MARKET ALLOCATIONS OF LOCATION CHOICE: AN EXAMPLE

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Market allocations of location choice: An example

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Abstract

The purpose of this paper is to make an example which, first, illustrates Starret's Spatial Imposibility Theorem, when agents have free mobility; and second, allowes us to get a competitive equilibrium with transportation when agents move only if there is a noticeable difference in utilities that justifies the change of location.

1 Introduction

This is a paper on General Equilibrium that makes explicit considerations of the agent's decision on where to be located.

In the last decade the work of M. Fujita, P. Krugmann, F. Thisse and many other authors has called the attention of theoretical economists about the study of location patterns that can arise as equilibria. More than forty years ago there also was a vivid interest on this subject as the work of Koopmans and Beckmann among others show. The example we present here has been constructed basically as an intent to understand an impossibility result stated by Starret (1978) and presented in Fujita and Thisse (2001), and to see what kind of changes in the economy can turn this into a possibility result.

The location, together with the physical characteristic of the good and its temporal dimension is, in the Arrow-Debreu set up, one of the distinctive features that are needed to identify the good. Therefore, it can be said that location has been taken into account on all abstract general equilibrium models. Nevertheless, most of those general equilibrium models only mention the location issue when defining the goods. Typically they do not need any special reference to a production

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activity devoted only to change good's location and nothing is said about where consumers and firms are located. General equilibrium models are basically spaceless. The recent work of Fujita, Krugmann and Thisse, among others, is devoted, on the contrary, to the study of the "Geography" that can turn out as consequence of economic decisions. The models those authors construct are also "General Equilibrium Models" because nothing is taken from outside, every variable needs to be explained and all economic interactions are important to determine the resulting equilibria. What it is important for us now, is that all those models explicitly reject the price taking behavior hypothesis. Thisse and Fujita (2001) argue that a non-competitive General Equilibrium approach is one of the three alternatives available (the others being the introduction of heterogeneous space and the study of externalities in production and consumption) for the study of spatial distribution, because of the "unsuspected" result established in Starret (1978) that they entitle Spatial Impossibility Theorem. They enunciate this theorem as

[Starret's] Spatial Impossibility Theorem*: Consider an economy with a finite number of agents and locations. If space is homogeneous, transport is costly and preferences are locally nonsatiated, then there is no competitive equilibrium involving transportation.*

The free mobility assumption explicitly mentioned by Starret is included in the definition of homogeneous space that basically requires that "consumer and producers have no intrinsic preferences for one location over the others" In his work Starret is "interested in analyzing how well the market handles the locational decisions in situations were people are reasonably free to move around if they see an economic incentive to do so". This question was already possed and answered in Koopmans and Beckmann (1957) and Starret presents his result as a generalization of what Koopmans and Beckmann call the *quadratic assignment problem* where *N* firms have to be allocated in *N* places in order to maximize the total production net of transportation costs. Koopmans and Beckmann's result could be stated as concluding "*Our main point here is that these* direct *physical interactions between production and/or consumption processes are by no means the only reasons for such a failure of the price system. The mere fact that scarce resources need to be utilized for the transportation of intermediate commodities between plans appears to be sufficient to deprive the price system of its ability to induce efficient decentralized allocative decisions*". In order to stablish his result Starret does not need economic agents to be maximizers, his result "*we have shown that under a wide set of cicumstances,(any) market configuration of prices must offer an economic incentive to move for some agents*" is based on the analysis of profits and incomes of the firms and consumers and is well aware that those "*move incentives (...) can be nullified by either fixed costs of moving or by perceived market power*".

When, at the Universidad Pública de Navarra, we started working on location economics we were not aware of those powerful results. We adopted the non-competitive approach because we simply understood it was descriptively more accurate than the competitive one.

