body of the literature analyzes the effects of having motivated agents in public organisations or in private organisations that serve collective goods [4, 16, 17, 27]. Wilson
explains how in the collective goods provision agencies, incentives are supplemented with a sense of mission based on a shared organizational culture. In Ghatak and Mueller organisations can reduce incentive payments when they contract intrinsically motivated agents. Thus, an organisation that adopts the non-for-profit status will attract motivated workers and will benefit from paying agents lower efficiency wages. Dewatripont, Jewitt and Tirole show that specialization and profesionalization of organisations raises the incentives of agents and create a sense of mission. They point out that “this paradigm can be fruitfully expanded, for example to a dynamic perspective where effort choices are repeated and where the evolution of mission design can be analysed (p. 216)”.

The above literature incorporates intrinsic motivation and the importance of the non-monetary incentives in principal-agent models. However, all these works have neglected the well established fact that incentives affect intrinsic motivation. Psychologists, and behavioural economists argue that under some specific conditions incentives crowd-out intrinsic motivation of agents. The crowding-out effect is one of the most important anomalies in economics, and it acts in a manner opposite to the fundamental economic ‘law’ that raising monetary incentives increases supply. Bowles and Polanía-Reyes classify the mechanisms accounting for crowding out. Our framework deals with three of these mechanisms: the informative value of incentives about principal’s intentions or type, the compromise of agents’ self determination or control aversion, and the agents’ preferences updating process.

However crowding-in also can occur. In sixteen out of the fifty experiments surveyed in Bowles and Polanía-Reyes they found evidence of crowding-in showing that well designed fines, subsidies, and the like, make incentives and intrinsic motivation complements rather than substitutes.

This work investigates the principal-agent relationship between managers and doctors, where the divergence in objectives between the principal’s performance measures and
the physicians’ mission is a source of conflict. It is assumed that principals in health care are primarily focused on health benefits. They focus heavily upon improving certain health performance measures that are easily observable by the electorate: for instance reducing the amount of time spent on waiting lists, increasing the number of operations conducted for common pathologies, increasing the infrastructure, buying new technology assets, reducing costs and saving resources, and enlarging the range of services supplied. In contrast, physicians’ goals are focused toward patients, a subset of all tax-payers, and also they have other interests in clinical and medical research, teaching and further education that taken together form what is called the doctors’ “mission”. One key fact of our approach is that incentives may make the action of providing health a less convincing signal of a doctors’ intrinsic motivation resulting in observers interpreting some generous acts as merely self-interested. This may crowd out doctors intrinsic motivation and they could shift from an ethical to a payoff maximizing frame [3, 7].

The contribution of our approach is threefold: first, following Dewatripont, Jewitt and Tirole [14] research program, we present a dynamical principal-agent model with intrinsically motivated agents and repeated effort and incentives choices to analyze the evolution of optimal contracts; second, we incorporate crowding effects in this dynamic model; and third, the proposed dynamical setting allows us to endogeneize changes in doctors’ preferences in response to the principal actions and therefore to evaluate how optimal contracts evolve and affect the outcomes of the game.

In the model, health managers have two options to motivate doctors: motivational investments and monetary incentives. We use the term *motivational investments* to refer to the resources devoted to well designed mechanisms, beyond the monetary incentives, oriented towards maintaining, recovering or enhancing doctors’ intrinsic motivation through a crowding-in effect. However, the use of pure monetary incentives may discourage doctors through a crowding-out effect, leading them to behave as payoff maximizers.
We discuss the conditions under which spending resources on motivational capital is optimal for the health organisation’s manager. Our results show that investing in motivational capital will be more efficient than monetary incentives in the long run. We will also prove that when doctors are risk-averse, it is more profitable for the health manager to invest in motivational capital.

The paper is organised as follows: section 2 presents the model, section 3 shows the results and section 4 summarises the work with some concluding remarks.

2 The Model

There are two players in the game: a doctor \( A \) (agent) and a health manager \( P \) (principal). We assume that \( A \) is intrinsically motivated. We also restrict the analysis to linear contracts.

The game is played for a finite number of periods \( t = 0, 1, \ldots, T, \ldots \). There is a health performance measure \( q_t \in \mathbb{R} \) — the number of QALYs for instance — that \( P \) wants to maximise. For all \( t \) let \( R_t(q_t) \) be a function \( R_t : \mathbb{R} \rightarrow \mathbb{R}_+ \) which assigns a monetary value to every \( q_t \).

Performance \( q_t \) is a function of doctor’s effort \( e_t \in \{e, \bar{e}\} \). Assume that \( q_t \in \{\bar{q}, q\} \) in which \( \bar{q} > q \). Take \( \bar{q} \) as \( P \)’s target for performance level and \( q \) as a failure to reach this target performance level. Let \( p(q_t = \bar{q}|e_t = 0) = \theta_1 \) be the conditional probability of high performance given \( A \)’s effort choice \( i = 0, 1 \) in which 0 indicates low effort \( e \) and 1 indicates high effort \( \bar{e} \). The probability distribution of \( q_t \) conditioned to \( e_t \) is given by: \( p(q_t = \bar{q}|e_t = 0) = \theta_1; \quad p(q_t = q|e_t = 0) = 1 - \theta_1 \) and, \( p(q_t = \bar{q}|e_t = 1) = \theta_0; \quad p(q_t = q|e_t = 1) = 1 - \theta_0 \). We assume that \( \theta_1 > \theta_0 \), which indicates that \( q_t \) is an informative signal of \( e_t \).

We denote the health expected revenue conditional to \( q_t \) with \( E[R_t(q_t)|\theta_i] \); \( \overline{R} \) and \( R \) will

\[\text{We use she and he to refer to the agent and the principal respectively, is conventional within the principal agent literature.}\]

\[\text{QALY stands for Quality Adjusted Live Years. For an estimation of the monetary value of a QALY see Pinto-Prades, Loomes and Brey (2009).}\]
stand for $R_t(\overline{q})$ and $R_t(q)$, respectively.

Let $w_t = \bar{w}(q_t)$ be the contingent monetary reward offered by $P$: $w(\overline{q}_t) = \bar{w}_t$ and $w(q_t) = w_t$, where $\overline{w}_t > w_t$. $E[w(q_t)|\theta_i]$ will then be the expected monetary cost for the health organization, or $P$. Let $s_0 \in \{0, S\}$ be the total initial investment in motivational capital. This investment generates a cost stream $C_t(s_0)$ that takes the value $C_0(S) = S$ or $C_0(0) = 0$ in $t = 0$ and gives the depreciation cost $C_t(S) = \gamma S$ for every $t \geq 1$ at a constant depreciation rate of $\gamma \in [0, 1)$. We assume, as in Murdock [24], that by having motivated doctors, $P$ should expect discounted future profits higher than the current cost of motivational incentives. $P$’s problem is to maximise the expected profit function.

$$E[\pi_t|\theta_i] = E[R_t(q_t)|\theta_i] - E[w_t(q_t)|\theta_i] - C_t(s_0)$$

We represent $A$’s preferences with the following overall expected utility function.

$$E[U_t|\theta_i] = E[u_t(w_t)|\theta_i] - \psi_t(e_t) + \phi_t(w_t, s_0)$$

The first term on the right hand side of the above expression $u_t(w_t)$, represents $A$’s utility from monetary incentives which “...complement the remuneration provided by the employer of the physician (p. 1)”, as in De Pouvoirville [26]. We assume that $A$ is risk-averse and that this utility function from monetary rewards satisfies the Inada conditions$^3$.

