A THEORY OF REFERENCE-DEPENDENT BEHAVIOR

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D.T.2004/02

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Abstract. There is extensive field and experimental evidence in a wide variety of environments showing that behavior depends on a reference point. This paper provides an axiomatic characterization for such behavior. Our approach is dual, we study choice behavior and preference relations. We proceed by gradually imposing more structure on behavior, requiring higher levels of rationality, that free the decision-maker from certain types of manipulations. Depending on the phenomena one wants to model, one degree of behavioral structure will be appropriate or another. We provide two applications of the theory: one to model the status-quo bias, and another to model addictive behavior.

Keywords: Individual rationality, reference-dependence, rationalization, path-independence, menu-independence, status-quo bias, addiction, habit formation.

1. Introduction

The last decades have accumulated extensive evidence to suggest that preferences in a variety of settings are influenced by a reference point. The reference point may be interpreted as the default choice as in *status quo bias* or *endowment effect* literature; the aspiration level as in *aspiration adaptation* models; the convention, norm, or belief about what one should choose, as, for example, in *cognitive dissonance* studies; the past consumption as in *addiction*, *habit formation*, *status-seeking*, or *brand loyalty* models; etc. What these cases have in common is that there is overwhelming evidence to document them, and that deviations from the standard rational choice theory are made in a predictable way.

In this paper we provide an axiomatically-based theory for the study of reference-dependent behavior. We focus on those cases where, if it exists, the reference point is in itself an element of the choice set.

Date: February, 2004.

We gratefully acknowledge financial support by the Spanish Commission of Science and Technology (CICYT, SEC2003-08105).

Furthermore, the reference point changes with each choice, in the sense that the last choice is the new reference point. Later, in section 5.3 we will allow the reference point to be a function of the complete choice history. Many of the above examples belong to this type of cases, although they by no means cover them all.[1]

Our axiomatic approach is dual. We study reference-dependent behavior by exploring properties on reference-dependent *choice correspondences*, and reference-dependent *preferences*. In the former case, a choice problem is defined as a pair (T, s) where T is a subset of X, the universal set of alternatives and, following Masatlioglu and Ok (2003), either $s \in T$, in which case we say that the choice problem has a reference point s, or $s \notin X$, in which case we say that (T, s) represents a choice problem without a reference point, paralleling the standard case. In the preference approach we assume a collection, or in the terminology of Tversky and Kahneman (1991), a *book* of preference relations, one for each of the elements in the universal set of alternatives X, and one binary relation that represents the preferences when there is no reference point, again paralleling the standard case. That is, we consider situations with and without a reference point. As a quick illustration consider the situation where agent A is about to buy a car. Her preferences over the set of feasible cars may depend on whether she currently owns, say, a classic SAAB-900, or no car at all. In the former case it may well be that A is biased to keep her classic car, and only if there is an exceptionally good car that beats the classic in all dimensions, will A may be willing to buy it. On the other hand, if A does not own the classic car, but it belongs to the set of feasible cars, A may well prefer a standard new car.

We proceed by gradually imposing more structure on reference-dependent behavior, requiring higher levels of rationality that free the decision-maker from certain types of manipulations. We start the analysis by studying the properties that are needed to rationalize reference-dependent choice correspondences. That is, we explore the conditions that a book of preference relations needs to satisfy in order to give the same choice behavior as the choice correspondence. We argue that this consistency requirement is a *minimal* rationality condition to be imposed on a theory of reference-dependent behavior. By extending properties α, β, and the weak axiom of revealed preferences to choice problems with reference points, we obtain that a choice correspondence

[1]See Rubinstein and Zhou (1999) for an example of a reference-dependent model where the reference point does not necessarily belong to the choice set. We will discuss this further in the conclusions.

satisfies properties α and β, if and only if, it satisfies the weak axiom, if and only if, there exists a book of complete preorder relations that rationalizes the choice correspondence. These results are in line with those in the classical theory. However, we will show that we deviate in some important points from the classical results. For example, we do not find uniqueness in the existence of the book of binary relations that rationalizes the choice correspondence.

From this minimal rationality requirement, we proceed to identify conditions that guarantee that a reference-dependent decision maker is invulnerable to certain types of manipulation. We first study *undominated behavior*. By undominated behavior we refer to situations where there are no cycles involving at least a strict link. For example, let an agent be confronted with a choice problem involving two alternatives, x and y, and let us assume that when she chooses from reference point x she strictly prefers y and vice versa. The agent will be switching from x to y, from y to x, and so on. Clearly, such erratic behavior is not consistent even with minimal rationality requirements. We identify the axioms on preferences and on choice correspondences that, in addition to those mentioned above, guarantee undominated behavior.

Furthermore, we say that behavior is *path-independent* if, given a choice set, final choices (to be formally defined below) cannot be influenced by the initial choice of reference point. Our next result, building on those already mentioned, characterizes what sort of choice correspondences and books of preferences satisfy path-independence.

Finally, a natural step further in the study of the rationality of reference-dependent behavior is the analysis of when behavior is not only independent of the initial reference point in a given choice set, but also independent of the way in which the choice problem is presented. That is, we will say that behavior is *menu-independent* whenever final choices do not depend on the sequence in which choice problems are presented.

After imposing all this structure the immediate question that arises is whether behavior is still reference-dependent, or collapses instead into a unique order, as in the classical setup. We will show by means of an example that in fact there is in fact still room for reference point dependence.

We end the paper by applying the theory derived here to two well-established phenomena; status quo bias and addiction. We show that, although their characterizations differ, choice behavior subject to the status quo bias and to addiction can be rationalized by a book of complete preorders, that satisfies undominated behavior, without being necessarily path-independent or menu-independent.

We close this introduction by connecting our paper to the theoretical literature. To the best of our knowledge Tversky and Kahneman (1991) provide the first explicit model of reference-dependent behavior. The model is an extension of Kahneman and Tversky's (1979) prospect theory to risk-less choices with the aim of modelling the status quo bias and related issues. They work with a book of reference-dependent preference relations on a two-dimensional commodity space, and *assume*, instead of *axiomatically deriving*, certain types of behavior (such as loss aversion) that lead to the status quo bias.

Munro and Sugden (2003) revise and extend Tversky and Kahneman's (1991) model to the n-dimensional commodity space. Their primary aim is to simplify Tversky and Kahneman's model in order to provide a formulation closer to the classical setup. Sagi (2002) and Sugden (2003) constitute further extensions of the original model. In the former, an axiomatic characterization of Tversky and Kahneman's model in risky choices is provided. Sugden, on the other hand, proposes a generalization of Savage's expected utility theory to the case where preferences are reference-dependent.

A second strand of the literature follows the revealed preference approach pioneered by Zhou (1997). He provides an axiomatic foundation of the status quo bias for choice *functions*. His approach has been followed by Bossert and Sprumont (2001), and by Masatlioglu and Ok (2003). Bossert and Sprumont consider status quo choice correspondences that select all those alternatives that are at least as good as the status quo. Masatlioglu and Ok provide a powerful characterization result of the status quo bias (see Theorem 5.1 and Corollary 5.2 below), with the innovation of also considering choice problems *without* a status quo. We adopt this innovation in our general framework of reference-dependent behavior.

Our paper is novel in studying reference-dependent behavior from a dual perspective, and in providing a general axiomatic baseline that starts by imposing minimal rationality assumptions, and then gradually incorporate more structure into reference-dependent behavior for riskless settings. It is our hope that the theory we present here will prove useful for the analysis of reference-dependent phenomena other than the status quo bias or endowment effect. As an illustration, we apply the theory to model addictive and habit formation behavior.

The paper is organized as follows. Section 2 introduces the notation that will be used subsequently. In section 3 we study the rationalization of reference-dependent choice correspondences and related issues. Section 4 contains the analysis of the different types of manipulations to which reference-dependent decision makers may be subject. In section 5 we develop two applications. Section 6 presents the conclusions and all the proofs are given in section 7.

2. Notation

Let X be a nonempty set of elements, representing the alternative space, and $\mathcal{X}$ the set of all nonempty subsets of X. For notational convenience, and following Masatlioglu and Ok (2003), we use $\diamond$ to denote an element such that $\diamond \notin X$. It will be convenient to define $X^* = X \cup \{\diamond\}$. A choice problem is a pair (T, s), where $T \in \mathcal{X}$, and either $s \in T$ or $s = \diamond$. When $s \in T$ we say that (T, s) is a choice problem that depends on a reference point, and when $s = \diamond$ we say that (T, s) is a choice problem that does not depend on a reference point. $\mathcal{C}(X)$ represents the set of all possible choice problems (T, s), while $\mathcal{C}_{\text{rd}}(X)$ denotes the set of all possible choice problems with a reference point.

A choice correspondence $c(T, s)$ assigns for every (T, s) a subset of T in $\mathcal{X}$. Hence, $c : \mathcal{C}(X) \to \mathcal{X}$, where $c(T, s) \subseteq T$ for all $(T, s) \in \mathcal{C}(X)$. Note that, as usual, c is by definition a non-empty valued mapping.

We now introduce reference-dependent preference relations. For every $s \in X^*$ denote by $\succeq_s$ a binary relation on X, $\succeq_s \subseteq X \times X$. To the collection of all such binary relations we denote $\{\succeq_s\}_{s \in X^*}$. We refer to this collection as the book of reference-dependent binary relations associated to X^*.

Binary relations $\succ_s$ and $\sim_s$ are the asymmetric and symmetric parts of $\succeq_s$, respectively. Hence, $x \succ_s y$ if and only if $x \succeq_s y$ and $\neg(y \succeq_s x)$, and $x \sim_s y$ if and only if $x \succeq_s y$ and $y \succeq_s x$. We will say that a binary relation is a preorder if it is reflexive and transitive, and a partial order if it is an antisymmetric preorder. For any (T, s), $M(T, \succeq_s)$ denotes the set of maximal elements in T with respect to $\succeq_s$, that is, $M(T, \succeq_s) = \{x \in T : y \succ_s x \text{ for no } y \in T\}$.

