LONG-RUN ANALYSIS IN ALTERNATIVE OPTIMIZING MONETARY MODELS

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Abstract

This paper explores the transmission channel from monetary variables to real variables in the steady-state equilibria of various neoclassical optimizing models with money. The existence of superneutrality is rejected for the four models at hand: time-cost transactions approach, output-cost transactions approach, money in the utility function model, and cash-in-advance model. However, the real effects of high rates of inflation are not large: output, consumption, and investment slightly fall with higher inflation rates. In addition, the welfare cost of inflation is calculated for annual rates of inflation ranging from 0% to 50%. Three of the models (all but the cash-in-advance model) agree on the following result: a 10% rate of inflation creates a permanent welfare cost equal to 0.3% of GDP per year.

Keywords: money, superneutrality, welfare cost of inflation.

JEL codes: E13, E41, E58.

1 Introduction

The treatment of money in optimizing macroeconomic models has been rather diverse in the recent literature. Following early contributions by Hicks (1935), Baumol (1952) and Sidrauski (1967), most of the works have in common the existence of a money demand based upon the role of money as a medium of exchange.[1] Money is employed by the economic agents as a mean of facilitating daily transactions. This property makes the consumer give a positive value to money. At the same time, there is a cost of holding money: the nominal interest rate forgone. Thus, transactions-facilitating money demand has been expressed as depending positively upon the consumption level (or total expenditures) and negatively upon the nominal interest rate.

The purpose of this paper is to study the long-term features of several optimizing models with transactions facilitating money. A Sidrauski-type (neoclassical) monetary general equilibrium model will be built up by using maximization criteria in four approaches that reflect distinctly the transactions-facilitating function of money.

In the first specification, the time-cost transactions approach (McCallum and Goodfriend (1987)) is analyzed by means of including a shopping time technology. Typically, actual shopping time will depend positively on the amount of consumption and negatively on the amount of real money holdings. Therefore, money services permit the consumers to economize the time spent during their shopping.

In addition, three more alternatives representing the role of money as a medium-of-exchange will be intorduced to collect the variety of monetary models commonly used in the literature: output-cost transactions model (Sims (1994)), money in the utility function models (Chari, Kehoe and McGrattan (1996), Kollman (1996) and Ireland (1997)), and cash-in-advance models (Woodford (1994)). The cash-in-advance models are based on the assumption that money is required to make purchases of consumption goods. Hence, the household must keep an amount of money at least equal to the desired consumption level creating the cash-in-advance constraint. In contrast, money in the utility function models does not include any explicit restriction in the transactions process. Money enters the utility function as an indirect way to represent the transactions-facilitating property of money. Finally, the output-cost transactions model incorporates a shopping technology in terms of output usages. Thus, the transaction cost function indicates the amount of output that the household needs to use in carrying out their purchases. This function is positively related to the

[1] There are two approaches to demand analysis that recognize only the store-of-value role of money, namely, the portfolio approach of Tobin (1958) and the overlapping generations (OG) approach of Wallace (1980). Since these ignore the medium-of-exchange function, they imply that no money would be demanded if there were other assets (of equal riskiness) yielding positive interest earnings to their holders. For a more extensive discussion, see McCallum and Goodfriend (1987).

consumption level and negatively to the amount of real money holdings.

Our attention is focused on how the deterministic steady state solution is altered by monetary conditions. In particular, we examine the effects of different values of the nominal money growth rate set by the monetary authorities. Having models that abstract from economic growth like ours implies that the steady state rate of nominal money growth equals the steady state rate of inflation. Therefore, we indistinctly refer to effects of steady state inflation or money-growth throughout the paper. The study of the long-term effects of inflation has recovered importance in the very last years (Feldstein (1995), Lucas (1995), Gillman (1995), Dotsey and Ireland (1996), Wolman (1996), Pakko (1998), Chadha, Haldane and Janssen (1998)). Here, there are two main issues to analyze: the long-term relationship between inflation and output, and the welfare cost of inflation. We will calibrate the models on the same criteria in order to compare the magnitudes obtained.

The general equilibrium problem derived here is meant to incorporate rational expectations under uncertainty. However, the stochastic elements of the model will be left out since they do not have any influence in the steady state solution reducing the system to a perfect foresight scenario. Aiming at long-run analysis, the purpose here is to explore the characteristics of the models in a deterministic economy.

A general equilibrium will be described for the four models. For the sake of clarity in the exposition, acronyms will be used to denote a particular model. Thus, TCT will refer to the time-cost transactions approach, OCT to the output-cost transactions approach, MIU to the money in the utility function model, and CIA to the model with the cash-in-advance constraint. To avoid repetition, we will display the common features only for the general equilibrium setup of the TCT model. Hence, the common equations among the four models at hand will be numbered without any subscript.

2 The time-cost transactions approach (TCT)

2.1 The model

Let the economy contain N alike households. The preferences of the infinitely-lived households, at a moment of time t, are expressed in an intertemporal utility function whose arguments are a consumption index and leisure time

$$U(c_t, l_t) + \beta U(c_{t+1}, l_{t+1}) + \beta^2 U(c_{t+2}, l_{t+2}) + ...,$$

where the discount factor is $\beta = \frac{1}{1+\rho}$ with $\rho > 0$ representing the intertemporal preference rate for the consumers. We assume positive first derivatives and negative second derivatives of the instantaneous utility function with respect to both consumption and leisure ($U_c > 0$, $U_l > 0$, $U_{cc} < 0$ and $U_{ll} < 0$). In addition, cross marginal effects are considered not to exist ($U_{cl} = 0$). Consumption units are an aggregate of many goods. Indexes were chosen for consumption and the price level as in Dixit and Stiglitz (1977).[2] Each household produces a different good. Being N the number of both households and goods, the i-th household produces $y(i)$ units of the i-th good according to the list of goods indexed in the consumption aggregate. Trading needs arise from the separability between what is produced (single good) and what is consumed (many goods) by the household. We assume that households put up a shop where they sell the good that they have produced. Therefore, all the other households willing to consume this good will manage to come to each shop and arrange the purchase of each good. In other words, every household must go shopping for all the other $N - 1$ goods. In this scenario, households can take advantage of using money as a medium-of-exchange within the existing transactions technology.

The production function for the i-th household gives the amount produced depending on the stock of capital ($k_t(i)$) undertaken from the previous production and on the labor force hired ($n_t^d(i)$)[3]

$$y_t(i) = f(n_t^d(i), k_t(i))$$
$$with \quad f_n > 0, f_k > 0, f_{nn} < 0, f_{kk} < 0 \; and \; f_{nk} > 0.$$

Technology shows decreasing marginal returns on both labor and capital. The latter is a necessary property to compute a finite and unique steady state equilibrium for the model. It is also assumed that the production function is homogeneous of degree 1, i.e., reports constant returns to scale

$$f(\varrho n_t^d(i), \varrho k_t(i)) = \varrho f(n_t^d(i), k_t(i)) \qquad for \; any \; \varrho > 0.$$

All the households have access to this technology. Acting as producers, they decide how much of their output should be employed as capital input in next period's production process. Hence,

[2] In particular, the consumption and price level indexes are

$$c_t = N^{\frac{1}{1-\vartheta}} \left(\sum_{i=1}^{N} c_t(i)^{\frac{\vartheta-1}{\vartheta}} \right)^{\frac{\vartheta}{\vartheta-1}} \qquad with \;\; \vartheta > 1$$
$$P_t = N^{-1} \left(\sum_{i=1}^{N} P_t(i)^{1-\vartheta} \right)^{\frac{1}{1-\vartheta}} \qquad with \;\; \vartheta > 1$$

being $i = 1, ..., N$ the distinct goods of the economy.

[3] Here and later d superscript denotes quantity demanded whereas s superscript denotes quantity supplied.

investment is defined as the increase in the stock of capital plus the amount accounting for replacing depreciated capital $(k_{t+1}(i)-(1-\delta)k_t(i))$. The rate of depreciation is given by δ and no adjustment costs of installing investment are assumed.

Nominal income is obtained from two sources: market value of production $(P_t(i)f(n_t^d,k_t))$ and net lump-sum nominal transfers from the government $(G_t(i))$. This is spent on consumption $(\sum P_t(i)c_t(i))$, on investment $(P_t(i)[k_{t+1}(i)-(1-\delta)k_t(i)])$, on payments for the labor hired at the market nominal wage $(W_t(n_t^d(i)-n_t^s(i)))$, on increasing the demand for nominal money holdings $(M_t^d(i)-M_{t-1}^d(i))$, and on net purchases of bonds $((1+R_t)^{-1}B_{t+1}^d(i)-B_t^d(i))$. Regarding the latter, let $(1+R_t)^{-1}B_{t+1}^d(i)$ be nominal purchases of government bonds bought by the household in period t. The amount agreed to be reimbursed to the household in period $t+1$ will be $B_{t+1}^d(i)$. Thus, R_t is the nominal interest rate. Accordingly, the budget constraint in period t for the i-th household is[4]

$$\begin{aligned}G_t(i)+P_t(i)f(n_t^d(i),k_t(i)) &= P_tc_t+P_t(i)[k_{t+1}(i)-(1-\delta)k_t(i)]+\\ W_t(n_t^d(i)-n_t^s(i)) &+ M_t^d(i)-M_{t-1}^d(i)+(1+R_t)^{-1}B_{t+1}^d(i)-B_t^d(i).\end{aligned}$$

Symmetric equilibrium is assumed, i.e., $P_t(i)=P_t$ for all i, and decisions are identical over the households throughout the rest of the paper and therefore no reference to one particular household will be made below. Dividing both sides of the budget constraint by the price level and using price symmetry, the budget constraint for any household can be expressed in real terms as follows

$$\begin{aligned}g_t+y_t &= c_t+k_{t+1}-(1-\delta)k_t+w_t(n_t^d-n_t^s)+\\ &\quad m_t^d-(1+\pi_t)^{-1}m_{t-1}^d+(1+r_t)^{-1}b_{t+1}^d-b_t^d,\end{aligned}$$

where g_t, w_t, m_t^d, m_{t-1}^d, b_{t+1}^d, b_t^d denote the output value of their respective capital letters, and the following definitions were used for inflation, the real interest rate, output, and the demand for real

[4] Note that in the budget constraint we have substituted total expenditures on consumption $(\sum P_t(i)c_t(i))$ for the product of the price level by the consumption aggregate (P_tc_t). These two figures are equivalent when using Dixit-Stiglitz indexes.

money balances[5]

$$\pi_t = \frac{P_t}{P_{t-1}} - 1, \tag{1}$$

$$1 + r_t = \frac{1 + R_t}{1 + \pi_{t+1}}, \tag{2}$$

$$y_t = f(n_t^d, k_t), \tag{3}$$

$$m_t^d = \frac{M_t^d}{P_t}. \tag{4}$$

In a TCT model, money appears affecting the transactions time technology. As being the medium of exchange, money is utilized by the household to economize the time spent on carrying out purchases of consumption goods. Without money, the household must negotiate a payment by credit or spend some time on transforming part of its income to money.[6] These alternatives to money suppose a higher amount of time spent on shopping. Besides, it seems obvious that a greater amount of consumption will require more shopping time. Thus, we present a generic shopping time function whose arguments are the consumption index and real money balances

$$s_t = s(c_t, m_t^d) \tag{5_{TCT}}$$

with $s_c > 0$, $s_m < 0$, $s_{cc} > 0$, $s_{mm} > 0$, $s_{cm} < 0$ and $s(0, m_t^d) = 0$, assuming rising marginal cost of consumption, diminishing marginal benefit of real balances and a negative crossed derivative. In addition, the shopping time function gives a zero value when there is no consumption. The transactions-facilitating property of money as a medium of exchange is represented through the signs $s_m < 0$ and $s_{cm} < 0$ which imply that the use of more monetary services reduces the total and marginal transactions costs. Taking into account the transactions time, households spend their time on three activities: working (n_t^s), shopping ($s(c_t, m_t^d)$) and leisure (l_t),

$$T = n_t^s + s(c_t, m_t^d) + l_t.$$

The optimal choices of the representative household in period t are those solving the optimizing problem (H_{TCT}) that consists of deciding $c_t, k_{t+1}, n_t^d, n_t^s, l_t, m_t^d, b_{t+1}^d$ so as to maximize the sum of current and discounted future utility values while satisfying the budget and time constraints:

$$\underset{c_t, k_{t+1}, n_t^d, n_t^s, l_t, m_t^d, b_{t+1}^d}{Max} \sum_{j=0}^{\infty} \beta^j U\left(c_{t+j}, l_{t+j}\right) \tag{H_{TCT}}$$

[5] Being a stock, the quantity of real money m_t^d is referred to be held at the beginning of period t.

