"PARAXIALITY CONSIDERATIONS OF HIGHER ORDER GAUSSAN MODES"

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ABSTRACT
Frequently, gaussian modes are used as free space wave equation solutions. This is not true since they are obtained assuming certain paraxial character in them.
In this paper, we present an original study of the condition for paraxiality of higher order gaussian modes solutions of the wave equations in free space. A bound for the product of number (k) in free space and the beam waist of a family of modes is obtained, in terms of the mode indexes - the bound being more restrictive the larger the order of the mode.

1.- INTRODUCTION

Nowadays, in millimeter applications, the quasi-optical transmision line system is prefered to drive the power from the source to the final experiment. Usually, the fundamental gaussian structure is chosen as carrier mode.
Nevertheless, in material processing and ceramic sintering applications, a uniform field distribution in the final cavity is required. This means, the particular field structure which carry the power through the line is irrelevant. These applications allow us to use another gaussian structures (higher order modes) as carrier.
Usually, the most used source in this kind of applications are the classical gyrotrons, which launch the power in a mixture of TE_{0m} circular waveguide modes. In reference [1] a optimum taper profile to obtain a conical gaussian structure from the TE_{01} circular waveguide mode was presented, and in [2] this conical gaussian structure has been used efficiently to carry the power through the quasi-optical transmision line.
It is important to remind, that the expresions used to define such gaussian structures, which are given in [3], have been obtained assuming paraxial behaviour for the solutions of the wave equation in the free space.

2.- PARAXIALITY

The expansion formula for the gaussian structure is,

$$\text{for} \quad z >> z_0 = \frac{k\varpi_0^2}{2} \qquad \varpi(z) = \varpi_0 \sqrt{1 + \left(\frac{2z}{k\varpi_0^2}\right)^2} \approx \frac{2z}{k\varpi_0} \tag{1}$$

k being the wave number and ϖ_0 the beam waist of the gaussian beam.
For lower values of $k\varpi_0$ product, the paraxiality of the solution will be very suspicious, because the asimptotic slope, the difraction, can be very high.
D.H. Martin in [4] presents the exact expresion for the far field pattern of a gaussian amplitude distribution with constant phase, S_e, and the paraxial aproximation using the paraxial expresions for the gaussian modes, S_a,

$$S_e(\theta) = \cos^2(\theta) \cdot e^{-2\,\mathrm{sen}^2(\theta)\left(\frac{k\varpi_0}{2}\right)^2} \qquad S_a(\theta) = \frac{1}{\cos^2(\theta)} \cdot e^{-2\tan^2(\theta)\left(\frac{k\varpi_0}{2}\right)^2} \tag{2}$$

As conclusion, D.H. Martin by simply observation of the two functions S_e and S_a, for differents values of $k\varpi_0$ proposes for the fundamental gaussian mode the subjective paraxial condition, $k\varpi_0>6$.
In this paper, in order to estimate the paraxiality condition for the higher order gaussian modes, we suggest an objective criteria based on the error function:

$$\varepsilon = \frac{\int_0^{2\pi} \left| S_e(\theta) - S_a(\theta) \right| d\theta}{\int_0^{2\pi} \left| S_e(\theta) \right| d\theta} \qquad (3)$$

Translating the paraxial condition proposed in [4] to this new criteria, we obtain the condition of $\varepsilon<3\%$. Then, we will assume the 3% of the error function, as the new criteria to establish the limit of the paraxial condition for the higher order modes.

To adapt S_e and S_a, for higher order modes, we only have to multiply these expresions by the normalized generating function with the square root of the exponential function argument.

Thus, calculating the error function for differents values of $k\varpi_0$ product for the Hermite Gauss modes (used in rectangular structures), we obtain the paraxial condition refered to the highest index of the mode (m) (figure 1),

$$k\varpi_0 >> 6(m+1) \qquad (4)$$

and for the Laguerre-Gauss modes (used in circular structures), the paraxiality condition will be,

$$k\varpi_0 >> 6(a+b+1) \qquad (5)$$

being a and b the indices of the mode under consideration.

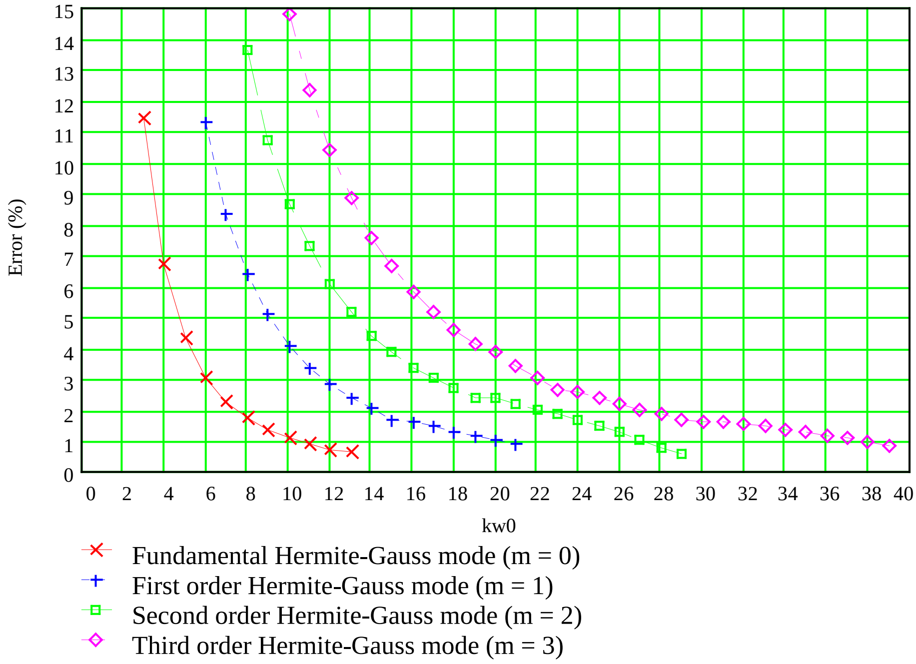

Figure 1.- Paraxial error for the Hermite-Gauss modes related to $k\varpi_0$ product.

3.- REFERENCES

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