SEEPAGE FLOW IN AQUIFERS WITH OPEN AND CLOSED BOUNDARIES

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1. Introduction

At previous gully erosion conferences, the effect of surface seal development on gully formation and growth was analyzed (Prasad and Römkens, 2003) and the effect of hydrological conditions on gully growth was discussed (Römkens and Prasad, 2005). In the latter study, it was suggested that seepage forces, i.e., exit gradients, may appreciable affect gully growth and that solutions of Laplace's equations for a quasi-steady state flow field might be helpful in assessing the effect of seepage on gully growth. In this presentation, seepage flow is discussed, based on selected studies that used conformal mapping procedures. These studies may be helpful in making assessments of the impact of seepage on gully erosion.

2. Theory

The stream flow area is described in terms of the complex spatial variable z, with x and y being the Cartesian coordinates, and the flow regime described by the complex potential ω where φ, is the potential function and ψ is the stream function. Thus:

$$z = x + iy \qquad \text{and} \qquad \omega = \varphi + i\psi \tag{1}$$

Solutions for the stream flow area are sought of the type $w = f(z)$ or $z = g(\omega)$, which either are obtained directly or indirectly through a complex variable $\zeta = \zeta + i\eta$, so that $\omega = f_1(\zeta)$ and $z = g_1(\zeta)$ facilitates calculations of the relationships $\omega = f(z)$ or $z = f(\omega)$. Also, in cases of free or open boundaries, the hodograph method is used, that describes the complex velocity field, given by

$$w(z) = u(x, y) + iv(x, y) = -\frac{d\omega}{dz} \tag{2}$$

where

$$\frac{\delta\varphi}{\delta x} = -u(x, y) \quad and \quad \frac{\delta\psi}{\delta x} = -v(x, y) \tag{3}$$

3. Discussion

The studies indicated are by:

1. van Deemter (1950), who discusses a number of 2-dimensional flow problems under steady state conditions in a homogeneous, isotropic aquifer with free as well as closed boundaries. Specifically, he analyzed, among others, the case of (i) drainage to a water free rectangular ditch with vertical, impervious, straight walls and (ii) the case of drainage to a partially filled ditch with vertical, pervious, walls. The solution obtained for case (ii) is:

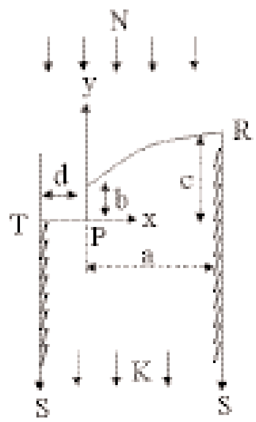

Fig. 1a. Flow field to be rectangular ditch without water and a permeable vertical wall with seepage (After v. Deemter).

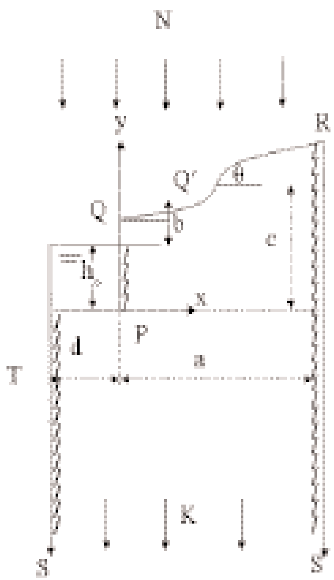

Fig. 1b. Flow field to be rectangular ditch water height h_o and impervious vertical wall (After v. Deemter).

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$$\left[\cosh\left[\pi\frac{\omega + i\left(kz + kd\right)}{k(a+d)}\right] - \lambda\right] \bullet \left[\cosh \pi \frac{k+N}{\frac{d\omega}{dz} + ik} - \cos\frac{N\pi}{k}\right] = \left(1+\lambda\right)\left(1 - \cos \pi \frac{N}{k}\right) \quad (4)$$

where

$$\lambda = \cos \pi \frac{\left(kd - Na\right)}{k(a+d)} \quad (5)$$

This solution shows an implicit relationship between z and ω.

2. Römkens (unpublished, 1964) analyzed seepage to a partially filled ditch with circular bottom, an impervious wall above the ditch water level, and a constant horizontal ground water table (infiltration problem in a ponded field) (Fig. 2). Solution obtained for the seepage flow rate, Q, is for an infinitely long aquifer with buffer zone of width c:

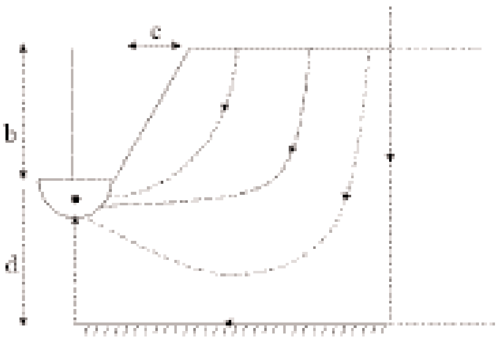

Fig. 2. Flow field to a partially filled ditch and an impervious side wall and a buffer zone with widcth c. Q is seepage per unit ditch length.

$$\omega = -\frac{Q}{\pi}\ln\left[\frac{-\sqrt{\cosh\frac{c\pi}{d} - \cosh z\frac{\pi}{d}} - \sqrt{\cosh\frac{c\pi}{d} - \cos\frac{b\pi}{d}}}{-\sqrt{\cosh\frac{c\pi}{d} - \cosh z\frac{\pi}{d}} + \sqrt{\cosh\frac{c\pi}{d} - \cos\frac{b\pi}{d}}}\right] \quad (6)$$

For c=0, this formula reduces to the case of a tile drainage problem.

3. Bakker (1997) discusses new solution procedures in analyzing groundwater flow problems, including flow with free boundaries using the hodograph method and flow problems over a horizontal impervious layer to a straight

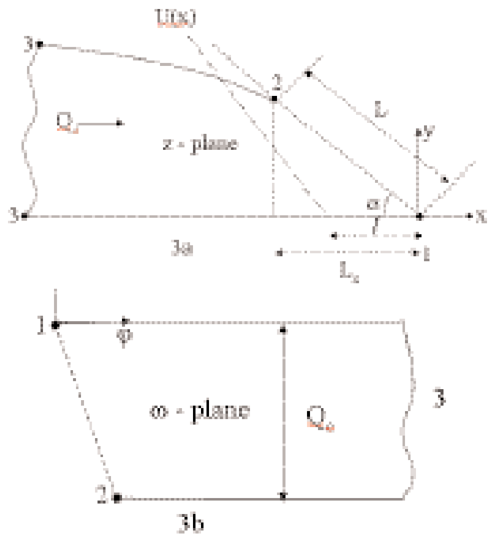

Fig. 3. Flow to a straight seepage surface. (a) The complex z-plane configuration, and (b) the complex potential-stream function representation.

seepage surface of angle ∇ with the horizontal using the Dupuis-Forehheimer model (Fig. 3). That relationship is:

$$\frac{dQx}{dx} = \frac{d^2U}{dx^2} = -N(x), \quad \text{where} \quad (7)$$

$$U = \int_0^h kydy - \frac{1}{2}Kh^2 = 0 \quad (8)$$

The various solutions obtained can now, in principle, be used to determine the potential and stream functions, and subsequently the exit gradients at the seepage surfaces. However, the calculations can be very complex and generally only be handled for simplified cases.

References

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