[6] This time is comparable to the so-called "trips for cash" pertaining to the shoe-leather cost literature.

subject to

$$\begin{gathered}
g_{t+j}+f(n^d_{t+j},k_{t+j})-c_{t+j}-k_{t+1+j}+(1-\delta)k_{t+j}-w_{t+j}(n^d_{t+j}-n^s_{t+j})- \\
m^d_{t+j}+(1+\pi_{t+j})^{-1}m^d_{t-1+j}-(1+r_{t+j})^{-1}b^d_{t+1+j}+b^d_{t+j}=0 \quad for\ all\ j\geq 0. \\
T-n^s_{t+j}-s(c_{t+j},m^d_{t+j})-l_{t+j}=0 \qquad for\ all\ j\geq 0.
\end{gathered}$$

The first order conditions coming up from (H_{TCT}) are:

$$U_c(c_t,l_t)-\lambda_{1t}-\lambda_{2t}s_c(c_t,m^d_t)=0, \tag{6_{TCT}}$$

$$-\lambda_{1t}+\beta\lambda_{1t+1}\left(1+f_k(n^d_t,k_t)-\delta\right)=0, \tag{7_{TCT}}$$

$$f_n(n^d_t,k_t)-w_t=0, \tag{8_{TCT}}$$

$$\lambda_{1t}w_t-\lambda_{2t}=0, \tag{9_{TCT}}$$

$$U_l(c_t,l_t)-\lambda_{2t}=0, \tag{10_{TCT}}$$

$$-\lambda_{1t}+\beta\lambda_{1t+1}(1+\pi_{t+1})^{-1}-\lambda_{2t}s_m(c_t,m^d_t)=0, \tag{11_{TCT}}$$

$$-\lambda_{1t}(1+r_t)^{-1}+\beta\lambda_{1t+1}=0, \tag{12_{TCT}}$$

$$\begin{gathered}f(n^d_t,k_t)+g_t-c_t-k_{t+1}+(1-\delta)k_t- \\ w_t(n^d_t-n^s_t)-m^d_t+(1+\pi_t)^{-1}m^d_{t-1}-(1+r_t)^{-1}b^d_{t+1}+b^d_t=0,\end{gathered} \tag{13_{TCT}}$$

$$T-n^s_t-s(c_t,m^d_t)-l_t=0, \tag{14_{TCT}}$$

where λ_{1t} and λ_{2t} are the Lagrange multipliers associated with the budget and time constraints respectively. The following transversality conditions must also be satisfied to rule out sequences that lead to non-finite or negative figures for capital and bonds respectively

$$\begin{aligned}
\lim_{j\to\infty} k_{t+1+j}\beta^{t-1+j}\lambda_{1t+j} &= 0, \\
\lim_{j\to\infty} b^d_{t+1+j}\beta^{t-1+j}\lambda_{1t+j} &= 0.
\end{aligned}$$

The government gives lump-sum transfers to the households that are financed through increasing the money supply ($M^s_t-M^s_{t-1}$) and by selling bonds to the households ($(1+R_t)^{-1}B^s_{t+1}-B^s_t$). In per household units, the government budget constraint becomes[7]

[7] Clearly, the government budget constraint can be expressed in total units by multiplying both sides of the relation by the number of households, N.

$$G_t = M_t^s - M_{t-1}^s + (1+R_t)^{-1} B_{t+1}^s - B_t^s,$$

that once divided by the price level P_t can be displayed in real terms as follows

$$g_t = m_t^s - (1+\pi_t)^{-1} m_{t-1}^s + (1+r_t)^{-1} b_{t+1}^s - b_t^s, \tag{15}$$

where it was used the inflation definition (1), the interest rate definition (2), and a definition of the amount of supply of real money balances such as

$$m_t^s = \frac{M_t^s}{P_t}. \tag{16}$$

In order to build a general equilibrium solution, we must include market clearing conditions. Regarding the labor, money, and bond markets, we have

$$n_t^d = n_t^s, \tag{17}$$

$$m_t^d = m_t^s, \tag{18}$$

$$b_{t+1}^d = b_{t+1}^s. \tag{19}$$

As for the final goods market, the set of equations presented so far already imply the existence of equilibrium in each of the N consumption goods markets. Specifically, substituting the market clearing conditions (17)-(19) and the government budget constraint (15) in the equation referring to the household budget constraint (12_{TCT}), one may reach

$$f(n_t^d, k_t) = c_t + k_{t+1} - (1-\delta) k_t,$$

which is applicable to each household and therefore to each individual good market. Finally, the monetary and fiscal policies are given exogenously. The instrument of monetary policy is the nominal money growth rate defined as

$$\mu_t = \frac{M_t^s}{M_{t-1}^s} - 1. \tag{20}$$

For the sake of simplicity, both fiscal and monetary policies are conducted by applying a constant rule

$$\mu_t = \mu \quad with \quad \mu \geq 0, \tag{21}$$

$$g_t = g \quad with \quad g \geq 0. \tag{22}$$

Our general equilibrium is defined by the set of twenty two equations ((1)-(4), (5_{TCT})-(14_{TCT}) and (15)-(22)) that together with the transversality conditions determine stable solution paths for the twenty two variables of the model c_t, k_{t+1}, n_t^d, n_t^s, l_t, m_t^d, b_{t+1}^d, g_t, w_t, y_t, s_t, m_t^s, R_t, r_t, π_t, μ_t, P_t, M_t^s, M_t^d, b_{t+1}^s, λ_{1t}, and λ_{2t}. A parallel general equilibrium problem may be expressed including producing firms as a separate entity that maximizes the expected profits outflow. In Appendix 1, this alternative setup is presented detailing the assumptions to make the two equilibria equivalent.

2.2 Superneutrality

Since there is no stochastic shock affecting the model and explosive paths were ruled out by including transversality conditions, the solution to the system of equations listed above converges (quickly enough) to the deterministic steady state solution. Here we will focus the attention on the effects of changing the steady state nominal money growth rate, μ, which coincides with the steady state rate of inflation, π. Since the real money balances do not grow in steady state (note that there is no economic growth in our setup), the growth rates for nominal money and prices must be equal to guarantee a constant m in steady state. The question to answer now is whether the model presents *superneutrality*. If the steady state value of output changes when the steady state nominal money growth (or inflation) does, a model is said not to have *superneutrality*.

The analysis of *superneutrality* will be a two-stage process. First of all, we will examine the money demand behavior to see the response of real money to a change in inflation. Second, we will look at how the real money holdings may affect the leisure/labor choice.[8] In general equilibrium, the amount of labor supplied (or labor demanded) is one of the variables entering the production function. Then, the effect of inflation on output will have the same sign as the effect on the labor supply. Yet, our production function is homogeneous of degree 1 and therefore the size of the effect on y will be the same as the one on n. Whenever the quantity of output (or labor force) is affected by the monetary conditions, *superneutrality* will not hold.

[8] Another transmission channel to have real effects could have been a change in the demand of capital due to changes in the real interest rate as in Tobin (1965) or in OG models. However, the steady state demand of capital is governed by the relation $r = f_k(n^d, k) - \delta$ and in our Sidrauski-type framework the steady state real interest rate r is independent from the inflation rate since it is tied to be equal to the rate of intertemporal preference ρ (see Appendix 2 for the proof). In turn, the only way to alter the stock of capital will happen to be coming from a new value of f_k when n changes.

Money demand

Substituting $\beta\lambda_{1t+1}$ from (12_{TCT}) and λ_{2t} from (9_{TCT}) into (11_{TCT}), and then using the real interest definition (2) leads to this money demand formulation in steady state

$$-ws_m = \frac{R}{1+R} \qquad (MD_{TCT})$$

where all the subscripts where dropped to denote steady state magnitudes. The left hand side is the marginal benefit of real money and the right hand side is its marginal cost. The latter is the interest rate forgone discounted one period to have it expressed in current period's units. The marginal benefit is the product of the amount of time saved due to the last unit of real money s_m multiplied by the output price (opportunity cost) of one unit of time, the real wage, w.

Under the properties assigned above to the production function, the real wage is constant in steady state and so it is independent from the rate of inflation (and from R, c, and m).[9] Considering the signs for the second order properties of the shopping time function (decreasing marginal returns on real money, $s_{mm} > 0$ and decreasing marginal cost of consumption in presence of real money, $s_{cm} < 0$), the money demand function MD_{TCT} is defined to be positively related to consumption and negatively to the nominal interest rate. The real wage enters the money demand formulation as part of the constant term

$$m = m(c, R) \qquad (MD_{TCT})$$
$$with\ m_c > 0 \quad and \quad m_R < 0.$$

Labor supply.

Using the household's first order conditions of consumption (6_{TCT}), labor supply (9_{TCT}), and leisure (10_{TCT}), it turns out this expression in steady state

$$\frac{U_l}{w} = \lambda_1 \qquad with\ \lambda_1 = \frac{U_c}{1+ws_c}$$

where the right hand side is occupied by the Lagrange multiplier attached to the household budget constraint (also called shadow price of consumption) and the left hand side is the marginal utility of leisure divided by the real wage which is the opportunity cost of one unit of time. The Lagrange

[9] See Appendix 2 for the proof. It is shown there that the real wage and the real interest rate in steady state only depend on the structural parameters of the model.

multiplier is the ratio between the consumption marginal utility and the output price of consumption. This expression indicates that households are choosing optimally when the marginal rate of substitution between consumption and leisure is equal to its relative price.

Under the second order assumptions $U_{cc} < 0$, $U_{ll} < 0$, $U_{cl} = 0$, $s_{cc} > 0$, and $s_{cm} < 0$ and a constant real wage in steady state, we obtain the following leisure function from the marginality condition

$$l = l(\lambda_1(c, m), c) = l(c, m)$$
$$with\ l_c > 0 \quad and \quad l_m < 0.$$

Let us concentrate on the sign of l_m. The use of more monetary services allow the households to reduce the marginal transaction cost ws_c that is in the denominator of λ_1.[10] This results in a rise of λ_1 that represents an increase in the extent of satisfaction from consuming goods.[11] When consumption is more satisfying households substitute more worktime for less leisure time in order to maintain $\frac{U_l}{w} = \lambda_1$ and consume more with the added labor income. Hence, more real money increases the Lagrange multiplier and decreases the amount of leisure. Taking into account this effect, a labor supply function LS_{TCT} can be then defined from the leisure function and the time constraint (14_{TCT})

$$n^s = T - l(c, m) - s(c, m) = n^s(c, m) \qquad (LS_{TCT})$$
$$with\ n_c^s < 0 \quad and \quad n_m^s > 0.$$

Labor supply rises with m in steady state due to the fall in both shopping and leisure times, $n_m^s > 0$. Now, let us see how the steady state equilibrium changes when there is more money growth to study the issue of *superneutrality*.

The appearance of a higher steady state money growth brings about the same increase in inflation and the nominal interest rate that result in a reduction of real money balances held via the money demand function MD_{TCT}. In this situation, the labor supply function LS_{TCT} would deliver a smaller number with both l and s being larger. As labor is one of the input factors in the production function and markets clear in steady state, there will be less labor entering the production function. The resulting level of output in a more inflationary economy will be smaller.

[10] Here we assume $s_{cm} < 0$ as defined above.

[11] The derivative of the Lagrange multiplier with respect to real money balances is
$\frac{\partial \lambda_1}{\partial m} = \frac{-U_{cl} s_m (1+ws_c) - ws_{cm} U_c}{(1+ws_c)^2} > 0$
with an unambiguous positive sign according to the signs assumed above for $U(.)$ and $s(.)$.

In conclusion, the transactions time model defined here does not present *superneutrality* and reports a steady state negative relationship between inflation and output.

Non-supernautrality in the TCT model. Transmission scheme.

$$\uparrow \mu \longrightarrow \uparrow \pi \longrightarrow \uparrow R \longrightarrow \downarrow m \longrightarrow \left\{ \begin{array}{l} \uparrow s_c \longrightarrow \downarrow \lambda_1 \overset{\frac{U_l}{w} > \lambda_1}{\longrightarrow} \uparrow l \downarrow n \\ \uparrow s \longrightarrow \downarrow n \end{array} \right\} \longrightarrow \downarrow y$$

Let $\eta_{a,b}$ denote the elasticity between variable a and variable b, according to the formula $\eta_{a,b} = \frac{\partial a}{\partial b}\frac{b}{a}$. The elasticity gives percent changes in one variable relative to percent changes in the other. Hence, we considered interesting to calculate the output-to-inflation elasticity for the models at hand so that the size of the *non-superneutrality* effect can be measured. In the shopping time model

$$\eta_{y,\pi} = \eta_{n,\pi} = [\eta_{n,s}\eta_{s,m} + \eta_{n,\lambda_1}\eta_{\lambda_1,m}]\eta_{m,\pi},$$

where $\eta_{y,\pi}$ is the sum of the two effects described above. First, the effect of less real money holdings on worktime because of the need to spend more shopping time, i.e., $\eta_{n,s}\eta_{s,m}\eta_{m,\pi}$. Secondly, the substitution effect due to a higher marginal transaction cost (and thus lower consumption satisfaction λ_1) that leads to reducing the labor supply for more leisure, $\eta_{n,\lambda_1}\eta_{\lambda_1,m}\eta_{m,\pi}$. Recall that $\eta_{y,\pi} = \eta_{n,\pi}$ when the production function is homogeneous of degree 1. Calculating the intermediate elasticities, it is obtained[12]

$$\eta_{y,\pi} = \left[\left(-1\frac{s}{n}\right)\left(s_m\frac{m}{s}\right) + \left(\frac{-U_l}{U_{ll}n}\right)\left(\frac{-ws_{cm}m}{1+ws_c}\right)\right]\left(\frac{s_m}{s_{mm}m}\frac{\pi}{R}\right).$$

The sign of $\eta_{y,\pi}$ is unambiguously negative since it can be observed that $\eta_{m,\pi} < 0$, $\eta_{n,s} < 0$, $\eta_{s,m} < 0$, $\eta_{n,\lambda_1} > 0$, and $\eta_{\lambda_1,m} > 0$ under the first and second order derivatives assumed for $U(.)$ and $s(.)$. The real effects of inflation will be larger with higher elasticity of inflation (or the nominal interest rate) in the money demand $\eta_{m,\pi}$, with higher elasticity of the shopping time with respect to real money balances $\eta_{s,m}$, with higher elasticity of the Lagrange multiplier in the labor supply function η_{n,λ_1}, and finally with a higher elasticity of real money holdings with respect to the Lagrange multiplier $\eta_{\lambda_1,m}$.

