

A structural analysis of US entry and exit dynamics

Technical Appendix

Miguel Casares¹, Hashmat Khan² and Jean-Christophe Poutineau³

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¹Departamento de Economía, Universidad Pública de Navarra, 31006, Pamplona, Spain. E-mail: mcasares@unavarra.es (Miguel Casares).

²Department of Economics, Carleton University, Ottawa, ON K1S 5B6, Canada. E-mail: Hashmat.Khan@carleton.ca (Hashmat Khan).

³CREM, UMR CNRS 6211, Université de Rennes I, Rennes, France. E-mail: jean-christophe.poutineau@univ-rennes1.fr (Jean-Christophe Poutineau).

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A. The optimizing programs of the model and other technical details.

Households

Following Smets and Wouters (2007), the preferences of the i^{th} representative household are defined by a non-separable utility function specification, which for period t reads

$$\left[\frac{(c_t(i) - hc_{t-1})^{1-\sigma_c}}{1-\sigma_c} \right] \exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} (l_t(i))^{1+\sigma_l} \right),$$

where $\sigma_c, \sigma_l > 0$ are the risk aversion and the inverse of Frisch elasticity, respectively; $0 < h < 1$ is the consumption (external) habit parameter, $c_t(i)$ is the household-level current consumption of bundles of goods, c_{t-1} is lagged aggregate consumption of these bundles, and $l_t(i)$ is the household-specific supply of labor.

The sources of household income are labor and capital earnings, equity return and the interest service of government bonds. The nominal wage is set by households as they have market power to supply a differentiated labor service. Thus, the representative household determines the nominal wage $W_t(i)$ constrained by its labor demand schedule. Labor income is $(W_t(i)/P_t^c) l_t(i)$, where the real wage is measured in consumption bundles at the Consumer Price Index (CPI), P_t^c . Capital income is $r_t^k u_t(i) k_{t-1}(i)$ where r_t^k is the market real rental rate, $u_t(i)$ is the variable capital utilization rate and $k_{t-1}(i)$ is the stock of capital installed in the previous period. Another source of income is equity ownership. Let d_t denote the average real dividend and v_t the average real equity value. The representative household gets $(n_t^s/n_{t-1}) d_t x_{t-1}(i)$ as the total dividends from her ownership of the share $x_{t-1}(i)$ of incumbent firms, and $(n_t^s/n_{t-1}) d_t n_t^E(i)$ from the entries of the previous period that do not fail in its first period of life. There is also some revenue from business destruction, which corresponds to both the liquidation value of the exit share, $(n_t^x/n_{t-1}) l v_t x_{t-1}(i)$, where $l v_t$ is the real liquidation value per business unit, and the liquidation of new goods that shut down after the first period of life, $(n_t^x/n_{t-1}) l v_t n_t^e(i)$. Gross income turns into net income when subtracting the amount of real tax payments, $t_t(i)$.

Net income is spent on purchases of bundles of consumption goods, $c_t(i)$, on investment on capital goods, $i_t(i)$, on portfolio investment on incumbents, $v_t (x_t(i) - (n_t^s/n_{t-1}) x_{t-1}(i))$, on net purchases of real government bonds, $(\exp(\varepsilon_t^b) (1 + r_t))^{-1} b_t(i) - b_{t-1}(i)$, where r_t is the real rate of return and ε_t^b is a risk-premium AR(1) shock, and on the cost of creating new goods, $\exp(\varepsilon_t^e) f^e + ec_t n_{t+1}^e(i)$, where ε_t^e is an AR(1) entry cost shock, f^e is the unit real cost of a license fee required by the government to begin the production of a new variety and ec_t is a variable entry cost to be defined below. In addition, there is some expenditure on covering the adjustment cost of variable capital utilization, $a(u_t(i)) k_{t-1}(i)$ where $a(u_t(i))$ is the adjustment cost variable described in Smets and Wouters (2007). As a result, the budget constraint of the representative household in period t

becomes,

$$\frac{W_t(i)}{P_t^e} l_t(i) + r_t^k u_t(i) k_{t-1}(i) + \left[\frac{n_t^s}{n_{t-1}} (d_t + v_t) + \frac{n_t^x}{n_{t-1}} l v_t \right] (x_{t-1}(i) + n_t^e(i)) - t_t(i) = c_t(i) + i_t(i) + a(u_t(i)) k_{t-1}(i) + v_t x_t(i) + \frac{b_t(i)}{\exp(\varepsilon_t^b)(1+r_t)} - b_{t-1}(i) + (\exp(\varepsilon_t^e) f^E + e c_t) n_{t+1}^e(i). \quad (\text{A1})$$