3. Rationalization of reference-dependent choice correspondences

By rationalization of a choice correspondence c on $\mathcal{C}(X)$ we mean that there is a book of binary relations $\{\succeq_s\}_{s \in X^*}$ such that, for any choice problem $(T, s) \in \mathcal{C}(X)$, $c(T, s) = M(T, \succeq_s)$. That is, the question of the rationalization of reference-dependent choice correspondences addresses the fundamental problem of *the existence of a book of binary relations that is consistent with choice behavior.*

In a classic article, Arrow (1959) shows that in the context of choice problems without a reference point, Samuelson's weak axiom of revealed preference (WARP) guarantees that there is a unique complete preorder relation $\succeq_\diamond$ that rationalizes $c(\cdot, \diamond)$. Based on the version of Mas-Colell, Whinston, and Green (1995), we now present a variant of the weak axiom within the context of choice problems with a reference point.

Weak Axiom of Revealed Preference (WARP): For any $(T, s) \in \mathcal{C}(X)$ with $x, y \in T$ and $x \in c(T, s)$, then for any $(V, s) \in \mathcal{C}(X)$ with $x, y \in V$ and $y \in c(V, s)$, it must be that $x \in c(V, s)$.

It is well-known that, in the standard context, the weak axiom is equivalent to Sen's (1969) properties α and β. Consider the following adaptation of properties α and β, due to Masatlioglu and Ok (2003).

Property α: For any $(T, s), (V, s) \in \mathcal{C}(X)$, if $y \in V \subseteq T$ and $y \in c(T, s)$, then $y \in c(V, s)$.

Property β: For any $(T, s) \in \mathcal{C}(X)$, if $z, y \in c(T, s)$, $T \subseteq V$, and $z \in c(V, s)$, then $y \in c(V, s)$.

We show that the versions of properties α and β for reference-dependent contexts are equivalent to the version of WARP introduced above. It turns out that properties α and β (and consequently WARP) imply the existence of a book of complete preorders that rationalizes c. Furthermore, we show that the existence of such a book of complete preorder relations implies that the choice correspondence c satisfies those variants of properties α and β.

We argue, therefore, that properties α and β constitute *minimal* rationality assumptions for a theory of reference-dependent behavior. These properties impose a minimal degree of consistency on choice behavior while, at the same time, allowing a great deal of freedom to the influence of the reference point on preferences. Consider the following example.

Example 3.1. *Let* $X = \{x, y\}$, $\{x\} = c(X, y)$ *and* $\{y\} = c(X, x)$.

While c clearly satisfies properties α and β, the orderings from x and y are opposite. In subsequent sections we will draw from this basic rationality assumptions to impose more structure on choice behavior.

In particular, the choice behavior implied in the above example will be forbidden on the grounds of manipulation-free arguments.

We can now establish our first result.

Theorem 3.2. *For any choice correspondence c on $\mathcal{C}(X)$, the following three statements are equivalent:*

- *c satisfies axiom WARP.*
- *c satisfies properties α and β.*
- *There exists a book $\{\succeq_s\}_{s\in X^*}$ of complete preorder relations that rationalizes c.*

It is interesting to note that from the proof of Theorem 3.2 it becomes clear that only weaker versions of transitivity and completeness are needed. We call these versions weak transitivity and weak completeness:

Weak Transitivity (WT): (i) For any $x, y, z, s \in X$ such that $\neg(s \succ_s x)$ and $\neg(s \succ_s z)$, if $x \succeq_s y \succ_s z$, then $x \succ_s z$. (ii) For any $x, y, z \in X$, and $s = \diamond$, if $x \succeq_\diamond y \succeq_\diamond z$, then $x \succeq_\diamond z$.

Weak Completeness (WC): (i) For any $x, z, s \in X$ such that $\neg(s \succ_s x)$ and $\neg(s \succ_s z)$, either $x \succeq_s z$, or $z \succeq_s x$, or both. (ii) For any $x, z \in X$, and $s = \diamond$, either $x \succeq_\diamond z$, or $z \succeq_\diamond x$, or both.

WT and WC require that, when there is no reference point, the standard conditions are imposed. However, when there is a reference point $s \in X$, WT and WC only impose restrictions when alternatives x and z are no worse than the reference point s. That is, in a way, WT and WC only apply to cases that are "above" the reference point. Transitivity and completeness of the book of binary relations could likewise be relaxed in all the results that follow.

It becomes clear from the proof and our previous comment that, contrary to the classic result of Arrow, the book of complete preorders rationalizing c need not be unique. Take the binary relation R_s defined in the proof of Theorem 3.2. That is, xR_sy if and only if $x \in c(\{x, y, s\}, s)$. Non-uniqueness arises as a result of the fact that, when alternatives $x, y, s \in X$ are all distinct, then it may well be that $\{s\} = c(\{x, y, s\}, s)$, and therefore $\neg(xR_sy)$ and $\neg(yR_sx)$. Hence, R_s may be non-complete, and therefore its completion admits different versions.

A final note on Theorem 3.2 is that it is easy to show that all axioms are independent. This is true for all the results contained in this paper. Since the proofs are very simple we omit them here, but we can provide them upon request.

We close this section by addressing an important related question that has to do with the relation between binary relations and choice behavior: when does a book of binary relations generate choice behavior that satisfies properties α and β? In line with classical results, the following proposition states that the only requirement is that the book be comprised of complete preorder relations. Since the proof of this result is straightforward we omit it.

Proposition 3.3. *If $\{\succeq_s\}_{s\in X^*}$ is a book of complete preorder relations, then M satisfies properties α and β.*

The following example shows that the converse is not generally true. Let $X = \{x, y, z, s\}$, and $s \succ_s x$, $s \succ_s y$, and $s \succ_s z$. Then, $M(\cdot, \succ_s)$ satisfies properties α and β, but it may well be that $x \succeq_s y \succeq_s z$ and $z \succ_s x$. This example reinforces the idea that when there is a reference point $s \in X$, the orderings "below" the reference point s are inessential, which could cause intransitivities.

4. On Manipulating a reference-dependent Decision-Maker

A decision maker whose behavior is reference-dependent may be subject to different types of manipulations. In this section we study three such types: dominated behavior, dependence on the starting point, and dependence on the sequence of presentation of choice problems. We will study the properties that guarantee behavior to be free of these kinds of manipulation. These properties gradually impose more structure on behavior by demanding higher levels of rationality. Contingent on the phenomena to which the theory is applied, it will be convenient to impose one type of behavior or another.

Typically, one of the first rationality demands is the requirement that behavior be undominated. Example 3.1 above shows a case of dominated behavior. Such a decision maker is never satisfied and hence may always be willing to change from x to y, from y to x, and so on ad infinitum. Clearly, such erratic behavior is not consistent even with minimal rationality requirements. In section 4.1 we identify the conditions that guarantee that a decision maker is free of this sort of cycle.

These are not the only traps that a reference-dependent decision maker may be involved in, however. The agent may exhibit undominated behavior, while being subject to the manipulation of an external agent that may exert some influence on her "final" choice (to be precisely defined below). Consider the following example.

Example 4.1. *Let* $X = \{x, y\}$*,* $\{y\} = c(X, y)$ *and* $\{x\} = c(X, x)$*.*

This time the decision maker exhibits undominated behavior but, when confronted with choice problem (X, x) selects x, and when presented with choice problem (X, y) she chooses y. That is, given the choice set X her choice depends on whichever reference point prevails. Anyone who is able to impose one reference point or the other has the power to dictate the final choice of the decision maker. When a subject is free of this sort of manipulation we say that her behavior is "path-independent". It is convenient to note, however, that this type of manipulation is not necessarily negative. That is, given that the behavior of a subject is path-dependent, she can be manipulated in such a way as to increase her well-being (see footnote 11). The properties that guarantee path-independence are analyzed in section 4.2.

Finally, while it is conceivable that a subject with undominated behavior also satisfies path-independence, her final choices may still depend on the particular sequence of presentation in the choice problems. In section 4.3 we will study this case under the rubric of "menu-independence".

The following definition will be helpful in the analysis that follows.

Definition 4.2. A sequence of reference-dependent choice problems $(T, s_i)_{i=0}^{n}$ with $s_i \in T$ is said to be an *RD-chain* if $s_i \in c(T, s_{i-1})$ for all $i = 1, 2, \ldots, n$. An RD-chain is *cyclical* if $s_0 \in c(T, s_n)$. An RD-chain is *strict* if there exists k in $\{0, 1, \ldots, n-1\}$ such that $s_k \notin c(T, s_k)$.

Clearly, any RD-chain with $i \geq 1$ in Example 3.1 is a strict, cyclical RD-chain, while any RD-chain in Example 4.1 is a cyclical but non strict RD-chain. The next example shows the case of a strict RD-chain that is non cyclical.

Example 4.3. *Consider the set* $X = \{0,1\} \times \mathbb{N}$ *where* $\mathbb{N}$ *is the set of natural numbers, and the function* $f : X \longrightarrow \mathcal{X}$ *defined as follows:*

$$f(x) = f((x_1, x_2)) = (\{x_1\} \times [\frac{x_2}{k}, kx_2]) \cap X,$$

where $k > 2$. *Take a choice correspondence such that for any* $(T, s) \in \mathcal{C}_{\text{rd}}(X)$, *it is* $c(T, s) = z$ *where* z *is the greatest vector in the set* $T \cap f(s)$.

The behavior described in the above example corresponds to an agent who, when endowed with a reference point $s = (s_1, s_2) \in X$: (i) follows a "social norm" or a "convention" $s_1 \in \{0, 1\}$, and (ii) maximizes the consumption of a "good" in a neighborhood of its reference point $s_2 \in \mathbb{N}$. Hence, it follows that any RD-chain with $i \geq 1$ is non-cyclical and strict.

Note that in Definition 5.6 it is stated that $s_i \in T$, hence one may wonder about RD-chains with $s_i = \diamond$. Note, however, that since $c(T, \cdot) \subseteq T$ then $s_i = \diamond$ is only possible when $i = 0$, but then, for our purposes, one can take the RD-chain following $s_0 = \diamond$.

Finally, we would like to stress that, implicit in the analysis that follows, it is assumed that the decision-maker is "myopic", that is, she does not anticipate how her choice behavior will react to current choices. Although this may be a strong assumption for a fully rational theory of decision, it seems a sensible assumption for a boundedly rational theory.