Now, the basic reason of our interest in a competitive general equilibrium model is that we still do not completely understand the reasons of the failure of the competitive mechanism in this particular set up. Certainly it seems that the existing *externalities* because "scarce resources need to be utilized for the transportation of intermediate commodities between plans" are the reason of the failure detected by Koopmans, Beckmann and Starret. But those *externalities* are also present in the simple model Mills (1970) constructed and yet he finds that the competitive mechanism yields a Pareto Optimum allocation when he dispenses with all the indivisibilities of the quadratic assignment problem as formulated by Koopmans and Beckmann. Probably the location issue is, after all, an issue where indivisibilities are unavoidable. In this work we will not be dealing with the Pareto Optimality of the competitive allocation, we just want to see if an allocation involving transportation where agents are price takers is possible. In our work it has been very helpful the work by Ellickson and Zame (1994) and our example is depends heavily on their construction.

2 Our example

As it has been just said our example is in the same spirit as the examples constructed in Ellickson and Zame (1994).

We will consider an economy with only two sites (1 and 2), three commodities, (land, labor and a consumption commodity) and a continuum of agents. There will be two types of consumers: a fraction θ are workers endowed with $\overline{h}$ units of time that can be devoted to leisure or to work, and a fraction $(1-\theta)$ are landowners that in addition to the endowment of time, have $\overline{l_1}$ units of land at site 1 and $\overline{l_2}$ units of land at site 2. Both types have the same utility function.

In this economy there are two productive sectors: in both sites there is the possibility of producing consumption commodity using land and labor as factors and there is also a transportation sector.

Our example illustrates Starret's theorem quite clearly because when free mobility is assumed then the only competitive equilibrium is an equilibrium with no transportation, regardless of the initial endowments at both sites. This, in our opinion, is an indication of how powerful this theorem is.

The main change we have made with respect to Ellickson and Zame is the introduction of an explicit transportation sector instead of the adoption of Samuelson's iceberg cost to capture that transportation is not free. We have introduced this transportation sector in order to be able to make straightforward comparisons with Starret's results.

We have also dispensed with free mobility assumption, explicitly considering some mobility costs as in Ellickson and Zame. Our way of modeling this cost is formally a special case of they way of modeling it. In our model a move from one place to another is only worthwhile when the improvement is noticeable. With free mobility, an equilibrium where identical people are located in different sites requires that the utilities attained in all those different sites are equal. In our example a noticeable difference is required for an agent to move. Formally, it is as if each agent had different preferences in the different places, this is clearly a case explicitly ignored in Starret (1978) and a special case in Ellickson and Zame.

The preferences of both workers and landowners are of the form

$$U(h_i, l_i, y_i) = h_i l_i y_i$$

where h_i is the amount of time devoted to leisure by consumer i, l_i is the amount of land used by this consumer, and y_i the amount of consumption good.

We will say that a consumer is in location j, $j \in \{1, 2\}$, when he uses land in j. When a consumer is in location j, he consumes y^i_j, i.e. the consumer has positive utility only if land, leisure and consumption commodity are being used at the same location.

In both sites it is possible to pruce comsumption good. The relation between inputs and output takes also the form of a very simple Cobb–Douglas function

$$y_i = (h^i_y)^{1/2}(l^i_y)^{1/2} \qquad i \in \{1, 2\}$$

We assume that the transportation of one unit of consumption good requires only α units of labor. If p_1 is the price of the consumption good at 1 and p_2 is the price of the consumption good at 2, the existence of a price taker transportation sector requires $p_2 = p_1 + \alpha w_1$ if the good is taken from location 1 to 2, and $p_1 = p_2 + \alpha w_2$ if the transported good goes from 2 to 1. This is a fundamental relation that has to be maintained in all competitive equilibria with transportation. With no loss of generality we will describe equilibria where the commodity is transported from 1 to 2.