The middle term $\psi_t(e_t)$ is the cost from effort in utility terms that depends positively upon effort: $\psi_t(\varepsilon) = 0$ and $\psi_t(\overline{\varepsilon}) = \Psi$. Thus, $\psi_t(e_t) \in \{0, \Psi\}$ where $\psi_t(\overline{\varepsilon}) = \{\Psi \in \mathbb{R}|\Psi > 1\}$.

The last term is $\phi_t(w_t, s_0) \in [0, \Phi]$, in which $\phi_t : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ captures $A$’s intrinsic motivation. Intrinsic motivation depends negatively on incentives $w_t$. This captures the crowding-out effect of incentives. Intrinsic motivation depends positively on $P$’s investment $s_0$.

$^3$Inada conditions: $du_t(w_t)/dw_t > 0$, $d^2u_t(w_t)/dw_t^2 < 0$, $u_t(0) = 0$, $\lim_{t \to \infty} \left[du_t(w_t)/dw_t\right] = 0$ and $\lim_{t \to 0} \left[du_t(w_t)/dw_t\right] = \infty$. 

7
in motivational capital $s_0$.

The incentives offered by $\mathcal{P}$ may affect the intrinsic motivation of $\mathcal{A}$ through crowding effects. The properties of this intrinsic motivation function and of crowding effects are summed up in the following assumptions:

A1: For any fixed value of $w_t(q_t) = \tilde{w}_t$ such that $\tilde{w}_t \in [0,\infty) \times [0,\infty)$ we have $\phi_t(\tilde{w}_t, S) - \phi_t(\tilde{w}_t, 0) > 0$.

A2: Intrinsic motivation depends negatively upon incentives:

$$\frac{\partial \phi_t(w_t, s_0)}{\partial w_t} < 0.$$  

A3: Crowding in: in the case that $\mathcal{P}$ chooses $s_0 = S$, $\phi_t$ increases over time:

$$\frac{d\phi_t(w_t, S)}{dt} > 0$$

A4: Crowding out: in case of $\mathcal{P}$ chooses $s_0 = 0$, $\phi_t$ decreases over time;

$$\frac{d\phi_t(w_t, 0)}{dt} < 0$$

Assumption A1 shows a fixed crowding effect. Assumption A2 states that in presence of intrinsic motivation, agents enjoy a higher reward from it if they perform at high effort norm. Assumption A3 tells that intrinsic motivation is negatively correlated with incentives (crowding out). Assumption A4 captures a crowding-in effect: when $\mathcal{P}$ chooses the $s_0 = S$, $\mathcal{A}$’s intrinsic motivation will increase period after period. Assumption A5 captures a crowding-out effect: when $\mathcal{P}$ chooses a $s_0 = 0$, $\mathcal{A}$’s intrinsic motivation will diminish period after period.

Physicians may have different degrees of intrinsic motivation at $t = 0$. The model captures this heterogeneity with a probability distribution function, $F_0(\phi_0)$ that is defined over the value of the intrinsic motivation at $t = 0$. For any $\phi^* \in [0, \Phi]$ the distribution
function calculates the probability $F_0(\phi^*) = \text{Prob}(\phi_0 \leq \phi^*)$. In the game, $\mathcal{P}$ knows $F_0(\phi_0)$. His offer at $t = 0$ affects $A$'s intrinsic motivation through crowding effects. We model crowding effects as time displacements of the distribution function conditional to $s_0 \in \{0, S\}$ and $w_t$ (for example, $F_t(\phi_t|w_t, s_0)$). Thus, for any $\phi_t = \phi^* \in [0, \Phi]$, the conditional distribution calculates the probability $F_t(\phi_t|w_t, s_0) = \text{Prob}(\phi_t \leq \phi^*)$.

Figure 1 shows how crowding effects affect the intrinsic motivation probability distribution function. At $t = 0$, $\mathcal{P}$ knows a given distribution function $F_0(\phi_0)$. His choice of incentives in $t = 0$ affects agents intrinsic motivation switching the distribution function at $t = 1, 2, \ldots, T, \ldots$.

![Diagram](image)

**Fig. 1: Stochastic Dominance.** The figure shows how crowding effects affect the intrinsic motivation probability distribution function in response to principal’s choice of incentive policy: motivational investments $s_0 = S$, or pure monetary incentives $s_0 = 0$. Motivational investments $s_0 = S$ cause crowding-in switching the distribution function to the right period after period. Pure monetary incentives $s_0 = 0$ cause crowding-out switching the distribution function to the left period after period. Stochastic dominance ensures that no curve cross each together.

As shown in the figure, if $\mathcal{P}$ chooses $s_0 = 0$, then the distribution function will shift to
In other words, if incentives are only monetary, then doctors will concentrate around lower values of intrinsic motivation \( \phi_t = 0 \). In contrast, if incentives are motivational \( s_0 = S \), then doctors will concentrate around higher values of intrinsic motivation \( \phi_t = \Phi \). In this latter case, the distribution function shifts period after period to the right in figure 1.

We assume stochastic dominance in distribution function time shifts. This property involves, as shown in figure 1, that \( \mathcal{P} \)'s choices of incentives affect every \( \mathcal{A} \) intrinsic motivation in the same way. As a result, stochastic dominance assumes that probability distributions do not intersect on another.

The game is a repeated dynamic re-contracting game. In each period of the game both players have to make new choices: \( \mathcal{P} \) must offer a new contract after updating his beliefs about \( \mathcal{A} \), and \( \mathcal{A} \) has to choose a new effort level. The choices made by \( \mathcal{P} \) affect \( \mathcal{A} \)'s intrinsic motivation, and changes in \( \mathcal{A} \)'s motivation affect the contract and equilibrium payments offered by \( \mathcal{P} \) in the next period.

Each period of the game consists of three stages: stage 0, stage 1, and stage 2. The timing of the within-period in each \( t = 0, 1, \ldots, T, \ldots \) is:

1. The principal \( \mathcal{P} \) knows the distribution of doctors’ intrinsic motivation \( F_0(\phi_0) \) at \( t = 0 \) or updates \( F_t(\phi_t|s_0, w_t) \) given \( w_t \) and \( s_0 \) at \( t = 1, 2, \ldots, T, \ldots \). He then offers a contract to \( \mathcal{A} \). This contract consists of a pair of stochastic contingent payments \( w_0(q_0) = \{w, \bar{w}\} \) and the choice to invest or not invest in motivational capital \( s_0: \{w(q_0), s_0\} \) at \( t = 0 \) and \( \{w(q_t), s_0\} \) at \( t = 1, 2, \ldots, T, \ldots \).

2. \( \mathcal{A} \) accepts or refuses the contract. If she accepts, then she chooses an action \( e_t \in \{e, \bar{e}\} \) at each \( t = 0, 1, 2, \ldots, T, \ldots \). If she refuses then she gets her reservation utility \( \bar{U} \).

