Handling multicriteria fuzzy decision-making problems based on intuitionistic fuzzy sets

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Abstract: In this paper we present a new technique for handling multicriteria decision making problems based on intuitionistic fuzzy sets. In this new technique we use the degree of reliability and of non-reliability of each criterion in relation to a set of alternatives. We also present a new score function in order to evaluate the degree of suitability of the choice of a certain alternative.

Keywords: Fuzzy set; intuitionistic fuzzy set; multicriteria fuzzy decision-making.

1. Introduction

Interval valued fuzzy sets IVFSs(X), ([9,10,11]) were introduced in the seventies, as a generalization of the concept of fuzzy set. K. Atanassov gave the definition of intuitionistic fuzzy set IFSs(X), in ([1]), and afterwards established that, IVFSs(X) and IFSs(X) ([2]), are equipollent generalizations of the concept of fuzzy set.

An intuitionistic fuzzy set ([1]) in X, X ≠ ∅ and Card(X) = K, is an expression A given by

\[ A = \{< x, \mu_A(x), \nu_A(x) > | x \in X \} \]

where

\[ \mu_A : X \rightarrow [0,1] \]
\[ \nu_A : X \rightarrow [0,1] \]

with the condition 0 ≤ \mu_A(x) + \nu_A(x) ≤ 1 for all x ∈ X

Numbers \mu_A(x) and \nu_A(x) denote respectively the degree of membership and the degree of non-membership of element x to set A. We will denote with IFSs(X)(X) the set of all the intuitionistic fuzzy sets in X. A complete study of these sets can be found in ([1],[2],[4],[6]).

The following expressions are defined in ([1]),([5]) for all A, B ∈ X

1. \( A \leq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) & \( \nu_A(x) \geq \nu_B(x) \) \( \forall x \in X \)
2. \( A \leq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) \& \( \nu_A(x) \leq \nu_B(x) \) \( \forall x \in X \)

3. \( A \vee B = \{ < x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) > | x \in X \} \)

4. \( A \wedge B = \{ < x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) > | x \in X \} \)

Operations 3 and 4 are easily generalizable to the case of \( n \) intuitionistic fuzzy sets.

5. \( A = B \) if and only if \( A \leq B \) \& \( B \leq A \)

6. \( A_c = \{ < x, \nu_A(x), \mu_A(x) > | x \in X \} \)

It is clear that \( \mu_A(x) \) is the degree of membership of element \( x \) to the intuitionistic fuzzy set \( A \) and \( \nu_A(x) \) is the degree of non-membership of \( x \) to set \( A \), note that for all \( x \in X \) it is verified that \( \mu_A(x) + \nu_A(x) = 1 \), then set \( A \) is fuzzy.

It has been proved in ([6]) that fuzzy sets, which we will represent in any of the following ways:

\[ A = \{ < x, \mu_A(x) > | x \in X \} = \{ < x, \mu_A(x), 1 - \mu_A(x) > | x \in X \} \]

are a particular case of the intuitionistic fuzzy sets.

In the past years fuzzy sets have been widely used for handling decision-making problems ([3], [7], [8], [12]). In this paper we present a new technique to for solving these problems using \( IFSs(X) \). We will consider that the set of alternatives and the sets of criteria are intuitionistic fuzzy sets. Also, we will use as score function the distance between the functions of membership and non-membership of an element in relation to the set of criteria and to the set of alternatives. For that reason we remind that the adaptation of the well known Hamming fuzzy distance to the \( IFSs(X) \) ([6]) gives us the following expression:

\[ 2 \cdot d_{IFSs(X)}(A, B) = \sum_{k=1}^{K} |\mu_A(x_k) - \mu_B(x_k)| + |\nu_A(x_k) - \nu_B(x_k)| \]

Let us consider \( A, B \in IFSs(X)(x) \) and let \( x_i \) be an element of \( X \), we will denote with \( d(x_i) \) the following expression:

\[ 2 \cdot d(x_i) = |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| \]

it is evident that:

\[ d_{IFSs(X)}(A, B) = \sum_{i=1}^{K} d(x_i) \]

The previous definition will be of great importance in this paper, as we will later see.