[12] For the sake of simplicity when calculating $\eta_{m,\pi}$, we approximated the money demand relation by a relation in which the nominal interest rate R took the place of the marginal cost of money $\frac{R}{1+R}$. We repeat this procedure below on calculating money demand elasticities for the OCT and MIU models.

2.3 Calibration

In order evaluate the long-term effects of inflation we need to utilize particular specifications for the generic functions presented with the model. A calibration process for the parameters of these functions will be carried out aiming at actual data. As for the production function, a Cobb-Douglas technology is used to produce output by employing labor and capital on the following fashion

$$f(n,k) = n^{1-\alpha}k^{\alpha}.$$

The calibration of the parameters is based on data taken from the US economy. The model is estimated in annual observations. The standard figure in the literature is assigned to the capital share parameter in the production function ($\alpha = 0.36$). The rate of intertemporal preference is set at 2% per year ($\rho = 0.02$), which implies $\beta = 0.9804$. The depreciation rate for capital δ will be 10% per year ($\delta = 0.1$). In a Sidrauski-type model, the rate of intertemporal preference coincides with the real interest rate in steady state. Hence, an annual 2% real interest rate is obtained by reaching a 12% gross marginal product of capital that becomes a 2% net marginal product of capital after subtracting for capital depreciation. The baseline steady state solution of the model is taken with a rate of inflation of 4% per year. The nominal interest rate implied is 6% per year. The total amount of time T is set equal to 3 with one third of it devoted to labor supply, $n = 1$, when the economy is in the baseline steady state.

The instantaneous utility function specification is

$$U(c_t, l_t) = \frac{c_t^{1-\sigma_{TCT}}}{1-\sigma_{TCT}} + \Upsilon_{TCT}\frac{l_t^{1-\gamma_{TCT}}}{1-\gamma_{TCT}}$$
$$with\ \sigma_{TCT}, \gamma_{TCT}, \Upsilon_{TCT} > 0.$$

A constant relative risk aversion (CRRA) function was selected with no cross marginal effects, i.e., $U_{cl}(c_t, l_t) = 0$. The value of σ_{TCT} is set to have an elasticity of intertemporal substitution of consumption (EIS from now on)[13] equal to 0.45. Ogaki and Reinhart (1998) recently estimated the EIS for US data obtaining this figure which is some bigger than previous studies. To calculate the EIS, we combined the first order conditions of consumption and bonds in the household's problem and loglinearized the resulting expression.[14] The parameter γ_{TCT} is calibrated in order to have

[13] The elasticity of intertemporal substitution (EIS) refers to:

$$\text{EIS}= \frac{\partial\left(\frac{\log c_{t+1}}{\log c_t}\right)}{\partial \log(1+r_t)}$$

and it was approximated by the expression:

$$\text{EIS}\simeq \frac{\partial\left(\frac{\log c_{t+1}}{\log c_t}\right)}{\partial r_t}$$

[14] See Uhlig (1998) for details regarding the loglinearization techniques used in this paper.

a elasticity of the labor supply with respect to the consumption Lagrange multiplier η_{n,λ_1} equal to 0.2. This elasticity is equal to the real wage elasticity in the labor supply function[15] and takes the value $\frac{l}{\gamma_{TCT} n}$. Taking l/n from the baseline steady state solution,[16] we calibrate γ such that $\frac{l}{\gamma_{TCT} n} = 0.2$. This figure is taken from Johnson and Pencavel (1984) where an extensive labor supply empirical study is conducted for US data. A low number of the labor supply elasticity has been generally utilized in the literature (see Pencavel (1986) for a survey). The leisure scale parameter in the utility function (Υ_{TCT}) is defined to give one third of the total time at work when the economy is in the baseline steady state ($n = 1$, $T = 3$).

The existing time-cost transactions technology is described by the shopping time function

$$s_t = s(c_t, m_t) = \left\{ \begin{array}{ll} 0 & if\ c_t = 0 \\ a_0 + a_1 \frac{c_t^{a_2}}{m_t^{a_3}} & if\ c_t > 0 \end{array} \right\}$$

$$with\ a_0, a_1 > 0, a_2, a_3 > 1.$$

Clearly, our shopping time specification presents a discontinuity when consumption becomes positive. The appearance of a constant term (a_0) in the shopping time function reflects the initial transaction costs due to a positive consumption. To justify this term, we rely on the construction of the consumption aggregate. A positive value of this aggregate implies that transactions were conducted in order to obtain a positive amount of all the goods of the economy. And this is true for a very small positive number of c_t. At that moment, households had to spend some time to get to all the shops of the other $N - 1$ households. We claim that this searching cost (a_0) is fixed. This cost is independent from the volume of transactions. It only depends on the number of goods in the economy N. Other transaction costs vary with the quantity of consumption and the money services (negotiation costs, transportation costs,...).

The scale parameter a_1 indicates the size of the variable time in the total shopping time. There are two other coefficients: the elasticity parameters a_2 and a_3. These two parameters inform about the elasticity of the transaction costs with respect to consumption and real money respectively. Actually, the elasticities are these two values multiplied by a reduction factor ($\eta_{s,c} = a_2(1 - \frac{a_0}{s})$ and $\eta_{s,m} = -a_3(1 - \frac{a_0}{s})$).[17] The higher is the constant searching cost, the lower is the impact of

[15] Combining the labor supply first order condition (9_{TCT}) with the leisure time first order condition (10_{TCT}), loglinearizing, and taking leisure out by entering the time constraint brings about this log-log labor supply function

$$\log n_t^s = \frac{l}{\gamma_{TCT} n} \log w_t + \frac{l}{\gamma_{TCT} n} \log \lambda_{1t} - \frac{s}{n} \log s_t$$

where the constant term was neglected. The same result is reached below for the other three models.

[16] In the baseline steady state solution with $\pi = 0.04$, we have $\frac{n}{T} = 0.333$, $\frac{l}{T} = 0.6563$, and $\frac{s}{T} = 0.0104$. As a result, $\frac{l}{n} = 1.97$.

[17] Loglinearizing our shopping time specification for positive figures of consumption it yields

consumption and money in the total shopping time. The calibrating process for the shopping time function coefficients was carried out through the money demand function implied by the model. We may fairly approximate equation (MD_{TCT}) with the following log-log money demand equation (assuming positive consumption)[18]

$$\log m = \tfrac{1}{1+a_3} \log \left(a_1 a_3 (1-\alpha) \left(\tfrac{1-\beta-\delta\beta}{\alpha\beta} \right)^{\frac{\alpha}{\alpha-1}} \right) + \tfrac{a_2}{1+a_3} \log c - \tfrac{1}{1+a_3} \log R. \qquad (MD_{TCT})$$

Ball (1998) and Meltzer and Rasche (1996) have recently estimated the long-run money demand function for the US economy. They found numbers around -0.2 for the interest rate elasticity and 0.5 for the consumption elasticity. The constant term will be such as the model have a ratio between real money and consumption in steady state equal to 0.25.[19] Accordingly, we set the values of a_1, a_2, and a_3 such that the elasticities in MD_{TCT} and the steady state money over consumption ratio are consistent with the empirical evidence. Finally, a_0 was calibrated by considering the shopping time, s, as 1.04% of the total time, T. This is equivalent to say that transaction costs are valuable 2% of output in steady state for the baseline inflation economy. All the calibrated parameters of the model are reported in Table 1.

2.4 Results

Long-term effects of inflation.

We will measure the real effects of inflation with two related instruments: the percent change in output for various rates of inflation and the elasticity between output and inflation. In particular, the calculations will be conducted for an economy with inflation rates from 0% to 50% per year. All the variables will be given in their steady state general equilibrium solution values. It is vital

$$\log s_t = a_2(1 - \tfrac{a_0}{s}) \log c_t - a_3(1 - \tfrac{a_0}{s}) \log m_t,$$

where the constant terms were neglected. Elasticities are easily calculated from this log-log formulation.

[18] The steady state value of the real wage appearing originally in MD_{TCT} was included now as part of the constant term. For a Cobb-Douglas production function, it is obtained $w = (1-\alpha) \left(\frac{1-\beta-\delta\beta}{\alpha\beta} \right)^{\frac{\alpha}{\alpha-1}}$.

[19] On quarterly US data, the ratio real money over consumption reported a sample mean equal to 1.03 in the period 1965:1-1997:4. M1 money stock seasonally adjusted (SA) was taken to represent nominal money. As for the price level, the GDP implicit deflator was chosen. Finally, consumption is given by the amount of (SA) real personal consumption expenditures. The base year was 1992 for both consumption and the price level. Source: Federal Reserve Bank of St. Louis.

Thus, the cash-in-advance constraint approximately holds in steady state when the model is quarterly calibrated. For the other three models, and to be consistent with the actual annual data, we assume a real money over consumption ratio of 0.25.

to realize that this is not a Phillips-curve analysis. On the contrary, the aim of the present work is to study the long-run effects of different monetary regimes.

The output to inflation elasticity defined above $\eta_{y,\pi}$was calculated for the utility function and shopping time specification calibrated above. Figures of this elasticity were obtained for rates of inflation moving from 0% to 50%. Table 2A reports the results. In the baseline inflation rate (4%), the TCT model presents an output-inflation elasticity equal to -0.003. Looking at the percent changes, it can be seen in Table 2A that output falls -0.22% when inflation moves from 0% to 4%. This drop is 0.48% when inflation reaches 10% and is 1.46% with a rate of inflation equal to 50% per year. We can affirm that there exists a negative real impact of inflation but the size of this effect is quite small. The significant changes in the model occur in the real money balances and the shopping time. The quantity of real money balances decreases 19.2% when inflation goes from 0% to 4%, 28.9% when inflation is 10% and 43.9% when inflation is 50%.

Shopping time increases because of the substantial reduction in the money services available that makes transactions more time-consuming. Thus, if steady state inflation went up from 0% to 4%, the transaction costs would be 7.6% higher in terms of shopping time. The increase would be 16.5% for a 10% rate of inflation and 50.1% in presence of a 50% inflation rate.

Welfare cost of inflation.

Recent papers by Lucas (1995), Gillman (1995), Dotsey and Ireland (1996) and Wolman (1997) have stressed the welfare effects of inflation in a general equilibrium framework. The traditional technique to calculate the welfare cost of inflation was based on the consumer surplus under the money demand function (Bailey (1956)). However, it has been abandoned recently because it is considered being within a partial equilibrium analysis. Following the technique described by Lucas (1995), we will estimate the welfare cost by calculating the amount of consumption needed to offset the loss of utility due to higher inflation. The household would be indifferent with the added consumption in a more inflationary scenario. This amount will be divided by the steady state output to be given as a fraction of output and to have a closer impression of its meaning.

An important part of this recently published work was focused on looking at the figures obtained in the proximities of the Chicago rule compared to the zero-inflation case. As the Chicago rule states, the optimal inflation rate is approximately the negative value of the real interest rate, $\pi^* = \frac{-r}{1+r}$. This deflation rate makes the cost of holding money, i.e. the nominal interest, be zero. Here our scope of analysis is wider and we compare high inflation economies with low inflation economies considering 0% as the (sub)optimal inflation rate.

To estimate the welfare cost of inflation we must look at the changes occurred in the two arguments of the utility function: consumption and leisure. Consumption falls at the same rates as output. Leisure slightly moves up. In turn, the low inflation of the baseline steady state (4%) produces a welfare loss equivalent to 0.16% of the output. A moderate inflation of 10% per year creates a welfare cost of 0.34% of output whereas this cost is a 1.03% of output with 50% as the annual inflation rate. Values are reported in Table 2B. Obviously, the welfare cost of inflation measured here is a permanent cost since we treat the model in steady state.

3 Output-cost transactions approach (OCT)

3.1 The model

The OCT approach assesses that some output resources of the household must be employed in carrying out the purchases of consumption goods. To reach the desired level of consumption c_t, the household must face some costs (transportation, searching costs, trading costs,...) by using units of output within the available transactions technology. Thus, transaction costs are now expressed in output units whereas in the TCT model they were given in time units. This creates a new element in the household budget constraint: a transactions cost function $h(c_t, m_t)$. More consumption requires more resources utilized to pay the transaction costs from the additional purchases. Real money holdings is the other entry of this function. Since money is the medium of exchange, it provides services that make transactions easier and decrease the costs associated with shopping. Hence, the transactions cost function is analogous to the shopping time function, being affected in a positive way by the consumption level and negatively by the amount of real money

$$h_t = h(c_t, m_t), \tag{5_{OCT}}$$

with $h_c > 0$, $h_m < 0$, $h_{cc} > 0$, $h_{mm} > 0$, $h_{cm} < 0$ and $h(0, m_t) = 0$. The partial and second derivatives represent the same behavior as in the shopping time function. They imply increasing marginal cost of consumption and decreasing marginal benefits of real money. The crossed derivative is negative, implying that the marginal cost of purchasing is lower in presence of more monetary services to facilitate transactions.