3. Finally, output is realised \( q_t \in \{q_t, \bar{q}_t\} \), payment is realised \( w(q_t) = \{w_t, \bar{w}_t\} \) and payoffs \( \pi_t \) and \( U_t \) are realized in each \( t = 0, 1, 2, \ldots, T, \ldots \).

\(^4\)For a more formal description of this property, see the mathematical appendix.
Fig. 2: Timing. The figure describes the stages of the game within each period, differentiating the starting period of the game \( t = 0 \), where no crowding effect has had place, from subsequent periods \( t = 1, 2, \ldots \) where \( P \)'s actions affects \( A \)'s intrinsic motivation through crowding effects.

Figure 2 shows the sequence of these stages in \( t = 0, 1, 2, \ldots, T, \ldots \).

Before solving the game, let us assume that \( P \) and \( A \) can not sign long term contracts at \( t = 0 \). As a result, they have to agree upon the rewards at every period \( t \). Once \( P \) has chosen \( s_0 = S \) in \( t = 0 \) he bears the depreciation cost \( C_t(s_0) = \gamma S \). We also assume that there is no contract renegotiation in the short term. In this game, the only way to agree upon a contract is to play the repeated game at every period \( t = 0, 1, \ldots, T, \ldots \) as a new game.

We can therefore write \( P \)'s problem as follows,

\[
Max_{(w_t(q_t), s_0)} \alpha_t \left( E[R_t(q_t)|\theta_0] - E[w_t(q_t)|\theta_0] \right) \\
+ (1 - \alpha_t) \left( E[R_t(q_t)|\theta_1] - E[w_t(q_t)|\theta_1] \right) - C_t(s_0) \tag{1}
\]
Subject to

\[ E[u_t(w_t)|\theta_1] - \psi_t(\bar{e}) + \bar{e} \cdot \phi_t(w_t, s_0) \geq E[u_t(w_t)|\theta_0] - \psi_t(\underline{e}) + \underline{e} \cdot \phi_t(w_t, s_0) \] (ICC) (2)

\[ E[u_t(w_t)|\theta_1] - \psi_t(\bar{e}) + \bar{e} \cdot \phi_t(w_t, s_0) \geq \bar{U} \] (PC) (3)

\[ u_t(w) \geq 0 \] (LLC) (4)

Where \( P_t(\phi_t < \bar{\phi}|s_0) = \alpha_t \) is the probability of having a doctor with an intrinsic motivation lower than the average conditional to \( \mathcal{P} \)'s choice of \( s_0 \), and \( P_t(\phi_t \geq \bar{\phi}|s_0) = 1 - \alpha_t \) is the probability of having a doctor with an intrinsic motivation higher or equal than the average conditional to \( \mathcal{P} \)'s choice of \( s_0 \). The objective function of \( \mathcal{P} \) is weighted by \( \alpha_t \) and \( 1 - \alpha_t \) because \( \mathcal{P} \) does not know the intrinsic motivation of each A. Therefore, he will offer a contract sufficient to incentivize the agent with average level of intrinsic motivation. Thus, those agents who are less intrinsically motivated than the average \( \phi_t < \bar{\phi}_t \) will shirk, and those who are equal or more intrinsically motivated than the average \( \phi_t \geq \bar{\phi}_t \) will exert high effort\(^5\). Condition (2) is \( \mathcal{A} \)'s incentive compatibility constraint (ICC) and ensures that the agent will prefer to exert high effort. (3) is the \( \mathcal{A} \)'s participation constraint (PC) and ensures that the agent will prefer to participate and accept the contract. Finally, (4) is a limited liability constraint (LLC) and ensures that the low utility payment will never fall below zero.

The solution to the above problem for each \( t \) is a pair of contingent payments \( \{\overline{w}, w\} \) associated with \( \overline{q} \) and \( q_t \), respectively. Let us show how we calculate the equilibrium of the game.

For notational simplicity we will write \( u_t(\overline{w}) = \overline{u} \) and \( u_t(w) = u \). Let \( h : u(w) \mapsto w \) be the inverse of the utility function \( h(u(w)) = (u(w))^{-1} = w \); then \( \overline{w} = h(\overline{u}) \) and \( w = h(u) \).

\(^5\)When crowding-out (crowding-in) effect entirely happens, all agents' intrinsic motivation will reach \( \phi_t = 0 \) (\( \phi_t = \Phi \)). Then, all agents will exert high effort because \( \phi_t = \overline{\phi}_t = 0 \) (\( \phi_t = \overline{\phi}_t = \Phi \)). As a consequence \( \mathcal{P} \)'s benefit function will be \( E\Pi_t = \theta_1 \cdot [R_t(q_t) - w_t(q_t)] - (1 - \theta_1) \cdot [R_t(q_t) - w_t(q_t)] - C_t(s_0) \).
Finally $\Delta \theta = (\theta_1 - \theta_0)$; and reservation utility is denoted by $\overline{U}$.

We rewrite $\mathcal{P}$’s problem as follows:

$$
\text{Max}_{(w(t),s_0)} \quad \alpha_t \cdot \left[ \theta_0 \left( \overline{R} - h(\overline{u}) \right) - (1 - \theta_0) \left( \underline{R} - h(\underline{u}) \right) \right]
$$

$$
+ (1 - \alpha_t) \cdot \left[ \theta_1 \left( \overline{R} - h(\overline{u}) \right) - (1 - \theta_1) \left( \underline{R} - h(\underline{u}) \right) \right] - C_t(s_0)
$$

Subject to

$$
\theta_1 \overline{u} + (1 - \theta_1) \underline{u} - \Psi + \phi_t \geq 0 \quad \text{(ICC)}
$$

$$
\theta_1 \overline{u} + (1 - \theta_1) \underline{u} - \Psi + \phi_t \geq \overline{U} \quad \text{(PC)}
$$

$$
\underline{u} \geq 0 \quad \text{(LLC)}
$$

Letting $\lambda$ and $\mu$ be the non-negative Khun-Tucker multipliers associated respectively to (ICC) and (PC) constraints. First-order conditions of this problem lead to:

$$
\frac{1}{u'(\overline{w})} = \mu + \frac{\Delta \theta}{\theta_1} \quad \text{(9)}
$$

$$
\frac{1}{u'(\underline{w})} = \mu - \frac{\Delta \theta}{(1 - \theta_1)} \quad \text{(10)}
$$

The equations (9) and (10) (jointly with (6) and (7)) form a system of four equations with four variables $(\overline{w}, \underline{w}, \mu, \lambda)$. Multiplying (9) by $\theta_1$ and (10) by $(1 - \theta_1)$ and adding those two modified equations, we obtain;

$$
\mu = \left( \theta_1/u'(\overline{w}) \right) + \left( 1 - \theta_1 \right)/u'(\underline{w}) > 0
$$

Therefore, $\mu > 0$ and the participation constraint (9) is binding. Using (11) and (9), we also obtain,
\[ \lambda = \left( (1 - \theta_1) \cdot \theta_1 / \Delta \theta \right) \cdot \left( (1/u'(\bar{w})) - (1/u'(\underline{w})) \right) > 0 \] (12)