2. Handling multicriteria fuzzy decision-making problems

Let \( A = \{ a_1, a_2, \ldots, a_m \} \) be an ordinary set of alternatives, we can write this set as an intuitionistic fuzzy set in the following way:

\[ A = \{ < a_1, 1.0, 0.0 >, < a_2, 1.0, 0.0 >, \cdots, < a_m, 1.0, 0.0 > \} \]

Let \( C_1, C_2, \cdots, C_n \) be sets of criteria, these sets are intuitionistic fuzzy sets on \( A \), that is
\[ C_i = \{ < a_j, \mu_{C_i}(a_j), \nu_{C_i}(a_j) > | a_j \in A \} \]

where \( i = 1, \ldots, n, j = 1, \ldots, m, \) and \( 0 \leq \mu_{C_i}(a_j) + \nu_{C_i}(a_j) \leq 1 \) for all \( a_j \in A \).

It is important to point out that \( \mu_{C_i}(a_j) \) indicates the degree in which the criterion \( C_i \) satisfies the alternative \( a_j \) and \( \nu_{C_i}(a_j) \) indicates the degree in which the criterion \( C_i \) does not satisfy the alternative \( a_j \).

We assume that there is a decision maker who wants to choose an alternative which satisfies criterion \( C_l \) and \( C_r \) and \( \cdots \) and \( C_p \) or which satisfies the criterion \( C_q \). In this case, we will call \( C \) the following intuitionistic fuzzy set:

\[ C = C_l \land C_r \land \cdots \land C_p \lor C_q \quad \text{where} \]

\[ \mu_C(a_j) = \mu_{C_l}(a_j) \land \mu_{C_r}(a_j) \land \cdots \land \mu_{C_p}(a_j) \lor \mu_{C_q}(a_j) \]

\[ \nu_C(a_j) = \nu_{C_l}(a_j) \lor \nu_{C_r}(a_j) \lor \cdots \lor \nu_{C_p}(a_j) \land \nu_{C_q}(a_j) \]

with \( j = 1, 2, \ldots, m \).

We will use as score function to evaluate the degree of suitability that an alternative satisfies the decision-maker’s requirement, the distance function \( d(a_j) \) (for each \( a_j \in A \)) between the degree of membership and the degree of non-membership of \( a_j \) to \( C \) and the degree of membership and the degree of non-membership of \( a_j \) to \( A \), that is

\[ 2 \cdot d(a_j) = |1.0 - \mu_C(a_j)| + |0.0 - \nu_C(a_j)| \]

This way, we have a fuzzy set associated to the set of alternatives \( A \) given by the following expression:

\[ D = \{ < a_j, d(a_j) > | a_j \in A \} \]

with \( j = 1, \ldots, m \).

The element of \( D \) that has the smallest membership, that is the least height, will mark the best choice of alternative, that is, the smallest of the values of \( d(a_j) \) will indicate that the corresponding alternative \( a_j \) is the one that best satisfies the decision-maker’s requirement, that is

\[ d(a_1) = s_1 \in [0, 1] \]

\[ d(a_2) = s_2 \in [0, 1] \]

\[ \cdots \]

\[ d(a_m) = s_m \in [0, 1] \]

if \( d(a_j) = s_j \in [0, 1] \) and \( s_j \) is the smallest value among the values \( s_1, s_2, \cdots, s_m \), then the alternative \( a_j \) is our best choice.

**Example**

Let \( A = \{ a_1, a_2, a_3, a_4 \} \) be an ordinary set of alternatives, this set can be represented as an intuitionistic fuzzy set in the following way:

\[ A = \{ < a_1, 1.0, 0.0 >, < a_2, 1.0, 0.0 >, < a_3, 1.0, 0.0 >, < a_4, 1.0, 0.0 > \} \]
and let $C_1, C_2$ and $C_3$ be three intuitionistic fuzzy sets of criteria on $A$.