Capital accumulation is costly as in Smets and Wouters (2007). Thus, the equation of motion for capital is,

$$k_t(i) = (1 - \delta_k) k_{t-1}(i) + \exp(\varepsilon_t^i) \left[1 - S\left(\frac{i_t(i)}{i_{t-1}(i)}\right) \right] i_t(i), \quad (\text{A2})$$

where δ_k is the constant rate of capital depreciation rate, $S(i_t(i)/i_{t-1}(i))$ is the investment adjustment cost function with the steady-state properties $S(1) = S'(1) = 0$ and $S''(1) = \varphi_k > 0$, and ε_t^i is an stochastic AR(1) shock to the price of investment relative to consumption goods.

Following Erceg *et al.* (2000), households can set the nominal wage of their specific labor service supplied, subject to a market signal that arrives with a constant probability as in Calvo (1983). Let $0 < \xi_w < 1$ represent the probability that the household is not able to set the optimal wage. In that case, the adjustment of the nominal wage would follow this indexation rule

$$W_t(\cdot) = W_{t-1}(\cdot) \left[(1 + \pi_{t-1}^c)^{\iota_w} (1 + \pi^c + \varepsilon_t^W)^{1-\iota_w} \right],$$

in which π_{t-1}^c is the lag of the rate of CPI inflation, $\pi_{t-1}^c = (P_{t-1}^c/P_{t-2}^c) - 1$, the steady-state CPI rate is π^c , there is an ARMA(1,1) stochastic component introduced through the wage-push shock ε_t^w , and $0 < \iota_w < 1$ is the parameter that determines the indexation share that mirrors lagged CPI inflation. As wage setters, households face the labor demand constraint *à la* Dixit and Stiglitz (1977)

$$l_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\theta_w} l_t, \quad (\text{A3})$$

where $W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}$ and $l_t = \left[\int_0^1 l_t(i)^{\frac{\theta_w-1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w-1}}$ are, respectively, the aggregate indices of nominal wages and labor with a constant elasticity of substitution $\theta_w > 0$. Assuming a constant discount factor per period, $\beta < 1$, the optimizing program of the household consists of maximizing

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\left[\frac{(c_{t+j}(i) - h c_{t-1+j})^{1-\sigma_c}}{1 - \sigma_c} \right] \exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} (l_{t+j}(i))^{1+\sigma_l} \right) \right)$$

subject to the budget constraint (A1), the capital accumulation constraint (A2), and the labor demand constraint (A3), for current period t and the expected expressions in all future periods. The first order conditions are computed with respect to the choice variables $c_t(i)$, $u_t(i)$, $k_t(i)$, $b_t(i)$, $W_t(i)$, $x_t(i)$, and $n_{t+1}^e(i)$. It should be noticed that the desired number of entries are decided one period in advance, which may capture time-to-build requirements. The behavioral equations for consumption, investment, and wage inflation are equivalent to those derived and described in Smets

and Wouters (2003), with just some differences in the wage inflation dynamics: we do not have a labor supply shock and the indexation rule is written in response to both CPI inflation and the cost-push shock on wages.¹ The first order conditions on the portfolio choices of equity, $x_t(i)$, and goods creation, $n_t^e(i)$, both unusual in DSGE models, are, respectively,

$$-\lambda_t v_t + \beta E_t \lambda_{t+1} \left[\frac{n_{t+1}^s}{n_t} (d_{t+1} + v_{t+1}) + \frac{n_{t+1}^x}{n_t} l v_{t+1} \right] = 0, \quad (\text{A4})$$

$$\beta E_t \lambda_{t+1} \left[\frac{n_{t+1}^s}{n_t} (d_{t+1} + v_{t+1}) + \frac{n_{t+1}^x}{n_t} l v_{t+1} \right] - \lambda_t (\exp(\varepsilon_t^e) f^e + e c_t) = 0, \quad (\text{A5})$$