4.1. **Undominated Behavior.** Undominated behavior is often imposed as a very first rationality requirement. For its motivation, consider the following "money-pump" argument. Take, in the context of the classical formulation: $x \succ y \succ z \succ x$. Suppose the agent starts with option x. Since $z \succ x$, the agent could pay a small enough sum to obtain z instead of x. After this trade, the agent could, by similar reasoning, obtain y and eventually return to x. The agent comes back to the original element, but with her money endowment diminished, and would again for a small enough sum be willing to trade x for z.[2]

The latter is a clear example of the possible irrationality of intransitive agents, who can be manipulated into sequences of trades. Of course, this argument can apply not only to triples but to any sequence of elements and also to cases in which only one of the links among the

[2]There are however many discussions on the general validity of the money-pump argument (see for example Raiffa 1968, Fishburn 1988, or Mandler 2001). In particular, it could be argued that in order for the money-pump argument to be valid, preferences should be defined over the Cartesian product of money and alternatives.

options is strict. Undominated behavior in the context of reference dependence can be directly stated in the form of the following property.

Crossed Transitivity (CT): For any collection $s_0, s_1, \ldots, s_n \in X$ such that $s_n \succsim_{s_{n-1}} s_{n-1} \succsim_{s_{n-2}} s_{n-2} \ldots s_1 \succsim_{s_0} s_0$ with at least one strict inequality, we must have $s_n \succ_{s_n} s_0$.

Notice that in this context a sequence of trades implies that the agent agrees to give up her current reference alternative. Crossed Transitivity establishes that when an agent's reference point is the last alternative in the sequence, namely s_n, she should strictly prefer the latter to the initial alternative s_0. Further, it is convenient to stress the generality of property CT. It involves finite sequences of any length, where all but one of the binary comparisons may be indifferent comparisons. Obviously, if all were indifferent binary comparisons, then no alternative is dominated.

In the next result we show the choice structure of an agent endowed with complete preorders that satisfy Crossed Transitivity. To do so we only need to discuss a particular class of choice sequences.

Non-Strict Cycles (NSC): There is no RD-chain associated to the choice correspondence c that is cyclical and strict at the same time.

Notice that the previous definition involves only choice problems *with the same set of available options*.[3] We show next that this is sufficient to guarantee that the book of complete preorders $\{\succsim_s\}_{s \in X^*}$ that rationalizes c satisfies CT.

Theorem 4.4. *For any choice correspondence c on $\mathcal{C}(X)$, the following two statements are equivalent:*

- *c satisfies properties α, β and NSC.*
- *There exists a book $\{\succsim_s\}_{s \in X^*}$ of complete preorder relations that rationalizes c, and that satisfies CT.*

Theorem 4.4 constitutes the second step in the imposition of a rationality structure on reference-dependent behavior. We now go a step further and study other sources of manipulation.

[3]Houthakker (1950) already proposed a property, "semitransitivity", that restricts the relation between the first and last elements of a chain. In his version, however, the choice set may change from one link to the other.

4.2. **Path-Independence.** We will say that behavior is path-independent if, for any given T in $\mathcal{X}$, "final" choices do not depend on an initial reference point. That is, path-independence implies that, for any given set of alternatives T, the fact that the RD-chain starts from $s \in T$ or from $x \in T$ is inessential. Clearly, path-independence imposes a great deal of consistency between the preference orders of different reference points. This leads naturally to the following question: Is it still the case that reference-dependent behavior that is rationalizable, and undominated, and that satisfies path-independence may depend on a reference point, or does it instead collapse into a unique order as in the classical setup? We will see that behavior may in fact still depend on reference points. This shows that behavior exhibiting a great deal of rationality may still show reference point dependency.

The following definition states precisely what we mean by "final" choices.

Definition 4.5. Set L is a *limit set* of problem $(T, s_0) \in \mathcal{C}(X)$ if the following two properties are satisfied:

- For any RD-chain $(T, s_i)_{i=0}^n$ containing a cyclical chain $(T, s_i)_{i=j}^n$, $j \in \{0, 1, \ldots, n\}$, it is $\{s_j, s_{j+1}, \ldots, s_n\} \subseteq L$.
- For any $z \in L$, there exists an RD-chain $(T, s_i)_{i=0}^n$ containing a cyclical chain $(T, s_i)_{i=j}^n$ and $z \in \{s_j, s_{j+1}, \ldots, s_n\}$.

A limit set of a choice problem (T, s_0) is the collection of alternatives in T within any RD-chain starting from (T, s_0) continues cyclically. Consider the following example.

Example 4.6. *Let* $X = \{x, y, z\}$, $c(X, x) = c(X, y) = \{x, y\}$, *and* $c(X, z) = \{y\}$.

Starting from (X, z) the following RD-chains are possible:

(1) $(X, z), (X, y), (X, y), \ldots, (X, y), \ldots$
(2) $(X, z), (X, y), (X, x), (X, y) \ldots, (X, x), (X, y), \ldots$
(3) $(X, z), (X, y), (X, x), (X, x), \ldots, (X, x), \ldots$

That is, the decision-maker may continue cyclically to choose alternative y when the reference point is y, or repeatedly switch back and forth between x and y, or stick with x. Clearly, no cyclical chain starting from (X, z) goes through z. Therefore, the limit set of (X, z) is $\{x, y\}$. Hence, the limit set L of a choice problem (T, s) can be regarded as the set of *possible final* choices.

In Example 4.3, for any $y \in X$ there exists no limit set of (X, y). This is because there exists no cyclical chain. The decision maker in Example 4.3 sticks to her norm and locally maximizes the consumption of a good, increasing its consumption level choice by choice. In Example 3.1 the limit sets of (X, x) and (X, y) are respectively $\{y\}$ and $\{x\}$, and in Example 4.1 the limit sets of (X, x) and (X, y) are $\{x\}$ and $\{y\}$.

Note that NSC implies that any set of alternatives V that belongs to a limit set L and that constitutes a cyclical chain must be non-strict, or put differently, every mind-change about the preferred alternative corresponds to an indifference comparison. This, however, it is not equivalent to the fact that all alternatives in V are indifferent from any reference point in V.

The next axiom formally states the path-independence condition on choice correspondences.

Limit Set Uniqueness (LSU): For any $T \in \mathcal{X}$ and for any $x, y \in T$, if L is a limit of (T, x), then L is a limit of (T, y).

On the side of the reference-dependent preferences we introduce the following condition, which constitutes a direct restriction on the difference between the orders of distinct reference points. For the moment, we will refer to it as ISQP. The choice of this term will be clarified in Section 5.1.

ISQP: For any $x, y \in X$, $x \succ_x y$ implies $x \succ_y y$.

We can now present our next result.

Theorem 4.7. *For any choice correspondence c on $\mathcal{C}(X)$, the following two statements are equivalent:*

- *c satisfies properties α, β, NSC and LSU.*
- *There exists a book $\{\succeq_s\}_{s \in X^*}$ of complete preorder relations that rationalizes c, and that satisfies CT and ISQP.*

To illustrate Theorem 4.7 consider Example 4.3. It is easy to see that it satisfies properties α, β and NSC, but it is not path-independent. Take any *finite* choice set T containing both elements of norm 0 (say $x = (0, x_2)$ where $x_2 \in \mathbb{N}$) and of norm 1 (say $y = (1, y_2)$, $y_2 \in \mathbb{N}$). Clearly, the limit set of any (T, x) corresponds to the maximum of all $x \in T$ according to the natural order $\leq$ with $x_1 = 0$, while the limit set of any (T, y) is the maximum of all $y \in T$ with $y_1 = 1$.

With regard to preferences, it is immediately clear that any book of reference-dependent preferences that rationalizes this choice structure does not satisfy ISQP. That is, from the point of view of norm 0, every alternative following norm 1 is inferior, and viceversa.

Examples 3.1 and 4.1 are not path-independent either. Final choices depend on initial reference points. Example 4.6 satisfies path-independence if and only if we have that $x \succ_z z$. Provided $x \succ_z z$ holds, then for any binary relation $\succeq_\diamond$[4] not only does it satisfy path-independence, but it also describes rationalizable and undominated behavior, while at the same time revealing differences in the orderings of $\succeq_x$ and $\succeq_z$.[5] This shows that while path-independence together with rationalizable and undominated behavior imposes a great deal of structure on reference-dependent behavior, there is still room for some degree of reference point influence.

4.3. **Menu-Independence.** So far we have studied the independence of final choices from initial reference points *given a set* $T \in \mathcal{X}$, but what happens when the choice set can also vary? We will tackle this issue here, but first let us introduce a last bit of notation.

The limit set of choice problem (T, s) will be denoted by $L(T, s)$. Hence, if behavior is path-independent we can write: for any $T \in \mathcal{X}$, and every $s, y \in T$, $L(T, s) = L(T, y) = L(T)$. We will say that behavior is *menu-independent* if, given the set of possible choice problems $\mathcal{C}(X)$, final choices do not depend on starting from a particular choice problem $(T, s) \in \mathcal{C}(X)$. Clearly, path-independence is a necessary condition for menu-independence. That is, if there are choice problems (T, s) and (T, y), with $s \neq y$, such that $L(T, s) \neq L(T, y)$, menu-independence is violated. Hence, we can formally state menu-independence as follows.

Plott Independence (PI): For any $T, V \in \mathcal{X}$, $L(T \cup V) = L(L(T) \cup L(V))$.

Clearly, our PI condition resembles the well-known Plott's (1973) independence condition.[6] PI states that the limit set, that is the set of possible final choices, depends neither on starting from a particular

[4] Actually, for any binary relation $\succeq_\diamond$ consistent with properties α and β.

[5] And since we also leave $\succeq_\diamond$ 'almost' free there are more sources of divergence between orders.

[6] A note on the terminology used in this paper. The distinction between *path*- and *menu*-independence is a useful distinction in the present framework with reference points. This was of course unnecessary in the classical setup.

reference point, nor from a particular decomposition of the choice set. We will now see that LSU is not only a necessary condition for PI, it also turns out that, in combination with NSC, it is a sufficient condition. To state the reverse implication we need to introduce a very weak property:

Weak-NSC: When the opportunity set has two elements, there is no RD-chain associated to the choice correspondence c that it is cyclical and strict at the same time.

In so far as the Weak-NSC property only applies to binary choice sets, it constitutes a considerable relaxation of NSC. We can now establish the following equivalence.