We assume that these $C_1, C_2$ and $C_3$ are represented by the intuitionistic fuzzy sets shown as follows:

$C_1 = \{< a_1, 0.0, 1.0 >, < a_2, 0.7, 0.1 >, < a_3, 0.2, 0.4 >, < a_4, 0.1, 0.9 >\}$

$C_2 = \{< a_1, 0.6, 0.4 >, < a_2, 1.0, 0.0 >, < a_3, 0.4, 0.5 >, < a_4, 0.3, 0.2 >\}$

$C_3 = \{< a_1, 0.5, 0.5 >, < a_2, 0.2, 0.6 >, < a_3, 0.9, 0.0 >, < a_4, 0.3, 0.3 >\}$

and we assume that the decision-maker wants to choose an alternative which satisfies the criteria $C_1$ and $C_2$ or which satisfies the criterion $C_3$, then the decision-maker's requirement can be expressed by $C = C_1 \land C_2 \lor C_3$, that is

$\mu_C(a_1) = \mu_{C_1}(a_1) \land \mu_{C_2}(a_1) \lor \mu_{C_3}(a_1) = 0.0 \land 0.6 \lor 0.5 = 0.5$

$\nu_C(a_1) = \mu_{C_1}(a_1) \lor \nu_{C_2}(a_1) \land \nu_{C_3}(a_1) = 1.0 \lor 0.4 \land 0.5 = 0.5$

$\mu_C(a_2) = \mu_{C_1}(a_2) \land \mu_{C_2}(a_2) \lor \mu_{C_3}(a_2) = 0.7 \land 1.0 \lor 0.2 = 0.7$

$\nu_C(a_2) = \mu_{C_1}(a_2) \lor \nu_{C_2}(a_2) \land \nu_{C_3}(a_2) = 0.1 \lor 0.0 \land 0.6 = 0.1$

$\mu_C(a_3) = \mu_{C_1}(a_3) \land \mu_{C_2}(a_3) \lor \mu_{C_3}(a_3) = 0.2 \land 0.4 \lor 0.9 = 0.9$

$\nu_C(a_3) = \mu_{C_1}(a_3) \lor \nu_{C_2}(a_3) \land \nu_{C_3}(a_3) = 0.4 \lor 0.5 \land 0.0 = 0.0$

$\mu_C(a_4) = \mu_{C_1}(a_4) \land \mu_{C_2}(a_4) \lor \mu_{C_3}(a_4) = 0.1 \land 0.3 \lor 0.3 = 0.3$

$\nu_C(a_4) = \mu_{C_1}(a_4) \lor \nu_{C_2}(a_4) \land \nu_{C_3}(a_4) = 0.9 \lor 0.2 \land 0.3 = 0.3$

therefore

$C = \{< a_1, 0.5, 0.5 >, < a_2, 0.7, 0.1 >, < a_3, 0.9, 0.0 >, < a_4, 0.3, 0.3 >\}$

Now we calculate $d(a_1), d(a_2), d(a_3)$ and $d(a_4)$.

$d(a_1) = \frac{|1.0 - 0.5| + |0.0 - 0.5|}{2} = 0.5$

$d(a_2) = \frac{|1.0 - 0.7| + |0.0 - 0.1|}{2} = 0.2$

$d(a_3) = \frac{|1.0 - 0.9| + |0.0 - 0.0|}{2} = 0.05$

$d(a_4) = \frac{|1.0 - 0.3| + |0.0 - 0.3|}{2} = 0.5$

With these values we construct the following fuzzy set:

$D = \{< a_1, 0.5 >, < a_2, 0.2 >, < a_3, 0.05 >, < a_4, 0.5 >\}$

we can observe that element $a_3$ has the smallest membership to set $D$, that is, the least height, therefore, in accordance with the previous, alternative $a_3$ is our best choice.
3. Conclusions

In this paper, we have presented a new technique for handling multicriteria fuzzy decision-making problems, where the sets of criteria are intuitionistic fuzzy sets. This new technique allows us to handle the degree of reliability and the degree of non-reliability of each criterion in relation to each alternative. On the other hand, we take as score function the distance, and among all the alternatives we choose the one that has associated to it the smallest distance.

4. Future research

In the method presented in this paper we have assumed that the set of alternatives \( A \) was an ordinary set, however, it seems logical to study the case in which said set is also intuitionistic fuzzy. On the other hand, with the score function we take, that is \( d(a_j) \), we construct a fuzzy set \( D \), it also seems logical to analyse the different numerical measures of information of this set (energy, entropy, etc...) as well as the correlation between \( A \) and \( D \).

References


