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WELFARE ANALYSIS IN A CASHLESS ECONOMY

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ABSTRACT:

The use of money can involve either an electronic adjustment or the exchange of cash. Phasing out cash has costs such as the disruption of privacy and the loss of seigniorage revenues, and benefits such as less criminal activities, reduction of tax evasion and an unconstrained monetary policy at the Zero Lower Bound. Nowadays, new technologies are challenging the use of cash, which implies rethinking the role of money to facilitate transactions. This paper makes a welfare analysis using a Real Business Cycle model with money to find the optimal share of electronic money. Additionally, the effects of tax evasion, transaction costs and the long-run inflation rate are estimated.

KEYWORDS:

Cash; Electronic Money; Real Business Cycle Model; Steady-State Solution; Transaction Costs; Tax Evasion; Zero Lower Bound.

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1. Introduction

Money is primarily used as a medium of exchange to facilitate transactions and as a store of value. The incipient digitalization allows a substantial increase in the use of electronic money in substitution of paper money because technology eases its use. Nonetheless, the world is drowning in cash because big amounts of it are still in circulation, with a surprisingly share of big-denominated bills (i.e. \$100, €500...). Notice that the European Central Bank has stopped issuing €500 bills, this is a first step to restraint the use of cash.

Therefore, cash is still there and due to its anonymity property in addition to its ease of transportation and storing, it implies that it is commonly used for criminal activities and tax evasion. Besides, cash constraints monetary policy as it is going to be analysed.

Phasing out cash would be beneficial because it would be a burden for tax evaders and those undertaking criminal activities. These gains are supposed to outweigh the losses of less privacy, the cost of the digitalization transformation and the loss of seigniorage revenues for governments.

That is why, this paper aims to make a welfare analysis of the optimal quantity of cash in a representative economy. This analysis is going to be undertaken with a Real Business Cycle with money model that has been calibrated to replicate the US economy. The optimizing program leads to 21 functions with 21 variables. The model is solved in steady-state equilibrium using MATLAB software, which it is helpful for the analysis of the long-run values of each variable of the model.

The elimination of cash should be seriously considered by governments. However, big-denominated coins such as €10 coin (they are more difficult to carry in big amounts) could be maintained in circulation in order to facilitate daily and small transactions and meanwhile, it is respectful with the privacy of individuals.

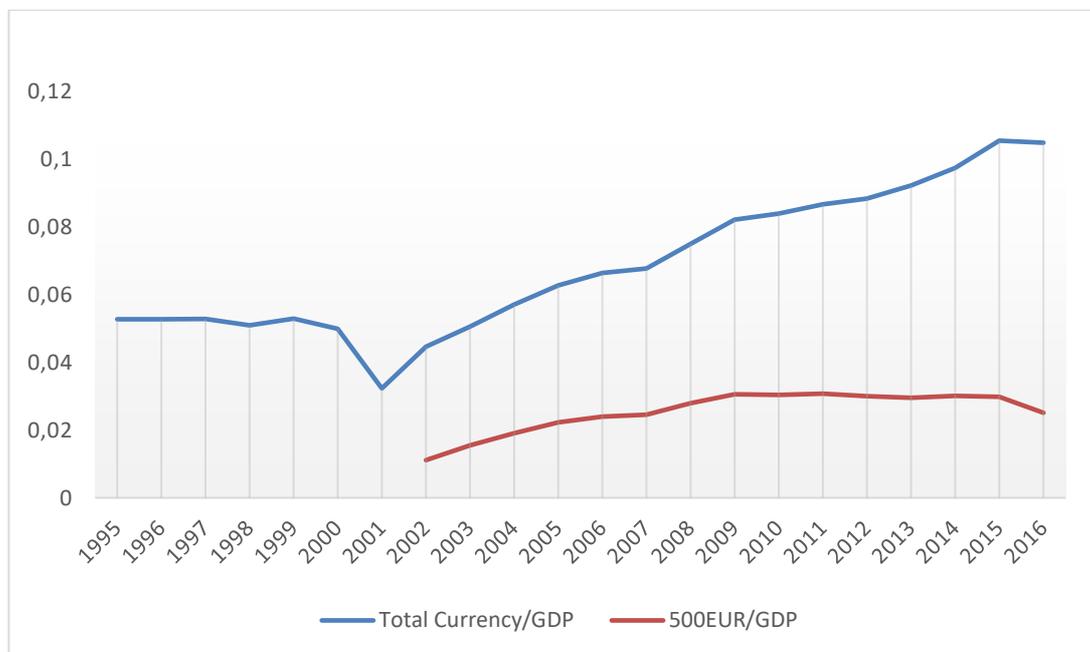
This paper has the following structure. In Section 2 the quantity of cash in circulation in both the Euro area and the US is reviewed. Section 3 deeply analyses, the cost and benefits from eliminating cash. Section 4 presents a plan for phasing out cash. Section 5 develops the Real Business Cycle Model with money, the optimizing programs and baseline calibration in Steady-State. Section 6 provides an analysis of the tax evasion effect, the electronic money transaction costs and the long-run inflation rate. Finally, some conclusions are given.

2. Quantity of cash in circulation

This section briefly describes how much cash is in circulation in both the European Union and United States. Having a view of the trend in the use of cash is important in order to know the departure point on this issue.

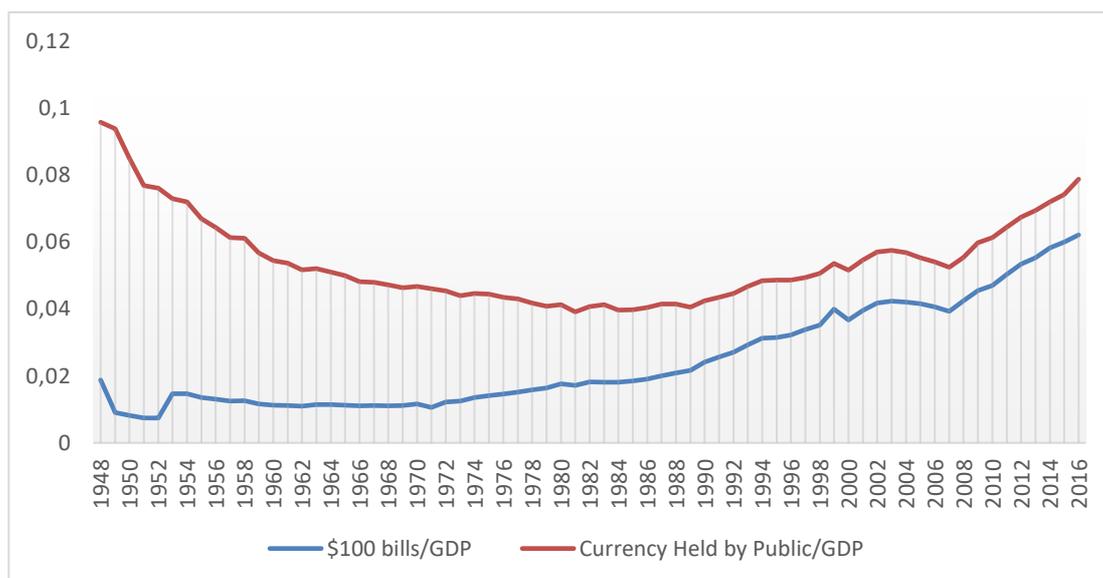
Kenneth Rogoff on its “The Curse of Cash” book gives some empirical evidence that is of concern. For example, in the US, 77% of total cash in circulation is demonstrated in \$100 bills and approximately half of them are held abroad and are out of government control. Within the European Monetary Union, the €500 bill accounts for 30% of total cash in circulation and it is held by few people. The following figures from Rogoff (2016) gives a perspective of how the use of cash relative to GDP has increased recently, in both the US and the European Monetary Union despite the increase of e-transactions.

Figure 1: *EURO Currency to GDP*



Source: Rogoff (2016), Chapter 3.

Figure 2: US Currency to GDP



Source: Rogoff (2016), Chapter 3.

The question is clear. Why does a monetary system need such big value bills? And what is being done with them? Corruption, terrorism and tax evasion are some of the problems that are involved in the use of cash because bills are easy to transport and store. The latter, gives an idea of how overwhelming and important the optimal use of cash in an economy is, and that its elimination have more gains than losses as it is going to be discussed in this paper.

3. Benefits and costs from phasing out cash

The quantity of cash circulating in any developed economy is supposedly higher than the cash that is being used, or at least the amount of cash used that is recorded by the authorities. Although it might be hard to quantify, most of the cash is being used in criminal activities as well as in the underground economy, and currencies such as the dollar and the euro are held abroad without any control. In this section the benefits and costs of eliminating cash from the economy will be briefly exposed.¹ Electronic money would decrease tax evasion and criminal activities, would protect public health from disease and let more room for monetary bodies in setting interest rates. On the other hand, it would harm privacy, require a huge capital investment and profits for governments from seigniorage would disappear.

¹ Following Chapter 5 of Rogoff (2016).

3.1 Benefits

3.1.1 The underground economy: Tax Evasion

The use of cash is heavily linked with its use for evading taxes. Cash transactions do not have to be registered to be done. Then, if no cash existed, it would be harder to evade taxes. This is a problem more common in small daily transactions or cash-intensive firms. The transaction could be made in exchange of cash, but it might never be recognized in the ledger or never reported to tax authorities. Schneider *et. al* (2010) found that the average shadow economy counts for about 17.1% of official GDP over 162 countries between 1999 and 2007. Notice that shadow economy is understood as the underground economy, where economic activities are undertaken with lack of legal supervision and therefore, do not report taxes. Policies to overcome this data are hard to assess, and furthermore, there is a positive link between the tax rate and the underreported taxes, that simultaneously marginal tax rates have a positive and significant influence on currency holdings. (Lynch, 1985).

This gives an idea of how much would the economy and tax authorities benefit from removing cash. Tax revenue could be collected effortless and the tax gap would be reduced because any to transaction would be registered and automatically reported. This would be a gain for the entire society since more collection of taxes would mean either higher budget for the government to spend or a lower tax rate to be applied. In fact, in India, there has been a withdrawn of 86% of their paper currency with the main target of eradicating tax evasion.²

Phasing out cash would improve equality of opportunities since everybody should report their entire income and then, nobody is cheating the system, which would be unfair for the ones who report taxes. This is one of the major benefits from removing cash.

3.1.2 Criminal activities

Within criminal activities, the main method of payment used is cash, which might be laundered by recognizing false profits at any firm. Cash is used in corruption, drugs, terrorism, human trafficking...etc. Almost any illegal activity you can imagine is cash-intensive.

² Found at the Irish Examiner, Hearne (2017). Note that India is not prepared to be cashless (93% of labour contracts are illegal), so they took a step back increasing supply again by 87% and then, corruption continues. (Sivabalan, 2017).

Phasing out cash would not mean that any criminal activity would disappear because there will be found other means of engaging in criminal activities. However, what it is true that in case that there were no cash, it would be much harder for criminals to undertake such activities and it would be a substantial reduction of them. Therefore, thanks to the traceability of electronic transactions, both the economy and the society would benefit from less criminal activities. Wright *et. al* (2017) found evidence in a study that there was a shrink in crime activities in the USA over several decades that comes simultaneously with a drop on cash usage.