where λ_t is the Lagrange multiplier of the budget constraint in period t . The first order condition of bonds implies $\lambda_t (\exp(\varepsilon_t^b) (1 + r_t))^{-1} = \beta E_t \lambda_{t+1}$, which can be inserted in (A4) to give the equilibrium condition for equity investment

$$v_t = \frac{1}{\exp(\varepsilon_t^b) (1 + r_t)} E_t \left[\frac{n_{t+1}^s}{n_t} (d_{t+1} + v_{t+1}) + \frac{n_{t+1}^x}{n_t} l v_{t+1} \right], \quad (\text{A6})$$

that implies an average equity value equal to the discounted sum of the expected returns when surviving, $(n_{t+1}^s/n_t) (d_{t+1} + v_{t+1})$, and the expected return when dying, $(n_{t+1}^x/n_t) l v_{t+1}$. Remarkably, the equilibrium equity value depends (positively) on the rate of business survival, n_{t+1}^s/n_t , as the weight for the return on surviving equity, and on the expected next-period liquidation value, $E_t l v_{t+1}$, as the anticipated return from the fraction of goods that are expected to have their production shut down.

Establishments (firms)

There are both single-good and composite-good establishments (firms) in the goods market. Single-good establishments combine labor and capital within a firm-specific production technology to supply heterogeneous consumption goods that are sold in a monopolistically competitive market to the composite-good firm. Single-good producers are price setters constrained by nominal rigidities and demand conditions. The composite-good firm aggregates all the varieties of consumption goods to make them available as consumption bundles in a fully-competitive market.

Single-good establishments

In period t , the representative establishment type ω produces a quantity $y_t(\omega)$ of this good using the Cobb-Douglas production technology,

$$y_t(\omega) = \exp(\varepsilon_t^a) z(\omega) k_t^\alpha(\omega) (\exp(\gamma t) l_t(\omega))^{1-\alpha}, \quad (\text{A7})$$

¹The wage inflation equation is displayed below in this technical appendix as part of the semi-loglinear set of dynamic equations of the model.

where $0 < \alpha < 1$ is the capital share parameter, $l_t(\omega)$ and $k_t(\omega)$ are respectively the demand for labor and capital at firm ω , ε_t^a is a labor-augmenting and economy-wide AR(1) technology shock, $z(\omega)$ is a firm-specific productivity level, and γ is the long-run rate of economic growth. While the shock ε_t^a is homogeneous to all firms, there is firm heterogeneity in productivity. Thus, the representative establishment gets $z(\omega)$ as its specific time-invariant productivity, which is taken as an individual draw from a Pareto distribution characterized by its lower bound z_{\min} and the shape parameter κ .²

Regarding market conditions, single-good firms operate in a monopolistically competitive market as in Dixit and Stiglitz (1977). Hence, the amount of firm-specific output, $y_t(\omega)$, is demand-determined in response to its relative price $P_t(\omega)/P_t^c$ and to the aggregate demand for bundles of consumption goods, y_t , as follows,

$$y_t(\omega) = \left(\frac{P_t(\omega)}{P_t^c} \right)^{-\theta_p} y_t, \quad (\text{A8})$$

where $\theta_p > 1$ is the constant elasticity of substitution across goods.

In addition, single-good establishments face rigidities on price setting determined by a fixed probability scheme as in Calvo (1983). Let $0 < \xi_p < 1$ denote the probability of not being able to set the optimal price. In such a case, the price adjustment would follow the indexation rule

$$P_t(\cdot) = P_{t-1}(\cdot) \left[(1 + \pi_{t-1})^{\iota_p} (1 + \pi + \varepsilon_t^P)^{1-\iota_p} \right],$$

in which π_{t-1} is the lagged rate of producer price inflation (measured at the average steady-state firm-level productivity \tilde{z}), π denotes the steady-state rate of producer price inflation, ε_t^P is an exogenous ARMA(1,1) price-push shock, and $0 < \iota_p < 1$ is the coefficient of the indexation share that responds to lagged inflation.