Theorem 4.8. *For any choice correspondence c on $\mathcal{C}(X)$ that satisfies properties α and β, the following two statements are equivalent:*

- *c satisfies properties NSC and LSU.*
- *c satisfies properties Weak-NSC and PI.*

Consider Example 4.6, let $x \succ_z z$ hold, and $\succeq_\diamond$ be consistent with properties α and β. Then, behavior is rationalizable, undominated, and satisfies path-independence. Menu-independence implies that, e.g., if we let our decision-maker first reach final choices from set $\{x, y\}$, then from set $\{y, z\}$, and then from the final choices resulting from the previous two problems, the conclusion will be the same as if we directly present her with the choice set $\{x, y, z\}$. And this is in fact the case: $L(L(\{x, y\}) \cup L(\{y, z\})) = L(\{x, y\}) = \{x, y\} = L(\{x, y, z\})$.

Hence, path-independence is quite a strong condition to impose on reference-dependent behavior. It not only requires that, in a given choice set T, reference points do not influence final choices, but also implies that the way in which the choice set is presented is inessential.

Note that since we are not imposing any extra property for menu-independence, when $x \succ_z z$ holds our conclusion for Example 4.6 remains valid. That is, behavior that is rationalizable, undominated, path-independent, and menu-independent does not necessarily collapse into a unique ordering; reference points still matter. It is interesting to note, however, that even in the case where the book of preference relations collapses into a unique ordering, the cardinal representation of preferences may differ between reference points. That is, preferences may be ordinally but not cardinally equivalent.

5. Applications

5.1. **Status quo bias.** There is a large and still growing experimental and field literature supporting the view that a decision maker typically values an alternative more highly when it is regarded as the status quo, than otherwise. Versions of this are called *status quo bias*, or *endowment effect*.[7] An illuminating experiment that nicely illustrates this bias is the following one, due to Knetsch (1989). A number of participants was *randomly* divided into three groups; call them group C, group M, and group N. Those in group C received from the experimenter a candy bar, those in group M received a mug, and those in group N got nothing. Then, participants in groups C and M were given the opportunity to change at zero cost their original object for the other. They simply had to express their wish to change the object, and the experimenter would immediately satisfy it. Participants in group N were simply given the opportunity to choose between a candy bar and a mug. The results are surprising. Preferences of subjects in group N were more or less evenly divided, while the great majority in groups C and M expressed no desire to change (90% of participants in both C and M), showing a strong status quo bias.

The many different versions of the above experiment that have been run have yielded similar results, which suggests a high level of robustness in this finding. The implications of the status quo bias are far more than anecdotal. This bias implies a significant discrepancy in willingness to pay and willingness to accept, as exemplified in the experiments of Kahneman, Knetsch, and Thaler (1990) and Bateman, Munro, Rhodes, Starmer, and Sugden (1997). It also implies a difference in the evaluation of opportunity costs, as opposed to costs of any other nature (see Thaler, 1980). Furthermore, the status quo bias implies that property rights influence the valuation of an object, and hence question the so-called Coase Theorem (see Kahneman, Knetsch, and Thaler, 1990).[8]

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[7] Thaler (1980) was the first to report this phenomenon. Other important experimental and field studies on the subject include Knetsch and Sinden (1984), Samuelson and Zeckhauser (1988), Knetsch (1989), Tversky, Slovic, and Kahneman (1990), Kahneman, Knetsch, and Thaler (1990, 1991), and Bateman, Munro, Rhodes, Starmer, and Sugden (1997). For reviews see Camerer (1995), and Rabin (1998).

[8] One wonders whether "presumption of innocence", which is common practice in the legal systems of many countries, is an implicit recognition of the role of the status quo bias in judgment. According to the status quo bias, the practice of the "presumption of innocence" would reduce the risk of convicting an innocent

In this section we apply the theoretical framework presented earlier to model the status quo bias. In particular, we will show that a choice correspondence that represents the status quo bias is rationalizable by a book of complete preorders that is consistent with undominated behavior, but not necessarily path-independent, or menu-independent.

In a recent paper, Masatlioglu and Ok (2003) nicely characterize a choice correspondence representing the status quo bias. Here, we will derive on the book of binary relations the set of axioms that are equivalent to theirs. For the sake of completeness in the exposition, we introduce Masatlioglu and Ok's (2003) axioms below. For their motivation, however, we refer to their paper.

Dominance (D): For any $(T, s) \in \mathcal{C}(X)$, if $\{y\} = c(T, s)$ for some $T \subseteq V$, and $y \in c(V, \diamond)$, then $y \in c(V, s)$.

Status-quo Irrelevance (SQI): For any $(T, s) \in \mathcal{C}_{\text{rd}}(X)$, if $y \in c(T, s)$ and there does not exist any non-empty $V \subseteq T$ with $V \neq \{s\}$ and $s \in c(V, s)$, then $y \in c(T, \diamond)$.

Status-quo Bias (SQB): For any $(T, s) \in \mathcal{C}(X)$, if $y \in c(T, s)$, then $\{y\} = c(T, y)$.

We now introduce the set of axioms needed to reproduce Masatlioglu and Ok's (2003) results on the book of binary relations. We start with an independence axiom.

Upper Independence (UI): For any $x, y \in X \backslash \{s\}$, such that $x \succ_s s$ and $y \succ_s s$, then $x \succeq_\diamond y$ if and only if $x \succeq_s y$.

This axiom restricts the influence of the reference point in evaluating pairs of alternatives. If x is evaluated to be at least as good as y when there is no reference point, then for a reference point s such that x and y are strictly preferred to s, x cannot deteriorate relative to y. Note, however, that the restrictions imposed by UI are limited. UI restricts the ordering of pairs of alternatives x and y when the qualifier "$x \succ_s s$ and $y \succ_s s$" holds.

Consider now the following two axioms.

person, which is typically regarded as a worse scenario than acquitting someone who is guilty.

Status-quo Positivity (SQP): For any $s \in X$ and $y \in X \backslash \{s\}$, if $s \succeq_\diamond y$ then $s \succ_s y$.

Status-quo Positivity 2 (SQP2): For any distinct $x, y, s \in X$, if $x \succ_s s \succ_s y$, then $x \succ_x y$.

These properties are easily justified by the empirical regularity that we attempt to model here. Property SQP establishes that if in the absence of a reference point, alternative s is valued to be as at least as good as y, when s happens to be the reference point its valuation with respect to y cannot only not deteriorate, it can only improve. Property SQP2 extends the influence of the status quo bias to some cases when there is a reference point. That is, if x is preferred to y from the perspective of the reference point s, only when the reference point s is in between of x and y, SQP2 states that the evaluation of x cannot deteriorate with respect to y when x is itself the reference point.

We now introduce the final axiom we need.

Status-quo Singularity (SQS): For any $s, y \in X$, if $s \sim_s y$, then $s = y$.

Property SQS strengthens the role of the status quo bias. It is a kind of antisymmetry that applies only to comparisons with the reference point. It imposes either that y is strictly preferred to the reference point s, or vice versa. SQS guarantees that, whenever the reference point is chosen, it is uniquely chosen.

We can now present the following result.

Theorem 5.1. *For any choice correspondence c on $\mathcal{C}(X)$, the following two statements are equivalent:*

- *c satisfies properties α and β, and axioms D, SQI, and SQB.*
- *There exists a book $\{\succeq_s\}_{s \in X^*}$ of complete preorder relations that rationalizes c, and that satisfies UI, SQP, SQP2, and SQS.*

Theorem 5.1, together with Masatlioglu and Ok's (2003) Lemma 1 and Theorem 1, immediately imply the following corollary.[9]

[9]For any $T \in \mathcal{X}$, $M(T, \succeq) = \{s \in T : y \succ s \text{ for no } y \in T\}$, and for any $s \in X$, $U_\succ(s) = \{y \in X : y \succ s\}$.

Corollary 5.2. *For any choice correspondence c on $\mathcal{C}(X)$, the first statement implies the second:*

- *There exists a book $\{\succeq_s\}_{s\in X^*}$ of complete preorder relations that rationalizes c, and that satisfies UI, SQP, SQP2, and SQS.*
- *There exist $\succeq \subseteq \succeq^*$, where $\succeq$ is a partial order and $\succeq^*$ is a completion of $\succeq$, such that $c(\cdot,\diamond) = M(\cdot,\succeq^*)$, and for all $(T,s)\in \mathcal{C}_{\mathrm{rd}}(X)$*

$$c(T,s) = \begin{cases} \{s\} & \text{if } s \in M(T,\succeq) \\ M(T\cap U_{\succ}(s),\succeq^*) & \text{otherwise} \end{cases}$$

Furthermore, if X is a non-empty finite set then the above two statements are equivalent.

For a detailed interpretation of the type of choice correspondence obtained in Corollary 5.2 we refer to Masatlioglu and Ok (2003). Here it is sufficient to note that when there is no reference point, the decision-maker selects an element from the set of maximal elements based on a complete preorder relation $\succeq^*$. Clearly, this is no more than the standard case. When there is a reference point s, the decision maker will uniquely choose s if it belongs to the set of maximal elements of a partial order $\succeq$, indicating a status quo bias. When the status quo bias is not strong enough to evaluate s as one of the best elements, then the decision maker will choose from the set of elements that are strictly better than s according to the complete preorder $\succeq^*$. Hence, in the latter case the reference point becomes inessential.

Theorem 5.1 clearly shows that any "status quo bias-choice correspondence" is rationalizable by a book of preorder relations. It is easy to show that behavior subject to the status quo bias is undominated. By way of contradiction and using NSC, let $(T,s_i)_{i=0}^n$ be a cyclical and strict RD-chain. Since it is cyclical it must be that $s_0 \in c(T,s_n)$, but then SQB implies that $\{s_0\} = c(T,s_0)$, which, since $(T,s_i)_{i=0}^n$ is strict, constitutes a contradiction.