Lastly, despite counterfeiting is already too difficult, with only electronic money or just small €10 coins in circulation, it would be almost impossible.

3.1.3 Monetary policy: Zero Lower Bound (ZLB)

Cash constraints room for monetary policies. As long as people had the opportunity to ensure that the value of their assets or bank deposits is not going to be reduced when negative interest rates take place, monetary authorities are constrained. The nature of cash makes it hard to penalize in nominal terms. That is why cash is a hindrance for monetary policy when negative interest rates would be necessary. This section is going to be given special attention since it is hard to incorporate into the upcoming model and it is an issue that must not be forgotten.

Currently, just when the worst financial crisis after the Great Depression is almost over, monetary authorities such as the European Central Bank (ECB) or the Federal Reserve Bank (Fed) are struggling with the stagnation of the economy, with low inflation and a slow recovery (the so-called Great Recession period). The way these authorities have fought the slow recovery is by bringing down the interest rates³ or issuing mass amounts of money, Quantitative Easing (QE) in order to increase liquidity in the economy. QE has been undertaken by purchasing assets of either governments or firms operating within the euro area. The latter has been used by the ECB because its inability to boost the economy with low interest rates until due to zero-bound and, that means, facing the liquidity trap. Another alternative used by central banks has been forward guidance, which is bringing down the real interest rate when reaching the zero-bound. Forward guidance is based on the credibility of

³ It is defined as the one-month Treasury bill rate that a given government pays to borrow from private markets.

any central bank, since they have to be believed that they will certainly target that inflation to pull down inflation expectations.

In an economy where money holdings and bonds can be substituted with each other, the preference for money depends on the nominal interest rate, which means that by lowering interest rates, the bonds yield is lower and therefore the demand for money holdings increases, which at the same time is an incentive for people to spend this money in other assets or goods and services, what would increase total output. The interest rate is also used as reference for lending and borrowing money between economic agents. In case of a long recession, lowering interest rates should boost the economy. Nonetheless, if this results to be ineffective, there exists the risk reaching the Zero Lower Bound (ZLB) or liquidity trap, where they cannot go further than zero because of the existence of cash.

That means that agents have already changed their preferences for holding money due to the lower yield of bonds, and however, it has been not enough stimulus for pushing output. The problem is bigger than it seems because the solution for this trap would be bringing interest rates below zero and with the existence of cash it is not easy because the negative interest rates make economic agents to exchange most of the assets they have for cash, because at least cash is not decreasing its face value and pays 0%. It sounds crazy. Negative interest rates would consist of a world where borrowers are awarded and savers are penalized. Although the neutrality of money in the long-run, the use of monetary policy to fix short-term issues is very useful.

The idea of negative interest rates would be effective only if cash is totally or partially phased out. If negative interest rates were possible and people could not exchange their bank deposits for cash, people would start consuming and investing more before they run out of savings. This can be also translated into a lower cost of borrowing than before, which fosters investment and consumption too, mostly durable goods. The decrease in interest rates also makes people with high valuable assets feel wealthier, and then, they would spend more as asset prices will go up.

*Intertemporal Consumption and Investment Effect: $\downarrow i \rightarrow \uparrow$ Investment and
 \uparrow Consumption $\rightarrow \uparrow$ Aggregate Demand*

Wealth Effect: $\downarrow i \rightarrow \uparrow$ Asset Prices $\rightarrow \uparrow$ Wealth $\rightarrow \uparrow$ Consumption

Similarly, a reduction of interest rates, the economy there is an improvement of the terms of trade, since lower interest rates make the home currency weaker against other currencies, which is translated into both more exports and less imports. The country undertaking negative interest rates would export more because their goods and services offered to the rest of the world are cheaper thanks for the depreciation of the exchange rate. Similarly, importing from the rest of the world is going to be more expensive due to the weak exchange rate, and therefore, many firms would be forced to stop importing from abroad and substitute it by local suppliers. These two effects improve the current account of the country, pushing up aggregate demand and then, total output.⁴

*Exchange Rate Effect: $\downarrow i \rightarrow \downarrow$ Home Exchange rate depreciates \rightarrow
 \uparrow Exports and \downarrow Imports $\rightarrow \uparrow$ Current Account Balance \rightarrow
 \uparrow Aggregate Demand*

Negative interest rates would work that well if, and only if the reduction into negative bounds work the same way as a reduction within positive bounds. It should be discussed whether they can cause instability or if it is possible to maintain financial stability. Stability is crucial due to the fact that for example it does not matter whether the rate of inflation is 2% or 8% if it is totally foreseeable and stable and the credibility of the central bank is not compromised. Wages and the whole economy can easily adapt to an expected and totally certain inflation.

According to Kenneth Rogoff, the bequest coming from the German mark discipline, has been transmitted to the Euro monetary policy and then, the ECB has been committed to attach their expected inflation to low rates, which is a target of 2%. This target is said to be inflexible to cope with changes in last years, which eventually have compromised the economy to reach the zero lower bound. He points out at least three reasons explaining why monetary authorities are currently dealing with the zero lower bound:

1. Setting the expected inflation to 2% might be too low because it needs lower interest rates than a higher expected inflation, say 6%, in order to maintain the real interest rate⁵ in the long-run, which finally, it is what really matters.

⁴ See Krugman *et al.* (2012) and Blanchard and Johnson (2015).

⁵ Note that the real interest rate is: $r = i - \pi^e$, being r the real interest rate, i the nominal interest rate and π^e

2. Secondly, the volatility of past years has been quite high, making central banks to change their interest rates with many cuts.
3. Finally, the fall of real interest rate below zero as well as below from normal levels. This could be explained by the high savings of emerging markets or the aging population in developed countries, which leads to slower growth, and then, it brings low interest rates in the short run equilibrium of credit markets.

Of course, it is well known that there exists the possibility of using fiscal policy to get out the zero lower bound. A permanent or temporary fiscal expansion will boost the economy depending on the magnitude of the fiscal expansion and its credibility. The main drawback with it is that this kind of policy would generate budget deficit that must be backed by future taxes. A smart idea to boost demand and fight against zero lower bound would be drone money. This idea consists of targeting low income individuals in an economy and giving them a certain amount of money, so they can spend. The reasons behind this policy is that, these people would save less than mid-income or high-income individuals, and therefore, demand would increase while inequality is also fought. However, this is only a second-best alternative after the abolition of cash and an alternative to QE.

Although phasing out cash is the most plausible idea, in the literature it is found other ways for breaking the ZLB:

- Stamp tax: It consists of making people pay periodically for a stamp on their paper currency, that is, making people pay interest rates for using currency. The idea comes from Gesell (1916) and it is known as “stamp money”. At Gesell’s time the idea was fascinating. In fact, during the Great Depression, some experiments were done such as in Wörgl, Austria, where its citizens had to buy every month stamps for 1% of the value to maintain its validity. There has been studied other variants to this idea such as expiration dates for papers or even lotteries, where the resulting serial numbers from that lottery, will lose their value. Nowadays, this crazy monetary policy could be feasible thanks to technological improvements. However, it is not elegant, and it seriously compromises the anonymity of people holding paper currency.
- Two-currency system: This first idea was first developed by an economist called Robert Eisler (1933). This is also some of the bequest that the Great Depression left.

the expected inflation rate set by a given monetary authority.

Eisler pointed out that in order to stabilize prices, a two-currency system within an economy may be possible. The idea is based on the existence of a “money banco” and “current money”, where one can be used only electronically and the other physically. Both currencies have an exchange rate between each other, that is managed by the respective central bank. These two currencies might be dollar bills on the one hand and electronic dollar on the other hand. This already exists, although the exchange rate between them is one. When the Fed would like to charge interest rates on electronic dollars (banco money), it only needs to set an exchange rate for a certain period. In case it is changed from one/one to say, one electronic dollar for 0.98 dollar, the central bank is charging a negative interest rate of 2%, that as long as both are set at the same time, people will not run into cash.

Despite what is generally thought about the ZLB, Jarrow (2013) claims that there does not actually exist a real ZLB. For Jarrow, the use of negative interest rates is possible given the existence of cash, so ZLB is a myth and not a reality. He points out that models applied to design fiscal and monetary policy contain the ZLB, which means that the outcome can be misleading and thus, the policy. One of the main arguments he uses is that the storage of cash is already too costly and risky for either firms or individuals and therefore, arbitrage opportunities between cash and bonds it is restrained to a given quantity.⁶ When reaching negative interest rates, the quantity of bonds traded in exchange for cash is restrained due to the huge risk of loss and credit risk and therefore no arbitrage exists.

The latter is contrary to what Kenneth Rogoff believes that in case negative interest rates were implemented and no major changes were made, nobody would benefit from borrowing from this policy because investors are unwilling to lend money to borrowers. There would be massive runs into cash much bigger than what already happened during the peak of the financial crisis in 2008. Rogoff talks about imposing a tax on large-scale cash storage. However, this would hardly discourage tax evasion or criminal activities. Besides, for banks it would be hard to pass interest rates to customers (mainly small) because of the fear of losing them. They prefer passing negative rates by charging higher commissions, which has already happened within the euro area and, in fact, it is believed that this practice reduces

⁶ Arbitrage opportunities are possible under negative interest because Jarrow assumes that both bonds and cash are riskless assets which cannot have different returns. In case bonds have negative interest and cash is just a mean for storing value, an arbitrage opportunity does exist.

even more the willingness to lend from commercial banks.

Although the possibility to apply negative interest rate during the financial crisis, it would be very hard to know if this would have worked well or if the economic recession would have been solved earlier than it has already been solved. In fact, Buiter (2010) suggests that according to the Taylor rule⁷, the interest rates in the US in 2009 should have been further below from 0%, between -5% and 7.5%, and thus the Zero Lower Bound has been hindering the recovery.

Eggertsson *et. al* (2013) suggests that the recovery during the financial crisis in the European Monetary Union (EMU) would have been much faster if peripheral countries would have implemented structural reforms. This is the only way that these countries can regain competitiveness and boost output. The policy undertaken to get out of the recession has consisted of depreciating the Euro and then, increasing the expectations of growth, so aggregate demand is increased. This policy does not fix the effects in the case that another crisis hits Europe. The structural reform is needed, but it would be only effective if it does not have to deal with the ZLB. It would deepen the recession while worsening deflation with a higher real interest rate. Without it, the impact of such reforms during an economic downturn is positive and much higher in magnitude.

When it comes to dealing with negative interest rates, the abolition of paper currency seems to be the best option against others such as a tax in paper currency or creating an electronic and a paper currency which have an exchange rate between each other. (See Buiter, 2010)

3.1.4 Public Health

Although this issue seems to be less important, public health would be slightly improved by the elimination of cash. The main point is that cash is dirty, and it is usually a way of spreading and transmitting diseases and harming public hygiene.

3.2 Costs

3.2.1 Privacy

Privacy is probably one of the most important downsides that may arise. Nobody wants to be totally controlled by the government and provide private information about any

⁷ The Taylor rule is defined as the positive relationship between the interest rate and the output gap and the excess of actual and expected inflation (Taylor, 1993).

transaction that is made. This “Big Brother” effect is impossible to avoid since every transaction is going to be recorded and available for the government and the civil freedoms are going to be compromised. Then, a balance between privacy rights and collect taxes and reduce crime should be reached.⁸

On the other hand, it could be argued that most people’s information and bank transactions are already disposed for governments and tax authorities recently known as Big Data. Information will be needed to the extent that it is useful to collect taxes. However, law and regulatory issues should be revisited so this private information is not badly used. That is why, there is no need to totally remove cash from the economy since few large coins could remain in circulation.