A competitive equilibria with free mobility is a vector of prices $(w_1, r_1, p_1, w_2, r_2, p_2)$, an allocation $(h^i_1, l^i_1, y^i_1, h^i_2, l^i_2, y^i_2)$ for all consumers i and a distribution of workers and landowners between the to sites (μ, γ) (where μ is the proportion of workers in site 1, and γ the proportion of land owners in 1) that satisfy the following conditions:

$$p_1 = 2\sqrt{r_1 w_1} \tag{1}$$

$$p_2 = 2\sqrt{r_2 w_2} \tag{2}$$

$$p_2 = p_1 + \alpha w_1 \tag{3}$$

$$(1-\theta)\bar{l}_1 = \sqrt{\frac{w_1}{r_1}} y_1 + \gamma(1-\theta)\frac{\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_1}{3r_1} + \mu\theta\frac{\bar{h} w_1}{3r_1} \tag{4}$$

being y_1 the total production in location 1.

$$(1-\theta)\bar{l}_2 = \sqrt{\frac{w_2}{r_2}} y_2 + (1-\gamma)(1-\theta)\frac{\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_2}{3r_2} + (1-\mu)\theta\frac{\bar{h} w_2}{3r_2} \tag{5}$$

being y_2 the total production in location 2.

$$y_1 = X + \gamma(1-\theta)\frac{\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_1}{3p_1} + \mu\theta\frac{\bar{h} w_1}{3p_1} \tag{6}$$

where X is the amount exported from site 1 to 2.

$$X + y_2 = (1-\gamma)(1-\theta)\frac{\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_2}{3p_2} + (1-\mu)\theta\frac{\bar{h} w_2}{3p_2} \tag{7}$$

$$(\gamma(1-\theta) + \mu\theta)\bar{h} = \sqrt{\frac{r_1}{w_1}} y_1 + \alpha X + \gamma(1-\theta)\frac{\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_1}{3w_1} + \mu\theta\frac{\bar{h}}{3} \tag{8}$$

$$((1-\gamma)(1-\theta)+(1-\mu)\theta)\bar{h} = \sqrt{\frac{r_2}{w_2}} y_2 + (1-\gamma)(1-\theta)\frac{\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_2}{3w_2} + (1-\mu)\theta\frac{\bar{h}}{3} \tag{9}$$

Equations (4) and (5) reflect the equality between supply and demand of land in each of the locations, equations (6) and (7) equalize supply and demand of the consumption, and equations (8) and (9) are equilibria condition for labor. Since for the economy as a whole Walras Law holds, only 5 of this last 6 equations are independent.

In order to capture Starret's result, under free mobility it is also required that, in an equilibrium, the utility of workers, and the utility of landowners at both locations be identical, that is:

$$\left(\frac{w_1\bar{h}}{3r_1}\right)\left(\frac{\bar{h}}{3}\right)\left(\frac{w_1\bar{h}}{3p_1}\right) = \left(\frac{w_2\bar{h}}{3r_2}\right)\left(\frac{\bar{h}}{3}\right)\left(\frac{w_2\bar{h}}{3p_2}\right) \tag{10}$$

$$\frac{(\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_1)^3}{27 r_1 w_1 p_1} = \frac{(\bar{l}_1 r_1 + \bar{l}_2 r_2 + \bar{h} w_2)^3}{27 r_2 w_2 p_2} \tag{11}$$

2.1 Ilustration of Starret's Theorem

In this section, we are going to demonstrate that the Starret's Theorem is truth in our example and in one extension of it.

2.1.1 Initial Model

In our model it is easy to see how strong Starret's Theorem is. As said before Starret's result does not deal explicitly with a competitive allocation, his result is valid for any price-taking configuration. In his proof, though, he assumes that all locations have the same endowment of land. In our illustration, we can dispense with this assumption if we focus only in the competitive allocation.

It is easy to verify that equations (10) and (11), with equations (1) and (2) imply that in equilibrium all prices must be equal between both sites, and that is incompatible with equation (3), namely, with transport activity.

In the examples we have used, for simplicity reasons, $\bar{h} = \bar{l}_1 = \bar{l}_2 = 1$, but the result is general.