Therefore, \( \lambda > 0 \) and the incentive compatibility constraint (6) is also binding. Thus, we can immediately obtain the values of \( \bar{w} \) and \( \underline{w} \) by solving a system with two equations and two unknowns. The result is shown below:

\[
\bar{w}_t = \bar{U} - \phi_t (w_t, s_0) + \left( (1 - \theta_0) / \Delta \theta \right) \Psi \\
\underline{w}_t = \underline{U} - \phi_t (w_t, s_0) - \left( \theta_0 / \Delta \theta \right) \Psi.
\]

Applying the variable change \( w_t(q_t) = h(u_t(w_t)) = (u_t(w_t))^{-1} \), we have the following payments,

\[
\bar{w}_t = h(\bar{u}_t) = \left( \bar{U} - \phi_t (w_t, s_0) + \left( (1 - \theta_0) / \Delta \theta \right) \Psi \right)^{-1} \\
\underline{w}_t = h(\underline{u}_t) = \left( \underline{U} - \phi_t (w_t, s_0) - \left( \theta_0 / \Delta \theta \right) \Psi \right)^{-1}.
\]

Thus, at every period of the game, \( P \) must offer to \( A \) the following expected payments,

\[
\bar{w}_t = \left( \bar{U} - E[\phi_t|w_t, s_0] + \left( (1 - \theta_0) / \Delta \theta \right) \Psi \right)^{-1} \] (13)
\[
\underline{w}_t = \left( \underline{U} - E[\phi_t|w_t, s_0] - \left( \theta_0 / \Delta \theta \right) \Psi \right)^{-1}. \] (14)

Using (13) and (14) \( P \)'s Expected Cost function \( EC_t \), Expected Revenue function \( ER_t \) and Expected Profit function \( E\Pi_t \) are calculated for every \( t \) as follows,

\[
EC_t = \alpha_t \cdot \left( \theta_0 \bar{w}_t + (1 - \theta_0) \underline{w}_t \right) + (1 - \alpha_t) \left( \theta_1 \bar{w}_t + (1 - \theta_1) \underline{w}_t \right) + C_t(s_0) \] (15)
\[
ER_t = \alpha_t \cdot \left( \theta_0 \bar{R}_t + (1 - \theta_0) \cdot \underline{R}_t \right) + (1 - \alpha_t) \left( \theta_1 \bar{R}_t + (1 - \theta_1) \underline{R}_t \right) + C_t(s_0) \] (16)
Let us use the superscript $s_0 \in \{0, S\}$ in $EC_{t}^{s_0}$ and $w_{t}^{s_0}$ to differentiate the expected cost function and expected payments when $P$ invests in motivational capital $s_0 = S$ from the no investment case $s_0 = 0$. We then write conditional to $s_0 \in \{0, S\}$ two Expected Cost functions, two Expected Revenue functions and two Expected Profit functions.

$$EC_{t}^{0} = \alpha_{t} \cdot \left( \theta_{0} \overline{w}_{t} + (1 - \theta_{0})w_{t} \right) + (1 - \alpha_{t}) \left( \theta_{1} \overline{w}_{t} + (1 - \theta_{1})w_{t} \right)$$  \hspace{1cm} (18)

$$EC_{t}^{S} = \alpha_{t} \cdot \left( \theta_{0} \overline{w}_{t} + (1 - \theta_{0})w_{t} \right) + (1 - \alpha_{t}) \left( \theta_{1} \overline{w}_{t} + (1 - \theta_{1})w_{t} \right) + C_{t}(S)$$  \hspace{1cm} (19)

$$ER_{t}^{0} = \alpha_{t} \cdot \left( \theta_{0} \overline{R}_{t} + (1 - \theta_{0})R_{t} \right) + (1 - \alpha_{t}) \left( \theta_{1} \overline{R}_{t} + (1 - \theta_{1})R_{t} \right)$$  \hspace{1cm} (20)

$$ER_{t}^{S} = \alpha_{t} \cdot \left( \theta_{0} \overline{R}_{t} + (1 - \theta_{0})R_{t} \right) + (1 - \alpha_{t}) \left( \theta_{1} \overline{R}_{t} + (1 - \theta_{1})R_{t} \right) + C_{t}(S)$$  \hspace{1cm} (21)

$$E\Pi_{t}^{0} = ER_{t}^{0} - EC_{t}^{0} = \alpha_{t} \cdot \left( \theta_{0} \overline{R}_{t} - \overline{w}_{t} \right) + (1 - \theta_{0}) \left( R_{t} - w_{t} \right)$$  \hspace{1cm} (22)

$$E\Pi_{t}^{S} = ER_{t}^{S} - EC_{t}^{S} = \alpha_{t} \cdot \left( \theta_{0} \overline{R}_{t} - \overline{w}_{t} \right) + (1 - \theta_{0}) \left( R_{t} - w_{t} \right) - C_{t}(S)$$  \hspace{1cm} (23)

As we have said in Section II, doctors’ intrinsic motivation can be considered another productive asset or capital of the health organization called Motivational Capital. The current net value ($CNV^{mk}$) of the return of an investment in motivational capital is:

$$CNV^{mk} = \sum_{t=0}^{T} \delta^{t} \left[ E\Pi_{t}^{S} - E\Pi_{t}^{0} \right]$$  \hspace{1cm} (24)

in which, $\delta^{t} = (1/(1+r))^{t}$ is the discount factor, and $r$ is the discount rate. We say that the principal has incentives to invest in motivational capital when $CNV^{mk} \geq 0$ and we say that, there is no incentive to invest in motivational capital when $CNV^{mk} < 0$.  

15
3 Results

We solve the principal’s problem under two alternative scenarios: when $P$ chooses $s_0 = S$ and when he chooses $s_0 = 0$. We calculate the solution for each case to show necessary and sufficient conditions for investing in motivational capital.

3.1 Motivational Incentives: Crowding In

First, we solve the model for the case in which the health manager chooses $s_0 = S$. In this case, $A’s$ spot utilities and spot payments in each $t$ are:

\[
\begin{align*}
\pi_t^S &= U - \phi_t(w_t, S) + \left((1 - \theta_0)/\Delta\theta\right)\Psi \\
\bar{w}_t^S &= U - \phi_t(w_t, S) - \left(\theta_0/\Delta\theta\right)\Psi \\
\bar{w}_t^S &= \left(U - E_t[\phi_t|w_t, S] + \left((1 - \theta_0)/\Delta\theta\right)\Psi\right)^{-1} \\
\bar{w}_t^S &= \left(U - E_t[\phi_t|w_t, S] - \left(\theta_0/\Delta\theta\right)\Psi\right)^{-1}.
\end{align*}
\]

Using (25) and (26) we calculate $A’s$ expected utility $E_t[U_t^S|\theta_1, \phi^h_t]$ for every period $t = 0, 1, 2, \ldots, T$. We differentiate two cases, $A^h$ or when agents have an intrinsic motivation above the average $\phi^h_t \geq \bar{\sigma}_t$ on the one hand, and $A^l$ or when agents who have an intrinsic motivation below the average $\phi^l_t < \bar{\sigma}_t$ on the other hand.

\[
\begin{align*}
E[U_t^{S,h}] &= E_t[U_t^{S,h}|\theta_1, \phi^h_t] = \left(\theta_1\pi_t^S + (1 - \theta_1)\bar{w}_t^S\right) - \Psi + \phi^h_t(w_t, S) \\
E[U_t^{S,l}] &= E_t[U_t^{S,l}|\theta_0, \phi^l_t] = \left(\theta_0\pi_t^S + (1 - \theta_0)\bar{w}_t^S\right) + \phi^l_t(w_t, S)
\end{align*}
\]