Omariba *et. al* (2012) point that the future of banking systems passes through the confidence of users that there is nothing to worry about. They also claim that there are security issues in electronic money and bank systems that could damage privacy issues that would affect consumer’s confidence.

3.2.2 Capital investment

This issue could be a bit controversial because it is impossible to predict that the transaction cost of managing cash will be lower than the cost of a cashless economy. It is true that phasing out cash from the economy needs from a huge initial investment that makes available e-transactions in any case or need. That needs from sophisticated software, electronic devices without forgetting the inclusion of the entire society into the system. Elderly people, low income individuals, homeless people...etc. may need special consideration. It should be neither forgotten investment in maintenance and security, the risk of cyber attacks is real but also the reduction of cash exposure to thefts.⁹

Transaction costs changes will be analysed deeper when explaining the model of this paper.

3.2.3 Seigniorage

Seigniorage is the profit that central banks obtain from issuing money in form of cash. Therefore, this is the only quantifiable cost that can be compared with gains and it is the

⁸ See Chapter 7 of Rogoff (2016)

⁹ There is a recent technology called Blockchain, which may allow to decrease transaction costs, but its analysis is not the aim of this paper. This technology would help optimize transaction costs and protect privacy issues thanks to its ledger.

main counterargument used for not phasing out cash. Stopping the issuance of cash would mean a loss for central banks and its government. However, this loss is insignificant with the gains above discussed (mainly with tax evasion). The estimated cost of giving up seigniorage according to Rogoff (2016) it would be between 0.4% and 0.8 % of GDP in the US economy. Note that this loss is bigger than it should be because of the underground economy, cash abroad and the use of cash for criminal activities.

4. A plan for phasing out cash

By using all the arguments above explained, Rogoff (2016) has already designed a plan for phasing out (or partially) cash which consists of the following stages:

1. Phasing out paper currency beginning with big bills and finishing with small coins. This phase could last forever since there is no need to phase totally out cash. There could be small coins of €10 for the use of daily transaction or small operations. The idea of using this kind of coins has to do with how difficult it might be to illegally transport huge quantities of physical money. The aim is to get rid of large notes.
2. Financial inclusion. This stage would consist of ensuring that everybody have access to debit or credit cards or any device or tool which allows to use electronic money. This is up to the government how to do it. This would focus specially on low-income and elderly people.
3. Privacy. There would be needed regulation and new law enforcement to ensure that the system works well. It must be discouraged any other means of transaction hidden from the government.
4. Ensure that transactions are made in real-time. A trustful payment infrastructure that allows safety and immediate payments.

Nonetheless, there is not an absolute truth. As Rogoff suggests, it might be that cash will never be completely withdrawn from the economy. Beretta (2014) strongly believes that cash would be necessary in any post-industrial economy because cash already plays a symbolic sense for people, that is objectively quantifying how much money they do have.

5. The Real Business Cycle model

In this section, a model has been created with the aim of approximately estimating the optimal quantity of electronic and paper currency, and also seeing how the different

advantages and drawbacks that electronic money has may affect welfare.

This is the main section of the paper and it consists of the development of a Real Business Cycle (RBC) model. It is a macroeconomic model based on micro-foundations generally used to analyse the operating characteristics of an economy and make decisions. Lucas (1976), claimed in his Lucas Critique that macroeconomics needed to be analysed from micro-foundations with resources constraints, preferences and parameters rather than just focusing on historical data.

This approach claims that the economy changes due to multiperiod fluctuations. Despite its simplicity, the fit of the model is surprisingly good. Kydland and Prescott (1982). This kind of models have as standpoint the utility function of a representative household as the basic economic unit. The utility function faces an optimizing problem subject to a budget constraint. An important characteristic is that in these models, in steady-state solution, a rise in monetary growth rate is equal to a rise in the inflation rate while it leads the utility level to decrease in steady-state. Sidrausky (1967). In the RBC model of this paper, money is going to be treated as transaction-facilitating, making transaction costs to fall. Goodfriend and McCallum (2007).

A RBC model is suitable for this paper proposes because it can be modified to incorporate both cash and electronic money as it is going to be seen. It allows a long-run analysis when it is solved in steady-state which will provide quantitative and qualitative analysis.

5.1 Model presentation

Electronic money is defined as the checking account balance that individuals can hold in commercial banks. Cash or paper currency is understood as the bills and coins that people hold either at home, wallet or elsewhere. As it has been discussed before, this money is said to be anonymous money since the privacy is not compromised.

In this economy people choose between two different means of holding the same currency, that is electronically m_t^e and paper currency m_t^c in each period t :

$$m_t = m_t^e + m_t^c$$

Where m_t stands for the total amount of currency in the economy and period t . Note that since consumers care mainly about the amount of goods they might afford throughout time, these quantities are in real terms rather than nominal; $m_t^e = \frac{M_t^e}{P_t}$; $m_t^c = \frac{M_t^c}{P_t}$. Note that P_t is

the aggregate price level.

Having these two types of holding currency it would be interesting to see them in terms of share, that is:

$$s_t = \frac{m_t^e}{m_t^e + m_t^c} = \frac{m_t^e}{m_t}$$

Where s_t is the share of electronic money held in this economy and in contrast, the term $(1 - s_t)$ is the share of paper money or cash in the economy.

Of course, holding money in different means implies different costs and benefits for individuals, which are presented as it follows:

On the one hand, holding electronic money instead of cash is beneficial for individuals since huge quantities of cash running within an economy eases criminal activities, and more specifically, the underground economy. On the other hand, with cash, making business out of the government control is easier and therefore, people will try to cheat the system in order to pay less taxes. When that is the case, tax authorities will need to increase either tax rate in order to keep collecting the same quantity of revenue and thus to hold the same budget. If individuals hold more electronic money, the tax evasion decreases, and therefore, as more people is paying taxes, tax authorities can lower tax rates to hold budget available constant. It leads to the following relation:

$$\tau_t = \tau_0 - \tau_1 s_t$$

Where τ_t is the income tax rate (ranging $0 < \tau_t < 1$) bore by both labour and capital income in period t. As discussed, the higher the share of electronic money, the lower the income tax rate that individuals must pay. Here, τ_0 is the fraction of τ_t that is not affected by the electronic money held by individuals and τ_1 is another parameter that sets to which extent the share of m_t^e in period t lowers the actual income tax rate.

On the other hand, if paper money were going to be totally phased out, the government control exerted over individuals would be much higher than before. There are many issues about how to deal with privacy which would need to be overcome in order to make it feasible. Indeed, this would mean an undoubtedly preference for individuals to hold paper currency instead of holding it electronically.

The following utility function is for an economy with identical households whose utility

depends positively on the amount of goods consumed at period t , c_t and negatively on both the amount of time they spend in working at period t that is n_t , and the share of electronic money in the economy at period t , s_t :

$$U(c_t, n_t, s_t) = \frac{c_t^{1-\varrho}}{1-\varrho} - \Psi \frac{n_t^{1+\gamma}}{1+\gamma} + b_0(1-s_t)^{b_1}$$

(+, -, -)

The utility function presents the following partial derivatives: $\frac{\partial U}{\partial n_t} < 0$, $\frac{\partial U}{\partial c_t} > 0$ and $\frac{\partial U}{\partial s_t} < 0$, with γ , ϱ , b_0 and b_1 as positive coefficients. The higher the number of goods consumed by households, the better meanwhile the higher either the quantity of labour and electronic money share, the worse. Parameters such as Ψ , b_0 and b_1 are collecting to which extent an increase in n_t and s_t are shaping the utility function respectively.

In this utility function, the parameters ϱ and γ are known as the Constant Relative Risk Aversion (CRRA) coefficient. They contribute to the utility function by shaping the preferences of households or individuals and with the property of a diminishing marginal utility for consumption and increasing labour marginal disutility, that is, $\frac{\partial^2 U}{\partial c_t^2} < 0$ and $\frac{\partial^2 U}{\partial n_t^2} > 0$. This is useful in such a way that household's utility does not increase at very fast rates when they enjoy a huge quantity of goods. The utility function is separable between consumption and work, cross effects are not considered.

The utility function is going to be used as an estimate of welfare of the economy in order to come up with the gains or losses that households would have with changes in money composition. Hence:

$$welfare_t = \sum_{j=0}^{\infty} E_t \beta^j \left[\frac{c_{t+j}^{1-\varrho}}{1-\varrho} - \Psi \frac{n_{t+j}^{1+\gamma}}{1+\gamma} + b_0(1-s_{t+j})^{b_1} \right]$$

Lastly, it must be defined a transaction cost function. The main propose of money either for both electronic and paper currency is to be use as a medium of exchange. Exchanging money for anything individuals might desire, carries a cost defined in the following function:

$$h_t(c_t, m_t, s_t) = h_0 + h_1 \left(\frac{c_t}{m_t^e + m_t^c} \right)^{h_2} + h_3 s_t$$

(+, -, +)

This is the transaction cost function, and it is going to set the cost of any transaction in this model. It is defined in such a way that the cost increases as long as the consumption is higher because more time and money is employed on that transaction. And then, $\frac{\partial h_t}{\partial c_t} > 0$, with h_0 , h_1 , h_2 and h_3 as positive coefficients.

Alternatively, when the quantity of real money (m_t) is higher, the cost incurred during transactions is lower.¹⁰ Since households have alternatives of holding value with other assets such as bonds or capital (as it is going to be seen), holding less money implies more income to transform this store of value into consumption goods. Households incur in commission fees, trips to banks' ATMs, transport costs, and therefore, shopping becomes costlier. That is why the higher the amount of money in circulation in the economy, the lower the transaction costs. Thus, $\frac{\partial h_t}{\partial m_t} < 0$.

The discussion concerning if the use of paper or electronic money increases or decreases the cost of transaction is more complicated. Then, let's discuss about the effect of s_t on transaction costs.

According to Humphrey (2010), the unit cost of transaction is different for the economic agents, for example for a merchant, cash is the cheapest mean of payment meanwhile for average consumers, credit cards, that is electronic money, it is the cheapest mean of payment. Bergman et al. (2007) in a research carried out for the Swedish Sveriges Riksbank claim that cash is the most efficient mean of payment when a payment amounts approximately eight euros. However, the approach considered by Humphrey is just taking into account the current state of art of different means of holding money. The approach of this model is different to the extent which the share of electronic money is changing throughout time and therefore, it implies a huge capital investment in electronic devices and software needed to carry electronic transactions in case of a huge share of electronic money is at the optimal equilibrium. This is why in subsection 6.2 the parameter h_3 is going to be changed to analyse its effects on the model.