We start by normalizing prices to $w_1 = 1$. With this normalization and equations (1) and (2), expression (10) gives us

$$\frac{w_1^2\bar{h}^3}{27r_1p_1} = \frac{w_2^2\bar{h}^3}{27r_2p_2} \quad \longrightarrow \quad \frac{1}{r_1^{3/2}} = \frac{w_2^{3/2}}{r_2^{3/2}}$$

and then

$$\frac{1}{r1} = \frac{w_2}{r_2} \tag{12}$$

By the same way, from equation (11)

$$\frac{(\bar{h} + r_1\bar{l}_1 + r_2\bar{l}_2)^3}{r_1^{3/2}} = \frac{(w_2\bar{h} + r_1\bar{l}_1 + r_2\bar{l}_2)^3}{r_2^{3/2}w_2^{3/2}}$$

or

$$\frac{\bar{h} + r_1\bar{l}_1 + r_2\bar{l}_2}{r_1^{1/2}} = \frac{w_2\bar{h} + r_1\bar{l}_1 + r_2\bar{l}_2}{r_2^{1/2}w_2^{1/2}}$$

Using (12) permit us to have

$$r_1^{1/2}\bar{l}_1 + \frac{r_2\bar{l}_2}{r_1^{1/2}} = \frac{r_1\bar{l}_1}{r_1^{1/2}w_2^{1/2}} + \frac{r_2^{1/2}\bar{l}_2}{w_2^{1/2}}$$

If we divide by $r_1^{1/2}$ we have

$$\bar{l}_1 + \frac{r_2\bar{l}_2}{r_1} = \frac{r_1^{1/2}\bar{l}_1}{r_1^{1/2}w_2^{1/2}} + \frac{r_2^{1/2}\bar{l}_2}{r_1^{1/2}w_2^{1/2}}$$

and using (12) once more

$$\overline{l}_1 + w_2\overline{l}_2 = \frac{\overline{l}_1}{w_2} + \overline{l}_2$$

Now it is easy to see that

$$w_2(\overline{l}_1 + w_2\overline{l}_2) = \overline{l}_1 + w_2\overline{l}_2 \quad \longrightarrow \quad w_2 = 1$$

But if $w_2 = 1$, then by (12) we have $r_1 = r_2$, and by (1) and (2), $p_1 = p_2$, which is a contradiction with (3). Then, it is not possible to satisfy (1)-(11), just as Starret's Theorem says.

An equilibrium with transport is not possible with free mobility if transport cost exists ($\alpha \neq 0$).

2.1.2 Model with intermediate product

In this part we are going to extend the previous framework supposing that we have got an intermediate product which can be produced in any region, or in both of them. Suppose that the production function has changed in the following form

$$y_i = (h_y^i)^{1/2}(m_y^i)^{1/2} \qquad i \in \{1, 2\}$$

where the new factor m_y^i represents an intermediate product, which is mobile between both regions. In order to produce one unit of m_y^i, we need, only, the land factor. For simplicity we have considered that the intermediate product's production function is :

$$m_i = \beta l^i \qquad i \in \{1, 2\}$$

The principal difference between this model and the last one, is that in this model the two production factors of y_i are mobile. Also, to transportate one unit of m_y^i we need incurre in a transport cost of α. Because of this, we are capable to find some differences in the previous equations. Notice that in this framework, the price equations have changed in the following form

$$p_1 = 2\sqrt{pm_1 w_1} \tag{13}$$

$$p_2 = 2\sqrt{pm_2 w_2} \tag{14}$$

$$pm_2 = pm_1 + \alpha \tag{15}$$

We can demonstrate that

$$pm_1 = \beta r_1 \tag{16}$$

$$pm_2 = \beta r_2 \tag{17}$$

Now, in the same sense of the previous section, the equations (13) and (14), with the expression (10) gives us

$$\frac{w_1^2\overline{h}^3}{54r_1^{3/2}\beta^{1/2}w_1^{1/2}} = \frac{w_2^2\overline{h}^3}{54r_2^{3/2}\beta^{1/2}w_2^{1/2}} \quad \longrightarrow \quad \frac{w_1^{1/2}}{r_1^{1/2}} = \frac{w_2^{1/2}}{r_2^{1/2}}$$

and then, by normalizaing prices to $w_1 = 1$, is easy to see that

$$\frac{1}{r1} = \frac{w_2}{r_2} \tag{18}$$

is the same that the (12).