Household's utility depends on consumption and leisure satisfying the same first and second order properties as in the TCT model. The time constraint states that the total amount of time is divided between working and leisure, i.e., there is no shopping time. The representative household decides their optimal choices for period t according to the maximization problem (H$_{OCT}$)

$$\underset{c_t, k_{t+1}, n_t^d, n_t^s, l_t, m_t^d, b_{t+1}^d}{Max} \sum_{j=0}^{\infty} \beta^j U\left(c_{t+j}, l_{t+j}\right) \tag{H$_{OCT}$}$$

subject to

$$g_{t+j} + f(n^d_{t+j}, k_{t+j}) - c_{t+j} - k_{t+1+j} + (1-\delta)k_{t+j} - w_{t+j}(n^d_{t+j} - n^s_{t+j}) -$$
$$m^d_{t+j} + (1+\pi_{t+j})^{-1} m^d_{t-1+j} - (1+r_{t+j})^{-1} b^d_{t+1+j} + b^d_{t+j} - h(c_{t+j}, m^d_{t+j}) = 0 \; for \; all \; j \geq 0$$
$$T - n^s_{t+j} - l_{t+j} = 0 \qquad for \; all \; j \geq 0$$

that turns into the following first order conditions

$$U_c(c_t, l_t) - \phi_{1t}(1 + h_c(c_t, m^d_t)) = 0, \tag{6_{OCT}}$$
$$-\phi_{1t} + \beta\phi_{1t+1}\left(1 + f_k(n^d_t, k_t) - \delta\right) = 0, \tag{7_{OCT}}$$
$$f_n(n^d_t, k_t) - w_t = 0, \tag{8_{OCT}}$$
$$\phi_{1t} w_t - \phi_{2t} = 0, \tag{9_{OCT}}$$
$$U_l(c_t, l_t) - \phi_{2t} = 0, \tag{10_{OCT}}$$
$$-\phi_{1t}(1 + h_m(c_t, m^d_t)) + \beta\phi_{1t+1}(1+\pi_{t+1})^{-1} = 0, \tag{11_{OCT}}$$
$$-\phi_{1t}(1+r_t)^{-1} + \beta\phi_{1t+1} = 0, \tag{12_{OCT}}$$
$$f(n^d_t, k_t) + g_t - c_t - k_{t+1} + (1-\delta)k_t - w_t(n^d_t - n^s_t) -$$
$$m^d_t + (1+\pi_t)^{-1} m^d_{t-1} - (1+r_t)^{-1} b^d_{t+1} + b^d_t - h(c_t, m^d_t) = 0, \tag{13_{OCT}}$$
$$T - n^s_t - l_t = 0, \tag{14_{OCT}}$$

where ϕ_{1t} and ϕ_{2t} represent the Lagrange multipliers associated to the budget and time constraints in period t.

The general equilibrium setup in the OCT model is formed by the following twenty two equations: relations (5_{OCT})-(14_{OCT}) from the household's optimal choices and the sets ((1)-(4)) and (15)-(22) that were introduced in the TCT model. Assuming satisfactory transversality conditions, stable solution paths can be derived for the twenty two endogenous variables c_t, k_{t+1}, n^d_t, n^s_t, l_t, m^d_t, b^d_{t+1}, g_t, w_t, h_t, y_t, m^s_t, R_t, r_t, π_t, μ_t, P_t, M^s_t, M^d_t, b^s_{t+1}, ϕ_{1t}, and ϕ_{2t}.

3.2 Superneutrality

Money demand.

When we substitute $\beta\phi_{1t+1}$ from (12_{OCT}) into (11_{OCT}) and then apply the real interest rate definition (2), it yields in steady state

$$-h_m = \frac{R}{1+R}. \qquad (MD_{OCT})$$

This money demand formulation resembles the one from the TCT model. However, they differ in the way money helps us in the shopping process. The monetary services represent saving on output under the OCT approach whereas they were saving on time within the TCT approach. Thus, the marginal return on money ($-h_m$) already gives income units and can be comparable to the discounted nominal interest rate standing on the other side of (MD_{OCT}). By using the signs for the second order derivatives in the transactions cost function ($h_{mm} > 0$ and $h_{cm} < 0$), a standard money demand expression can be obtained with a positive consumption elasticity and a negative nominal interest rate elasticity

$$m = m(c, R) \qquad (MD_{OCT})$$
$$with\ m_c > 0 \quad and \quad m_R < 0.$$

Labor Supply

The household's optimal choices imply the satisfaction of this relation in steady state

$$\frac{U_l}{w} = \phi_1 \qquad with\ \phi_1 = \frac{U_c}{1+h_c},$$

that indicates that the leisure marginal utility divided by its price is equal to the consumption marginal utility divided by its price. Note that the price of each unit of consumption in terms of output is equal to $1 + h_c$, i.e., reflects the existence of a marginal transaction cost. Since the real wage w is independent from c, l, and m in steady state, we can reach the leisure function

$$l = l(\phi_1(c, m), c) = l(c, m)$$
$$with\ l_c > 0 \quad and \quad l_m \lesseqgtr 0,$$

according to the sign of the second order derivatives $h_{cc} > 0$, $h_{cm} < 0$, $U_{cc} < 0$, and $U_{ll} < 0$. As shown, there is a dual possible outcome in the sign of l_m. This is a consequence of the existence of a direct link between money and consumption in the steady state overall resource constraint. In the OTC setup the quantity of real money balances appears in the household budget by entering the transactions cost function $h(c, m)$. In the steady state general equilibrium, consumption is the amount of output left after renovating capital and paying the transaction costs, $c = y - \delta k - h(c, m)$.[20] Therefore, more money will bring about more consumption via lower transaction costs paid off. This effect tends to reduce the value of the Lagrange multiplier since the marginal utility of consumption decreases (recalling $U_{cc} < 0$). However, the denominator of the Lagrange multiplier also falls because of the negative impact of money on the marginal transaction cost (assuming $h_{cm} < 0$). As a result, both the numerator and the denominator of the Lagrange multiplier expression change in the same direction. The net result will depend on the size of these two effects. The first derivative allows us to measure the net effect.[21] If $U_{cc}h_m(1 + h_c)$ is smaller than $-h_{cm}U_c$ the Lagrange multiplier ϕ_1 is positively related to the quantity of real money held by the household and consequently $l_m < 0$. By contrast, if $U_{cc}h_m(1 + h_c)$ exceeds $-h_{cm}U_c$ we will get $l_m < 0$. In any event, the labor supply function is obtained by introducing the time constraint (14_{OCT})

$$n^s = T - l(c, m) = n^s(c, m) \qquad (LS_{OCT})$$
$$with\ n^s_c < 0 \quad and \quad n^s_m \lesseqgtr 0,$$

where again the sign of n^s_m is tied to the sign of $\dfrac{\partial \phi_1}{\partial m}$. The two possible outcomes are described in the following box

Non-supernautrality in the OCT model. Transmission scheme with $\dfrac{\partial \phi_1}{\partial m} > 0$.

$$\uparrow \mu \longrightarrow \uparrow \pi \longrightarrow \uparrow R \longrightarrow \downarrow m \longrightarrow \left\{ \begin{array}{l} \uparrow h \longrightarrow \downarrow c \\ \uparrow h_c \end{array} \right\} \longrightarrow \downarrow \phi_1 \overset{\frac{U_l}{w} > \phi_1}{\longrightarrow} \uparrow l \downarrow n \longrightarrow \downarrow y.$$

Non-supernautrality in the OCT model. Transmission scheme with $\dfrac{\partial \phi_1}{\partial m} < 0$.

$$\uparrow \mu \longrightarrow \uparrow \pi \longrightarrow \uparrow R \longrightarrow \downarrow m \longrightarrow \left\{ \begin{array}{l} \uparrow h \longrightarrow \downarrow c \\ \uparrow h_c \end{array} \right\} \longrightarrow \uparrow \phi_1 \overset{\frac{U_l}{w} < \phi_1}{\longrightarrow} \downarrow l \uparrow n \longrightarrow \uparrow y.$$

If it is the case that $\dfrac{\partial \phi_1}{\partial m} > 0$, there will be a negative steady state relationship between inflation and output and no superneutrality whereas when $\dfrac{\partial \phi_1}{\partial m} < 0$ there exists a positive steady state relationship between inflation and output and no superneutrality.

[20] The steady-state overall resource constraint in the OCT model is $y = c + \delta k + h(c, m)$. It is reached by combining in steady state the household budget constraint (13_{OCT}), the government budget constraint (15), and the market clearing conditions (17) – (19).

[21] One may reach
$\dfrac{\partial \phi_1}{\partial m} = \dfrac{-U_{cc}h_m(1 + h_c) - h_{cm}U_c}{(1 + h_c)^2} \gtrless 0.$

The elasticity between steady state output and inflation can be decomposed for the OCT model to yield

$$\eta_{y,\pi} = \eta_{n,\pi} = \eta_{n,\phi_1}\eta_{\phi_1,m}\eta_{m,\pi}.$$

After deriving η_{l,ϕ_1}, $\eta_{\phi_1,m}$ and $\eta_{m,\pi}$, we obtain the following expression for $\eta_{y,\pi}$

$$\eta_{y,\pi} = \left(\frac{-U_l}{U_{ll}n}\right)\left(\frac{-U_{cc}h_m}{U_c}m - \frac{h_{cm}}{1+h_c}m\right)\left(\frac{h_m}{h_{mm}m}\frac{\pi}{R}\right).$$

The sign of $\eta_{y,\pi}$ cannot be determined unambiguously. With $\eta_{n,\phi_1} < 0$ and $\eta_{m,\pi} < 0$ as implied by the first and second order derivatives of $U(.)$ and $h(.)$, the direction of the output-to-inflation relationship will be ultimately rely on the sign of $\eta_{\phi_1,m}$. As seen above, $\eta_{\phi_1,m}$can be either positive or negative.

3.3 Calibration

The utility function and the transactions technology function specified for the OCT model are

$$U(c_t, l_t) = \frac{c_t^{1-\sigma_{OCT}}}{1-\sigma_{OCT}} + \Upsilon_{OCT}\frac{l_t^{1-\gamma_{OCT}}}{1-\gamma_{OCT}}$$
$$with\ \sigma_{OCT}, \gamma_{OCT}, \Upsilon_{OCT} > 0,$$

$$h(c_t, m_t) = \left\{\begin{array}{ll} 0 & if\ c_t = 0 \\ b_0 + b_1\frac{c_t^{b_2}}{m_t^{b_3}} & if\ c_t > 0 \end{array}\right\}$$
$$with\ \ b_0, b_1 > 0,\ \ b_2, b_3 > 1.$$

The interpretation of the coefficients is the same as in the shopping time function. Both specifications represent a transactions technology. The selected functional form $h(c_t, m_t)$ yields the following log-log version of the steady state money demand function

$$\log m = \frac{1}{1+b_3}\log b_1 b_3 + \frac{b_2}{1+b_3}\log c - \frac{1}{1+b_3}\log R, \qquad (MD_{OCT})$$

where again the elasticities only depend upon the coefficients powering consumption and money in the transactions technology specification $h(c_t, m_t)$. In order to conduct comparisons among the models, the calibration process will be based, as much as possible, on the same criteria for each model. Thus, the transactions technology elasticities, b_2 and b_3, are calibrated to fit the results of recent long-term estimates of the money demand elasticities for the US economy. In addition, we figured out the value of b_1 to imply that $\frac{m}{c} = 0.25$. Finally, the constant term in $h(c_t, m_t)$ was set to yield a transactions technology value equal to 2% of the total income for the baseline steady state solution. This is equivalent to say that 1.04% of the total time is devoted to carry out transactions. Regarding σ_{OCT}, γ_{OCT} and Υ_{OCT}, they also were calibrated the same manner as in the TCT approach. The calibration figures appear in the Table 1.

3.4 Results

Long-term effects of inflation.

Table 3A reports the results in percent changes inflation rates from 0 to 50%. The most quantitatively important effects of inflation are a fall in real money balances held and, consequently, a rise in transaction costs. Real money holdings drop by 28.9% when inflation rises from 0% to 10% and by 43.9% when inflation reached 50% per year. As for transaction costs, the reduction in transactions-facilitating money increases transaction costs by 18.2% when inflation is 10% and by 55.5% when inflation is 50%. In contrast, output reports a very small negative response to inflation. Actually, our calibrated OCT model is close to yield *superneutrality* reporting $\eta_{y,\pi} = -0.000086$ for a rate of inflation of 4%. The existence of two opposite effects of real money holdings on the Lagrange multiplier almost offset each other. The net effect is still positive so as to have $\dfrac{\partial \phi_1}{\partial m} > 0$ and therefore a negative output-to-inflation elasticity.[22] In percent units, output decreases 0.017% in value when inflation rises from 0% to 10% and 0.048% when the rate of inflation is 50%. In other words, the response of output is quantitatively very small; less than one twentieth of 1% when inflation is 50%.

Welfare cost of inflation.

Regarding the welfare analysis, we must look at the effects in the arguments of the utility function: consumption and leisure. A very small negative response of output is due to the same small negative response of work time and, consequently, a very small increase in leisure. A change in steady state inflation from 0% to 50% brings about an increase in leisure time by 0.023%. This barely affects utility values. However, consumption behaves much more sensitive to inflation because

[22] After substituting our specified $U(.)$ and $h(.)$ in $\eta_{\phi_1,m} = \frac{-U_{cc}h_m}{U_c}m - \frac{h_{cm}}{1+h_c}m$, it can be shown that $\eta_{y,\pi}$ is positive when $-\sigma + \frac{b_2}{1+b_2 b_1 c^{b_2-1} m^{-b_3}}$ is negative. Accordingly, a lower elasticity of intertemporal substitution (higher σ) or a lower consumption elasticity in the money demand (lower b_2) tend to make the sign of $\eta_{y,\pi}$ positive.

of the appearance of transaction costs $h(c,m)$ in the household budget constraint. In steady state, consumption is output minus depreciated capital and transaction costs, $c = y - \delta k - h(c,m)$. Thus, higher transaction costs in a more inflationary economy (with less monetary services) lead to a fall in consumption. Values are reported in Table 3B. For example, steady state consumption decreases by 0.43% when inflation is 10% and by 1.33% when inflation is 50% compared to the zero inflation economy. This drop in consumption mostly explains the figures calculated for the welfare cost of inflation. The range of numbers is noticeably similar to the TCT. A low inflation rate (4%) has a cost associated equal to 0.13% of output. In a economy with moderate inflation (10%) this cost rises to almost 0.3%. In a highly inflationary economy (π =50%) the welfare cost is a permanent 0.88% of output.