Although there already exist some technologies in order to clear person to person (P2P) payments such as Google Wallet, Paypal or contactless credit cards, which are in ways of

¹⁰ Note that $m_t = m_t^e + m_t^c$

improving, it is going to keep it simple it is going to be assumed that with a higher share of electronic money in the economy, transaction costs are going to be much higher due to the huge fixed investment needed at the beginning and the maintenance costs incurred. That is why Rogoff (2016). supports the idea of indefinitely leaving small bills and coins in circulation (or even just coins with a value of 10 euros or dollars), higher share of paper money, lower transaction costs, which is the same that we are assuming. Hence, h_3 is the parameter defining by how much transaction costs are going to be increased when the share of electronic money is increased and $\frac{\partial h_t}{\partial s_t} > 0$ to guarantee the transaction-facilitating role of money.

In conclusion, when it comes to transaction costs depending on the mean of holding money, there exists two effects for both paper money and electronic money. In case of paper money, it has negative effect, because costs are lower when more money is in circulation, and on the other hand, it also decreases the transaction costs because of the higher the share of paper money (lower s_t) the lower the cost. This implies that $\frac{\partial h_t}{\partial m_t^e} < 0$. Electronic money has two opposite effects. The first one is the same as paper currency, with the same effect. The second effect is positive with higher transaction costs when the share of electronic money is higher. It is assumed that first effect offsets the second and, the overall effect is negative, implying that $\frac{\partial h_t}{\partial m_t^e} < 0$.¹¹

5.2 The optimizing programs

5.2.1 Households

Once it is known both potential benefits and downsides of holding electronically more or less money, the optimizing program for a representative household subject to budget constraints can be written as follows:

$$\max_{c_t, n_t, k_{t+1}, b_{t+1}, m_t^e, m_t^c} \sum_{j=0}^{\infty} E_t \beta^j \left[\frac{c_{t+j}^{1-\varrho}}{1-\varrho} - \Psi \frac{n_{t+j}^{1+\gamma}}{1+\gamma} + b_0 (1 - s_{t+j})^{b_1} \right]$$

¹¹ This is because taking the first derivative of transaction cost respect to electronic money, the money effect is higher than the electronic money share effect $h_1 h_2 \left(\frac{c_t}{m_t^e + m_t^c} \right)^{h_2 - 1} \left(\frac{c_t}{(m_t^e + m_t^c)^2} \right) > h_3 \frac{m_t^c}{(m_t^e + m_t^c)^2}$

Subject to:

$$\begin{aligned}
& (1 - \tau_{t+j})w_{t+j}n_{t+j} + (1 - \tau_{t+j})r_{t+j}^k k_{t+j} + g_{t+j} \\
& = c_{t+j} + [k_{t+j+1} - (1 - \delta)k_{t+j}] + [(1 + r_{t+j})^{-1}b_{t+1+j} - b_{t+j}] \\
& + [(m_{t+j}^e + m_{t+j}^c) - (1 + \pi_{t+j})^{-1}(m_{t-1+j}^e + m_{t-1+j}^c)] + [h_0 \\
& + h_1(\frac{c_{t+j}}{m_{t+j}^e + m_{t+j}^c})^{h_2} + h_3s_{t+j}]
\end{aligned}$$

For $\forall j = 1, 2, \dots, \infty$

All variables are expressed in real terms as they have been divided by the aggregate price level.

The term E_t is the rational expectation operator, which sets how expectations are evolving over time. Besides, β is the discount factor of the perception of a household individual about de future in each period. The higher the discount, the less individuals care of future periods. By definition, it is: $\beta = \frac{1}{1+\rho} < 1$, with a positive rate of intertemporal preference $\rho > 0$.

In the budget constraint we can find different elements defining the sources of income of household individuals and different means of holding and expending these revenues either, goods, capital, currency (m_t^e or m_t^c), transaction costs or assets such as bonds.

There are three sources of income. The first one is the revenues coming from individuals employing n_t units of time in supplying labour and earning a real wage w_t per unit of time. Hence, the wage multiplied by labour is the total revenue from supplying labour to firms, and it is subject to the income tax rate τ_t in order to obtain the net revenues coming from labour.

Households also have the opportunity of getting a return from lending capital to firms k_t . Individuals gain r_t^k for any unit of capital supplied in period t, which gives a gross profit of r_t^k times k_t . This is also subject to the same income tax rate as labour to obtain the net return coming from lending capital goods to the firms. The last source of income is the government transfers denoted as g_t .

Households can spend their revenue on different uses, which are represented on the right-hand side of the budget constraint. The first one is the consumption of goods c_t which are produced by firms and sold in the competitive goods market.

The following three terms could be understood as the portfolio choice where individuals can decide how to allocate their wealth. First, the amount of capital goods they are deciding to acquire next period k_{t+1} , minus the amount of capital they are currently holding that needs to get subtracted the depreciation rate (usage rate) δ . That is the net amount capital that is going to be increased this period. Next, it is found the amount of wealth that individuals decide to hold in bonds in the next period b_{t+1} , which need to be divided by the real interest rate $(1 + r_t)$ that bonds are yielding, in order to have it in present-period terms. The quantity of bonds b_t is subtracted to the amount of bonds individuals are deciding to hold next period, to obtain the net purchases of bond assets. Lastly, individuals can also allocate their wealth increasing the quantity of real money (either electronic or paper), taking into account the real money held in period t-1 suffers from a lower purchasing power, and that is why it is divided by the price inflation rate $(1 + \pi_t)$.

Finally, the transaction cost bore by individuals when they purchase consumption goods using money as a medium of exchange is the last use of households spending.

In order to maximize the utility function, the Lagrangian function is written in the following way:

$$\begin{aligned}
& \mathcal{L}_t(c_t, n_t, k_{t+1}, b_{t+1}, m_t^e, m_t^c) \\
&= \sum_{j=0}^{\infty} E_t \beta^j \left[\frac{c_{t+j}^{1-\varrho}}{1-\varrho} - \Psi \frac{n_{t+j}^{1+\gamma}}{1+\gamma} + b_0 (1 - s_{t+j})^{b_1} \right. \\
&+ \lambda_{t+j} \left[(1 - \tau_{t+j}) w_{t+j} n_{t+j} + (1 - \tau_{t+j}) r_{t+j}^k k_{t+j} + g_{t+j} - c_{t+j} \right. \\
&- [k_{t+j+1} - (1 - \delta)k_{t+j}] - [(1 + r_{t+j})^{-1} b_{t+1+j} - b_{t+j}] \\
&- [(m_{t+j}^e + m_{t+j}^c) - (1 + \pi_{t+j})^{-1} (m_{t-1+j}^e + m_{t-1+j}^c)] - [h_0 \\
&+ h_1 \left(\frac{c_{t+j}}{m_{t+j}^e + m_{t+j}^c} \right)^{h_2} + h_3 s_{t+j} \left. \right] \left. \right]
\end{aligned}$$

The first order conditions are computed as the partial derivative of the Lagrangian function with respect to the choice variables. The consumption first order condition is:

$$\frac{\partial \mathcal{L}_t}{\partial c_t} = c_t^{-\varrho} - \lambda_t - \lambda_t h_1 h_2 \left(\frac{c_t}{m_t^e + m_t^c} \right)^{h_2-1} \left(\frac{1}{m_t^e + m_t^c} \right) = 0$$

Where λ_t that is the Lagrange multiplier associated to the budget constraint in period t, solving for λ_t leads to the shadow value of consumption¹²:

$$\lambda_t = \frac{c_t^{-\rho}}{1 + h_{c_t}}$$

This shadow value tells us the marginal impact or increase of consumption in the utility function divided by the marginal transaction cost of one more unit of consumption, which has sense because individuals must incur in transaction costs when they purchase goods. Then, the shadow price gathers the total variation of one more unit of consumption on the optimal solution of the maximizing problem. First order condition with respect to labour is:

$$\frac{\partial \mathcal{L}_t}{\partial n_t} = -\Psi n_t^\gamma + \lambda_t(1 - \tau_t)w_t = 0$$

By plotting λ_t above it is get that:

$$\Psi n_t^\gamma = \frac{c_t^{-\rho}}{1 + h_{c_t}} (1 - \tau_t)w_t$$

This leads to an equilibrium condition that equalizes the marginal utility of labour to the Lagrange multiplier times the net real wage. So, what does it happen if the quantity of electronic money in this economy increases? Undoubtedly, a higher s_t reduces the income tax rate. Therefore, individuals will pay less taxes and will dispose of more net real wage to consume goods, which increases the labour that individuals supply. If the negative marginal utility of labour is lower than the net return of labour, people would work more because of the availability of enjoying more income without decreasing the utility function by a big amount.

Then, if it is solved for n_t , it can be seen the labour supply function:

$$n_t = \left[\frac{1}{\Psi} \frac{c_t^{-\rho}}{1 + h_{c_t}} (1 - \tau_t)w_t \right]^{\frac{1}{\gamma}}$$

The First Order Conditions for bonds and capital in period t+1 are the following:

¹² Let us define $h_{c_t} = \frac{\partial h_t}{\partial c_t} = h_1 h_2 \left(\frac{c_t}{m_t^\xi + m_t^\zeta} \right)^{h_2 - 1} \left(\frac{1}{m_t^\xi + m_t^\zeta} \right)$

$$\frac{\partial \mathcal{L}_t}{\partial b_{t+1}} = -\lambda_t(1+r_t)^{-1} + \beta E_t \lambda_{t+1} = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial k_{t+1}} = \beta E_t \lambda_{t+1}(1-\tau_t)r_t^k - \lambda_t + \beta E_t \lambda_{t+1}(1-\delta) = 0$$

Putting together both First Order Conditions, it is obtained:

$$\lambda_t(1+r_t)^{-1}(1-\tau_t)r_t^k + \lambda_t(1+r_t)^{-1}(1-\delta) = \lambda_t$$

And solving for r_t :

$$r_t = (1-\tau_t)r_t^k - \delta$$

This equation is important because it shows that the net return (after taxes and minus depreciation) coming from the use of capital (right-hand side) and the bond or financial asset real interest rate must be the same.