When in Section 4.2 we introduced axiom ISQP we promised further explanation of the term. Consider SQP again, and note that it implies that when $x \succ_y y$ then $\neg(y \succ_\diamond x)$. By completeness it must be that $x \succ_\diamond y$, and by again applying SQP we obtain $x \succ_x y$. Hence, SQP together with completeness implies that if $x \succ_y y$, then $x \succ_x y$. Remarkably, ISQP involves precisely the reverse implication, namely if $x \succ_x y$, then $x \succ_y y$. We, therefore, named this axiom *Inverse* Status Quo Positivity (ISQP). The implication of the latter on the relation between path-independence and status quo bias is clear; a choice correspondence representing the status quo bias need not be path-independent. The following example represents the case of a "strong" status quo bias that does not satisfy path-independence. It can be argued that the example is an abstract representation of Knetsch's experiment. Let $X = \{x, y\}$, where $x \succ_x y$, $y \succ_y x$, and $x \sim_\diamond y$. When there is no reference point, both alternatives are evaluated by the decision-maker as indifferent, but as soon as the decision-maker has a reference point, she unambiguously prefers the reference point to the other alternative. However, path-independence is clearly violated; the final choice depends on which reference point is taken first.

Hence, a decision maker that manifests the status quo bias nevertheless exhibits choice behavior that can be rationalized through a book of complete preorder relations, and is undominated. This suggests that there is still a great deal of rationality in those who are subject to this bias.[10] However, such a decision maker could easily be manipulated in situations where an external agent may exercise some influence on the prevalence of a specific reference point, or on the way the choice problem is presented. In this sense, the final choice depends on the discretion of the agent.[11]

5.2. **Addiction.** Our second application concerns addictive behavior and habit formation models. We chose this application for three reasons: (i) It shows that the theory we are proposing has a considerable range of application, (ii) it illustrates that the theory can be directly applied to richer contexts than those considered so far, and (iii) it suggests possible directions for future research.

Since Duesenberry (1949), Pollak (1970), and Ryder and Heal (1973) the literature concerning models of habit formation and addiction has attracted the attention of economists. The former is used in variants of growth models to explore life-cycle consumption plans, and is being applied to a wide variety of cases (see, e.g., Campbell 1999). With regard to the latter, the work of Becker and Murphy (1988) has generated an enlightening discussion on how to approach the modelling

[10]Interestingly, Huck, Kirchsteiger, and Oechssler (2003) theoretically show that decision makers that exhibit a status quo bias are favored by evolution.

[11]Camerer, Issaharoff, Loewenstein, O'Donoghue, and Rabin (2003) argue that the status quo bias constitutes a perfect example for the possibility of what they call *"paternalistic regulation"*. That is, regulation that could result in great benefits to those incurring in this error, while causing little harm to those that are not subject to it.

of addictive behavior (see, e.g., Herrnstein and Prelec 1992, and Elster 1999). The modelling approaches to both types of phenomena are very similar. It is typically assumed that an increase in past consumption increases present and future consumption, and hence the focus is mainly on increasing consumption patterns.

To be more specific, we will interpret the behavior arising from the axioms below on the grounds of addiction, and leave aside the habit formation interpretation. We will say that behavior is addictive if (1) the current reference point is the last level of consumption; (2) given the actual reference point, the individual either maintains the current level of consumption or increases it; (3) there is a saturation level over which increments in consumption are not positively evaluated; (4) the higher the reference point, the higher the saturation level; and (5) for any reference point, every alternative between two selected alternatives is also selected.

We now turn to the characterization we propose. We begin by introducing a linearly ordered universal set of alternatives $(X, \leq)$. That is, $\leq$ is a transitive, complete and antisymmetric binary relation over the non-empty alternative space X, and $<$ is the asymmetric part of $\leq$. To illustrate, consider that $X = [0, k]$ represents the feasible set of grams of some addictive substance, where a natural linear order arises. We start with the characterization of choice correspondences.

Monotonicity (M): For any $(T, s) \in \mathcal{C}(X)$, if $y \in c(T, s)$ then $s \leq y$.

This axiom simply states that addictive behavior leads to consumption levels *above* the reference point, where 'above' is interpreted according to $\leq$. In a recent paper Bossert and Sprumont (2001) formulated a similar property, called Non-deteriorating Choice. Their interpretation is that $\leq$ is a preference relation, and hence, the agent behaves by choosing a *better* alternative than the reference point. Here, however, since we are describing addictive patterns we maintain a more general interpretation of $\leq$.

Saturation Monotonicity (SM): For any pair $(T, s), (T, r) \in \mathcal{C}(X)$, if $s \leq r$ then, for all $p \in c(T, r)$ there exists $q \in c(T, s)$ such that $q \leq p$.

SM imposes a weak property on the monotonicity of the sets of possible consumption levels selected. It states that if p is chosen from r, then from the perspective of any reference point below r, say s, the set of possible choices should start before p. In other words, the saturation level of s (a consumption level above which increments according to $\leq$ are not evaluated as strictly positive) is lower than that of r.

Convexity (C): For all $(T, s) \in \mathcal{C}_{\text{rd}}(X)$, if $V \subseteq T, v \in c(V, s), t \in c(T, s)$, if there is $p \in V$ such that $v \leq p \leq t$ or $t \leq p \leq v$ then $p \in c(V, s)$.

Whenever $T = S$, C simply means that any alternative in between two selected ones should also be selected. If $T \neq S$ and $t \in V$, then if properties α and β apply, it must also be that $t \in c(V, s)$. However, it could be the case that $t \notin V$. In such cases, C imposes that any available level in between v and t should be selected when V is the choice set.

We now introduce a set of axioms over a book of binary relations that characterize addictive behavior.

Left Status-quo Positivity (LSQP): For any $s \in X$ and $y \in X \backslash \{s\}$, if $y \leq s$ then $s \succ_s y$.

Notice the analogy between LSQP and axiom SQP, used in the previous application. If the linear order $\leq$ describes the same ranking as $\succeq_\diamond$, both axioms are equivalent. This could be the case if, say, a "clean" person who has not yet consumed any level of the addictive substance (e.g. cigarettes) were to contemplate that "more cigarettes are better than less". However, it can perfectly be the case that preferences without a reference point $\succeq_\diamond$ are very different. In such cases, both axioms diverge. LSQP establishes that the evaluation of the reference point cannot deteriorate for those initiating in addictive behavior.

Status-quo Monotonicity (SQM): For any $x \leq y \leq u \leq v$ elements of X, such that $v \succ_x u \succeq_x x$ it must be that $v \succ_y u$.

Suppose that the greater element v is clearly preferred to u whenever the reference point is x, and both are significant with respect to x. If we move the reference point nearer to the pair u, v, this comparison must necessarily remain the same.

Absence of Jumps (AJ): If $y \succeq_x x$, $x \leq h \leq y$ then $h \succeq_x x$.

If an alternative is preferred to the reference point, intermediate levels should also be preferred to the reference point.

Weak single peakedness (WSP): If $y \succ_x z \succeq_x x, y \leq z \leq h$ implies $z \succeq_x h$.

WSP establishes that, once an agent has decided not to positively evaluate improvements over $\leq$, this pattern is maintained. Notice that standard single peaked preferences satisfy stronger versions of this property. First of all, with single peaked preferences the above restriction is imposed over the entire set X, while we only impose it for elements above the reference point. Secondly, single peakedness would require a strict preference whenever $z < h$, which is not our case.

Theorem 5.3. *For any choice correspondence c on $\mathcal{C}(X)$, the following two statements are equivalent:*

- *c satisfies properties α and β, and axioms M, SM and C.*
- *There exists a book $\{\succeq_s\}_{s\in X^*}$ of complete preorder relations that rationalizes c, and that satisfies LSQP, SQM, AJ and WSP.*

Addictive behavior is obviously a type of undominated behavior. By M, any RD-chain $(T, s_i)_{i=0}^n$ must satisfy $s_i \leq s_{i+1}, i = 0, 1, \dots, n-1$. In order to be strict, it must be that $s_k < s_{k+1}$ for some k, and therefore, it cannot be cyclical. On the other hand, the following example shows that it is neither necessarily path-independent, nor, therefore, menu-independent.

Example 5.4. *Let $X = \{x, y, z\}$, $x \leq y \leq z$, $y \succ_x x \succ_x z$, $z \succ_y y \succ_y x$, and $z \succ_z y \succ_z x$.*

If we let $T = \{x, z\}$, then it is clear that LSU does not hold. But the above example says more. It describes a *gradual* pattern of addiction, and shows that if the choice set could be manipulated, the addiction level of the individual could be kept to a minimum. That is, if the agent starts with reference point x (e.g., $x = 0$ grams of cocaine) and the choice set is X, then he will end up consuming the maximum feasible quantity z, whereas if he is confronted with choice set T, from the perspective of x he may find z too much and will then keep "clean". This is an illustration of why educational initiatives emphasizing "all or nothing" consumption patterns may be successful in preventing addiction (for a discussion see Herrnstein and Prelec 1992).

We end this section by addressing points (ii) and (iii) mentioned in the motivation for this section. In the above mentioned models of habit formation and addiction it is typically assumed that the instantaneous utility function at time t depends on the current level of consumption c_t and on the stock of consumption $s_t = (1-\gamma)s_{t-1} + \gamma c_t$, $\gamma \in (0,1]$. Then $u(c_t, s_t)$ is increasing in the difference $c_t - s_t$. In our context, we could interpret the stock of consumption as the reference point. Therefore, this suggests that the current reference point need not be the last choice as we have assumed so far, but a function of the history of past consumption. In this line, consider the following formulation.

Let $\Psi = X \times (\cup_{n=1}^{\infty} X^n)$. An element $(s_0, c_1, \ldots, c_k) \in \Psi$ reflects an initial reference point $s_0 \in X$ and a sequence of consumption levels $\bar{c} = (c_1, \ldots, c_k) \in X^k \subset \cup_{n=1}^{\infty} X^n$. A reference point could then be a function of the vector $(s_0, c_1, \ldots, c_k)$. We impose two mild conditions:

Definition 5.5. We say that $f : \Psi \to X$ is a reference point function (single-valued) if for any $s_0 \in X$ and for any $\bar{c} \in \cup_{n=1}^{\infty} X^n$:

- $f(s_0, \bar{c}, f(s_0, \bar{c})) = f(s_0, \bar{c})$, and
- $f(s_0, \bar{c}, \bar{c}, \ldots, \bar{c}, c_1, \ldots, c_p) = f(s_0, c_1, \ldots, c_p)$, where $\bar{c}$ is equal to $(c_1, \ldots, c_p, \ldots, c_k)$.