By the same way, from equation (11)

$$\frac{(\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2)^3}{\beta^{1/2}r_1^2} = \frac{(w_2\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2)^3}{r_2^2 w_2\beta^{1/2}}$$

or

$$\frac{\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2}{r_1^{2/3}\beta^{1/6}} = \frac{w_2\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2}{r_2^{2/3}w_2^{1/3}\beta^{1/6}}$$

Then we are able to rewrite this equation as

$$\frac{(\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2)^3}{r_1^{2/3}} = \frac{(w_2\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2)^3}{r_2^{2/3}w_2^{1/3}}$$

Using (18), and a little computation, permit us to have

$$\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2 = \frac{w_2\overline{h} + r_1\overline{l}_1 + r_2\overline{l}_2}{w_2}$$

Now it is easy to see that

$$w_2(r_1\overline{l}_1 + r_2\overline{l}_2) = r_1\overline{l}_1 + r_2\overline{l}_2 \quad \longrightarrow \quad w_2 = 1$$

Then, if $w_2 = 1$ we can see by (18) that we have got $r_1 = r_2$, by (16) and (17), we have found that $pm_1 = pm_2$, which is a contradiction with (15), and by this outcome we have $p_1 = p_2$ again, such as the Starret's Theorem predicts. Once more, we have to conclude saying that the equilibrium which involves the transport is not possible with free mobility if transport cost exists ($\alpha \neq 0$).

2.2 Introducing mobility costs. Two equilibria

The equality of utilities captured by equations (10) and (11) before, are replaced by equations

$$\frac{1}{\lambda_w} \leq \frac{\left(\frac{w_1\bar{h}}{3r_1}\right)\left(\frac{\bar{h}}{3}\right)\left(\frac{w_1\bar{h}}{3p_1}\right)}{\left(\frac{w_2\bar{h}}{3r_2}\right)\left(\frac{\bar{h}}{3}\right)\left(\frac{w_2\bar{h}}{3p_2}\right)} \leq \lambda_w, \quad \lambda_w \geq 1 \tag{19}$$

$$\frac{1}{\lambda_l} \leq \frac{\frac{(\bar{l}_1 r_1+\bar{l}_2 r_2+\bar{h}w_1)^3}{27 r_1 w_1 p_1}}{\frac{(\bar{l}_1 r_1+\bar{l}_2 r_2+\bar{h}w_2)^3}{27 r_2 w_2 p_2}} \leq \lambda_l, \quad \lambda_l \geq 1 \tag{20}$$

In our new equilibria differences in utility of both workers and landowners in sites 1 and 2 are allowed provided those differences are not too big. Only if the utility at 1 is noticeable bigger than the utility at 2 will an agent decide to change from 2 to 1. The required difference to do the change is captured by the λ's. In the limit, when $\lambda_w = \lambda_l = 1$, we are back to the free mobility case.

It should be clear that if λ_w and λ_l are conveniently chosen, equilibriua with transportation where agents are price takers must exist; If we arbitrarily start from a distribution of workers and landowners and resolve the system of equations (1) to (9) we will find an equilibrium where agents in different places have different utilities. The arbitrarily given distribution will not be an equilibrium distribution if agents are able to move, but if the λ are sufficiently big then we can get get as an equilibrium –conditioned to the selected λ– almost any arbitrary initial distribution.