From the bonds' First Order Condition it is obtained the consumption Euler equation¹³ that is the optimal intertemporal allocation of consumption in period t and t+1:

$$\frac{c_t^{-\rho}}{(1+h_{c_t})(1+r_t)} = \beta E_t \frac{c_{t+1}^{-\rho}}{1+h_{c_{t+1}}}$$

The relation with the bond First Order Condition is that bonds represent the future consumption of goods and they are similar to savings, since they serve as a store of value that yields r_t , and that is why the consumption in period t is divided by the interest rate of bonds. These bonds are borrowed by individuals by the government. In case the effect of consumption in the utility is higher in period t than in period t+1, the c_t increases in exchange for less bonds in period t+1, that means less savings in period t+1 and therefore c_{t+1} turns lower. The opposite effect happens when consumption in period t+1 is higher than the consumption in period t. Therefore, the consumption marginal utility is penalized twice, when a purchase of goods is done through transaction cost and with the opportunity cost of not transforming money into bonds. The First Order Conditions with respect to electronic real money and real currency are as they follow:¹⁴

¹³ Note that $\lambda_{t+1} = \frac{c_{t+1}^{-\rho}}{1+h_{c_{t+1}}}$

¹⁴ Let's use the following definitions: $S m_t^e = \frac{\partial s_t}{\partial m_t^e} = \frac{-m_t^e}{(m_t^e + m_t^c)^2} = S m_t^c$; $\frac{\partial s_t}{\partial m_t^e} = \frac{m_t^c}{(m_t^e + m_t^c)^2}$,

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial m_t^e} &= -b_0 b_1 (1 - s_t)^{b_1 - 1} \frac{m_t^c}{(m_t^e + m_t^c)^2} + \lambda_t \tau_1 \frac{m_t^c}{(m_t^e + m_t^c)^2} (w_t n_t + r_t^k k_t) - \lambda_t \\ &\quad + E_t \beta \lambda_{t+1} (1 + \pi_{t+1})^{-1} \\ &\quad - \lambda_t \left[h_1 h_2 \left(\frac{c_t}{m_t^e + m_t^c} \right)^{h_2 - 1} \left(\frac{-c_t}{(m_t^e + m_t^c)^2} \right) + h_3 \frac{m_t^c}{(m_t^e + m_t^c)^2} \right] = 0 \\ \frac{\partial \mathcal{L}_t}{\partial m_t^c} &= -b_0 b_1 (1 - s_t)^{b_1 - 1} \frac{-m_t^e}{(m_t^e + m_t^c)^2} + \lambda_t \tau_1 \frac{-m_t^e}{(m_t^e + m_t^c)^2} (w_t n_t + r_t^k k_t) - \lambda_t \\ &\quad + E_t \beta \lambda_{t+1} (1 + \pi_{t+1})^{-1} \\ &\quad - \lambda_t \left[h_1 h_2 \left(\frac{c_t}{m_t^e + m_t^c} \right)^{h_2 - 1} \left(\frac{-c_t}{(m_t^e + m_t^c)^2} \right) + h_3 \frac{-m_t^e}{(m_t^e + m_t^c)^2} \right] = 0 \end{aligned}$$

The First Order Condition taken from electronic real money combined with that of bonds leads to the demand of electronic real money:¹⁵

$$\frac{c_t^{-\varrho}}{1 + h_{c_t}} \tau_1 s_{m_t^e} (w_t n_t + r_t^k k_t) - \frac{c_t^{-\varrho}}{1 + h_{c_t}} h_{m_t^e} = b_0 b_1 (1 - s_t)^{b_1 - 1} s_{m_t^e} + \frac{c_t^{-\varrho}}{1 + h_{c_t}} \left(\frac{R_t}{1 + R_t} \right)$$

Besides, applying analogous steps, the paper real money demand function obtained in period t is:

$$-b_0 b_1 (1 - s_t)^{b_1 - 1} s_{m_t^c} - \frac{c_t^{-\varrho}}{1 + h_{c_t}} h_{m_t^c} = -\frac{c_t^{-\varrho}}{1 + h_{c_t}} \tau_1 s_{m_t^c} (w_t n_t + r_t^k k_t) + \frac{c_t^{-\varrho}}{1 + h_{c_t}} \left(\frac{R_t}{1 + R_t} \right)$$

It can be observed from the two last equations that the right-hand side represents the gains of using either paper or electronic money and the left-hand side represents the losses coming from the use of each kind of holding money.

By the electronic money demand function, it can be concluded that it has two gains. The first one has to do with the gains of paying less income taxes from capital and labour when

$$\begin{aligned} h_{m_t^c} &= \frac{\partial h_t}{\partial m_t^c} = h_1 h_2 \left(\frac{c_t}{m_t^e + m_t^c} \right)^{h_2 - 1} \left(\frac{-c_t}{(m_t^e + m_t^c)^2} \right) + h_3 \frac{-m_t^e}{(m_t^e + m_t^c)^2}; \\ h_{m_t^e} &= \frac{\partial h_t}{\partial m_t^e} = h_1 h_2 \left(\frac{c_t}{m_t^e + m_t^c} \right)^{h_2 - 1} \left(\frac{-c_t}{(m_t^e + m_t^c)^2} \right) + h_3 \frac{m_t^c}{(m_t^e + m_t^c)^2} \end{aligned}$$

¹⁵ In order to get to $\left(\frac{R_t}{1 + R_t} \right)$ term, note that R_t is the nominal interest rate and in this model, it is assumed that the Fisher effect holds. that is: $(1 + R_t) = (1 + r_t) E_t (1 + \pi_t)$ where r_t is the real interest rate and π_t is the inflation rate.

the share of electronic money increases and as it is already known, the income tax rate decreases. The second gain it is commonly shared with paper money, so regardless of its nature it simply states that the use of money either cash or electronic, decreases transaction costs significantly¹⁶. When it comes to losses associated to the use of electronic money, the first one is the marginal utility loss of it due to the loss of privacy that electronic money generates. The last term, is a loss that does not depend on the kind of money that individuals are holding. The latter is the opportunity cost that individuals bear because of holding money instead of investing it on other assets such as capital or bonds. The opportunity cost is $\left(\frac{R_t}{1+R_t}\right)$, that is the nominal return of bonds expressed at face value, at period t. This is an opportunity cost because by consuming more goods in period t, individuals' utility increases, as bonds have a yield differential of $\frac{R_t}{1+R_t}$ with respect to the 0% money return.

Likewise, the paper money demand function has similar structure. The common advantages or disadvantages of money are the same as electronic money, lower transaction costs due to the use of money as medium of exchange and the opportunity cost associated to holding money. However, the use of paper money has the disadvantage of paying more taxes from income.¹⁷ On the other hand, the marginal utility of paper money is positive, and increasing the share of paper money results in a higher utility for individuals.

In conclusion, apart from the common properties of both paper and electronic money, the benefits related to holding paper money are the drawbacks of electronic money and vice versa. Therefore, a trade-off between both types of money is faced, and an optimal quantity of money as well as the optimal share of electronic money in the model must be found.

Then, if both money demand functions are considered, it is obtained the demand for electronic money share:¹⁸

$$s_t = 1 - \left[\frac{c_t^{-\rho}}{1 + h_{c_t}} \left(\frac{\tau_1 (w_t n_t + r_t^k k_t) - h_3}{b_0 b_1} \right) \right]^{\frac{1}{b_1 - 1}}$$

This is the optimal share of electronic money in the economy in period t. It must be considered that it is only the optimal quantity of electronic money relative to the total amount

¹⁶ Recall that $h_{m_t^e} < 0$ by assumption

¹⁷ Recall that: $s_{m_t^c} < 0$ and $h_{m_t^c} < 0$ by assumption.

¹⁸ To reach to this, notice that: $s_{m_t^e} - s_{m_t^c} = h_{m_t^e} - h_{m_t^c} = \frac{1}{m_t^e + m_t^c}$

of money and not the optimal quantity of any kind of money. The latter, basically shows how the share of electronic money is affected by several factors. Affecting positively, it is found the gain from paying less taxes and the Lagrange multiplier, and negatively, the increase in transaction costs, h_3 , and the negative effect on the utility function due to loss of anonymity and the increase of authorities control.

Since the Fisher relationship holds, it is known that in this economy the nominal interest rate in period t R_t is equal to the real interest rate in period t subject to inflation in period t :

$$(1 + R_t) = (1 + r_t)E_t(1 + \pi_{t+1})$$

Note that π_{t+1} is the next-period inflation rate. The current inflation rate π_t is assumed to be equal to the rate of growth of the monetary base μ_t , since it is known that in the long-run equilibrium money growth is strongly associated with inflation¹⁹. Thus:

$$\mu_t = \frac{M_t}{M_{t-1}} - 1 = \pi_t$$

5.2.2 Firms

In this economy, firms are economic agents producing an amount of output y_t , with two inputs that borrow from households, labour n_t and capital k_t to be combined with the available production technology. Moreover, firms are demanding some amounts of these two inputs in order to maximize profits and are paying w_t per unit of labour and r_t^k per unit of capital.

Firms will maximize profits subject to a production constraint that is a Cobb-Douglas function. The latter will work as Production Possibility Frontier (PPF):

$$\max_{n_t, k_t} y_t - w_t n_t - r_t^k k_t$$

Subject to:

$$y_t = k_t^\alpha n_t^{1-\alpha}$$

The main objective of maximizing profit firms is to get the demand functions for both labour and capital. The Lagrange function in period t is:

¹⁹ The trend component in Consumer Price Index inflation is entirely due to the growth component of monetary base growth. i. e. (Benati 2005) and (Crowder 1998).

$$\mathcal{L}_t(n_t, k_t) = y_t - w_t n_t - r_t^k k_t + \lambda_t (y_t - k_t^\alpha n_t^{1-\alpha})$$

The First Order Conditions for labour and capital demand are:

$$\frac{\partial \mathcal{L}_t}{\partial n_t} = -w_t - \lambda_t (1 - \alpha) k_t^\alpha n_t^{-\alpha} = 0$$

$$\frac{\partial \mathcal{L}_t}{\partial k_t} = -r_t^k - \lambda_t \alpha k_t^{\alpha-1} n_t^{1-\alpha} = 0$$

From this maximization process it can be taken that in equilibrium, the marginal revenue for either capital or labour must be equal to its marginal productivity:

$$w_t = (1 - \alpha) \frac{y_t}{n_t}$$

$$r_t^k = \alpha \frac{y_t}{k_t}$$

This a condition that must be satisfied if firms want to maximize profits. The cost of labour w_t cannot be higher than the revenue that the firm gets with that increase in labour. The same happens with the cost of capital r_t^k for firms. Therefore, firms must hire workers and borrow labour until the cost for the firm is equal to the revenue they get from employing these inputs.

5.2.3 Government constraint

Last but not least, it is needed to be found the overall resources constraint of this economy. This constraint captures together, the household budget constraint and the government budget constraint. The government is responsible of issuing bonds and money and collecting taxes in period t . Therefore, g_t is financed as follows:

$$g_t = (1 + r_t)^{-1} b_{t+1} - b_t + m_t - (1 + \pi_t)^{-1} m_{t-1} + \tau_t (w_t n_t + r_t^k k_t)$$

The government can get itself indebted by issuing bonds in period t , $(1 + r_t)^{-1} b_{t+1} - b_t$, that is the variation of bonds from period $t-1$ to t . The second way to get financed is by money creation, where the term $m_t - (1 + \pi_t)^{-1} m_{t-1}$, is the money created in period t , that is the total monetary base change from period $t-1$ to period t . The last term shows the income coming from the collection of taxes in period t . The government income tax rate is exerted over income from labour $w_t n_t$ and capital $r_t^k k_t$.

Plugging g_t on the household budget constraint, it can be easily taken the overall resources

constraint:

$$(w_t n_t + r_t^k k_t) = c_t + k_{t+1} - (1 - \delta)k_t + h_t$$

Since it is known that $w_t = (1 - \alpha) \frac{y_t}{n_t}$ and $r_t^k = \alpha \frac{y_t}{k_t}$ hold in equilibrium, the production in this economy is going to be the following:

$$y_t = c_t + k_{t+1} - (1 - \delta)k_t + h_t$$

5.3 Competitive equilibrium: The Steady-State solution

After presenting and explaining the whole model, a system of 21 equations with 21 different variables has been obtained that need to be expressed in the steady-state solution. These 21 equations contain the 21 variables to be found:

$n_t, c_t, h_{c_t}, \tau_t, w_t, r_t^k, r_t, s_{m_t^e}, k_t, h_{m_t^e}, s_t, R_t, s_{m_t^e}, h_{m_t^e}, y_t, m_t, m_t^e, m_t^c, \lambda_t, h_t$ and *welfare*_t

Although the general competitive equilibrium could be solved using either a dynamic model or a steady-state (deterministic) model, the steady state equilibrium is the most suitable methodology to follow for the main proposition of this paper. Since this paper aims to find the long-run optimal quantity of electronic money circulating in an economy, this is going to be solved into a steady-state form. This implies that variables in period t are going to be the same in every period, so there is no growth and that the technology progress is constant over time. There does not exist uncertainty, and that everything that can happen in between is known since the long-run is the only thing that matters. The model is going to be solved using MATLAB software. It is also going to be used for the different simulations of the model parameters. Table 1 contains the 21 equations expressed in the General Competitive form and transformed into the Steady-State Competitive form.