4 Money in the utility function approach (MIU)

4.1 The model

Another form of considering a monetary model is just by including money as one of the arguments of the utility function. The standard assumptions are added for the new element, $U_m > 0$ and $U_{mm} < 0$. With respect to the cross derivatives, it is assumed that $U_{cm} > 0$ and $U_{cl} = U_{ml} = 0$. The first of them, $U_{cm} > 0$, is justified with the existence of money as means of facilitating transactions. More money services turn out in a higher marginal utility of consumption. In addition, leisure cross marginal effects are neglected. Most of the MIU models used in the recent literature satisfy these two assumptions (see for examples Hairault and Portier (1993), Benassy (1995), Chari, Kehoe and McGrattan (1996), Kollman (1996), and Ireland (1997)).

Unlike the time-cost transactions approach and the output-cost transactions approach, the fact of carrying out transactions is not costly in terms of either time or output in a MIU model. Money is valuable *per se*, not for saving some output resources or time. There is no intuitive sense to accept intrinsical value of money and some authors criticize this approach for including money inside the utility function. Others see it as a different and simpler way of reflecting the capacity of money to facilitate transactions in a many-goods economy. In fact, MIU models frequently appear in the literature. Brock (1974), and McCallum (1983) viewed MIU models as an equivalent way of expressing the transactions time approach. The TCT model can be represented as a particular case in the MIU model by substituting the time constraint for leisure in the utility function. As a result, utility would depend on consumption, real money and worktime. Then, the assumptions taken to define the shopping time function are transmitted to the utility function. For example, if the cross derivative is negative in the shopping time function ($s_{cm} < 0$), the utility function with money should be non-separable in consumption and real money and with a positive cross marginal utility ($U_{cm} > 0$).[23] Likewise, the connections between the MIU and the transactions technology approaches were described in Feenstra (1986).

Let us describe the optimizing program for the representative household under the MIU approach. In the time constraint, the total time is distributed on worktime supplied and leisure time with no shopping time. The household budget constraint does not include transaction costs either. Hence, the optimal decision choices from the households come from solving (H_{MIU})

$$\underset{c_t,k_{t+1},n_t^d,n_t^s,l_t,m_t^d,b_{t+1}^d}{Max} \sum_{j=0}^{\infty} \beta^j U\left(c_{t+j}, m_{t+j}^d, l_{t+j}\right) \quad (\mathrm{H}_{MIU})$$

subject to

$$\begin{gathered} f(n_{t+j}^d, k_{t+j}) + g_{t+j} - c_{t+j} - k_{t+1+j} + (1-\delta)k_{t+j} - w_{t+j}(n_{t+j}^d - n_{t+j}^s) - \\ m_{t+j}^d + (1+\pi_{t+j})^{-1} m_{t-1+j}^d - (1+r_{t+j})^{-1} b_{t+1+j}^d + b_{t+j}^d = 0 \quad for\ all\ j \geq 0. \\ T - n_{t+j}^s - l_{t+j} = 0 \qquad for\ all\ j \geq 0. \end{gathered}$$

The first order conditions coming up from (H_{MIU}) are

$$U_c(c_t, m_t^d, l_t) - \psi_{1t} = 0, \quad (6_{MIU})$$

$$-\psi_{1t} + \beta\psi_{1t+1}\left(1 + f_k(n_t^d, k_t) - \delta\right) = 0, \quad (7_{MIU})$$

$$f_n(n_t^d, k_t) - w_t = 0, \quad (8_{MIU})$$

$$\psi_{1t} w_t - \psi_{2t} = 0, \quad (9_{MIU})$$

$$U_l(c_t, m_t^d, l_t) - \psi_{2t} = 0, \quad (10_{MIU})$$

$$U_m(c_t, m_t^d, l_t) - \psi_{1t} + \beta\psi_{1t+1}(1+\pi_{t+1})^{-1} = 0, \quad (11_{MIU})$$

$$-\psi_{1t}(1+r_t)^{-1} + \beta\psi_{1t+1} = 0, \quad (12_{MIU})$$

$$\begin{gathered} f(n_t^d, k_t) + g_t - c_t - k_{t+1} + (1-\delta)k_t - w_t(n_t^d - n_t^s) - \\ m_t^d + (1+\pi_t)^{-1} m_{t-1}^d - (1+r_t)^{-1} b_{t+1}^d + b_t^d = 0, \end{gathered} \quad (13_{MIU})$$

$$T - n_t^s - l_t = 0. \quad (14_{MIU})$$

[23] If $U(c_t, l_t)$ is the utility function of a TCT model, we have
$U(c_t, l_t) = U(c_t, T - n_t^s - s(c_t, m_t)) = \widetilde{U}(c_t, m_t, n_t^s)$
It can be demonstrated that $\widetilde{U}_{cm} = -U_{cl}s_m + U_{ll}s_m s_c - U_l s_{cm}$. Considering the signs $U_{cl} = 0$, $s_m < 0$, $U_{ll} < 0$, $s_c > 0$, $U_l > 0$, and $s_{cm} < 0$, it yields $\widetilde{U}_{cm} > 0$.

where ψ_{1t} and ψ_{2t} are now the Lagrange multipliers associated with the budget and time constraints respectively.

Equations (6_{MIU})-(14_{MIU}) and the sets (1)-(4) and (15)-(22) from the TCT model amount to twenty one equations. Consequently, the presented general equilibrium model can be solved containing the following twenty one variables c_t, k_{t+1}, n_t^d, n_t^s, l_t, m_t^d, b_{t+1}^d, g_t, w_t, y_t, m_t^s, R_t, r_t, π_t, μ_t, P_t, M_t^s, M_t^d, b_{t+1}^s, ψ_{1t}, and ψ_{2t}. Solution paths will be convergent if transversality conditions are satisfied.

4.2 Superneutrality

Money demand.

The demand for money behavior is governed by the following relation in steady state

$$\frac{U_m}{U_c} = \frac{R}{1+R} \qquad (MD_{MIU})$$

which can be obtained from the household first order conditions (6_{MIU}), (11_{MIU}), (12_{MIU}), and the definition of the real interest rate (2). In the MIU model, the marginal benefit of holding real balances is the ratio between the marginal utility of money and the marginal utility of consumption. The marginal cost is again the discounted nominal interest rate. The signs of the second derivatives in the utility function will determine how the variables are related in (MD_{MIU}). We supposed decreasing utility returns on both money ($U_{mm} < 0$) and consumption ($U_{cc} < 0$). Then, the money demand will decrease with a rise in the interest rate and with a greater level of consumption

$$m = m(c, R) \qquad (MD_{MIU})$$
$$with\ m_c > 0 \quad and \quad m_R < 0.$$

Labor supply.

Rearranging relations (6_{MIU}), (9_{MIU}), and (10_{MIU}), it yields in steady state

$$\frac{U_l}{w} = \psi_1 \qquad with\ \ \psi_1 = U_c.$$

The household's optimal choice of leisure depends exclusively on the marginal utility of consumption provided a constant real wage w in steady state. Therefore, the influence of money here is linked to the sign of the cross marginal utility U_{cm}. Assuming the use of transactions-facilitating real money balances, we expect $U_{cm} > 0$ so as to have the leisure function

$$l = l(\psi_1(c,m)) = l(c,m)$$
$$with\ l_c > 0 \quad and \quad l_m < 0,$$

under the second order assumptions $U_{cc} < 0$ and $U_{ll} < 0$. Inserting our leisure function in the time constraint (14_{MIU}) gives rise to the labor supply function

$$n^s = T - l(c,m) = n^s(c,m) \qquad (LS_{MIU})$$
$$with\ n^s_c < 0 \quad and \quad n^s_m > 0.$$

The analysis of *superneutrality* in the MIU model rests on the sign of the cross derivative in the utility function U_{cm}. By construction, this sign is expected to be positive reflecting the characteristics of money as a medium of exchange. Therefore, higher inflation and lower real balances would imply a reduction in the marginal utility of consumption. The leisure optimal choice would be then higher to satisfy the leisure first order condition. In turn, less work time would be supplied by the household and the output level of the general equilibrium economy would drop. Under the assumptions taken here, *superneutrality* does not hold in the MIU model and inflation and output are negatively related in steady state. *Superneutrality* would hold if the marginal utility of consumption were independent from the quantity of real money balances, i.e. $U_{cm} = 0$, and thus there were separability in the utility function between consumption and real money balances.

Non-supernautrality in the MIU model. Transmission scheme.

$$\boxed{\uparrow \mu \longrightarrow \uparrow \pi \longrightarrow \uparrow R \longrightarrow \downarrow m \longrightarrow \downarrow U_c \longrightarrow \downarrow \psi_1 \overset{\frac{U_l}{w} > \psi_1}{\longrightarrow} \uparrow l \downarrow n \longrightarrow \downarrow y}$$

The output-to-inflation elasticity in the MIU model is

$$\eta_{y,\pi} = \eta_{n,\pi} = \eta_{n,\psi_1}\eta_{\psi_1,m}\eta_{m,\pi}$$

which turns into the following expression when the intermediate elasticities are calculated

$$\eta_{y,\pi} = \left(\frac{-U_l}{U_{ll}n}\right)\left(\frac{U_{cm}m}{U_c}\right)\left(\frac{1}{\frac{U_{mm}m}{U_m} - \frac{U_{cm}m}{U_c}}\frac{\pi}{R}\right).$$

Maintaining the crucial assumption $U_{cm} > 0$, the sign of $\eta_{y,\pi}$ is unambiguously negative in the MIU model since $\eta_{n,\psi_1} > 0$, $\eta_{\psi_1,m} > 0$, and $\eta_{m,\pi} < 0$. However if there were separability in the utility function such that $U_{cm} = 0$ *superneutrality* would hold with $\eta_{\psi_1,m} = 0$.

4.3 Calibration

The MIU framework can be specified through any utility function that contains real money balances as one of its entries. When we presented the decision-making process, it was utilized a utility function with real money, consumption, and leisure time. Now, we take that generic representation and choose a particular function to examine the behavior of the model. For the sake of consistency with the transactions-facilitating role of money, the utility function should be non-separable between consumption and real money with a positive U_{cm}. Hence, we present the following MIU utility function specification:

$$U(c_t, m_t, l_t) = \frac{[(\varpi c_t^{\frac{\nu}{\tau}} + (1-\varpi) m_t^{\upsilon})^{\frac{1}{\upsilon}}]^{1-\sigma_{MIU}}}{1-\sigma_{MIU}} + \Upsilon_{MIU} \frac{l_t^{1-\gamma_{MIU}}}{1-\gamma_{MIU}}$$
$$\textit{with} \qquad \sigma_{MIU}, \tau, \gamma_{MIU}, \Upsilon_{MIU} > 0, \nu < 1 \qquad \textit{and} \qquad 0 < \varpi < 1,$$

in which U_{cm} is positive by construction. This specification has a generalized CES utility index for consumption and real money, commonly used in the literature. The parameter τ is incorporated to derive a money demand expression such that the elasticity of consumption may take a value different from 1. When it was the case that $\tau = 1$, i.e. the standard CES case, the money demand would imply a unitary consumption elasticity and the utility function would have constant elasticity of substitution between consumption and money. In any event, the steady state log-log money demand (MD_{MIU}) derived for our utility function specification is

$$\log m = \frac{1}{\upsilon - 1} \log \left(\frac{\varpi}{(1-\varpi)\tau} \right) + \frac{\frac{\upsilon}{\tau} - 1}{\upsilon - 1} \log c + \frac{1}{\upsilon - 1} \log R. \qquad (MD_{MIU})$$

We set all the parameters of the model following the criteria described for the TCT model. Hence, MD_{MIU} will have a consumption elasticity equal to 0.5 and an interest rate elasticity equal to -0.2. In addition, the elasticity of the labor supply with respect to the Lagrange multiplier is 0.2, the EIS is 0.45, the ratio of worktime over total time is 1/3, and m/c=0.25 when the model is in the baseline steady state with 4% inflation. The figures obtained for σ_{MIU}, υ, τ, ϖ, γ_{MIU}, and Υ_{MIU} are reported in Table 1.

4.4 Results

Long-term effects of inflation.

Once calibrated the model, the figure found for $\eta_{y,\pi}$ was negative but very close to zero and so to the *superneutrality* case. When steady state inflation is 4%, $\eta_{y,\pi} = -0.0001$. As discussed above, the sign of $\eta_{y,\pi}$ is determined by the sign of U_{cm}. For our utility function specification, the sign of the latter depends on the value of $1 - \sigma_{MIU} - \nu$.[24] If this magnitude is positive both U_{cm} and $\eta_{\psi_1,m}$ are positive, and $\eta_{y,\pi}$ is negative. Hence, a lower EIS (higher σ_{MIU}) or a higher nominal interest rate elasticity in the money demand (lower υ in absolute value) could turn the value of $\eta_{y,\pi}$ on the positive side.