Note that throughout the paper $f(s_0, \bar{c}) = c_k$ where c_k is the last component of $\bar{c}$. It is clear that in general, given $T \subset X$, $f(s_0, \bar{c}) \notin T$. However, under some particular restrictions the use of the reference point function may be appropriate. As an illustration consider the case where the family of choice sets is restricted to budget sets in a linear space. Let $T = [0, Y]$ be such a budget set. In this case given a $s_0 \in T$ and $\bar{c} = (c_1, \ldots, c_i)$ with $c_k \in T$, for any $1 \le k \le i$, it is clear that:

$$s_{i+1} = f(s_0, c_1, \ldots, c_i) = \gamma c_i + (1-\gamma) f(s_0, c_1, \ldots, c_{i-1}) = \gamma c_i + (1-\gamma) s_{i-1}$$

is also in T. Therefore, the reference point function of the habit formation (and addiction) models is well defined.

When the reference point function is well defined we can establish the following more general definitions of chains.

Definition 5.6. Given a reference point function f, $< (T, s_i)_{i=0}^{n}, (c_i)_{i=1}^{n} >$ is an ERD-*chain* (extended chain) if $s_0 \in T$, $s_i = f(s_0, c_1, \ldots, c_i)$ for all $i = 1, \ldots, n$, and $c_i \in c(T, s_{i-1})$ for all $i = 1, \ldots, n$. An ERD-chain is *cyclical* if $s_0 = f(s_0, c_1, \ldots, c_{n+1})$, where $c_{n+1} \in c(T, s_n)$. An ERD is *strict* if there exists $k \in \{0, 1, \ldots, n-1\}$ such that $s_k \notin c(T, s_k)$.

Hence, the theory we propose in this paper can be extended to deal with richer contexts. It remains a task for future research, though, to characterize reference-dependent behavior when the reference point may be a function of all the past history.

6. Conclusion

This paper is a reaction to the accumulated empirical evidence suggesting that behavior is reference-dependent in a variety of environments. It has been our aim to provide an axiomatic characterization of such behavior, general enough to be applicable to the modelling of a wide range of specific phenomena. We believe that the incorporation of these types of well-established and predictable phenomena into economic theory is a natural step that will eventually help towards the better understanding of economic behavior.

We have studied choice behavior and preference relations, and have gradually imposed rationality demands on them. We have shown that reference-dependent behavior may satisfy a great deal of rationality demands, while still being dependent on reference points.

We have axiomatized reference-dependent behavior when the reference point, if there is one, belongs to the choice set. We chose this as the best approach to certain problems, such as those we have modelled here: status quo bias and addiction. Of course, in some situations it may be more appropriate not to impose that the reference point be in the choice set. Rubinstein and Zhou (1999) constitute an example of the latter. They characterize choice functions that select the closest point to the reference point in the choice set (they axiomatize a "minimal distance" choice function). This suggests what we believe to be a promising line for future research, namely, the characterization of reference-dependent behavior for phenomena where the reference point may not belong to the choice set.[12]

We further suggest that the application of the proposed theory may prove fruitful for the modelling of other reference-dependent phenomena, thus opening up another line for future research.

7. Proofs

Proof of Theorem 3.2. Let us first show that when c satisfies axiom WARP, then c satisfies properties α and β. Consider a choice correspondence satisfying WARP. Suppose, by way of contradiction

[12]Note however that, as Rubinstein and Zhou state, considering choice problems where the reference point does not necessarily belong to the choice set does not make the theory more general, since the required axioms will be more demanding.

that α is not satisfied. In this case, there exist a pair of sets $V \subset T$ and a pair of elements $x \in V$, $s \in V$ or $s = \diamond$ such that $x \in c(T, s)$ but $x \notin c(V, s)$. Since c is non-empty valued by definition, there exists $y \in c(V, s)$, $y \in V \subset T$, thus contradicting WARP. Property β is trivial, and therefore, this part of the proof is omitted.

We now show that if $c(T, s)$ satisfies properties α and β, then there exists a book of complete preorder relations $\{\succeq_s\}_{s \in X^*}$ such that $c(T, s) = M(T, \succeq_s)$. Let c satisfy properties α and β, and define for all s in X^* the binary relation R_s on X by xR_sy if and only if $x \in c(\{x, y, s\}, s)$. In order to condense the proofs, in the previous definition and throughout the paper, we interpret that whenever $s = \diamond$, $\{x, y, \diamond\} = \{x, y\}$. Let $\succeq_s \equiv R_s \cup N_s$, where N_s is the non-comparable part of binary relation R_s. That is, xN_sy if and only if $\neg(xR_sy)$ and $\neg(yR_sx)$. Since $\succeq_s$ is trivially complete, we need only show that it is a transitive relation. First note that whenever $s = \diamond$, we can apply standard results that guarantee the transitivity of $\succeq_\diamond$. Then, for all $x, y, z, s \in X$:

- Case 1: If xR_syR_sz, then xR_sz. Let xR_syR_sz. If $x \in c(\{x, y, z, s\}, s)$ by applying property α we directly obtain $x \in c(\{x, z, s\}, s)$. If s is in $c(\{x, y, z, s\}, s)$, by property α, $s \in c(\{x, y, s\}, s)$, then by property β, $x \in c(\{x, y, z, s\}, s)$, and by applying property α again we get $x \in c(\{x, z, s\}, s)$. If y is in $c(\{x, y, z, s\}, s)$, then a procedure analogous to the previous one shows that $x \in c(\{x, z, s\}, s)$. If $z \in c(\{x, y, z, s\}, s)$, applying property α implies that $z \in c(\{y, z, s\}, s)$, then by property β, $y \in c(\{x, y, z, s\}, s)$, which was shown above to imply $x \in c(\{x, z, s\}, s)$.
- Case 2: If xN_syN_sz, then xN_sz. Let $\{s\} = c(\{x, y, s\}, s)$, $\{s\} = c(\{y, z, s\}, s)$, and by way of contradiction, $\{s\} \neq c(\{x, z, s\}, s)$. If $x \in c(\{x, z, s\}, s)$, by property α, $x \in c(\{x, s\}, s)$, and by property β, $x \in c(\{x, y, s\}, s)$, which contradicts xN_sy. If $z \in c(\{x, z, s\}, s)$, by property α, $z \in c(\{z, s\}, s)$, by property β, $z \in c(\{y, z, s\}, s)$, which contradicts yN_sz. Hence it must be that $\{s\} = c(\{x, z, s\}, s)$.
- Case 3: If xR_syN_sz, then xR_sz. Let xR_syN_sz. Then note that $y, z \notin c(\{x, y, z, s\}, s)$, because otherwise by property α, yN_sz would be contradicted. If $\{s\} = c(\{x, y, z, s\}, s)$, then by properties α and β, $\{s\} = c(\{x, y, s\}, s)$, which can only hold when $x = s$. Then, if $x = s$, since $\{x\} = c(\{x, y, z\}, x)$, property α implies that $x \in c(\{x, z\}, x)$. Finally, if $x \neq s$, it must be that $x \in c(\{x, y, z, s\}, s)$, and by property α, $x \in c(\{x, z, s\}, s)$, as desired.
- Case 4: We now show that xN_syR_sz cannot hold. If x or y are in $c(\{x, y, z, s\}, s)$, by property α, $x \in c(\{x, y, s\}, s)$ or $y \in c(\{x, y, s\}, s)$, which contradicts xN_sy. If $z \in c(\{x, y, z, s\}, s)$, by properties α and β, $y \in c(\{x, y, z, s\}, s)$, which contradicts xN_sy. Hence, $\{s\} = c(\{x, y, z, s\}, s)$, but by properties α and β, $\{s\} = c(\{y, z, s\}, s)$, which contradicts yR_sz.

We now show that if $y \in c(T,s)$ then $y \in M(T,\succeq_s)$. By property α, $y \in c(T,s)$ implies $y \succeq_s h$ for all $h \in T$, and hence it must be that $y \in M(T,\succeq_s)$. Now let $y \in M(T,\succeq_s)$, which implies that $y \succeq_s h$ for all $h \in T$. Take any $z \in c(T,s)$. By property α, $z \in c(\{s,y,z\},s)$. Since $y \succeq_s z$, it must also be that $y \in c(\{s,y,z\},s)$, and hence property β guarantees that $y \in c(T,s)$. This concludes the second part of the proof.

We finally show that if there is a book $\{\succeq_s\}_{s\in X^*}$ of complete preorders that rationalizes c, then c satisfies WARP. Standard results apply to the case of $s = \diamond$. Now we study the case where $s \neq \diamond$. Let, by reduction to the absurd, $y \in c(T,s)$, $s,x,y \in T \cap V$, $x \in c(V,s)$ and $y \notin c(V,s)$. Since $y \in c(T,s) = M(T,\succeq_s)$, there is no $h \in T$ such that $h \succ_s y$, but since $y \notin c(V,s) = M(V,\succeq_s)$, then there is a $k \in V$ such that $k \succ_s y$. By transitivity and completeness, since x is maximal in V, we must have $x \succ_s y$ contradicting $y \in c(T,s)$. □

The following lemma will be useful for our next characterization result.