Where $\phi^h_t(w_t, S)$ is the amount of intrinsic motivation of $k$-est agent whose intrinsic motivation is above the average and $\phi^l_j(w_t, S)$ is the amount of intrinsic motivation of $j$-est agent whose intrinsic motivation is below the average.
Finally, using (23), (29) and (30), we calculate the current value of the sum of all periods expected profits ($\Gamma^S$), the sum of the all periods expected utilities ($\Lambda^S$) and the current value of the total surplus ($TS^S$) when the action of $P$ is $s_0 = S$.

$$\Gamma^S = \sum_{t=0}^{T} \delta^t \Pi^S_t$$

$$\Lambda^S = \sum_{t=0}^{T} \delta^t \left[ \alpha_t \cdot E\mathcal{U}^S_{t, l} + (1 - \alpha_t) \cdot E\mathcal{U}^S_{t, h} \right]$$

$$TS^S = \Lambda^S + \Gamma^S$$

(31)

### 3.2 Motivational Incentives: Crowding Out

The second case is $s_0 = 0$, when $P$ uses pure monetary rewards and causes the crowding out of intrinsic motivation. In this case, $A$’s spot utilities and spot payments in each $t$ are:

$$\bar{u}^0_t = U - \phi_t(w_t, 0) + \left((1 - \theta_0)/\Delta \theta\right)\Psi$$

$$u^0_t = U - \phi_t(w_t, 0) - \left(\theta_0/\Delta \theta\right)\Psi$$

$$\bar{w}^0_t = \left(U - E_t[\phi_t|w_t, 0] + \left((1 - \theta_0)/\Delta \theta\right)\Psi\right)^{-1}$$

$$w^0_t = \left(U - E_t[\phi_t|w_t, 0] - \left(\theta_0/\Delta \theta\right)\Psi\right)^{-1}.$$  

Using (32) and (33) we calculate $A$’s expected utility $E_t[\mathcal{U}^0_t|\theta_t, \phi_t]$ for every period $t = 0, 1, 2, \ldots, T$. We differentiate two cases, $A^h$ or when agents have an intrinsic motivation above the average $\phi^h_t \geq \phi_t$ on the one hand, and $A^l$ or when agents who have an intrinsic motivation below the average $\phi^h_t < \phi_t$ on the other hand.

$$E\mathcal{U}^{0,h}_t = E_t[\mathcal{U}^{0,h}_t|\theta_t, \phi^h_t] = \left(\theta_1 \bar{u}^0_t + (1 - \theta_1)u^0_t\right) - \Psi + \phi^h_t(w_t, 0)$$

(36)
\[ EU_t^{0,l} = E_i[U_t^{0,l} | \theta_0, \phi_{t}^l] = \left( \theta_0 \pi_t^0 + (1 - \theta_0) \omega_t^0 \right) + \phi_{t}^l (w_t, 0) \]  

(37)

Where \( \phi_{t}^{hk}(w_t, 0) \) is the amount of intrinsic motivation of \( k \)-est agent whose intrinsic motivation is above the average and \( \phi_{t}^{lj}(w_t, 0) \) is the amount of intrinsic motivation of \( j \)-est agent whose intrinsic motivation is below the average.

Finally, using (22), (36) and (37), we calculate the current value of the sum of all periods expected profits (\( \Gamma^S \)), the sum of the all periods expected utilities (\( \Lambda^0 \)) and the current value of the total surplus (\( TS^0 \)) when the action of \( P \) is \( s_0 = 0 \).

\[ \Gamma^0 = \sum_{t=0}^{T} \delta^t \text{E} \Pi_t^0 \]

\[ \Lambda^0 = \sum_{t=0}^{T} \delta^t \left[ \alpha_t \cdot \text{E} U_t^{0,l} + (1 - \alpha_t) \cdot \text{E} U_t^{0,h} \right] \]

\[ TS^0 = \Lambda^0 + \Gamma^0 \]  

(38)

3.3 Comparative Statics

A health manager who is considering to invest in doctors motivation anticipates that to benefit from this, even in the long run, the additional profits of having intrinsically motivated agents must overcome the additional costs of motivate them somewhere in time.

Our model shows that an intrinsically motivated doctor is willing to work for lower overall pay. Thus \( P \)'s benefits will be increasing when he decides to motivate doctors \( s_0 = S \) and decreasing when he decides not to motivate doctors. Therefore, motivational capital profitability requires that the following condition holds once the crowding effects have entirely happened.

\[ w_t^0 - w_t^S + \theta_1 (\Delta w^0 - \Delta w^S) > \gamma S \]  

(39)
Changes in each parameter of the model will affect the profitability of such the investment on Motivational Capital. Then to study how these parameters affects the benefits of investing in motivation will be a key question to find the conditions under which a health manager may benefit from motivational capital. in the nexte sections we analyze different cases.

Motivational Capital and Optimal Contracts

We want to establish a decision rule for $P$. He will take an action over $s_0 = \{0, S\}$ depending upon the total present profit that he can extract from each. Our analysis of $P$’s behaviour then begins with a comparison of the different values of the contracts that he gets in with each decision. Let $T$ be the number of periods that the game is going to be played. We then have:

$$\Gamma^S - \Gamma^0 = \sum_{t=0}^{T} \delta^t [E\Pi^S_t - E\Pi^0_t]$$

Looking at the above expression, the decision rule for $P$ will be to choose $s_0 = 0$ (pure monetary reward incentives) when $\Pi^S - \Pi^0 < 0$ and to choose $s_0 = S$ when $\Pi^S - \Pi^0 > 0$.

As we can see, the above expression equals the expression (24), which reflects the current net value of an investment made by $P$ to generate motivation $CNV^{mk}$. $P$ will then choose $s_0 = S$ in the case that $CNV^{mk} > 0$ and will choose $s_0 = 0$ in the case that $CNV^{mk} \leq 0$.

We then establish the following result:

**Proposition 1.** Let $T$ be the number of periods that the game will be played. Let $L^0 < T$ and $L^S < T$ be the minimum number of time periods enough to allow crowding effects entirely happen for $s_0 = 0$ and $s_0 = S$ respectively. If $\left[ \bar{w}^0_t - \bar{w}^S_t + \theta_1(\Delta w^0 - \Delta w^S) \right] > \gamma S$ with $\Delta w^0_t = \bar{w}^0_t - \bar{w}^0_{t-1}$ and $\Delta w^S_t = \bar{w}^S_t - \bar{w}^S_{t-1}$, then there exists a threshold $t^*$ such that:

$$CNV^{mk} = \Gamma^S - \Gamma^0 = \sum_{t=0}^{t^*} \delta^t [E\Pi^S_t - E\Pi^0_t] = 0$$
and for which

i. If $t^* < T$ then $CNV^{mk} > 0$ and $\mathcal{P}$ finds it profitable to invest in motivational capital and choose the $s_0 = S$ strategy.

ii. If $t^* \geq T$ then $CNV^{mk} \leq 0$ and $\mathcal{P}$ finds it profitable to not invest in motivational capital and chooses the $s_0 = 0$ strategy.