The magnitudes found for percent changes in output at increasing inflation rates show the proximity to *superneutrality*. When inflation rises from 0% to 4%, output drops only by 0.008%. When inflation reaches 10% the output fall is 0.017% and in an economy with an annual 50% inflation output is 0.051% less than in a zero inflation economy. With no measure of transaction costs in the MIU model, the only variable significantly affected by inflation is real money balances. There is almost a 30% reduction in real money holdings when inflation moves up from 0% to 10%. This reduction becomes 43.6% when π=50%. Of course, all the changes are permanent since this is steady state analysis. Table 4A informs about these results.

Welfare cost of inflation.

To consider the welfare cost of inflation, it is necessary to look at how consumption, money and leisure evolve with increasing inflation. The three variables are entering the utility function. Consumption decreases at the same very low rates as output (it is a linear function of output), money has a moderately high reduction and leisure hardly takes higher values. By plugging the three responses in the utility function, the welfare cost obtained is 0.14% for a 4% inflation, 0.31% for a 10% inflation and 0.96% for a 50% inflation (see Table 4B). These numbers are similar to results reached in the TCT and in the OCT models. Rather than a decrease of consumption, the existence of a significant welfare cost reflects here the fact that real money has a direct influence on the level of utility by entering the utility function.

[24] The output-to-inflation elasticity depends on the elasticity of the Lagrange multiplier with respect to real money balances $\eta_{\psi_1,m} = \frac{U_{cm}m}{U_c}$. For our utility function specification, $\eta_{\psi_1,m} = (1 - \sigma_{MIU} - \nu)\frac{(1-\varpi)m^{\nu}}{\varpi c^{\upsilon/b}+(1-\varpi)m^{\upsilon}}$, implies that $\eta_{\psi_1,m} > 0$ when $1 - \sigma_{MIU} - \nu > 0$ since the second part of $\eta_{\psi_1,m}$ is definitely positive. After calibration, it was found $1 - \sigma_{MIU} = -3.255$ and $\nu = -4$ for the baseline model, implying a slightly positive figure for $\eta_{\psi_1,m}$.

All the conclusions reached for the MIU model are subject to the properties assumed in the particular utility function used in the analysis. Results may change by considering a different specification. For instance, as it was pointed out in footnote 24, the TCT model could have even been considered as one particular case of the MIU model. Our intention here is just to estimate some long-term indicators of a representative MIU model to compare them with the other approaches. Aiming at this goal, the utility function analyzed was chosen because of its tractability, consistency with the other models, and frequent use in the literature.

5 Cash-in-advance approach (CIA)

5.1 The model

Here we have another view of the role of money as a medium-of-exchange. Money as something necessary to conduct any purchase. It is the only counterpart to any transaction. This is the reason why the cash-in-advance constraint emerges. The household must keep an amount of money at least equal to the desired consumption expenditures

$$M_{t+j}^d \geq \sum P_{t+j}(i)c_{t+j}(i) \qquad for\ all\ j \geq 0,$$

that using $\sum P_{t+j}(i)c_{t+j}(i) = P_{t+j}c_{t+j}$ and dividing by the price level, it becomes in real terms

$$m_{t+j}^d \geq c_{t+j} \qquad for\ all\ j \geq 0.$$

This inequality will be binding in our general equilibrium because nominal interest rate are always greater than zero[25]. Money is only demanded to satisfy the cash-in-advance constraint. Neither does it enter the utility function nor does it have any direct influence in the time allocation for the household. No direct transaction costs are considered in the budget constraint. The utility function depends again on both consumption and leisure with the same characteristics as for the models above. However, the cash-in-advance is one more constraint attached to the maximization problem (H$_{CIA}$):

$$\underset{c_t,k_{t+1},n_t^d,n_t^s,l_t,m_t^d,b_{t+1}^d}{Max} \sum_{j=0}^{\infty} \beta^j U\left(c_{t+j}, l_{t+j}\right) \qquad (\mathrm{H}_{CIA})$$

[25] Nominal interest rates are positive due to non-negative money-growth ($\mu_t \geq 0$) and positive marginal productivity of capital ($f_k(n_t^d, k_t) > 0$).

subject to

$$f(n^d_{t+j}, k_{t+j}) + g_{t+j} - c_{t+j} - k_{t+1+j} + (1-\delta)k_{t+j} - w_{t+j}(n^d_{t+j} - n^s_{t+j}) -$$
$$m^d_{t+j} + (1+\pi_{t+j})^{-1} m^d_{t-1+j} - (1+r_{t+j})^{-1} b^d_{t+1+j} + b^d_{t+j} = 0 \ for \ all \ j \geq 0,$$
$$T - n^s_{t+j} - l_{t+j} = 0 \qquad for \ all \ j \geq 0,$$
$$m^d_{t+j} - c_{t+j} = 0 \qquad for \ all \ j \geq 0.$$

The first order conditions obtained from (H_{CIA}) are

$$U_c(c_t, l_t) - \theta_{1t} - \theta_{3t} = 0, \tag{5_{CIA}}$$
$$-\theta_{1t} + \beta\theta_{1t+1}\left(1 + f_k(n^d_t, k_t) - \delta\right) = 0, \tag{6_{CIA}}$$
$$f_n(n^d_t, k_t) - w_t = 0, \tag{7_{CIA}}$$
$$\theta_{1t} w_t - \theta_{2t} = 0, \tag{8_{CIA}}$$
$$U_l(c_t, l_t) - \theta_{2t} = 0, \tag{9_{CIA}}$$
$$-\theta_{1t} + \beta\phi_{1t+1}(1+\pi_{t+1})^{-1} + \theta_{3t} = 0, \tag{10_{CIA}}$$
$$-\theta_{1t}(1+r_t)^{-1} + \beta\theta_{1t+1} = 0, \tag{11_{CIA}}$$
$$f(n^d_t, k_t) + g_t - c_t - k_{t+1} + (1-\delta)k_t - w_t(n^d_t - n^s_t) -$$
$$m^d_t + (1+\pi_t)^{-1} m^d_{t-1} - (1+r_t)^{-1} b^d_{t+1} + b^d_t = 0, \tag{12_{CIA}}$$
$$T - n^s_t - l_t = 0, \tag{13_{CIA}}$$
$$m^d_t - c_t = 0. \tag{14_{CIA}}$$

where θ_{1t}, θ_{2t} and θ_{3t} are the Langrange multipliers of the budget, time, and cash constraints respectively in period t.

As we did for the other models, the CIA model can be solved in general equilibrium by having the system of twenty two equations: (1)-(4), (5_{CIA})-(14_{CIA}), and (15)-(22) to reach solution paths for the twenty two variables c_t, k_{t+1}, n^d_t, n^s_t, l_t, m^d_t, b^d_{t+1}, g_t, w_t, y_t, m^s_t, R_t, r_t, π_t, μ_t, P_t, M^s_t, M^d_t, b^s_{t+1}, θ_{1t}, θ_{2t}, and θ_{3t}. Stability in the solution sequences is guaranteed when the transversality conditions are held.

5.2 Superneutrality

Money demand.

The money demand function is governed by the requirement of satisfying the cash-in-advance constraint (14_{CIA})

$$m = c. \qquad (MD_{CIA})$$

By taking logarithms on both sides, this expression supposes a money demand function with unitary consumption elasticity and zero interest rate elasticity. Put in another way, the CIA model exhibits unitary consumption velocity of money for any nominal interest rate.

Labor supply.

Using the household's first order conditions of consumption (5_{CIA}), labor supply (8_{CIA}), leisure time (9_{CIA}), and bonds (10_{CIA}), and the nominal interest rate definition (2), it turns out this expression in steady state

$$\frac{U_l}{w} = \theta_1 \qquad with\ \theta_1 = \frac{U_c}{1 + \frac{R}{1+R}},$$

where, like in the other three models, the marginal utility of leisure divided by its output price w is equal to the marginal utility of consumption divided by its output price $1 + \frac{R}{1+R}$. In this regard, the CIA supposes that one more unit of real money balances whose opportunity cost is $\frac{R}{1+R}$ must be held to purchase one more unit of consumption and therefore the output price of consumption is $1 + \frac{R}{1+R}$. The leisure function implied by the above expression is

$$l = l(\theta_1(c, R)) = l(c, R)$$
$$with\ l_c > 0 \quad and \quad l_R > 0.$$

A higher nominal interest discourages the household to consume due to the more costly price of holding the required money balances. The shadow price of consumption θ_1 falls and therefore leisure time increases to recuperate the equilibrium condition (assuming $U_{ll} < 0$). A similar argument can be stated to explain $l_c > 0$. More consumption reduces the marginal utility of consumption (assuming $U_{cc} < 0$) and θ_1 falls so as to bring about an increase in leisure. The labor supply function implied by the leisure function and the time constraint (13_{CIA}) is

$$n^s = T - l(c,R) = n^s(c,R) \qquad (LS_{CIA})$$
$$with\ n^s_c < 0 \quad and \quad n^s_R < 0.$$

As argued above, the response of leisure time, and thus worktime, to a new inflation in the CIA model is by means of a direct effect of the nominal interest in the Lagrange multiplier θ_1. In the presence of a new steady state higher inflation, a rise in the interest rate makes the consumption shadow price smaller due to the increase in the output price of consumption $(1+\frac{R}{1+R})$. The optimal choice from the households will be an increase in the amount of leisure time. Then, less time will be spent at work and less output will be produced. *Superneutrality* does not hold in steady state and there is a long-term negative relationship between inflation and output.

Non-superneutrality in the CIA model. Transmission scheme.

$$\boxed{\uparrow \mu \longrightarrow \uparrow \pi \longrightarrow \uparrow R \longrightarrow \downarrow \theta_1 \overset{\frac{U_l}{w} > \theta_1}{\longrightarrow} \uparrow l \downarrow n \longrightarrow \downarrow y}$$

The elasticity of output with respect to inflation for the CIA model is[26]

$$\eta_{y,\pi} = \eta_{n,\pi} = \eta_{n,\theta_1}\eta_{\theta_1,\pi} = \left(\frac{-U_l}{U_{ll}n}\right)\left(\frac{-\pi}{1+R}\right).$$

The sign $\eta_{y,\pi}$ is always negative with $\eta_{n,\theta_1} > 0$ and $\eta_{\theta_1,\pi} < 0$.

5.3 Calibration

The CIA model was solved for the same CRRA type of utility function as the TCT and OCT models depending on consumption and leisure

$$U(c_t, l_t) = \frac{c_t^{1-\sigma_{CIA}}}{1-\sigma_{CIA}} + \Upsilon_{CIA}\frac{l_t^{1-\gamma_{CIA}}}{1-\gamma_{CIA}}$$
$$with \qquad \sigma_{CIA}, \gamma_{CIA}, \Upsilon_{CIA} > 0.$$

The log-log money demand formulation is obtained from the binding cash-in-advance constraint with a unitary consumption elasticity and a zero interest rate elasticity:

$$\log m = \log c. \qquad (MD_{CIA})$$

[26] Here again we used the approximation $R \simeq \frac{R}{1+R}$ to calculate $\eta_{\theta_1,\pi}$.

The CIA model is calibrated here in quarterly terms because the cash-in-advance constraint (or money demand) obliges to stand a unitary value of $\frac{m}{c}$. This number is implied by the actual data whit quarterly observations (see footnote 20). Since there are no coefficients in the money demand representation of the CIA model, we only need to calibrate the coefficients of the utility function. Here again, the risk aversion coefficient was set to imply a intertemporal elasticity of substitution equal to 0.45. In addition, γ_{CIA} and Υ_{CIA} take respectively the values such that the elasticity of the Lagrange multiplier in the labor supply is 0.2 and the working time is one third of the total in the steady state solution for the baseline inflation rate $\pi = 4\%$ per year. Table 1 reports the selected values for the parameters σ_{CIA}, γ_{CIA}, and Υ_{CIA}.

5.4 Results

Long-term effects of inflation.

The CIA model presents unrealistic features. Money demand is not directly affected by a change in the inflation rate because it has a zero nominal interest rate elasticity. The cash-in-advance constraint obliges money to be tied to consumption. So, the only response of money to higher of inflation is by means of the consumption-leisure substitution that takes place through the labor supply optimal choice. A higher steady state nominal interest rate makes consumption more costly because of the cash-in-advance requirement. This fact gives rise to less consumption and more leisure. In steady state with a Cobb-Douglas production function, consumption, capital and output are linear functions of the working time. Therefore, these three variables, and real money in order to satisfy the cash-in-advance constraint. decrease at the same rate when inflation goes up. As a result, money appears to respond very little, and both consumption and output very much. Table 5A reports some of these values. Comparing to a 0% inflation economy, real money and output fall by 0.33% with a 10% inflation rate and by 1.44% with a 50% inflation rate. These numbers cannot be supported by the empirical evidence since the response of real money is much lower than expected. The value of $\eta_{y,\pi}$ when π=4% is -0.0075, the largest in absolute value among the models at hand. In conclusion, the CIA model provides a very restrictive money demand function that turns out into unrealistic long-run relationships.

Welfare cost of inflation.

According to the time constraint (13_{CIA}), leisure is the complementary to working time. Thus, we find a falling consumption and a growing leisure time entering the utility function at higher inflation rates. These two effects tend to compensate resulting in quite a low welfare loss for the households. Indeed, the CIA model reports the lowest figures for the welfare cost of inflation (see

Table 5B for the magnitudes and Figure 2 for a comparison). A 10% rate of inflation has a 0.024% of output as its welfare cost. This amount rises to 0.078% for a 30% inflation and to 0.14% for a 50% inflation per year.

