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ON THE INTUIOTIONISTIC FUZZY RELATIONS

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ABSTRACT:

Intuitionistic fuzzy relations which generalize the already existing such relations are introduced and their basic properties are shown.

The concept of the Intuitionistic Fuzzy Relation (IFR) is based on the definition of the Intuitionistic Fuzzy Sets (IFSs) [1]. It is introduced in different forms and different ways, and practically independently, in [2-9]. We must note that the approaches in the various IFR definitions are different in researches by different authors. The approach from [7-9] was in some sense the most general. In the present form it includes Buhaescu's [4], Stoyanova's [6], Burillo and Bustince's [8, 9] results as particular cases.

Let everywhere below X, Y and Z be ordinary finite non empty sets (universes).

An IFS A^* in X is an object with the form:

$$A^* = \{<x, \mu_A(x), \gamma_A(x)>/x \in E\},$$

where the functions $\mu_A : E \to [0, 1]$ and $\gamma_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \le \mu(x) + \gamma(x) \le 1.$$

Below we shall write A instead of A^*. We shall denote by IFS the set (or class, in the sense of the NBG set theory) of all IFSs and by FS - of all fuzzy sets.

We shall call Intuitionistic Fuzzy Norm (IFN) in [0, 1] x [0, 1] every couple <S, T> of two mappings S, T: [0, 1] x [0, 1] -> [0, 1] satisfying the properties (cf. [7-10])

i) Boundary conditions, T(x, 1) = x, T(x 0) = 0, S(x, 1) = 1 and S(x, 0) = x for every $x \in [0, 1]$;

ii) Monotony, $T(x, y) \le T(z, t)$ and $S(x, y) \le S(z, t)$ iff $x \le z$ and $y \le t$, where $x, y, z, t \in [0, 1]$;

iii) Commutative, T(x, y)= T(y, x) and S(x, y) = S(y, x) for every $x, y \in [0, 1]$;

iv) Associative, T(T(x, y), z) = T(x, T(y, z)) and S(S(x, y), z) = S(x, S(y, z)) for every $x, y, z \in [0, 1]$;

v) Connection, $S(x, y) + T(z, t) \le 1$ for every $x, y, z, t \in [0, 1]$, such that $x + z \le 1$ and $y + t \le 1$.

We shall note that when S(x, y) + T(x, y) = 1 we obtain the definition from [8] with some corrections, where T is called a norm and S - a conorm.

An IFR is called every IFS $R \subset X \times Y$ with the form (cf. [8, 9]):

$$R = \{<<x, y>, \mu_R(x, y), \gamma_R(x, y)>/x \in X \ \& \ y \in Y\},$$

where $\mu_R : X \times Y \to [0, 1]$, $\gamma_R : X \times Y \to [0, 1]$ are degrees of membership and non-membership as the ordinary IFSs (see [1]) or degrees of validity and non-validity of the relation R; and for every $<x, y> \in X \times Y$:

$$0 \le \mu_R(x, y), \ \gamma_R(x, y) \le 1.$$

We shall denote with IFR(X, Y) the set of all IFRs over X x Y.

This new relation, which obviously is an IFR, we shall call inverse relation to R.

Let $P, R \in IFR(X, Y)$ and let below $(x, y) \in X \times Y$. We shall define the following relations and operations over IFRs (ch. [1, 8]).

$P \subset R$ iff $(\forall (x, y) \in X \times Y)(\mu_P(x, y) \le \mu_R(x, y) \ \& \ \gamma_P(x, y) \ge \gamma_R(x, y))$;

$P = R$ iff $P \subset R \ \& \ R \subset P$;

$\bar{P} = \{<x, \gamma_P(x, y), \mu_P(x, y)>/(x, y) \in X \times Y\}$

$P \cap R = \{<<x, y>, \min(\mu_P(x, y), \mu_R(x, y)), \max(\gamma_P(x, y), \gamma_R(x, y))>/ (x, y) \in X \times Y\}$

$P \cup R = \{<<x, y>, \max(\mu_P(x, y), \mu_R(x, y)), \min(\gamma_P(x, y), \gamma_R(x, y))>/ (x, y) \in X \times Y\}$