Figure 3 illustrates the result using a particular case. The left side shows $\mathcal{P}$’s expected profit functions for $s_0 = 0$ and $s_0 = S$. The right side shows the value of the $CNV^{mk}$ as a function of time $t$. The $t^*$ threshold determines the critical point which determines the best strategy for $\mathcal{P}$.

Fig. 3: Current Net Value of Motivational Capital. The graph shows together the expected profit functions $E\Pi^S_t$ and $E\Pi^0_t$, joint with the current net value of motivational capital $CNV^{mk}$. In $t^0$ and $t^S$ the crowding effects of $s_0 = 0$ and $s_0 = S$ are completed. The motivational investments profitability threshold $t^*$ shows the point at which the $CNV^{mk}$ becomes positive and therefore investing in motivational capital $s_0 = S$ is the best choice for $\mathcal{P}$.

$CNV^{mk}$ depends on $\mathcal{P}$’s time preference, which is captured in the model by the parameter $\delta$. Lower values indicate that the health manager puts more weight on the present. Impatience therefore makes $s_0 = S$ less attractive.

Remark. A lower value of $\delta$ means that the health manager is more focused on the short
term. This implies that $t^*$ will be larger, consequently making any investment of resources in motivational capital (i.e., implementing the $s_0 = S$ strategy) less attractive to him.

This simple observation leads to an important discussion: the need for politically independent managerial positions in health. The political cycle forces politicians and consequently managers in health, to set short-term goals. They have a low $\delta$ because they put a lot of weight in the profits earned during the legislature. In contrast, doctors are career professionals who have long-term goals in health provision. As a result, politicians usually prefer to implement control and command policies and monetary incentives rather than implementing motivational incentives or investing in motivational capital (both of which are initially costly).

### Depreciation Cost and Motivational Capital

A high depreciation cost $\gamma$ may make investments in motivational capital no optimal at all.

**Proposition 2.** Let $t^0 < T$ and $t^S < T$ be the minimum number of periods enough to allow crowding effects entirely happen for $s_0 = 0$ and $s_0 = S$ respectively. Then, taking $S$ as constant, if $\gamma \geq \left[ \frac{w_0^0 - w_0^S + \theta_1(\Delta w_0^0 - \Delta w_0^S)}{S} \right]$, then $CNV_{mk} < 0$ for all $t = 1, 2, \ldots$ and $P$ never will find profitable to invest in motivational capital.

Proposition 2 states that when the depreciation cost is a higher fraction of the initial invested amount in motivational capital than the fraction that expected additional profits are over the same invested amount, then health manager never will invest in motivational capital because he will not expect any profit from this, neither in the short nor in the long run.

### Risk Aversion and Motivational Capital

In the model, agents are risk-averse and thereby receive contingent rewards linked to performance $q_t$. As $A$’s intrinsic motivation increases, fewer incentives and less variation in
payments are required in order to encourage him to exert high effort. Less variation in payments indicates that \( A \) can be compensated with a lower risk premium, and this constitutes another cost-saving source for the health organization.

Proposition 3 formally states that investing in motivational capital is more profitable in the presence of risk-averse \( A \):

**Proposition 3.** Investing in motivational capital is more profitable for \( P \) in presence of risk-averse agents. Let \( A^1 \) and \( A^2 \) be a pair of agents with \( \phi^1 \) and \( \phi^2 \) intrinsic motivation respectively. If the agents are risk-averse and \( \phi^1 < \phi^2 \), then the risk premium will be lower in the case of \( A^2 \) than in the case of \( A^1 \). This additional advantage in costs shortens \( t^* \) and consequently \( CNV_{t^*}^{mk} \) will earlier become positive.

The intuition behind this result is that incentives must be greater in order to encourage high effort from agents without much intrinsic motivation. However, these higher incentives raise the range between the low \( w \) and the high \( \bar{w} \) payments. Given that \( A \) is risk averse, the risk premium that \( P \) should offer to make the incentive contract attractive for \( A \) will be higher. Analogously, intrinsically motivated agents required fewer incentives to exert high effort. Consequently, she has to bear a lower variance over payments and has to be compensated with a lower risk premium.

**Doctors’ Outside Options and Motivational Capital**

When doctors have less options to employ out of the organization then, intuitively, we may expect that investments in motivational capital would become less attractive for the health manager. This is so because with less outside options \( ( \bar{U} = 0 ) \), the low incentive payment \( w_{r}^{s0} \) required to incentivize doctors’ effort is equal to 0 due to limited liability constraint (8).

**Proposition 4.** Investments in motivational capital will be less likely to be optimal to the principal (health manager) when \( \bar{U} = 0 \) and no outside options are available to agents (doctors).
Proposition 4 states that in contexts with no outside options or with less likely ones, incentivize high effort from doctors is less costly. Then, health managers will find less attractive to invest in motivational capital and they will be more focused in using only monetary incentives.

![Diagram](image)

**Fig. 4: Information and intrinsic motivation.** Four different curves appear in the figure. \( h(\bar{u}, \phi_1) \) and \( h(u, \phi_1) \) show incentives for every value of \( \theta_0 \) when the agent has \( \phi_1 \) intrinsic motivation. \( h(\bar{u}, \phi_2) \) and \( h(u, \phi_2) \) show incentives for every value of \( \theta_0 \) when the agent has \( \phi_2 \) intrinsic motivation. In the figure we represent the case in which \( \phi_2 > \phi_1 \).

**Information and Motivational Capital**

Information affects motivational capital. Higher values of \( \theta_0 \), or closer to \( \theta_1 \) may change the conditions under which investing in motivational capital is optimal. We study how a poor correlation between effort and performance affects the decision of investing in motivational
Higher values of \( \theta_0 \) increase incentive payments in expected terms. This is so, by twofold reason: first because a poorer (more random) signal of doctors’ effort increases doctors’ rent extraction power and, second, because doctors are risk averse. In this context a reduction in incentive payments coming from having more intrinsically motivated doctors, result in a higher expected savings. Therefore, as mean as performance becomes more random signal of effort, the profitability of investing in motivational increase.

**Proposition 5.** *Whenever the LLC condition is applied, higher values of \( \theta_0 \), more close to \( \theta_1 \) entail higher values of \( CNV^{mk} \).*

Figure 4 illustrates proposition 5. The figure show how incentives respond to an increment on \( \theta_0 \). As it can be seen in the figure, an increment of the same amount on \( \theta_0 \) (\( \Delta \theta_0 \)), generates an increment in incentive payment of lower magnitude when the agent is more intrinsically motivated (\( \Delta w_{t,\phi_1} > \Delta w_{t,\phi_2} \)). Then, investing in motivational capital is more profitable in cases in which performance is a more random signal of the agent’s effort.

## 4 Conclusion

The following conclusions summarize the results of this work.

Results show that in the long run, to dedicate resources to crowd in doctors intrinsic motivation, although costly at inception, will result more efficient than the use of monetary incentives. However, if health care managers are focused on the short run (legislative period), then they will have a tendency to choose purely monetary rewards.