The two equilibria we present here are just equilibria with transportation when prices are taken parametrically by all agents. Those equilibria have been found "after a little calculation" that we have done with the powerfull help of *MATHEMATICA*®. The above examples are equilibria when λ_w and λ_l are greater respectively that the ratios of utilities shown in the tables. The way we have made the numeric calculations does not permit us to say that the equilibrium foun is the only associated to those λ's.

In the tables notice that quantities in "Production and factors demands" are total quantities for each market, but quantities in "Demands" sections are individual quantities (and number of people in each group is normalized to one).

Table 1: Free and no–free mobility between two sites of equal size.

Common parameter values			
$\alpha = 0.1 \quad \theta = 2/3 \quad w_1 \quad \overline{l}_1 = 1 \quad \overline{l}_2 = 1 \quad \overline{h} = 1$			
Free mobility		No–Free mobility	
Distribution	(endogenous)	Distribution	(exogenous)
$\mu = 0.5$	$\gamma = 0.5$	$\mu = 0.4$	$\gamma = 0.4$
Equilibrium Prices		Equilibrium Prices	
$w_1 = 1$	$w_2 = 1$	$w_1 = 1$	$w_2 = 0.852$
$r_1 = 1.5$	$r_2 = 1.5$	$r_1 = 1.195$	$r_2 = 1.534$
$p_1 = \sqrt{6}$	$p_2 = \sqrt{6}$	$p_1 = 2.187$	$p_2 = 2.287$
Production and factors demands		Production and factors demands	
$y_1 = 1/(3\sqrt{6})$	$y_2 = 1/(3\sqrt{6})$	$y_1 = 0.132$	$y_2 = 0.139$
$X = 0$	$h_x = 0$	$X = 0.015$	$h_x = 0.002$
$l_y^1 = 1/9$	$l_y^2 = 1/9$	$l_y^1 = 0.120$	$l_y^2 = 0.104$
$h_y^1 = 1/6$	$h_y^2 = 1/6$	$h_y^1 = 0.144$	$h_y^2 = 0.186$
Demands		Demands	
$l_w^1 = 2/9$	$l_w^2 = 2/9$	$l_w^1 = 0.279$	$l_w^2 = 0.185$
$h_w^1 = 1/3$	$h_w^2 = 1/3$	$h_w^1 = 1/3$	$h_w^2 = 1/3$
$y_w^1 = 1/(3\sqrt{6})$	$y_w^2 = 1/(3\sqrt{6})$	$y_w^1 = 0.152$	$y_w^2 = 0.124$
$l_l^1 = 8/9$	$l_l^2 = 8/9$	$l_l^1 = 1.040$	$l_l^2 = 0.778$
$h_l^1 = 4/3$	$h_l^2 = 4/3$	$h_l^1 = 1.243$	$h_l^2 = 1.017$
$y_l^1 = 2\sqrt{6}/9$	$y_l^2 = 2\sqrt{6}/9$	$y_l^1 = 0.568$	$y_l^2 = 0.522$
Utilities		Utilities	
$\left.\begin{array}{l} U_w^1 = 0.272 \\ U_w^2 = 0.272 \end{array}\right\}$	$\Rightarrow \frac{U_w^1}{U_w^2} = 1$	$\left.\begin{array}{l} U_w^1 = 0.383 \\ U_w^2 = 0.207 \end{array}\right\}$	$\Rightarrow \frac{U_w^1}{U_w^2} = 1.848$
$\left.\begin{array}{l} U_l^1 = 0.645 \\ U_l^2 = 0.645 \end{array}\right\}$	$\Rightarrow \frac{U_l^1}{U_l^2} = 1$	$\left.\begin{array}{l} U_l^1 = 0.735 \\ U_l^2 = 0.569 \end{array}\right\}$	$\Rightarrow \frac{U_l^1}{U_l^2} = 1.291$

Table 2: Free and no–free mobility between two sites with site 1 bigger (double size) than 2.