Lemma 7.1. *Let c be defined on $\mathcal{C}(X)$ and satisfy properties α, β, and NSC. Let $\{\succeq_s\}_{s\in X^*}$ be a book of complete preorders that rationalizes c. Take $s_n \succeq_{s_{n-1}} s_{n-1} \succeq_{s_{n-2}} s_{n-2} \dots s_1 \succeq_{s_0} s_0$ with at least one strict inequality. Then, for all $p \in \{1,2,\dots,n\}$, for all $t \in \{1,2,\dots,p\}$, we must have $s_p \in c(\{s_p, s_{p-1},\dots,s_{p-t}\}, s_{p-1})$.*

Proof of Lemma 7.1. If $t = 1$, then, for any $p \in \{1,2,\dots,n\}$, using $s_p \succeq_{s_{p-1}} s_{p-1}$, we conclude $s_p \in c(\{s_p, s_{p-1}\}, s_{p-1})$. We will prove now the following induction hypothesis: If the result is valid for any $t = 1,2,\dots,k$ with $k < p$ (otherwise the induction is complete for this value of p), it is also valid for $t = k+1$. By contradiction, let $s_p \notin c(\{s_p, s_{p-1},\dots,s_{p-k-1}\}, s_{p-1})$. In this case, consider the set $T = \{s_p, s_{p-1}, s_{p-2},\dots,s_{p-k-1}\}$. If for some $j \in \{1,2,\dots,k\}$, we have $s_{p-j} \in c(T, s_{p-1})$ then by property α, we have $s_{p-j} \in c(T \setminus \{s_{p-k-1}\}, s_{p-1})$ and using the induction hypothesis and property β, we obtain $s_p \in c(T, s_{p-1})$, which is absurd. Then, we must have $\{s_{p-k-1}\} = c(T, s_{p-1})$, and by properties α and β $\{s_{p-k-1}\} = c(T \setminus \{s_p\}, s_{p-1})$. By hypothesis it must also be $s_{p-1} \in c(T \setminus \{s_p\}, s_{p-2})$. Since $s_{p-k} \succeq_{s_{p-k-1}} s_{p-k-1}$, we can find an element s_{l_1} such that $s_{l_1} \in c(T \setminus \{s_p\}, s_{p-k-1})$ and $l_1 > p-k-1$. If $l_1 \neq p-1$ proceed in an analogous way considering an element s_{l_2} such that $s_{l_2} \in c(T \setminus \{s_p\}, s_{l_1})$ and $l_2 > l_1$ (note that such an l_2 exists because the induction hypothesis is valid for the set $T \setminus \{s_p\}$, smaller than T). Since set T is finite, we will find an element $l_m = p-1$ such that $s_{l_m} \in c(T \setminus \{s_p\}, s_{l_{m-1}})$. The sequence of reference-dependent choice problems

$(T\backslash\{s_p\}, s_{p-1}), (T\backslash\{s_p\}, s_{p-k-1}), (T\backslash\{s_p\}, s_{l_1}), \ldots, (T\backslash\{s_p\}, s_{l_{m-1}})$ constitutes a strict RD-chain. However, $s_{p-1} = s_{l_m} \in c(T \setminus \{s_p\}, s_{l_{m-1}})$, contradicting NSC. Therefore, the induction is valid for $t = k + 1$. Thus, the lemma is proved. □

Proof of Theorem 4.4. We first show that, given a choice correspondence c satisfying properties α, β and NSC, we can find a book of preferences satisfying the requirements. Consider the binary relations $\succeq_s$ defined in the proof of Theorem 3.2. They turn out to be not only transitive but complete, and therefore, to conclude the proof we only need to observe that they satisfy CT. Let $s_n \succeq_{s_{n-1}} s_{n-1} \succeq_{s_{n-2}} s_{n-2} \ldots s_1 \succeq_{s_0} s_0$ with at least one strict inequality. Suppose, by way of contradiction, that $s_0 \succeq_{s_n} s_n$. Consider the set $V = \{s_0, s_1, \ldots, s_n\}$. Select a group of elements by means of the following inductive argument:

(1) By hypothesis, there exists an integer $k \in \{0, 1, \ldots, n-1\}$ such that $s_k \in c(V, s_n)$. Select the smallest integer in such group and denote it by p_1.

(2) If $p_j = n$, stop the inductive process. Otherwise, to obtain p_{j+1}, consider the problem (V, s_{p_j}). Lemma 7.1 guarantees that s_{p_j+1} exists in $c(\{s_0, \ldots, s_{p_j+1}\}, s_{p_j})$. Therefore, there exists an integer $k \in \{p_j + 1, \ldots, n\}$ such that $s_k \in c(V, s_{p_j})$. Let p_{j+1} denote the smallest integer with such property.

The inductive reasoning gives us a collection of natural numbers p_1, $p_2, \ldots, p_m$ where $m \geq 2$. The sequence of problems $(V, s_n), (V, s_{p_1}), \ldots,$ $(V, s_{p_{m-1}})$ constitutes an RD-chain. Moreover, we will now show that this chain is strict. To see this, consider the strict comparison $s_{r+1} \succ_{s_r} s_r$ with $r \in \{0, 1, \ldots, n-1\}$. We need to study two cases:

• $p_q \leq r < p_{q+1}$. We will show that $p_q \notin c(V, p_q)$. To this end we need to analyze two cases. First, if $p_{q+1} - p_q = 1$ the preference between these two elements is strict, and then $p_q \notin c(V, p_q)$. Second, if the difference is bigger, then suppose by way of contradiction that $p_q \in c(V, p_q)$. In this case, $p_q + 1$ is also in $c(V, p_q)$, contradicting the definition of p_{q+1}.

• $p_1 > r$. In this case, it is easy to see that $s_n \notin (V, s_n)$. Otherwise, $0 = p_1$, which using $s_0 \succeq_{s_n} s_n$ contradicts $p_1 > r$.

We have proved that the RD-chain is strict. Therefore, by NSC, it must be $s_n \notin c(V, s_{p_{m-1}})$, contradicting $s_{p_m} = s_n \in c(V, s_{p_{m-1}})$. Hence $s_0 \succeq_{s_n} s_n$ cannot hold and, $\succeq_{s_n}$ being complete, we obtain $s_n \succ_{s_n} s_0$. This proves that the book of preferences satisfies CT.

We now prove that if there is a book of reference-dependent preferences satisfying the conditions, the corresponding choice mapping satisfies properties α, β and NSC. Taking into account Theorem 3.2, we only need to prove NSC. Consider a strict chain $(T, s_i)_{i=0}^n$. Since the chain is strict, there must exist $0 \le k < n$ and $y \in T$ such that $y \succ_{s_k} s_k$. Since $s_i \in c(T, s_{i-1})$, and given completeness, we must have $s_i \succeq_{s_{i-1}} h$, $i = 1, 2, \ldots, n$, for all $h \in T$. In particular, we have $s_{k+1} \succeq_{s_k} y$ and by transitivity, we obtain $s_{k+1} \succ_{s_k} s_k$. We therefore obtain a sequence of comparisons

$$s_n \succeq_{s_{n-1}} s_{n-1} \succeq_{s_{n-2}} s_{n-2} \ldots s_{k+1} \succ_{s_k} s_k \ldots s_1 \succeq_{s_0} s_0$$

and by applying CT we must have $s_n \succ_{s_n} s_0$ which shows that s_0 is not in $M(T, \succeq_{s_n}) = c(T, s_n)$, as desired. $\square$

Proof of Theorem 4.7. We begin by showing that if c satisfies properties α, β, NSC and LSU, then there exists a book $\{\succeq_s\}_{s \in X^*}$ of complete preorder relations that rationalizes c, and that satisfies CT and ISQP. Take the book of complete preorders defined in the proof of Theorem 3.2. By considering Theorem 4.4, we only need to check ISQP. Consider by way of contradiction two elements $x, y \in X$ such that $x \succ_x y$ but $y \succeq_y x$. Since the book rationalizes c, we must have $\{x\} = c(\{x, y\}, x)$ and $y \in c(\{x, y\}, y)$. Thus, the limit set of $(\{x, y\}, x)$ is $\{x\}$, while the limit of $(\{x, y\}, y)$ must contain y, contradicting LSU.

We now prove the converse statement. Consider Theorem 4.4, then we only need to check LSU. Suppose, by way of contradiction, the existence of a subset T and elements $x, y \in T$ such that

- L is the limit set of (T, x),
- L' is the limit set of (T, y), and

with $S \neq S'$.

Without loss of generality, suppose $z \in L$ and $z \notin L'$, and take $w \in L'$. By NSC, for all $s \in L \cup L'$ we must have $s \in c(T, s)$ (otherwise, the cyclical subchain containing s would be strict, which is absurd). Then, it is $z \in c(T, z)$ and $w \in c(T, w)$, implying $z \succeq_z w$ and $w \succeq_w z$. By ISQP we must have $z \sim_z w$ and $w \sim_w z$. Thus, $z \in c(T, w)$ and $w \in c(T, z)$. There exists a cyclical RD-chain $(T, s_i)_{i=0}^k$ such that $s_0 = y$ and $s_k = w$. Then, the chain $(T, s_i)_{i=0}^{k+1}$, with $s_{k+1} = z$ implies that $z \in L'$, which is absurd. $\square$

Proof of Theorem 4.8. That path-independence is a necessary condition for PI is trivial. To see that NSC is also a necessary condition consider by way of contradiction a cyclical and strict RD-chain where the reference set T contains at least 3 elements. Since the chain is strict, there exists a pair of elements x, y such that $x \notin c(T, x)$, $y \in c(T, x)$, and $x, y \in L(T)$. Properties α and β guarantee that $\{y\} = c(\{x, y\}, x)$ and therefore, using Weak-NSC, it is clear that $L(\{x, y\}) = \{y\}$. Consider the non-empty sets $A = \{x, y\}$ and $B = T \setminus A$. We chose $x \in L(T)$, but note that neither $x \in L(A)$, nor $x \in L(B)$, and hence $x \notin L(L(A) \cup L(B))$. Therefore, PI does not hold, a contradiction. Finally Weak-NSC concludes the proof.

We now show that path-independence and NSC are sufficient to imply PI and Weak-NSC. Since the second property is trivial, let us prove Menu Independence. Let T and V be any two sets in $\mathcal{C}(X)$. First, let $z \in L(T \cup V)$ and, without loss of generality, let $z \in T$. Then, by NSC it must be that $z \in c(T \cup V, z)$. Further, by property α we have that $z \in c(T, z)$, and hence by LSU, $z \in L(T)$. Hence, we can apply property α again to get $z \in c(L(T) \cup L(V), z)$, and by LSU, $z \in L(L(T) \cup L(V))$. Then we have that $L(T \cup V) \subseteq L(L(T) \cup L(V))$.

Now let $z \in L(L(T) \cup L(V))$. The case where $z \in T$ and $z \in V$ is trivial. So consider, without loss of generality, that $z \in T$ and $z \notin V$. By NSC, $z \succeq_z h$ for all $h \in T \cup L(V)$. Let there exist a $t \in V \setminus L(V)$ such that $t \succ_z z$. By LSU there are $s_0, s_1, \ldots, s_n \in V$ and $x \in L(V)$ such that $x \succeq_{s_n} s_n \succeq_{s_{n-1}} s_{n-1} \succeq_{s_{n-2}} s_{n-2} \ldots s_1 \succeq_{s_0} s_0 \succeq_t t \succ_z z$. Then, by Theorem 4.7 CT holds, which implies that $x \succ_x z$, and by ISQP we have that $x \succ_z z$, a contradiction. Hence, $z \succeq_z h$ for all $h \in T \cup V$, and then $L(L(T) \cup L(V)) \subseteq L(T \cup V)$, as desired. □

Proof of Theorem 5.1. We first show that if there is a book $\{\succeq_s\}_{s \in X^*}$ of complete preorders that satisfies WT, UI, SQP, SQP2, SQS, and $c(T, s) = M(T, \succeq_s)$, then c satisfies properties α and β, and axioms D, SQI, and SQB. The implication on properties α and β is shown in Theorem 3.2.