In order to analyze optimal pricing, let us assume that the representative establishment ω in period t receives the Calvo market signal to set the optimal price. Then, it will choose $P_t(\omega)$ to maximize the expected stream of real dividends conditional to the lack of future optimal pricing

$$\sum_{j=0}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \xi_p^j \left(\left(\frac{P_t(\omega) \Pi_{t,t+j}^p}{P_{t+j}^c} \right)^{1-\theta_p} y_{t+j} - w_{t+j} l_{t+j}(\omega) - r_{t+j}^k k_{t+j}(\omega) \right),$$

where $\beta_{t,t+j}$, $s_{t,t+j}(\omega)$ and $\Pi_{t,t+j}^p$ denote, respectively, the stochastic discount factor, the probability of survival and the price indexation factor all of them between periods t and $t+j$. In addition, $w_{t+j} = W_{t+j}/P_{t+j}^c$ is the aggregate real wage in any period $t+j$.³ The stochastic discount factor in

²The probability distribution function and the cumulative distribution function of $z(\omega)$ are respectively $g(z(\omega)) = \kappa z_{\min}^{\kappa} / z(\omega)^{\kappa+1}$ and $G(z(\omega)) = 1 - (z_{\min}/z(\omega))^{\kappa}$. The shape parameter κ must be higher than $(\theta_p - 1)$ to have a well-defined average productivity.

³The price indexation factor between t and $t+j$ consistent with the indexation rule is computed as follows $\Pi_{t,t+j}^p = \prod_{k=0}^j [(1 + \pi_{t+k})^{\iota_p} (1 + \pi + \varepsilon_{t+1+k}^P)^{1-\iota_p}]$.

equilibrium is $\beta_{t,t+j} = \prod_{k=1}^j \left(e^{\varepsilon_{t+k}^b} (1 + r_{t+k}) \right)^{-1}$ and the probability of continuation between periods t and period $t + j$ is given by the accumulated survival rate

$$s_{t,t+j}(\omega) = \left\{ \begin{array}{ll} 1 & \text{if } j = 0 \\ \prod_{k=1}^j (n_{t+k}^s(\omega) / n_{t+k-1}(\omega)) & \text{if } j = 1, 2, 3, \dots \end{array} \right\}.$$

The optimal choices of the firm must be subject to the expected schedule of Dixit-Stiglitz demand constraints,

$$\exp(\varepsilon_{t+j}^a) z(\omega) k_{t+j}^\alpha(\omega) (\exp(\gamma(t+j)) l_{t+j}(\omega))^{1-\alpha} = \left(\frac{P_t(\omega) \Pi_{t,t+j}^p}{P_{t+j}^c} \right)^{-\theta_p} y_{t+j}, \text{ for } j = 0, 1, 2, \dots$$

The first order conditions with respect to the price, $P_t(\omega)$, labor demand, $l_t(\omega)$, and capital demand, $k_t(\omega)$, are,

$$\begin{aligned} E_t^\xi \sum_{j=0}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \xi_p^j \left(\begin{array}{l} (1 - \theta_p) \left(\frac{P_t(\omega) \Pi_{t,t+j}^p}{P_{t+j}^c} \right)^{-\theta_p} \frac{y_{t+j} \Pi_{t,t+j}^p}{P_{t+j}^c} \\ + m c_{t+j}(\omega) \theta_p \left(\frac{P_t(\omega) \Pi_{t,t+j}^p}{P_{t+j}^c} \right)^{-\theta_p - 1} \frac{y_{t+j} \Pi_{t,t+j}^p}{P_{t+j}^c} \end{array} \right) &= 0, \\ -w_t + (1 - \alpha) m c_t(\omega) \exp(\varepsilon_t^a) \exp((1 - \alpha)t) (k_t(\omega) / l_t(\omega))^\alpha &= 0, \\ -r_t^k + \alpha m c_t(\omega) \exp(\varepsilon_t^a) \exp((1 - \alpha)t) (l_t(\omega) / k_t(\omega))^{1-\alpha} &= 0, \end{aligned}$$