Common parameter values			
$\alpha = 0.1$ $\theta = 2/3$ w_1 $\overline{l}_1 = 2$ $\overline{l}_2 = 1$ $\overline{h} = 1$			
Free mobility		No–Free mobility	
Distribution	(endogenous)	Distribution	(exogenous)
$\mu = 0.5$	$\gamma = 0.5$	$\mu = 0.6$	$\gamma = 0.3$
Equilibrium Prices		Equilibrium Prices	
$w_1 = 1$	$w_2 = 1$	$w_1 = 1$	$w_2 = 0.739$
$r_1 = 1$	$r_2 = 1$	$r_1 = 0.731$	$r_2 = 1.108$
$p_1 = 2$	$p_2 = 2$	$p_1 = 1.710$	$p_2 = 1.810$
Production and factors demands		Production and factors demands	
$y_1 = 2/9$	$y_2 = 1/9$	$y_1 = 0.275$	$y_2 = 0.051$
$X = 0$	$h_x = 0$	$X = 0.127$	$h_x = 0.013$
$l_y^1 = 2/9$	$l_y^2 = 1/9$	$l_y^1 = 0.321$	$l_y^2 = 0.042$
$h_y^1 = 2/9$	$h_y^2 = 2/9$	$h_y^1 = 0.235$	$h_y^2 = 0.063$
Demands		Demands	
$l_w^1 = 1/3$	$l_w^2 = 1/3$	$l_w^1 = 0.456$	$l_w^2 = 0.222$
$h_w^1 = 1/3$	$h_w^2 = 1/3$	$h_w^1 = 1/3$	$h_w^2 = 1/3$
$y_w^1 = 1/6)$	$y_w^2 = 1/6)$	$y_w^1 = 0.195$	$y_w^2 = 0.136$
$l_l^1 = 4/3$	$l_l^2 = 4/3$	$l_l^1 = 1.628$	$l_l^2 = 0.995$
$h_l^1 = 4/3$	$h_l^2 = 4/3$	$h_l^1 = 1.190$	$h_l^2 = 0.815$
$y_l^1 = 2/3$	$y_l^2 = 2/3$	$y_l^1 = 0.696$	$y_l^2 = 0.609$
Utilities		Utilities	
$\left.\begin{array}{r} U_w^1 = 0.5 \\ U_w^2 = 0.5 \end{array}\right\} \Rightarrow \frac{U_w^1}{U_w^2} = 1$		$\left.\begin{array}{r} U_w^1 = 0.800 \\ U_w^2 = 0.272 \end{array}\right\} \Rightarrow \frac{U_w^1}{U_w^2} = 2.940$	
$\left.\begin{array}{r} U_l^1 = 1.185 \\ U_l^2 = 1.185 \end{array}\right\} \Rightarrow \frac{U_l^1}{U_l^2} = 1$		$\left.\begin{array}{r} U_l^1 = 1.349 \\ U_l^2 = 0.905 \end{array}\right\} \Rightarrow \frac{U_l^1}{U_l^2} = 1.489$	

3 Concluding comments

Our example shows that it is possible to have a competitive equilibrium with transportation, when the non-homogeneity of the space, in the form of mobility costs is introduced. All the authors we have mentioned in this paper we aware of this possibility and the all have assumed free mobility in their models. An explanation to do that can be found in Starret (1978): "*This [free mobility] assumption is necessary if we are going to discuss first-best welfare issues, since the firstbest problem involves a free initial choice of location for all agents. Now one may argue that from a descriptive point of view, much of observed locational configurations are historical in nature; that is, agents make initial locational choices at some point of time and then are locked into those choices over time due to the high costs of moving. There is undoubtedly truth to this position, and we could probably attribute some of the inefficiencies of our presen configuration to historical factors. However there are always the new location decisions made by "new" agents;these would consist of new families or firms and intramarginal units who are forced to move due to transfers, bankrupcies, etc. If there are enough of these marginal and intramarginal agents, and the market allocated them properly, we might still expect to achieve a desirable outcome*". We think that costs of mobility together with a price taking behaviour can be helpful to understand the historical configuration of the present economic geography.

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