Table 1 *General Competitive Equilibrium in the short-run and in the long-run.*

Dynamic Equilibrium (short-run)	Steady-State Equilibrium (long-run)
$n_t = \left[\frac{1}{\Psi} \lambda_t (1 - \tau_t) w_t \right]^{\frac{1}{\gamma}}$	$n = \left[\frac{1}{\Psi} \lambda (1 - \tau) w \right]^{\frac{1}{\gamma}}$
$r_t = (1 - \tau_t) r_t^k - \delta$	$r = (1 - \tau) r^k - \delta$
$\frac{c_t^{-\rho}}{(1 + h_{c_t})(1 + r_t)} = \beta E_t \frac{c_{t+1}^{-\rho}}{1 + h_{c_{t+1}}}$	$r = \rho$

$\lambda_t \tau_1 s_{m_t^e} (w_t n_t + r_t^k k_t) - \lambda_t h_{m_t^e}$ $= b_0 b_1 (1 - s_t)^{b_1 - 1} s_{m_t^e}$ $+ \lambda_t \left(\frac{R_t}{1 + R_t} \right)$	$\lambda \tau_1 s_{m^e} (w n + r^k k) - \lambda h_{m^e}$ $= b_0 b_1 (1 - s)^{b_1 - 1} s_{m^e}$ $+ \lambda \left(\frac{R}{1 + R} \right)$
$-b_0 b_1 (1 - s_t)^{b_1 - 1} s_{m_t^e} - \lambda_t h_{m_t^e}$ $= -\lambda_t \tau_1 s_{m_t^e} (w_t n_t + r_t^k k_t)$ $+ \lambda_t \left(\frac{R_t}{1 + R_t} \right)$	$-b_0 b_1 (1 - s)^{b_1 - 1} s_{m^e} - \lambda h_{m^e}$ $= -\lambda \tau_1 s_{m^e} (w n + r^k k)$ $+ \lambda \left(\frac{R}{1 + R} \right)$
$w_t = (1 - \alpha) \frac{y_t}{n_t}$	$w = (1 - \alpha) \frac{y}{n}$
$r_t^k = \alpha \frac{y_t}{k_t}$	$r^k = \alpha \frac{y}{k}$
$y_t = k_t^\alpha n_t^{1 - \alpha}$	$y = k^\alpha n^{1 - \alpha}$
$h_{c_t} = h_1 h_2 \left(\frac{c_t}{m_t} \right)^{h_2 - 1} \left(\frac{1}{m_t} \right)$	$h_c = h_1 h_2 \left(\frac{c}{m} \right)^{h_2 - 1} \left(\frac{1}{m} \right)$
$\tau_t = \tau_0 - \tau_1 s_t$	$\tau = \tau_0 - \tau_1 s$
$s_{m_t^e} = \frac{-m_t^e}{(m_t)^2}$	$s_{m^e} = \frac{-m^e}{(m)^2}$
$s_{m_t^e} = \frac{m_t^e}{(m_t)^2}$	$s_{m^e} = \frac{m^e}{(m)^2}$
$h_{m_t^e} = h_1 h_2 \left(\frac{c_t}{m_t} \right)^{h_2 - 1} \left(\frac{-c_t}{(m_t)^2} \right) + h_3 \frac{-m_t^e}{(m_t)^2}$	$h_{m^e} = h_1 h_2 \left(\frac{c}{m} \right)^{h_2 - 1} \left(\frac{-c}{(m)^2} \right) + h_3 \frac{-m^e}{(m)^2}$
$h_{m_t^e} = h_1 h_2 \left(\frac{c_t}{m_t} \right)^{h_2 - 1} \left(\frac{-c_t}{(m_t)^2} \right) + h_3 \frac{m_t^e}{(m_t)^2}$	$h_{m^e} = h_1 h_2 \left(\frac{c}{m} \right)^{h_2 - 1} \left(\frac{-c}{(m)^2} \right) + h_3 \frac{m^e}{(m)^2}$
$h_t = h_0 + h_1 \left(\frac{c_t}{m_t} \right)^{h_2} + h_3 s_t$	$h = h_0 + h_1 \left(\frac{c}{m} \right)^{h_2} + h_3 s$
$m_t = m_t^e + m_t^c$	$m = m^e + m^c$
$s_t = \frac{m_t^e}{m_t^e + m_t^c} = \frac{m_t^e}{m_t}$	$s = \frac{m^e}{m}$
$\lambda_t = \frac{c_t^{-\varrho}}{1 + h_{c_t}}$	$\lambda = \frac{c^{-\varrho}}{1 + h_c}$
$(1 + R_t) = (1 + r_t) E_t (1 + \pi_t)$	$(1 + R) = (1 + r) (1 + \pi)$
$y_t = c_t + k_{t+1} - (1 - \delta) k_t + h_t$	$y = c + \delta k + h$
$welfare_t = \sum_{j=0}^{\infty} E_t \beta^j \left[\frac{c_{t+j}^{1-\varrho}}{1-\varrho} - \Psi \frac{n_{t+j}^{1+\gamma}}{1+\gamma} + b_0 (1 - s_{t+j})^{b_1} \right]$	$welfare = \frac{1}{1-\beta} \left[\frac{c^{1-\varrho}}{1-\varrho} - \Psi \frac{n^{1+\gamma}}{1+\gamma} + b_0 (1 - s)^{b_1} \right]$

Source: Own Elaboration

Additionally, the model counts with 15 parameters, which are going to be calibrated next

based on different criteria in order to solve for each variable.

5.4 Baseline calibration

The main objective of the baseline calibration is to define the parameters, so they can replicate a representative economy. Once, the baseline calibration, the parameters defining the behaviour of the model can be modified to analyse the effects on an economy. The calibration of the model is needed mainly in the specific parameters that have been introduced by the model, $\tau_1, b_0, b_1, h_0, h_1, h_2$ and h_3 . The rest of parameters are non-specific parameters of this model.

The model parameters have been calibrated to replicate data from the US economy taken from FRED database. The US economy has been chosen because of the wide availability of economic data. The calibration assumes a time period of one year.

Table 2: *Baseline Calibration of parameters*

$\Psi = 0.4820$	$\alpha = 0.36$	$\tau_1 = 0.035$
$\delta = 0.08$	$\varrho = 1.5$	$b_0 = 0.02$
$\pi = 0.03 = \mu$	$\gamma = 2$	$b_1 = 0.75$
$\rho = 0.015$	$h_0 = 0.01$	$h_1 = 0.01$
$\tau_0 = 0.35$	$h_2 = 2.25$	$h_3 = 0.02$

Source: Own elaboration.

This calibration leads to the following results for each of the twenty-one variables:

Table 3: *Steady-State solution in baseline calibration*

$n = 1$	$s_{m^e} = 0.1176$	$y = 1.7044$
$c = 1.3059$	$k = 4.3982$	$m = 0.9823$
$h_c = 0.0327$	$h_{m^e} = -0.0411$	$m^e = 0.8688$
$\tau = 0.3190$	$s = 0.8845$	$m^c = 0.1134$
$w = 1.0908$	$R = 0.0455$	$\lambda = 0.6489$
$r^k = 0.1395$	$s_{m^c} = -0.9005$	$h = 0.0467$
$r = 0.0150$	$h_{m^c} = -0.0615$	$welfare = -129.0303$

Source: Own elaboration. Results from MATLAB

The non-specific parameters have been calibrated as any other real business cycle papers in the literature.²⁰ It is important to notice that δ sets the yearly depreciation of capital, and that

²⁰ See Casares (2007).

is why its value is higher than other papers that usually set its value as a quarterly depreciation. ρ is set so the perception that individuals have from the next period is $\beta = \frac{1}{1+0.015} = 0.985$, that is the utility discount factor for period $t + 1$. Besides, since $\rho = 0.015$, it implies that the real annual interest rate, r in this model is 1.5%.

In case of Ψ , it has been calibrated in order to obtain that the labour supply is normalized at 1.00 in the baseline steady-state solution.

On the other hand, τ_1 has been set, so in the steady-state solution, the share of electronic money decreases the income tax rate by approximately 3%, which would be quite realistic. It has also been considered that by setting that value, the income tax rate cannot be lowered more than 3.5%, which would happen in a corner solution, $s = 1$, which corresponds to a cashless economy.

The transaction cost function is going to be calibrated considering that this function represents the state of art of transactions technology, which is going to be important when setting the value of h_3 . The fixed transaction cost h_0 is calibrated to 0.01, so in the steady-state solution, the total transaction cost is a reasonable percentage over the real output. This ratio is $\frac{h}{y}$, which happens to be 2.74%. The scale parameter h_1 has been calibrated to bring a ratio $\frac{m}{y}$ at 0.57 in the steady-state solution, which approximates to the average ratio of the US economy from 1980 to 2017, being m the monetary aggregate, M2²¹ in real terms, and y is the real GDP. This average happens to be 0.54 and 0.57 in the model.²² Besides, h_2 is 2.25 so the convexity in the transaction cost function is moderate and it also holds stable the ratio of real money stock to real output. Finally, h_3 has been set to 0.02. This parameter is a bit difficult to calibrate because of its meaning. It is important to notice that it is a positive value, because the model starts from the assumption that the existence of more electronic money incurs in higher costs such as investment in new technology and new capital (new apps, new hardware, new security systems...) as well as its maintenance and replacement of devices. Both sign and impact of this parameter is going to be analysed later. In the baseline

²¹ M2 is preferred over M1 because it is the narrowest concept of money and M2 clusters more liquid assets that can be understood as electronic money, such as savings deposits, small-denomination time deposits and balances in retail money market mutual funds.

²² The way to calibrate this parameter has been taken from Casares et al. (2018).

calibration, h_3 leads to having the transaction cost that has to do with electronic money is close to 35% of total transaction costs, that is the ratio $\frac{h_3 s}{h} = 0.37$.

The parameter b_0 has a value so the disutility of electronic money or utility of paper money, (the Big Brother effect), is close to 0.25% of the utility coming from consumption. Then the ratio $\frac{b_0(1-s)^{b_1}}{\frac{c^{1-\varrho}}{1-\varrho}}$ is 0.22%, which it is considered the big brother effect is significantly less important for the wellbeing of individuals than consumption or labour. The concavity, b_1 has been set to 0.75 in order to be moderate too.

Additionally, the model replicates and approximates to other ratios of the US economy. According to the data available in FRED, the average historical ratio $(1 - \frac{\text{Currency in circulation}}{M2})$ from 1984 to 2017 should not vary too much from the model of this paper, that is the variable s . The real ratio is 89% whereas in the baseline calibration, s takes the value of 88.45% which is a good approximation to the optimal share of electronic money. Similarly, the model is also close to the average ratio obtained from FRED, deflated currency in circulation to real GDP, $\frac{m^c}{y}$. The real ratio is 5.67%, and in the model, it is 6.65%.