where E_t^ξ is the rational expectation operator conditional to the lack of optimal pricing, and $m c_{t+j}(\omega)$ is the Lagrange multiplier of the demand constraint in period $t + j$ (i.e., the firm-specific real marginal cost). The ratio $P_t(\omega) / P_{t+j}^c$ can be decomposed in the following way

$$P_t(\omega) / P_{t+j}^c = \left(P_t(\omega) / \tilde{P}_{t+j} \right) \left(\tilde{P}_{t+j} / P_{t+j}^c \right) = \left(P_t(\omega) / \tilde{P}_{t+j} \right) \tilde{\rho}_{t+j},$$

by introducing \tilde{P}_{t+j} as the Producer Price Index (PPI): the average price across all firms that have the steady-state average productivity \tilde{z} (and they differ due to their specific Calvo pricing histories). We also introduce $\tilde{\rho}_{t+j}$ as their relative price in period $t + j$ obtained as the ratio between the referential PPI and the CPI

$$\tilde{\rho}_{t+j} = \tilde{P}_{t+j} / P_{t+j}^c.$$

Using such decomposition in the pricing first order condition, the optimal price $P_t(\omega)$ becomes

$$P_t(\omega) = \frac{\theta_p}{\theta_p - 1} \left[\frac{E_t^\xi \sum_{j=0}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \xi_p^j m c_{t+j}(\omega) \left(\tilde{P}_{t+j} \right)^{\theta_p} \left(\Pi_{t,t+j}^p \tilde{\rho}_{t+j} \right)^{-\theta_p} y_{t+j}}{E_t^\xi \sum_{j=0}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \xi_p^j \left(\tilde{P}_{t+j} \right)^{\theta_p - 1} \left(\Pi_{t,t+j}^p \tilde{\rho}_{t+j} \right)^{-\theta_p + 1} y_{t+j}} \right], \quad (\text{A9})$$

where the real marginal cost is firm-specific due to the constant firm-level productivity $z(\omega)$

$$m c_{t+j}(\omega) = \frac{w_{t+j}^{1-\alpha} (r_{t+j}^k)^\alpha}{\alpha^\alpha (1 - \alpha)^{(1-\alpha)} \exp(\varepsilon_{t+j}^a) z(\omega)}.$$

Since

$$\widetilde{mc}_{t+j} = \frac{w_{t+j}^{1-\alpha} (r_{t+j}^k)^\alpha}{\alpha^\alpha (1-\alpha)^{(1-\alpha)} \exp(\varepsilon_{t+j}^a) \widetilde{z}}$$

we can write an analogous expression to (A9) for the optimal price, \widetilde{P}_t^* , set by the firm that operates with the steady-state average productivity \widetilde{z}

$$\widetilde{P}_t^* = \frac{\theta_p}{\theta_p - 1} \left[\frac{E_t^\xi \sum_{j=0}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \xi_p^j \widetilde{mc}_{t+j} \left(\widetilde{P}_{t+j}\right)^{\theta_p} \left(\Pi_{t,t+j}^p \widetilde{\rho}_{t+j}\right)^{-\theta_p} y_{t+j}}{E_t^\xi \sum_{j=0}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \xi_p^j \left(\widetilde{P}_{t+j}\right)^{\theta_p-1} \left(\Pi_{t,t+j}^p \widetilde{\rho}_{t+j}\right)^{-\theta_p+1} y_{t+j}} \right]. \quad (\text{A10})$$

Comparing (A9) and (A10) and noticing that $mc_{t+j}(\omega) = \widetilde{mc}_{t+j} \frac{\widetilde{z}}{z(\omega)}$ yields

$$P_t^*(\omega) = \frac{\widetilde{z}}{z(\omega)} \widetilde{P}_t^* \quad (\text{A11})$$

In loglinear terms, the optimal price equation (A10) for the firm with average productivity brings the following relative price

$$\widetilde{P}_t^* - \widehat{P}_t = (1 - \beta\gamma s \xi_p) E_t \sum_{j=0}^{\infty} (\beta\gamma s \xi_p)^j \left(\widehat{mc}_{t+j} - \widehat{\rho}_{t+j} + \sum_{k=1}^j (\pi_{t+k} - \iota_p \pi_{t-1+k} - (1 - \iota_p) \varepsilon_{t+k}^p) \right). \quad (\text{A12})$$