6 Conclusions

Four of the most used optimizing monetary models based on the concept of money as a medium-of-exchange were examined for their long-term features. The variety of procedures observed in the literature reflects the fact that money can play its role of facilitating transactions in different ways. Money may affect the time allocation (TCT approach), money may affect the income allocation (OCT approach), money may affect directly utility (MIU approach), and money may be required in advance for carrying out purchases (CIA approach).

This paper describes how the transactions facilitating property of money can be rationalized to study long-term properties. Thus, models were solved under optimizing criteria to obtain the steady state solutions in general equilibrium. None of the models satisfied *superneutrality*. In steady state, the nominal money growth rate affects the values of real variables: output, investment, consumption, and capital. We explained how this takes place in a twofold process. First, the money demand function gave the response of the amount of real money balances held to a change of the nominal interest rate. Second, the labor supply choice was altered by the amount of monetary services available for the household. In monetary economies with money used to carry out transactions, the quantity of real money affects the degree of satisfaction of consumption, and this motivates a leisure/labor substitution.

Money demand expressions derived from optimization criteria are alike for three of the approaches: time-cost transactions model (TCT), output-cost transactions model (OCT), and money in the utility function model (MIU). In fact, these models can be parameterized to imply the same elasticities in their money demand function. By contrast, the cash-in-advance model (CIA) generates a money demand with fixed elasticities: unitary consumption elasticity and zero nominal interest rate elasticity.

Labor supply functions reflect leisure/labor substitutions that take place when steady state money-growth is altered. Unlike the money demand functions, a different impact in each model was reported based on the various ways money can be used to conduct purchases. Thus, the output-inflation relationship was defined distinctly among the four models at hand.

The long-term features examined were the steady state effects of inflation and the welfare cost of inflation. The models were calibrated with US data and solved for a wide range of rates of inflation. For the sake of consistency, the same elasticity of intertemporal substitution, elasticities in the money demand function (if possible), and elasticities in the labor supply function were set in the four approaches.

The results obtained were similar except for the cash-in-advance setup. In this particular model, real money balances and output change equally in size when inflation rises. This feature seems to be very unrealistic. In the other three models, there is an important reduction in real money balances as the main consequence of inflation. The real money holdings decrease in value by around 30% when inflation moves from zero to 10%, and there almost exists a 45% decrease when inflation gets to 50%. Another significant effect of inflation is the increase in the transaction costs due to the decrease in the money services available for the household. These costs are collected in the TCT model in terms of time and in the OCT model in terms of output. The latter approach reported a increase in the transaction costs of 16% when steady state inflation moves from 0% to 10% and 55% when inflation reaches 50% per year. In terms of time, the TCT model informed of a 16.5% increase in shopping time for a 10% inflation and a 50% increase for a 50% inflation.

Output adjusts by much less in the presence of more inflation and less real money balances. The OCT model and the MIU model were on the verge of *superneutrality*, and they featured output dropping around 0.02% when annual inflation rises from 0 to 10%. By contrast, the TCT and CIA showed a more significant decrease though still quite small. If steady state inflation changes from 0% to 10% per year, there is a permanent reduction in output of 0.48% and 0.33%, respectively. A permanent high inflation at 50% per year generates a fall in output by around 1.45% in both models. Figure 4 compares the responses of steady state output to rates of inflation moving from zero to 50% for the four models at hand.

Values of the welfare cost of inflation were also calculated for the models at hand and results can be compared in Figure 5. Surprisingly, even though the transmission mechanisms for long-run effects of money-growth are somewhat different, three of the models (TCT, OCT, and MIU) agree to report very similar numbers as Figure 5 shows. In the TCT and OCT model, both of them incorporating transaction costs, the welfare cost comes from decreasing consumption since leisure moves very slightly up. The welfare losses in the MIU model are caused by a very significant reduction in real money holdings that enter the utility function. The CIA model reports different figures, quite smaller than the others because consumption increases and leisure decreases in such a way that there is very little final effect. Hence, in the TCT, OTC, and MIU models, the loss of utility derived from a 10% inflation was measured to be about 0.3% of the total output with respect to the zero inflation economy. A moderately high inflation (30%) produces a welfare cost around 0.7% of output. The highest inflation rate studied (50%) supposed welfare cost values around 1%.

Our welfare cost results are smaller than those reported in Lucas (1996) or in Pakko (1998), but closer to the figures in Chadha, Haldane and Janssen (1998). However, these cited papers emphasize results obtained at low inflation rates. Our results, by contrast, emphasize differences and similarities resulting from several modelling approaches, with a substantial range of inflation rates considered.

APPENDIX 1. A equivalent general equilibrium with firms.

Let the economy be formed with N households and N firms. Households own of the stock of capital which is supplied to the firms in exchange of a rental price. The contract is made one period in advance at the rental price of the capital market in real terms ($\mathfrak{r}$). Thus, the capital income received by the household in period t is $\mathfrak{r}_{t-1}k_t^s$. Households also work for the firms and receive in return the market real wage per unit of time. Firms decide the labor and capital demands through the profit maximization criterion. The utility function and the production function have the same properties described in the main text for the TCT model. Symmetric equilibrium holds.

- Households decision:

$$\underset{c_t,k_{t+1}^s,n_t^s,l_t,m_t^d,b_{t+1}^d}{Max} \qquad \sum_{j=0}^{\infty} \beta^j U\left(c_{t+j}, l_{t+j}\right)$$

subject to

$$\begin{gathered} g_{t+j} + w_{t+j}n_{t+j}^s + \mathfrak{r}_{t-1+j}k_{t+j}^s - c_{t+j} - k_{t+1+j}^s + (1-\delta)k_{t+j}^s - \\ m_{t+j}^d + (1+\pi_{t+j})^{-1}m_{t-1+j}^d - (1+r_{t+j})^{-1}b_{t+1+j}^d + b_{t+j}^d = 0 \qquad for\ all\ j \geq 0. \\ T - n_{t+j}^s - s(c_{t+j}, m_{t+j}^d) - l_{t+j} = 0 \qquad for\ all\ j \geq 0. \end{gathered}$$

First order conditions from the representative household in period t

$$U_c(c_t, l_t) - \lambda_{1t} - \lambda_{2t}s_c(c_t, m_t^d) = 0, \qquad (A1.1)$$

$$-\lambda_{1t} + \beta\lambda_{1t+1}\left(1 + \mathfrak{r}_t - \delta\right) = 0, \qquad (A1.2)$$

$$\lambda_{1t}w_t - \lambda_{2t} = 0, \qquad (A1.3)$$

$$U_l(c_t, l_t) - \lambda_{2t} = 0, \qquad (A1.4)$$

$$-\lambda_{1t} + \beta\lambda_{1t+1}(1+\pi_{t+1})^{-1} - \lambda_{2t}s_m(c_t, m_t^d) = 0, \qquad (A1.5)$$

$$-\lambda_{1t}(1+r_t)^{-1} + \beta\lambda_{1t+1} = 0, \qquad (A1.6)$$

$$\begin{gathered} g_t + w_t n_t^s + \mathfrak{r}_{t-1}k_t^s - c_t - k_{t+1}^s + (1-\delta)k_t^s - \\ m_t^d + (1+\pi_t)^{-1}m_{t-1}^d - (1+r_t)^{-1}b_{t+1}^d + b_t^d = 0, \end{gathered} \qquad (A1.7)$$

$$T - n_t^s - s(c_t, m_t^d) - l_t = 0. \qquad (A1.8)$$

- Firms decision

$$\underset{n_t^d, k_t^d}{Max} \qquad \sum_{j=0}^{\infty} \beta^j \left(y_{t+j} - w_{t+j} n_{t+j}^d - \mathfrak{r}_{t-1+j} k_{t+j}^d \right)$$

subject to

$$y_{t+j} = f\left(n_{t+j}^d, k_{t+j}^d\right) \qquad for\ all\ j \geq 0.$$

First order conditions from the representative firm in period t

$$f_n\left(n_t^d, k_t^d\right) - w_t = 0, \tag{A1.9}$$

$$f_k\left(n_t^d, k_t^d\right) - \mathfrak{r}_{t-1} = 0, \tag{A1.10}$$

$$y_t = f\left(n_t^d, k_t^d\right). \tag{A1.11}$$

- Government budget constraint

$$g_t = m_t^s - (1+\pi_t)^{-1} m_{t-1}^s + (1+r_t)^{-1} b_{t+1}^s - b_t^s. \tag{A1.12}$$

- Definitions

$$\pi_t = \frac{P_t}{P_{t-1}} - 1, \tag{A1.13}$$

$$\mu_t = \frac{M_t^s}{M_{t-1}^s} - 1, \tag{A1.14}$$

$$1 + r_t = \frac{1+R_t}{1+\pi_{t+1}}, \tag{A1.15}$$

$$m_t^s = \frac{M_t^s}{P_t}, \tag{A1.16}$$

$$m_t^d = \frac{M_t^d}{P_t}. \tag{A1.17}$$

- Shopping time function

$$s_t = s(c_t, m_t^d). \tag{A1.18}$$

- Monetary policy rule

$$\mu_t = \mu \quad with \quad \mu \geq 0. \tag{A1.19}$$

- Fiscal policy

$$g_t = g \quad with \quad g \geq 0. \tag{A1.20}$$

- Market clearing conditions

$$n_t^d = n_t^s, \tag{A1.21}$$

$$k_{t+1}^d = k_{t+1}^s, \tag{A1.22}$$

$$m_t^d = m_t^s, \tag{A1.23}$$

$$b_{t+1}^d = b_{t+1}^s. \tag{A1.24}$$

The set of twenty four equations $(A1.1) - (A1.24)$ together with transversality conditions determine stable solution paths for the twenty four variables of the model c_t, k_{t+1}^s, n_t^s, l_t, m_t^d, b_{t+1}^d, g_t, w_t, $\mathfrak{r}_{t-1}$, s_t, y_t, n_t^d, k_{t+1}^d, m_t^s, R_t, r_t, π_t, μ_t, P_t, M_t^s, M_t^d, b_{t+1}^s, λ_{1t}, and λ_{2t}. The resulting general equilibrium solution is equivalent to the general equilibrium developed in the main text for the twenty two variables of the TCT model. Now we have two new variables k_{t+1}^d(or k_{t+1}^s) and $\mathfrak{r}_{t-1}$, two new equations $(A1.10)$ and $(A1.22)$, one more market (capital market), and the following implied relationship: $\mathfrak{r}_t = r_t + \delta$ reached combining $(A1.2)$ and $(A1.5)$.

APPENDIX 2.

This appendix consists of a proof that the steady state real interest rate and the steady state real wage of the TCT model developed in the main text only depend on the structural parameters of the model.

- Real interest rate.

As for the real interest rate, the demonstration is straightforward. The first order conditions of bonds (12_{TCT}) implies in steady state

$$\frac{1}{\beta} = 1 + r. \tag{A2.1}$$

Provided that the discount factor β was defined as the inverse of $1+\rho$, $(A2.1)$ yields

$$\rho = r. \tag{A2.2}$$

The steady-state real interest rate r is equal to the rate of intertemporal preference of the households ρ.

- Real wage.

The production function is homogeneous of degree 1. Thus, applying Euler theorem

$$f(n,k) = f_n(n,k)n + f_k(n,k)k. \tag{A2.3}$$

Next, the first order condition due to the labor demand (8_{TCT}) implies that the real wage is equal to the marginal product of labor in steady state. Using this in $(A2.3)$ and rearranging it is obtained

$$w(n,k) = \frac{f(n,k) - f_k(n,k)k}{n}, \tag{A2.4}$$

from which the real wage depends on both labor and capital. Let $\overline{k} = \frac{k}{n}$ be the stock of capital per unit of work time. The production function reports constant returns to scale. Accordingly, equation $(A2.4)$ can be expressed as follows

$$w(n,k) = f\left(\overline{k}\right) - f_k(n,k)\overline{k}. \tag{A2.5}$$

Partial derivatives of the real wage in $(A2.5)$ with respect to its two arguments, labor and capital, are calculated to yield

$$\frac{\partial w(n,k)}{\partial n} = f_{\overline{k}}(\overline{k})\left(\frac{-k}{n^2}\right) - f_{kn}(n,k)\overline{k} - f_k(n,k)\left(\frac{-k}{n^2}\right), \tag{A2.6a}$$

$$\frac{\partial w(n,k)}{\partial k} = f_{\overline{k}}(\overline{k})\left(\frac{1}{n}\right) - f_{kk}(n,k)\overline{k} - f_k(n,k)\left(\frac{1}{n}\right). \tag{A2.6b}$$

Since $f(n,k)$ is homogeneous of degree 1, $f_k(n,k)$ is homogeneous of degree 0. Homogeneity of degree 0 implies

$$f_{\overline{k}}(\overline{k}) = f_k(n,k). \tag{A2.7}$$

Equations (7_{TCT}) and (12_{TCT}) from the f.o.c. of the households yield

$$r = f_k(n,k). \tag{A2.8}$$

Recalling ($A2.2$), it is obtained that in steady state $f_{kn}(n,k) = f_{kk}(n,k) = 0$. Inserting this result and ($A2.7$) in the partial derivatives of the real wage $A2.6a$ and $A2.6b$

$$\frac{\partial w(n,k)}{\partial n} = f_k(n,k)\left(\frac{-k}{n^2}\right) - f_k(n,k)\left(\frac{-k}{n^2}\right) = 0, \tag{A2.6a'}$$

$$\frac{\partial w(n,k)}{\partial k} = f_k(n,k)\left(\frac{1}{n}\right) - f_k(n,k)\left(\frac{1}{n}\right) = 0. \tag{A2.6b'}$$

The real wage is constant in steady state for any given structural parameters.