• D: Let $y \neq s \neq \diamond$. Let $\{y\} = c(T, s) = M(T, \succeq_s)$, and $y \in c(V, \diamond) = M(V, \succeq_\diamond)$, $T \subseteq V$. Then $y \succ_s h$ for all $h \in T \setminus \{y\}$, and $y \succeq_\diamond h$ for all $h \in V$. For all $t \in V \setminus \{s, y\}$, either $t \succ_s s$ or $s \succeq_s t$. When $t \succ_s s$, given that $y \succ_s s$, then by UI, $y \succeq_\diamond t$ implies $y \succeq_s t$. If $s \succeq_s t$, by SQS $s \succ_s t$. Now, assume, by way of contradiction, that $t \succ_s y$. Then, since $y \succ_s s$ and $s \succ_s t$, $M(\{s, y, t\}, \succeq_s) = \emptyset$, a contradiction. Then it must be that $y \succeq_s t$, and therefore, $y \in M(V, \succeq_s) = c(V, s)$. Now let $y = s \neq \diamond$. Then $y \in c(V, \diamond) = M(V, \succeq_\diamond)$ implies $y \succeq_\diamond h$ for all $h \in V$. By SQP $y \succ_y h$ for all $h \in V \setminus \{y\}$, and hence $y \in M(V, \succeq_y) = c(V, y)$. Finally, the case when $s = \diamond$ is trivial, and therefore omitted.

• SQI: Let $y \in c(T, s) = M(T, \succeq_s)$, and assume that there exists no non-empty $V \subseteq T$ with $V \neq \{s\}$ and $s \in c(V, s)$. Then $y \succeq_s h$ for all $h \in T$, and $h \succ_s s$ for all $h \in T \setminus \{s\}$. By applying UI we get $y \succeq_\diamond h$ for all $h \in T \setminus \{s\}$. Finally, it must be that $y \succ_\diamond s$, since otherwise $s \succeq_\diamond y$ implies, by SQP, that $s \succ_s y$, a contradiction. Hence, $y \in M(T, \succeq_\diamond) = c(T, \diamond)$.

• SQB: Let $y \neq s \neq \diamond$ and $y \in c(T, s) = M(T, \succeq_s)$. Then, $y \succeq_s h$ for all $h \in T$. By SQS, $y \succeq_s s$ implies $y \succ_s s$. For all $z \in T \backslash \{s, y\}$, either $s \succeq_s z$ or $z \succ_s s$. If $s \succeq_s z$, by SQS $s \succ_s z$. Then $y \succ_s s \succ_s z$, by SQP2, implies $y \succ_y z$. If $z \succ_s s$, then $y \succeq_s z$, by UI, implies $y \succeq_\diamond z$, and SQP implies $y \succ_y z$. Since it must also be that $y \succ_\diamond s$, by SQP, $y \succ_y s$. Hence $\{y\} = M(T, \succeq_y) = c(T, y)$. Now let $y \neq s = \diamond$. Then if $y \in c(T, \diamond) = M(T, \succeq_\diamond)$, $y \succeq_\diamond h$ for all $h \in T$, by SQP $y \succ_y h$ for all $h \in T \backslash \{y\}$, and then $\{y\} = M(T, \succeq_y) = c(T, y)$. Finally, let $y = s \neq \diamond$. If $y \in c(T, y) = M(T, \succeq_y)$, $y \succeq_y h$ for all $h \in T$, by SQS $y \succ_y h$ for all $h \in T \backslash \{y\}$, and then $\{y\} = M(T, \succeq_y) = c(T, y)$.

To show the reverse implication we will use the book of binary relations defined in the proof of Theorem 3.2, where it was shown that if c satisfies properties α and β, then $\{\succeq_s\}_{s \in X^*}$ is a book of complete preorders, and that $c(T, s) = M(T, \succeq_s)$.

• UI: Let $x \succ_s s$, $y \succ_s s$. We will show both implications for the case of $x \neq y$; the case of $x = y$ is trivial. Let $x \succeq_s y$, then by definition, either $x \in c(\{x, y, s\}, s)$ or $\{s\} = c(\{x, y, s\}, s)$. If $\{s\} = c(\{x, y, s\}, s)$, then properties α and β imply that $\{s\} = c(\{x, s\}, s)$, which contradicts $x \succ_s s$. Then, it must be that $x \in c(\{x, y, s\}, s)$. Note that $s \notin c(\{x, s\}, s)$, and $s \notin c(\{y, s\}, s)$. If $s \in c(\{x, y, s\}, s)$ then by property α we get $s \in c(\{x, s\}, s)$, a contradiction. Hence we can apply SQI to get $x \in c(\{x, y, s\}, \diamond)$. By property α, $x \in c(\{x, y\}, \diamond)$, as desired. Let us now assume that $x \succeq_\diamond y$, and by way of contradiction, $\neg(x \succeq_s y)$. Then $x \in c(\{x, y\}, \diamond)$, and $\{y\} = c(\{x, y, s\}, s)$. By SQI we get $y \in c(\{x, y, s\}, \diamond)$. By applying properties α and β, $x \in c(\{x, y, s\}, \diamond)$. Now, since $\{x\} = c(\{x, s\}, s)$ and $x \in c(\{x, y, s\}, \diamond)$, by D, $x \in c(\{x, y, s\}, s)$, a contradiction. Hence it must be that $x \succeq_s y$.

• SQP: Let $s \succeq_\diamond y$, then it must be that $s \in c(\{s, y\}, \diamond)$. Applying SQB we get $\{s\} = c(\{s, y\}, s)$, as desired.

• SQP2: Let $x \succ_s s \succ_s y$, by transitivity, $x \succ_s y$, which by definition implies that $x \in c(\{x, y, s\}, s)$. By SQB $\{x\} = c(\{x, y, s\}, x)$, and by applying properties α and β, we get $\{x\} = c(\{x, y\}, x)$.

• SQS: Let $s \sim_s y$. Since choice correspondences are non-empty value by definition, the case of no comparability between s and y does not apply here. Then $s \in c(\{y, s\}, s)$ and $y \in c(\{y, s\}, s)$. By SQB $\{s\} = c(\{y, s\}, s)$, and hence $y = s$. □

Proof of Theorem 5.3. We first show that if c satisfies the mentioned properties, we can find a book of preferences as described. Consider the book of complete preorders that rationalizes c in the proof of Theorem 3.2. If $x > y$, by M, we cannot have $y \in c(\{x, y\}, x)$ and therefore, $x \succ_x y$, which proves LSQP.

To show SQM, consider $x \leq y \leq u \leq v$ with $v \succ_x u \succeq_x x$. First of all, we will prove that $v \succ_x y$. Otherwise, $y \succeq_x v \succ_x u \succeq_x$ and therefore, $y \in c(T, x)$ where $T = \{x, y, u, v\}$. At the same time $v \in c(V, x)$ with $V = \{x, u, v\}$. Since $y \leq u \leq v$ and $u \in V$ by applying C we must have $u \in c(V, x)$ which is absurd because $v \succ_x u$. Thus, $v \succ_x y$. Given the hypothesis, we must have $\{v\} = c(T, x)$. Applying SM leads to $\{v\} = c(T, y)$ and therefore, $v \succ_y u$.

To show AJ, suppose $y \succeq_x x \succ_x h$ with $x \leq h \leq y$. Consider the sets $T = \{x, y, h\}$ and $V = \{x, h\}$. By hypothesis, $y \in c(T, x)$ and $\{x\} = c(V, x)$. The application of C leads to $h \in c(V, x)$. However, this is absurd, since we cannot have $h = x$.

To prove WSP, suppose $y \succ_x z \succeq_x x$ and $y \leq z \leq h$. By way of contradiction, let us suppose that $h \succ_x z$. There are two possibilities:

- If $y \succeq_x h$, then consider $T = \{x, y, z, h\}$ and $V = \{x, z, h\}$. It is obvious that $y \in c(T, x)$ while $h \in c(V, x)$. By C it must be also $z \in c(V, x)$ which is absurd, because $h \succ_x z$.
- If $h \succ_x y$, consider the sets $T = \{x, y, z, h\}$ and $V = \{x, y, z\}$. Then we have $h \in c(T, x)$ and $y \in c(V, x)$. The application of C again leads to $z \in c(V, x)$ which is absurd because $y \succ_x z$.

This concludes the first part of the proof.

Now suppose that there exists a book satisfying the requirements. To show M is straightforward. To prove SM, consider $x \leq y$ and $p \in c(T, y)$. By M we must have $x \leq y \leq p$. If $p \in c(T, x)$, the proof is complete. If not, consider any $q \in c(T, x)$ such that $p < q$. In this case, $q \succ_x p$ and $q \succeq_x x$. AJ guarantees that $p \succeq_x x$. Then, applying SQM, we have $q \succ_y p$ and therefore, $p \notin c(T, y)$ which is absurd. Therefore, there are no such elements and any $h \in c(T, x)$ must fulfil $h < p$, which proves SM.

To end the proof, we only need to show that C is satisfied. Consider $V \subseteq T$, $v \in c(V, x)$ and $t \in c(T, x)$. Let $p \in V$, with $v \leq p \leq t$ and suppose, by way of contradiction, that $p \notin c(V, x)$. In this case, we must have $v \succ_x p$ and $x \leq v \leq p \leq t$, otherwise by LSQP $x \succ_x v$, a contradiction. Since $t \succeq_x x$ AJ implies that $p \succeq_x x$. Therefore, $v \succ_x p \succeq_x x$ and $v \leq p \leq t$ guarantees $p \succeq_x t$ by application of WSP. Therefore $p \in c(T, x)$ and by property α, $p \in c(V, x)$, which is absurd. The case in which $t \leq p \leq v$ is very similar and is, therefore, omitted. □

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