A health manager considering to invest or not in motivation will compute the present value of the expected returns of both alternatives. As long as health manager will have a lower discount factor more weight will put in the short run and less attractive will find to invest in doctors’ motivation. This, strengthen the previous conclusion.
In the model investments in motivational capital entails depreciation costs. If the cost of depreciation is so large that the benefit from doctors' intrinsic motivation cannot compensate it, then an investment in doctors’ motivation will not be optimal at all for health manager. 

Doctors outside options affect positively to the optimality of making investments in motivational capital. Then, in public health, investments in motivational capital will be more attractive in those medical specialties which have better outside options in the private sector.

When doctors are risk-averse, investments in motivational capital are more likely to be profitable for the health manager. When doctors are intrinsically motivated they are paid with lower uncertainty to exert high effort and this result in a benefit for the health manager.

Whenever doctors own a large amount of private information or performance is a poor signal of effort and contexts in which doctors’ private information is higher make more investments in motivational capital more profitable. This case is particularly interesting because in health, outcomes are hard to measure and often the factors which determine them are not only doctors’ effort. Furthermore, physicians are highly qualified professionals in areas of advanced and complex knowledge. As a consequence, they own a large amount of private information. Our results predict that health organizations are excellent candidates to benefit from motivational capital investments due to the informational features that characterize them.

Finally, other parameters and elements present in the model can offer information to determine when motivational investments will be optimal for health manager. The total surplus of contracts depend on the doctors’ intrinsic motivation distribution function. The form of this distribution and how the crowding effects affect it, are crucial to determine the effects produced by incentives—economic or motivational—on the total welfare of all the members of the health organization. This is a field to explore in depth that we consider for further research.
A Mathematical Appendix

Stochastic Dominance

Crowding effects move the distribution of doctors intrinsic motivation with a stochastic dominance. Then for any fixed value of $\phi_t = \phi^*$ we have,

$$ F_t(\phi_t = \phi^* | w_t, 0) \geq F_{t-1}(\phi_{t-1} = \phi^* | w_{t-1}, 0) \geq \cdots \geq F_0(\phi_0) $$

$$ \geq \cdots \geq F_{t-1}(\phi_{t-1} = \phi^* | w_{t-1}, S) \geq F_t(\phi_t = \phi^* | w_t, S) $$

Assume that $F_t(\phi_t | w_t, S)$ converges to the upper bound of intrinsic motivation $\phi_t = \Phi$ and that $F_t(\phi_t | w_t, 0)$ converges to the lower bound of intrinsic motivation $\phi_t = 0$.

$$ \lim_{t \to \infty} F_t(\phi_t | w_t, S) = \rho $$

where

$$ \rho = \begin{cases} 1 & \text{if } \phi_t = \Phi \\ 0 & \text{otherwise} \end{cases} $$

and

$$ \lim_{t \to \infty} F_t(\phi_t | w_t, 0) = 1, \text{ for every } \phi_t \in [0, \Phi]. $$

Let $E_t[\phi_t | s_0, w_t]$ be the mathematical expectation in $t$ of the value of $\phi_t$ given the incentive policy $s_0$ and incentives $w_t$. Consequently, stochastic dominance on $E_t[\phi_t | ·]$ assumes:

$$ \forall t = 0, 1, \ldots, T, \ldots \quad E_{t+1}[\phi_{t+1} | w_{t+1}, 0] < E_{t}[\phi_t | w_t, 0] $$

$$ \forall t = 0, 1, \ldots, T, \ldots \quad E_{t+1}[\phi_{t+1} | w_{t+1}, S] > E_{t}[\phi_t | w_t, S] $$

$$ \forall t = 0, 1, \ldots, T, \ldots \quad E_{t}[\phi_t | w_t, 0] < E_{t}[\phi_t | w_t, S] $$

In which:

$$ E_t[\phi_t | w_t, s_0] = \int_0^\Phi \phi_t f(\phi_t | w_t, s_0) d\phi_t $$

Proof of Proposition 1
We have to study the sign of the following expression:

\[
\Gamma^S - \Gamma^0 = \sum_{t=0}^{T} \delta^t \left[ \alpha_t \cdot \left( \theta_0 \left( R_t - \bar{w}^S_t \right) + (1 - \theta_0) \left( R_t - \bar{w}^S_t \right) \right) + (1 - \alpha_t) \cdot \left( \theta_1 \left( R_t - \bar{w}^S_t \right) + (1 - \theta_1) \left( R_t - \bar{w}^S_t \right) \right) \right] - \sum_{t=0}^{T} \delta^t \left[ \alpha_t \left( \theta_0 \left( R_t - \bar{w}^0_t \right) + (1 - \theta_0) \left( R_t - \bar{w}^0_t \right) \right) + (1 - \alpha_t) \left( \theta_1 \left( R_t - \bar{w}^0_t \right) + (1 - \theta_1) \left( R_t - \bar{w}^0_t \right) \right) \right] - \sum_{t=0}^{T} \delta^t C_t(S)
\]

We have to show that there is a given threshold \( t^* \in \{0, 1, 2, \ldots, T, \ldots \} \) such that,

\[
\Gamma^S - \Gamma^0 = 0
\]

Crowding effects, stochastic dominance, and (39) imply that for all \( \hat{t} \in \{0, 1, \ldots, T, \ldots \} \) such that \( \hat{t} \geq t^0 \) and \( \hat{t} \geq t^S \), the following condition holds,

\[
E\Pi^S_{\hat{t}} - E\Pi^0_{\hat{t}} > 0
\]

As we know in \( t = 0 \),

\[
E\Pi^S_0 - E\Pi^0_0 < 0
\]

Then there exists a \( \bar{t} \) such that \( 0 < \bar{t} < \hat{t} \) in which,

\[
E\Pi^S_{\bar{t}} = E\Pi^0_{\bar{t}}
\]
and then,

\[
\sum_{t=0}^{\bar{t}} \left[ E\Pi_t^S - E\Pi_t^0 \right] < 0 \quad (41)
\]

\[
\sum_{t=\bar{t}}^T \left[ E\Pi_t^S - E\Pi_t^0 \right] > 0 \quad (42)
\]

where (41) results in a negative and finitely bounded value and (42) is unbounded and only finds its limit when the game ends. Formally,

\[
\sum_{t=0}^{\bar{t}} \left[ E\Pi_t^S - E\Pi_t^0 \right] = -M \quad (43)
\]

\[
\lim_{t \to \infty} \sum_{t=\bar{t}}^\infty \left[ E\Pi_t^S - E\Pi_t^0 \right] = \infty \quad (44)
\]

From (40) and (44) we now that there exists a \( t^* \in \{\bar{t} + 1, \ldots, T, \ldots \} \) such that,

\[
\sum_{t=t^*}^{T} \left[ E\Pi_t^S - E\Pi_t^0 \right] = M
\]

And therefore the following holds,

\[ \Gamma^S - \Gamma^0 = 0 \]

**Proof of Proposition 2**

Immediate. The analogous case of proposition 1. Proof available from the authors upon request.