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Table 1. Calibration of parameters.

TCT model	OCT model	MIU model	CIA model
$\sigma_{TCT} = 2.21$	$\sigma_{OCT} = 2.21$	$\sigma_{MIU} = 4.255$	$\sigma_{CIA} = 2.222$
$\gamma_{TCT} = 9.844$	$\gamma_{OCT} = 10$	$\gamma_{MIU} = 10$	$\gamma_{CIA} = 10$
$\Upsilon_{TCT} = 519.87$	$\Upsilon_{OCT} = 710.47$	$\Upsilon_{MIU} = 254.95$	$\Upsilon_{CIA} = 258.35$
$a_0 = 0.0274$	$b_0 = 0.024$	$\upsilon = -4$	
$a_1 = 0.00002236$	$b_1 = 0.00002119$	$\tau = 8/3$	
$a_2 = 2.5$	$b_2 = 2.5$	$\varpi = 0.99996$	
$a_3 = 4$	$b_3 = 4$		

Table 2A. TCT model.

Changes in real money, transaction costs, and output when annual inflation moves up from 0% (percent changes in steady state).

Elasticity of output with respect to inflation ($\eta_{y,\pi}$).

Inflation	Real money	Trans.costs($s(c,m)$)	Output	$\eta_{y,\pi}$
2	-12.6	4.1	-0.12	-0.001
4	-19.2	7.6	-0.22	-0.003
6	-23.5	10.8	-0.32	-0.004
8	-26.6	13.7	-0.40	-0.005
10	-28.9	16.5	-0.48	-0.006
20	-36.1	27.9	-0.81	-0.010
30	-39.8	36.8	-1.07	-0.013
40	-42.2	44.0	-1.28	-0.016
50	-43.9	50.1	-1.46	-0.018

Table 2B. TCT model.

Welfare cost of inflation from 0% inflation (percentage of output).

Changes in consumption and leisure when annual inflation moves up from 0% (percent changes in steady state).

Inflation	Welfare cost	Consumption	Leisure
2	0.08	-0.12	0.0003
4	0.16	-0.22	0.0009
6	0.22	-0.32	0.0013
8	0.28	-0.40	0.0016
10	0.34	-0.48	0.0021
20	0.57	-0.81	0.0036
30	0.75	-1.07	0.0042
40	0.91	-1.28	0.0046
50	1.03	-1.46	0.0052

Table 3A. OCT model.

Changes in real money, transaction costs, and output when annual inflation moves up from 0% (percent changes in steady state).

Elasticity of output with respect to inflation ($\eta_{y,\pi}$).

Inflation	Real money	Trans.costs($h(c,m)$)	Output	$\eta_{y,\pi}$
2	-12.6	4.5	-0.005	-0.000048
4	-19.2	8.4	-0.008	-0.000086
6	-23.5	11.9	-0.011	-0.000103
8	-26.6	15.2	-0.013	-0.000146
10	-28.9	18.2	-0.017	-0.000171
20	-36.1	30.8	-0.026	-0.000267
30	-39.8	40.7	-0.034	-0.000331
40	-42.2	48.8	-0.042	-0.000378
50	-43.9	55.5	-0.048	-0.000413

Table 3B. OCT model.

Welfare cost of inflation from 0% inflation (percentage of output).

Changes in consumption and leisure when annual inflation moves up from 0% (percent changes in steady state).

Inflation	Welfare cost	Consumption	Leisure
2	0.07	-0.10	0.0005
4	0.13	-0.19	0.0025
6	0.19	-0.28	0.0040
8	0.24	-0.36	0.0055
10	0.29	-0.43	0.0065
20	0.49	-0.74	0.0085
30	0.64	-0.97	0.0130
40	0.78	-1.17	0.0170
50	0.88	-1.33	0.0230

Table 4A. MIU model.
Changes in real money and output when annual inflation moves up from 0% (percent changes in steady state).
Elasticity of output with respect to inflation ($\eta_{y,\pi}$).

Inflation	Real money	Output	$\eta_{y,\pi}$
2	-12.6	-0.004	-0.0001
4	-19.1	-0.008	-0.0001
6	-23.3	-0.011	-0.0001
8	-26.4	-0.014	-0.0002
10	-28.8	-0.017	-0.0002
20	-35.9	-0.028	-0.0004
30	-39.5	-0.037	-0.0005
40	-41.9	-0.045	-0.0006
50	-43.6	-0.051	-0.0006

Table 4B. MIU model.
Welfare cost of inflation from 0% inflation (percentage of output).
Changes in consumption, real money, and leisure when annual inflation moves up from 0% (percent changes in steady state).

Inflation	Welfare cost	Consumption	Real money	Leisure
2	0.08	-0.004	-12.6	0.002
4	0.14	-0.008	-19.1	0.004
6	0.20	-0.011	-23.4	0.006
8	0.26	-0.014	-26.4	0.007
10	0.31	-0.017	-28.8	0.008
20	0.53	-0.028	-35.9	0.014
30	0.70	-0.037	-39.5	0.019
40	0.84	-0.045	-41.9	0.023
50	0.96	-0.051	-43.6	0.026

Table 5A. CIA model.
Changes in real money and output when annual inflation moves up from 0% (percent changes in steady state).
Elasticity of output with respect to inflation ($\eta_{y,\pi}$).

Inflation	Real money	Output	$\eta_{y,\pi}$
2	-0.07	-0.07	-0.0038
4	-0.14	-0.14	-0.0075
6	-0.20	-0.20	-0.0111
8	-0.27	-0.27	-0.0146
10	-0.33	-0.33	-0.0179
20	-0.63	-0.63	-0.0330
30	-0.92	-0.92	-0.0400
40	-1.19	-1.19	-0.0572
50	-1.44	-1.44	-0.0671

Table 5B. CIA model.
Welfare cost of inflation from 0% inflation (percentage of output).
Changes in consumption and leisure when annual inflation moves up from 0% (percent changes in steady state).

Inflation	Welfare cost	Consumption	Leisure
2	0.004	-0.07	0.03
4	0.009	-0.14	0.06
6	0.014	-0.20	0.10
8	0.019	-0.27	0.13
10	0.024	-0.33	0.16
20	0.049	-0.63	0.32
30	0.078	-0.92	0.46
40	0.107	-1.19	0.59
50	0.137	-1.44	0.72

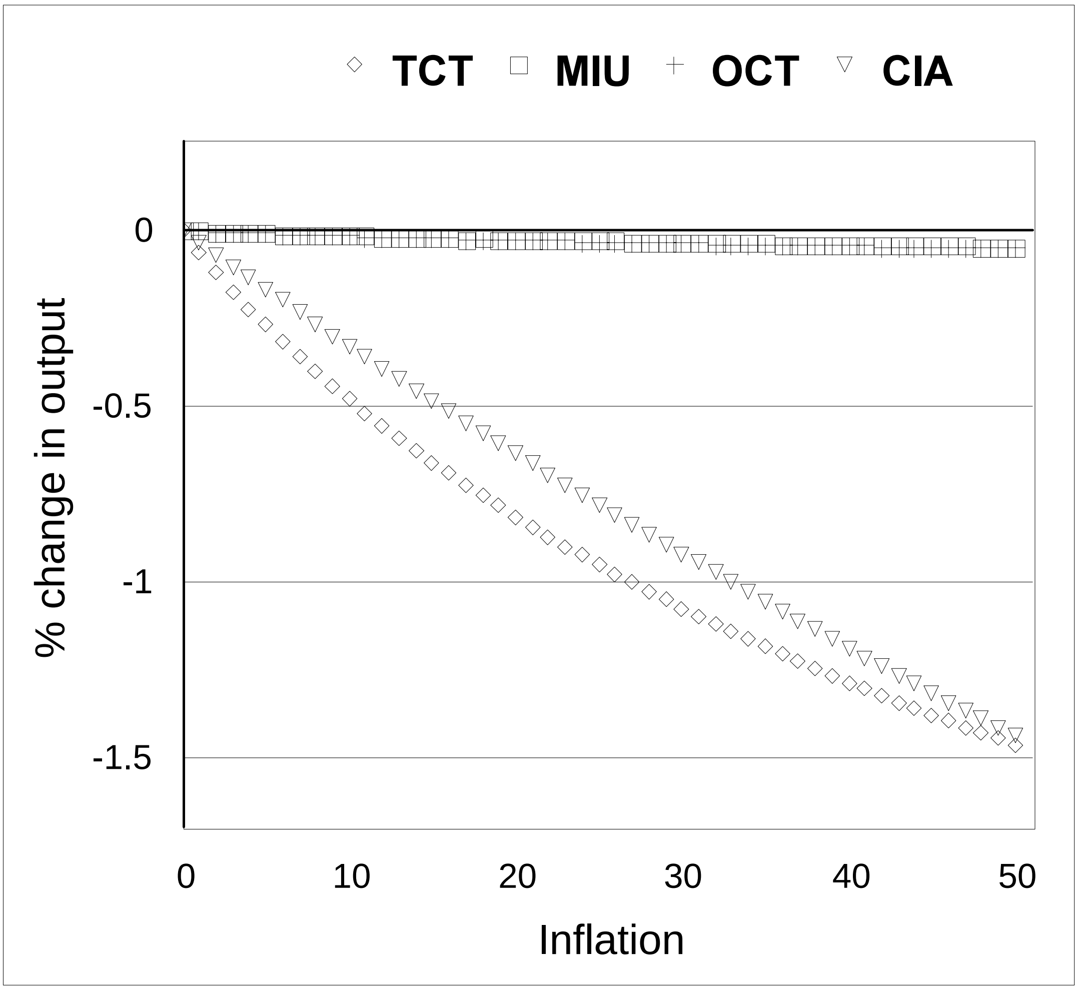

Figure 1: Comparison among the models I. Percent change in steady-state output when steady-state inflation rises from 0% to 50% per year.

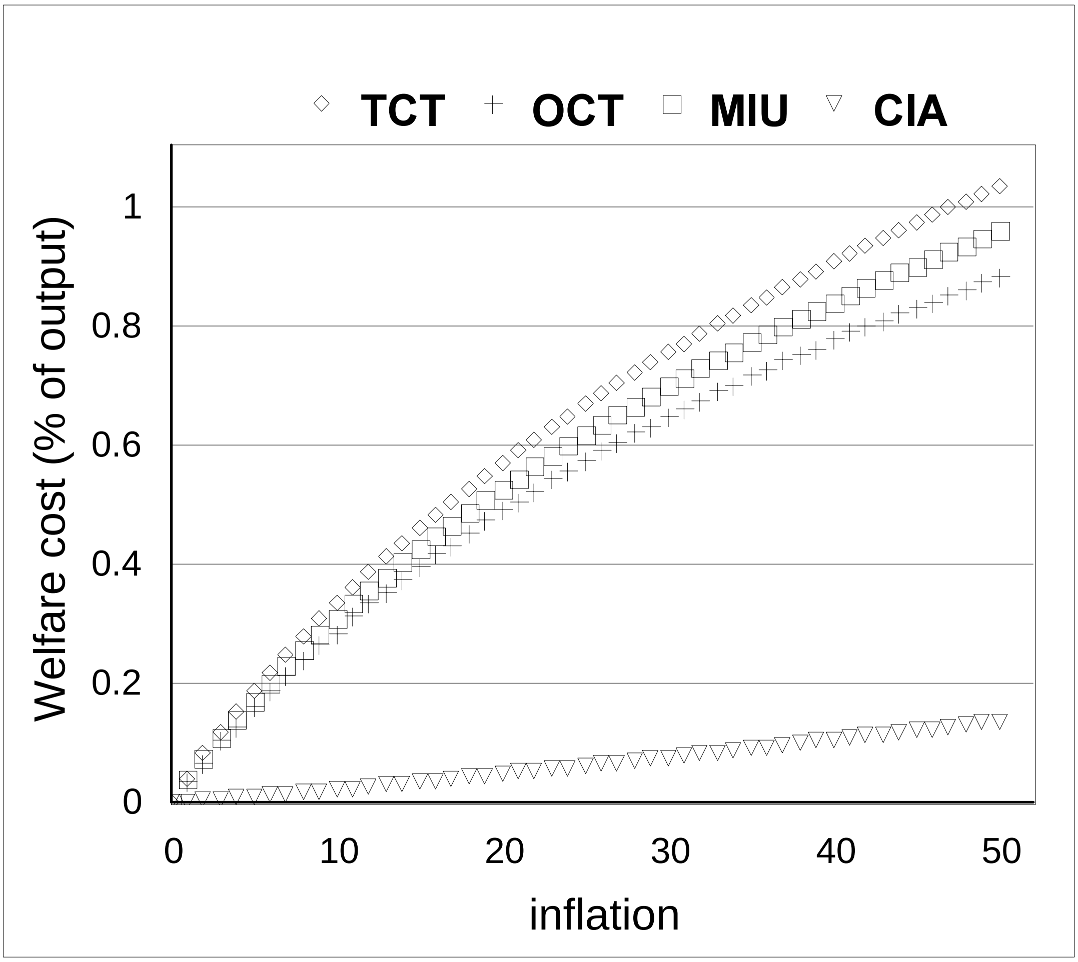

Figure 2: Comparison among the models II. Welfare cost of inflation as percentage of output in steady-state when steady-state inflation rises from 0% to 50% per year.