Finally, the ratio $\frac{\delta k}{y}$ has been thought to be 20.64% and the stock of capital, k is approximately 2.5 times real output, y . Notice that μ is going to be treated as an exogenous variable in the model, so is it inflation. Inflation has been set to 3% per year, which is close to the average US annual inflation from 1968 to 2018.

6. Model simulations and analysis

In this section of this paper, the model is going to be deeply analysed, that aims to see how changes in different parameters in the baseline calibration is going to change the results of the model through several simulations from MATLAB.

First τ_1 is going to be modified from 0.0266 to 0.071, changing 0.0025 each movement. Second, h_3 is going to change from -0.0275 to 0.0345, increasing by 0.005 apart from -0.029 and 0.0327 because these latter values are close to the corner solutions of s , either 1 or 0. Eventually, the behaviour of the model with inflation changes due to a different monetary policy target is going to be seen, moving $\pi = \mu$ from -0.01477 to 0.08. Since welfare is measured in terms of utility, the permanent loss can be expressed in terms of consumption

because it is a better proxy for a real economy. For this, the consumption equivalence c_e is defined as follows²³:

$$c_e = \left[(wel^{\max}(1 - \beta)) + \Psi \frac{n^{1+\gamma}}{1 + \gamma} - b_0(1 - s)^{b_1}(1 - \rho) \right]^{\frac{1}{1-\rho}}$$

The latter contains the fixed variable wel^{\max} that is calculated as the maximum welfare reached when the parameters are altered. Values for n and s differ depending on the point of the simulation that the loss wants to be estimated. The percentage permanent and annual will be computed as percentage of output in the following way:

$$\frac{c_e - c}{y} \cdot 100$$

Where c and y are the steady-state values of consumption and real output at the point of the simulation where the loss wants to be estimated.

6.1 Tax evasion effect

The aim of this sub-section is to see what happens to different variables when the negative impact of the electronic money share on income tax rate is changed. This can be measured by an increase of τ_1 , that paper money decreases and electronic money increases, so the share also increases making the income tax rate lower and thus, labour and capital usage increases fostering real output supply side and thus, consumption from the demand side. These effects should lead to an improvement of welfare as long as τ_1 is bigger thanks to more consumption that offsets the increase on labour and the decrease of paper money. In this case the maximum welfare is reached when τ_1 has the biggest impact on the reduction of income taxes, that is 0.071 and the income tax becomes 27.29%. This is very close to the corner solution because the share of electronic money is 99.74%.

Therefore, Figure 3 shows how the variable s , rapidly increases as long as τ_1 is higher and then, it slows down when the parameter approaches 0.035. This is important to take into account, because it says that for an economy to prefer almost totally or partially the use of electronic money over cash, with $s=99.74\%$ ²⁴ (when $\tau_1 = 0.071$), the income tax rate should be decreased from 0.3190 in the baseline calibration to 0.2792. When τ_1 is 0.0266 the share

²³ The consumption equivalence equation is obtained from solving welfare in the steady-state solution for consumption.

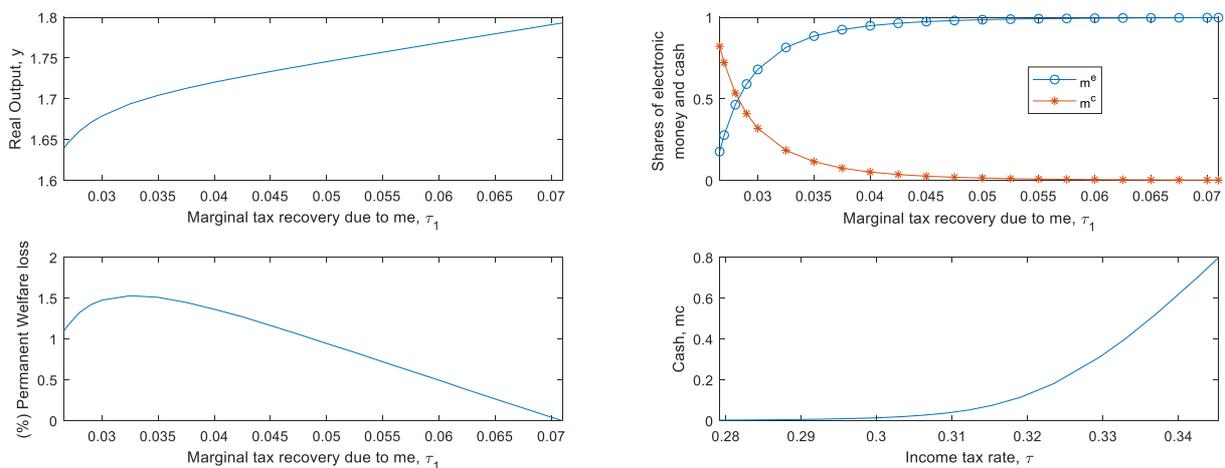
²⁴ This value of s is consistent with the argument of Rogoff that it might happen that only an insignificant amount of coins or small-value bills would remain in circulation.

of electronic money is very low, 17.76%. This result also states the significant weight that τ_1 has on the choice of holding currency. Notice that the quantity of real money barely changes. Moreover, note the relationship between τ and m^e . Whenever the income tax rate becomes bigger, individuals prefer to hold more cash in exchange for electronic money, so tax evasion and criminal activities are more likely to take place. This happens due to the fact that the lower the tax evasion the effect, the less the incentives that exist for giving up cash in exchange of electronic money.

As expected, real output becomes bigger thanks to a lower income tax rate. The reason is that with a higher tax evasion effect, people obtain more resources from labour and capital returns, w and r^k because taxes exerted over them are lower, so more n and k are supplied in the economy, which increases output production. In fact, real output in steady-state solution varies 9.36% from the minimum to the maximum value of τ_1 in the simulation.

When it comes to welfare, a higher τ_1 improves welfare much more than a lower one. In steady state welfare is -129.0303 and it can be increased until its maximum -127.8802, where there is 0% permanent welfare loss with the minimum τ_1 in this simulation, 0.0226. This reaffirms the fact that more electronic money is beneficial for the economy because it fosters consumption and increases welfare. It can be seen in the bottom-left graph, how the permanent welfare loss evolves while τ_1 changes. Notice that it is hump-shaped because with the marginal decrease of s is big enough to start creating some benefits from enhanced privacy that outweigh the marginal decrease of consumption in the social welfare function.

Figure 3: τ_1 Effects on Real Output, Share of Electronic Money, Welfare and Cash.



Source: Own Elaboration. MATLAB output

After doing this simulation, it has been found that the maximum welfare loss is 1.51% of

real output in baseline calibration. In fact, the biggest loss is obtained when τ_1 takes the value of 0.0325 within the simulated interval in this subsection. The economy could be better off whenever the reduction of income tax rate due to more electronic money is higher. As long as τ_1 approaches the maximum welfare, the welfare loss reduction becomes bigger.

6.2 Transaction cost of electronic money

A change on the transaction cost of electronic money h_3 is going to be analysed. This may have effects on the economy if either the share of electronic money increases (positive value of h_3) or drops (negative value of h_3). Over this simulation, it is expected that when h_3 rises, transaction costs increase making the marginal transaction cost electronic money bigger. Hence, the demand for electronic money would drop because its transaction costs turn higher than with cash. This would lead to a decrease of s which makes the income tax rate bigger and slows down the economy due to less production. Cash decreases transaction costs more than electronic money and this is mostly caused by a higher h_3 .

In this simulation, the maximum welfare possible is reached when h_3 makes the use of electronic money cheaper in transaction costs terms than paper currency. This is when it takes the value of -0.0275. This is a corner solution because the share of electronic money is 99.42%.

Therefore, Figure 4 is composed of four different plots that describe the responses when there is a change on the value of h_3 . Marginal transaction cost of either m^e and m^c evolves with changes of h_3 . It demonstrates that the only reason for both marginal transaction costs to be different is the sign of h_3 . Thus, when $h_3 = 0$ both h_{m^e} and h_{m^c} are equal and they take the value of -0.0435 and total transaction costs happen to be 0.0291.²⁵ Transaction costs become 14.52% higher when h_3 is 0.0345 than when it is negative at 0.0275.

The latter is a concern for the optimal value of electronic money share s . When electronic money makes transaction costs easier than paper money ($h_3 < 0$), the drawback that m^e had in the baseline calibration becomes an advantage since it drops transaction costs further than cash, and therefore, the share of electronic money becomes 99.42% when $h_3 = -0.0275$. Contrary, when $h_3 = 0.01$ and the difference in the marginal transaction cost between cash

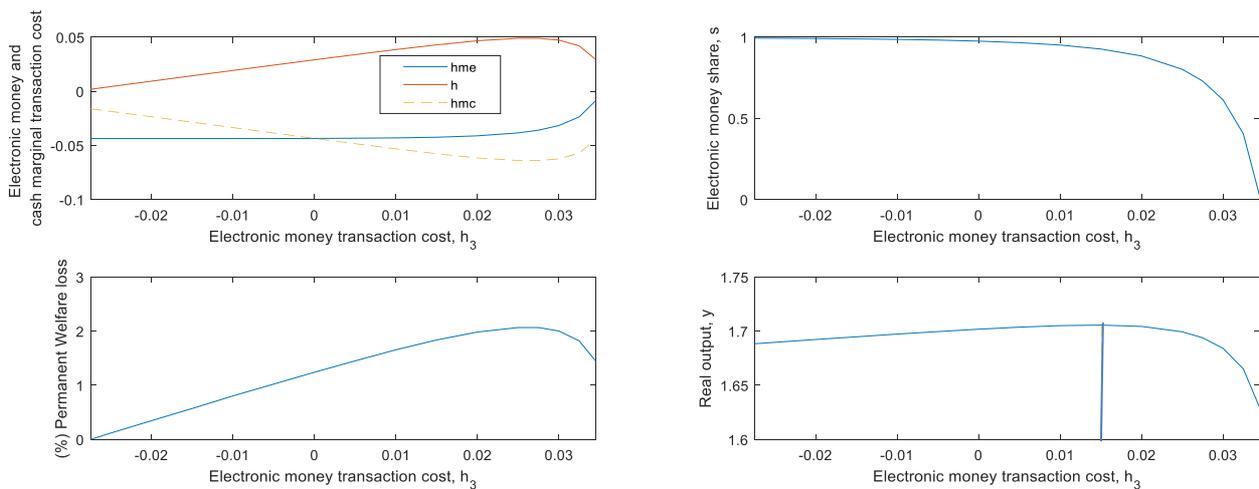
²⁵ Note that:

$$h_{m^c} = h_1 h_2 \left(\frac{c}{m}\right)^{h_2-1} \left(\frac{-c}{(m)^2}\right) + h_3 \frac{-m^e}{(m)^2} = h_1 h_2 \left(\frac{c}{m}\right)^{h_2-1} \left(\frac{-c}{(m)^2}\right) + h_3 \frac{m^c}{(m)^2} = h_{m^e} \text{ if } h_3 \text{ takes the value of 0.}$$

and electronic money starts becoming bigger, the share of electronic money shrinks rapidly to 2.37% when $h_3=0.0345$. This huge increase in cash holdings caused by higher transaction costs for m^e , makes income tax rate higher due to the increase of cash in the economy and therefore, it leads to a lower real output, y . However, transaction costs do affect the level of output, so the maximum real output is not reached when the share of electronic money is at its maximum because at that point, h_3 makes transaction costs very low reducing demand and thus, real output. Thus, the maximum level of real output (blue line on the bottom right plot), 1.7056 is when there exists an equilibrium between both effects, that is when $h_3=0.015$ and $s=0.9272$. Then it decreases by 4.46% when $h_3=0.0345$ and by 1.00% when $h_3=-0.0275$. Besides, it could be said that whenever h_3 is between 0 and 0.015, real output does not suffer major changes. However, for the economy it is better to be $h_3 < 0.02$ than $h_3 > 0.02$, because the increase of electronic money share in the second case, makes real output suffer more due to the increase it causes on the income tax rate. Therefore, the effect of electronic money share fosters output more than it decreases through smaller transaction costs.