Next, let us recall the Dixit-Stiglitz price aggregator with Calvo-style stickiness and the indexation rule

$$\widetilde{P}_t = \left[(1 - \xi_p) \left(\widetilde{P}_t^*\right)^{1-\theta_p} + \xi_p \left((1 + \pi_{t-1})^{\iota_p} (1 + \pi + \varepsilon_t^P)^{1-\iota_p} \widetilde{P}_{t-1} \right)^{1-\theta_p} \right]^{1/(1-\theta_p)},$$

which can be log-linearized, using $(\pi_t - \pi) = \widehat{P}_t - \widehat{P}_{t-1}$ for the rate of PPI, to obtain

$$\widehat{P}_t^* - \widehat{P}_t = \frac{\xi_p}{1-\xi_p} \left((\pi_t - \pi) - \iota_p (\pi_{t-1} - \pi) - (1 - \iota_p) \varepsilon_t^p \right). \quad (\text{A13})$$

Combining (A12) and (A13) results in the inflation equation

$$\begin{aligned} & (\pi_t - \pi) - \iota_p (\pi_{t-1} - \pi) - (1 - \iota_p) \varepsilon_t^p = \\ & \frac{(1 - \beta s \xi_p)(1 - \xi_p)}{\xi_p} \sum_{j=0}^{\infty} (\beta\gamma s \xi_p)^j \left(\widehat{mc}_{t+j} - \widehat{\rho}_{t+j} + \sum_{k=1}^j \left((\pi_{t+k} - \pi) - \iota_p (\pi_{t-1+k} - \pi) - (1 - \iota_p) \varepsilon_{t+k}^p \right) \right), \end{aligned}$$

where, by doing $(\pi_t - \pi) - \beta\gamma s \xi_p E_t (\pi_{t+1} - \pi)$, simplifies to the hybrid New Keynesian Phillips curve

$$\begin{aligned} (\pi_t - \pi) = & \frac{\iota_p}{(1 + \beta\gamma s \iota_p)} (\pi_{t-1} - \pi) + \frac{\beta\gamma s}{(1 + \beta\gamma s \iota_p)} E_t (\pi_{t+1} - \pi) \\ & + \frac{(1 - \beta\gamma s \xi_p)(1 - \xi_p)}{\xi_p (1 + \beta\gamma s \iota_p)} \left(\widehat{mc}_t - \widehat{\rho}_t \right) + \frac{(1 - \iota_p)}{(1 + \beta\gamma s \iota_p)} \left(\varepsilon_t^p - \beta E_t \varepsilon_{t+1}^p \right). \end{aligned}$$

Composite-good firms

Composite-good firms act as packers of single goods and sell the final bundles of consumption goods in a competitive flexible-price market. The representative composite-good firm produces bundles of consumption using the production technology that combines each of the n_t single varieties produced at the establishments as follows,

$$y_t = \left[\int_0^{n_t} y_t(\omega)^{\frac{\theta_p-1}{\theta_p}} d\omega \right]^{\frac{\theta_p}{\theta_p-1}}, \quad (\text{A14})$$

where the elasticity of substitution of across single goods in the aggregate production function (θ_p) is the same as the elasticity of substitution between individual goods in household consumption. The amount of consumption bundles produced is not indexed for any specific firm because symmetric equilibrium holds across all the identical composite-good firms. Thus, the corresponding price of one consumption bundle can also be expressed in economy-wide terms as obtained from the Dixit-Stiglitz aggregator,

$$P_t^c = \left[\int_0^{n_t} P_t(\omega)^{1-\theta_p} d\omega \right]^{\frac{1}{1-\theta_p}}. \quad (\text{A15})$$

B. Short-run and long-run equilibria in the DSGE model with endogenous entry and exit

Set of log-linearized (64) dynamic equations for fluctuations around the detrended steady state in the short-run equilibrium:

Law of motion for total number of establishments:

$$\widehat{n}_t = \widehat{n}_t^s + \delta_n (\widehat{n}_t^e - \widehat{n}_{t-1}), \quad (\text{B1})$$

where $\delta_n = n^x/n$ is the steady-state exit rate. Decomposition between surviving and exiting establishments:

$$\widehat{n}_{t-1} = (1 - \delta_n) \widehat{n}_t^s + \delta_n \widehat{n}_t^x. \quad (\text{B2})$$