**Proof of Proposition 3**

Using (13), (14) and \( h(u) : u \to w \) we have that incentive payments are,

\[
w_t = h \left( \bar{U} - E[\phi_t|w_t, s_0] + \left( (1 - \theta_0) / \Delta \theta \right) \Psi \right) \quad (45)
\]

\[
w_s = h \left( \bar{U} - E[\phi_t|w_t, s_0] - \left( \theta_0 / \Delta \theta \right) \Psi \right) . \quad (46)
\]
From (45) and (46) we now that for any pair of expected values of intrinsic motivation $\phi^1, \phi^2 \in \phi(w_t, s_0)$ such that $\phi^1 > \phi^2$ we have that,

$$w^1_t = h \left( U - \phi^1 + \left( (1 - \theta_0)/\Delta \theta \right) \Psi \right) < h \left( U - \phi^2 + \left( (1 - \theta_0)/\Delta \theta \right) \Psi \right) = w^2_t \tag{47}$$

$$w^1_t = h \left( U - \phi^1 - \left( \theta_0/\Delta \theta \right) \Psi \right) < h \left( U - \phi^2 - \left( \theta_0/\Delta \theta \Psi \right) \right) = w^2_t \tag{48}$$

Assume first that agents are risk neutral to set a benchmark case. That is to say that $u_t(w_t) = w_t$. Then $P$ will pay lower incentives to the higher motivated $A$,

$$w^1_t - w^2_t = \phi^2 - \phi^1$$

$$w^1_t - w^2_t = \phi^2 - \phi^1 \quad \text{if} \quad w^1_t > 0 \quad \text{and} \quad w^2_t > 0$$

$$w^1_t - w^2_t = \eta \quad \text{where} \quad \eta = U - \phi^2 - \left( \theta_0/\Delta \theta \right) \Psi \quad \text{if} \quad w^1_t = 0 \quad \text{and} \quad w^2_t > 0$$

$$w^1_t - w^2_t = 0 \quad \text{if} \quad w^1_t = 0 \quad \text{and} \quad w^2_t = 0$$

In the model agents are risk-averse, $h' > 0$ and $h'' > 0$. Then for the cases above we have that,

$$h \left( U - \phi^1 + \left( (1 - \theta_0)/\Delta \theta \right) \Psi \right) - h \left( U - \phi^2 + \left( (1 - \theta_0)/\Delta \theta \right) \Psi \right) > \phi^2 - \phi^1$$

$$h \left( U - \phi^1 - \left( \theta_0/\Delta \theta \right) \Psi \right) - h \left( U - \phi^2 - \left( \theta_0/\Delta \theta \right) \Psi \right) > \phi^2 - \phi^1 \quad \text{if} \quad w^1_t > 0 \quad \text{and} \quad w^2_t > 0$$

$$h \left( U - \phi^2 - \left( \theta_0/\Delta \theta \right) \Psi \right) - U - \phi^2 - \left( \theta_0/\Delta \theta \right) \Psi \quad \text{if} \quad w^1_t > 0 \quad \text{and} \quad w^2_t = 0$$

$$0 \quad \text{if} \quad w^1_t = 0 \quad \text{and} \quad w^2_t = 0$$

and then, the costs saved by $P$ because $A$s are intrinsically motivated are higher when agents are risk averse and therefore $CNV^mk_t$ will earlier reach a positive value.

**Proof of Proposition 4**

Setting $U = 0$ we have that $w^1_t = 0$ and $w^2_t = 0$. 

29
From now on we proof the proposition 4 in two steps.

i.- \[ \left[ w^0_t - w^S_t + \theta_1(\Delta w^0 - \Delta w^S) \right] > \left[ \theta_1(\Delta w^0 - \Delta w^S) \right] \] and condition (39) becomes harder to hold than in the case of \( \bar{U} > 0 \) and having a \( CNV_{mk}^t < 0 \) for all \( t = 1, 2, ... \) is more likely.

ii.- For the case in which (39) holds, reasoning as in the proof of proposition 3, it is straightforward to see that \( \bar{w}_t^0 - \bar{w}_t^S \) is the unique source of savings for \( P \) and in any case will be lower (as much equal if LLC applies) than in the case of \( \bar{U} > 0 \). So thus, \( CNV_{mk}^t \) will become positive earlier when \( \bar{U} > 0 \).

**Proof of Proposition 5**

By (13) and (14) we know that,

\[
\begin{align*}
\bar{u}_t &= \bar{U} - \phi_t(w_t, s_0) + \frac{(1 - \theta_0)}{\Delta \theta} \cdot \Psi \\
u_t &= \bar{U} - \phi_t(w_t, s_0) - \frac{\theta_0}{\Delta \theta} \cdot \Psi.
\end{align*}
\]

Applying function \( h(u) = w \) in order to calculate payments we have that,

\[
\begin{align*}
\bar{w}_t &= h\left( \bar{U} - \phi_t(w_t, s_0) + \frac{(1 - \theta_0)}{\Delta \theta} \cdot \Psi \right) \\
w_t &= h\left( \bar{U} - \phi_t(w_t, s_0) - \frac{\theta_0}{\Delta \theta} \cdot \Psi \right).
\end{align*}
\]

Differentiating (49) and (50) with respect to \( \theta_0 \) we have,

\[
\begin{align*}
\frac{\partial \bar{w}_t}{\partial \theta_0} &= h'(\bar{u}_t) \cdot \frac{(1 - \theta_1)}{(\theta_1 - \theta_0)^2} \Psi > 0 \\
\frac{\partial w_t}{\partial \theta_0} &= -h'(u_t) \cdot \frac{\theta_1}{(\theta_1 - \theta_0)^2} \Psi < 0.
\end{align*}
\]

and then higher \( \theta_0 \) implies higher \( \bar{w}_t \) and lower \( w_t \). Differentiating (51) and (52) with respect
to \( \phi_t \) we have that,

\[
\frac{\partial^2 w_t}{\partial \theta_0 \partial \phi_t} = -h''(u_t) \cdot \frac{(1 - \theta_1)}{(\theta_1 - \theta_0)^2} \Psi < 0 \tag{53}
\]

\[
\frac{\partial^2 w_t}{\partial \theta_0 \partial \phi_t} = h''(u_t) \cdot \frac{\theta_1}{(\theta_1 - \theta_0)^2} \Psi > 0 \tag{54}
\]

Interpreting the signs of (51) and (53) we have that, as mean as higher is \( A \)'s intrinsic motivation, an increase of same magnitude in \( \theta_0 \) provokes an increase in \( \overline{w}_t \) of lower magnitude. From the signs of (52) and (54) we interpret that as mean as higher is \( A \)'s intrinsic motivation, an increase of equal magnitude in \( \theta_0 \) provokes an decrease in \( \overline{w}_t \) of lower magnitude. Let \( \theta_0^{LLC} \in (0, \theta_1) \) be the minimum value of \( \theta_0 \) such that \( LLC \) is applied. Then we have that,

\[
\left. \frac{\partial w_t}{\partial \theta_0} \right|_{\theta_0 \geq \theta_0^{LLC}} = 0
\]

This establishes that, for higher values of \( \theta_0 \) the benefits from choosing \( s_0 = S \) come only from the lower impact the higher \( \theta_0 \) has on \( \overline{w}_t \) when agents intrinsic motivation is higher. Thus, with \( \theta_0 \) more close to \( \theta_1 \) more intrinsically motivated agents imply higher values of \( CNV_{mk}^t \) for every \( t = 0, 1, 2, \ldots, T, \ldots \)

References