Lastly, welfare effects of the simulation show how welfare is higher when the share of electronic money is as much as possible. It is -129.0303 in baseline and it is reduced to negative 127.5262 when h_3 is -0.0275 and the share 99.42%, a 0% permanent welfare loss. Undoubtedly the permanent welfare loss follows a similar path as the bottom-left graph shows. Note that its hump-shape is caused due to the same reasons of tax evasion changes.

Figure 4: h_3 Effect on Transaction Costs, Electronic Money Share, Real Output and Welfare



Source: Own Elaboration. MATLAB output.

After doing this simulation, it has been found that the permanent loss is equivalent to 1.98%

of real output in baseline calibration. This result is close to the maximum loss, which means that the economy could be better off whenever the transaction costs of electronic money are optimized and make transaction costs cheaper and easier. As long as h_3 approaches the maximum welfare, the loss reduction becomes bigger and real output barely drops.

6.3 Welfare cost of inflation

This section shows how different inflation targets of monetary authorities affect the model and different variables. In this case the annualized rate of inflation is going to change from -1.47% to 8%. This is analyzed because both means of holding money have the common burden of the opportunity cost for holding money instead of bonds that yield the nominal interest rate R , which is dependent on inflation since Fisher relationship holds. In the steady-state solution of the model, an increase of the rate of nominal money growth μ , increases one-to-one inflation and the nominal interest rate. Subsequently, the opportunity cost of money holdings is higher and therefore the quantity of real money demand decreases, making transaction costs and the marginal transaction cost of consumption higher. The increase on h_c reduces the supply of labor and capital and therefore, output decreases by the supply side. The reasoning behind the setting of the simulation from inflation at -1.47% is that it is precisely the optimal inflation rate (π^*) according to the Chicago Rule of Milton Friedman (1969) to maximize social welfare in the household optimizing program: “Our final rule for the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal rate of interest equal to zero”. Since the Fisher relationship holds in the model, according to the Chicago rule, the nominal interest rate, R should be 0 and the optimal inflation rate becomes.²⁶

$$\pi^* = \mu^* = \frac{-\rho}{1 + \rho} = \frac{-0.015}{1 + 0.015} = -0.0147 = -1.47\%$$

As expected, the maximum welfare is reached when inflation or money growth are at their optimal level, therefore, Chicago Rule holds in this model. However, since there are negative opportunity costs when holding money, the quantity of real money either paper currency or electronic currency relative to real output $\frac{m}{y}$, becomes extremely high, 9179.34% and recall that this value is 57.63% in baseline calibration. This may also represent a liquidity trap, $R =$

²⁶ Recall that $r = \rho$ in the steady state solution and the Fisher relationship is solved for inflation when nominal interest rate is 0.

0 when the monetary policy to boost output is ineffective regardless of the quantity of money issued. The latter meets somehow with the situation lived in the financial crisis. As it can be seen in Figure 1 and Figure 2, in both Euro area and USA the quantity of money (paper) in circulation increased because monetary policies were trying to boost output issuing money and decreasing nominal interest rates.

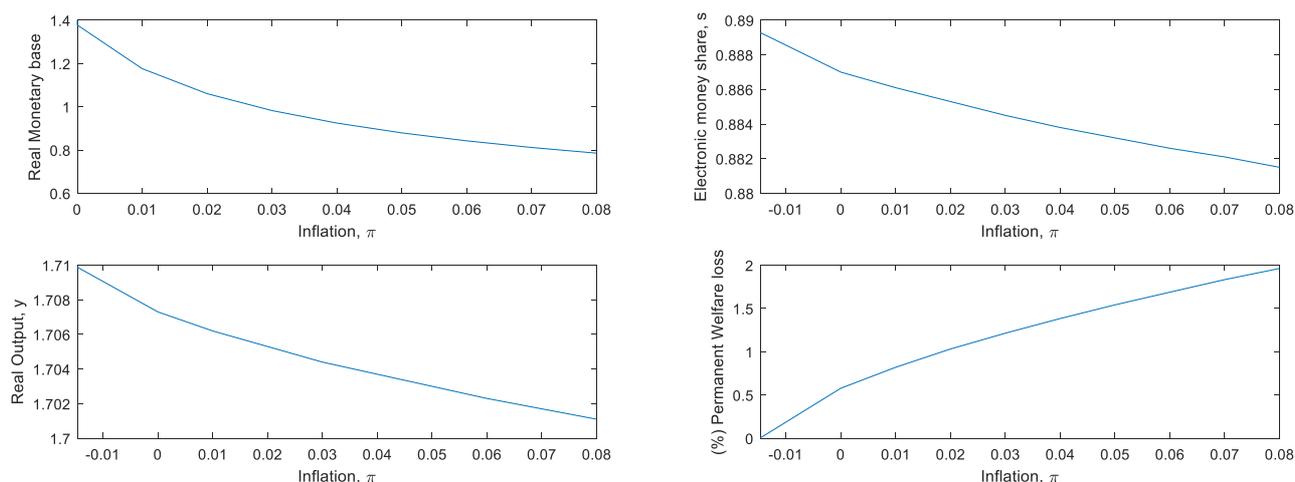
π	-0.0147	0	0.01	0.02	0.03	0.05	0.08
$\frac{m}{y}$	9179.34%	80.70%	68.92%	62.18%	57.63%	51.62%	46.17%

As long as the quantity of money is reduced, transaction costs tend to increase, being 1.626% of real output at the optimal inflation rate and becoming 3.427% of real output when inflation is 8%. Whenever inflation is higher, transaction costs increase and real output drops.

π	-0.0147	0	0.01	0.02	0.03	0.05	0.08
$\frac{h}{y}$	1.62%	2.15%	2.37%	2.56%	2.74%	3.04%	3.42%

Now, inflation changes the long-run model solution are seen. Then, Figure 5 shows that the only major changes on the model when inflation is moved from -1.47% to 8.00%, are on the real money and the transaction costs of the economy. Share of electronic money does not change significantly (although less inflation leads to more electronic money preference) and output slightly decreases with higher inflation. As it could be expected, the real money increases whenever steady state inflation is lower because the opportunity cost of receiving a yield instead of holding money becomes smaller and hence, the loss of purchasing power when money is held becomes lower. It shows how both means of holding money follow similar paths and how the real money varies significantly. Note that the simulation of -1.47% inflation has been omitted in the graph for real money because of the huge quantity of real money in steady-state would visually disrupt the graph. Figure 6 also confirms that the higher the inflation rate, the lower is social welfare for households.

Figure 5: Steady-State Inflation Changes



Source: Own Elaboration. MATLAB output

Table 4 shows how real output becomes lower as long as inflation rate turns bigger. The annual and permanent welfare loss also becomes bigger when the inflation rate deviates more from the optimal value of -1.47%, which is also shown in Figure 5.

Table 4: Changes on real output and welfare loss if inflation deviates from Chicago Rule.

π	$\frac{\Delta y}{y} \cdot 100$ Permanent output loss.	Annual Permanent welfare loss. $\frac{c_e - c}{y} \cdot 100$
-1.47%	0.000%	0.00%
0.00%	-0.152%	0.58%
1.00%	-0.216%	0.82%
2.00%	-0.269%	1.03%
3.00%	-0.322%	1.21%
4.00%	-0.363%	1.38%
5.00%	-0.404%	1.54%
6.00%	-0.444%	1.69%
7.00%	-0.480%	1.83%
8.00%	-0.515%	1.96%

Source: Own elaboration. MATLAB output.

After doing this simulation, it has been found that the permanent welfare loss results to be a 1.21% of real output when the long-run inflation rate is 3%. This loss becomes bigger as long as the inflation target is higher and vice versa. Likewise, a 3% rate of inflation real output loss amounts to 0.322% when it deviates from Chicago Rule.

7. Conclusion

Nowadays, new transaction technologies ease the use of electronic money. Therefore, the use of cash must be reconsidered since it has many disadvantages related to tax evasion, constrained monetary policy and criminal activities. Despite privacy issues, financial inclusion or the loss of seigniorage revenues, a cashless society has ever been so likely, where

just big denominated coins could remain in circulation for daily transactions. These issues are a real challenge for both authorities and regulators to be implemented. Thus, this paper makes a welfare analysis of a cashless economy taking into account positive and negative features of electronic money and paper money.

This welfare analysis is developed by a Real Business Cycle model with money introduced as a transaction-facilitating tool. The model has been optimized for both firms and households and then turned into the long-run solution, the Steady-State. The optimizing programs result in 21 equations and 21 variables to be solved in MATLAB. The model has been calibrated to closely replicate the US economy in the long-run. The model has been useful to estimate changes in tax evasion, transaction costs and the annual rate of inflation.

The results of the model simulations indicate different qualitative and quantitative conclusions. Inflation does negatively affect welfare and a low inflation helps to reduce the share of electronic money. At the Chicago Rule, real money holdings increase significantly, and a 3% inflation rate creates a 1.21% permanent welfare loss of output when it deviates from Chicago Rule. Electronic money transaction costs greatly affect the share of electronic money and if electronic transactions happened to be easy, welfare would rise. In case it makes transaction costs cheaper $h_3 < 0$, social welfare would increase from 0.74% up to 1.98% according to the simulation. One of the most important advantages of electronic money is the drop of tax evasion, which should be totally enough by its own to totally outweigh other related losses. Reducing the income tax rate due to less tax evasion by 7.1% instead of 3.5% in baseline calibration would increase permanent social welfare by 1.51% according to the model simulation.

Note that no simulation has been able to lead the model to a 100% share of electronic model, which meets the statement of Rogoff (2016) that cash would not need to be totally phased out. These brand-new coins should be harder to handle, store and carry for criminal activities and tax evasion proposes.

This RBC with money has been very useful to analyse the effects of electronic money on the economy and estimate welfare effects. Nonetheless, the model used in this paper presents disadvantages due to the complexity of the economy and diversity of individuals. It is well-known that the economy is not composed by equal individuals and equal production structures, with welfare measured in terms of consumption, labour and privacy. Furthermore, the steady-state solution used in this paper does not allow to analyse how

electronic money would behave with shocks that causes adjustment over time. It would be interesting to further develop the model in the dynamic solution and incorporating other costs such as seigniorage and revenues such as unconstrained monetary policy.

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