Entry decision:

$$\widehat{n}_{t+1}^e = \widehat{n}_t + \varsigma^{-1} \left(\frac{v}{ec} \widehat{v}_t - \frac{f^e}{ec} \varepsilon_t^e \right) \quad (\text{B3})$$

Liquidation value:

$$\widehat{l}v_t = \varepsilon_t^x. \quad (\text{B4})$$

Exit decision:

$$\widehat{n}_t^x = \widehat{n}_{t-1} + \kappa \left(\frac{1 - \delta_n}{\delta_n} \right) \widehat{z}_t^{cr}. \quad (\text{B5})$$

Productivity cutoff point:

$$\begin{aligned} \widehat{z}_t^{cr} = & \left(\beta\gamma s + \frac{\kappa(1-\beta\gamma s)(\tilde{\rho}-\Omega)}{\Omega} \right) E_t \widehat{z}_{t+1}^{cr} + \\ & (1 - \beta\gamma s) E_t \left(\widehat{m}c_{t+1} - \frac{(\tilde{\rho}-\Omega)}{\Omega} (\widehat{y}_{t+1} - (R_t - E_t \pi_{t+1}^c + \varepsilon_t^b)) - \frac{\tilde{\rho} - (\tilde{\rho}-\Omega)\theta_p}{\Omega} \widehat{\rho}_{t+1} \right) + \frac{(\tilde{\rho}-\Omega)}{\beta s \Omega} \left(\widehat{l}v_t - \beta\gamma s E_t \widehat{l}v_{t+1} \right), \end{aligned} \quad (\text{B6})$$

with $\Omega = \widetilde{m}c \frac{\tilde{z}}{z^{cr}}$ evaluated in steady state. Relative prices as a function of number of goods:

$$\widehat{\rho}_t = \frac{1}{\theta_p - 1} \widehat{n}_t. \quad (\text{B7})$$

Variety effect from producer price inflation to consumer price inflation:

$$\pi_t^c = \pi_t - \widehat{\rho}_t + \widehat{\rho}_{t-1}. \quad (\text{B8})$$

Output decomposition between intensive and extensive margin of fluctuations:

$$\widehat{y}_t = \widehat{n}_t + \widehat{\rho}_t + \widehat{y}_t. \quad (\text{B9})$$

Equity accumulation equation (portfolio investment):

$$\widehat{v}_t = \beta\gamma v_1 E_t \widehat{v}_{t+1} + \beta\gamma v_2 E_t \widehat{d}_{t+1} + \beta\gamma (v_1 + v_2) E_t \widehat{n}_{t+1}^s + \beta\gamma v_3 E_t \left(\widehat{n}_{t+1}^x + \widehat{l}v_{t+1} \right) - (R_t - E_t \pi_{t+1}^c + \varepsilon_t^b) \widehat{n}_t, \quad (\text{B10})$$

where $v_1 = \frac{v}{(1-\delta_n)(d+v)+\delta_nlv}$, $v_2 = \frac{d}{(1-\delta_n)(d+v)+\delta_nlv}$ and $v_3 = \frac{\delta_nlv/(1-\delta_n)}{(1-\delta_n)(d+v)+\delta_nlv}$.

Firm-level average dividend:

$$\widehat{d}_t = \widehat{y}_t + \theta_p \widehat{\rho}_t - (\theta_p - 1) \widehat{mc}_t. \quad (\text{B11})$$

New-Keynesian Phillips curve from Calvo (1983)-type sticky pricing with indexation:

$$\begin{aligned} \pi_t - \pi = & \frac{\iota_p}{(1+\beta\gamma s\iota_p)} (\pi_{t-1} - \pi) + \frac{\beta\gamma s}{(1+\beta\gamma s\iota_p)} E_t (\pi_{t+1} - \pi) \\ & + \frac{(1-\beta\gamma s\xi_p)(1-\xi_p)}{\xi_p(1+\beta\gamma s\iota_p)} \left(\widehat{mc}_t - \widehat{\rho}_t \right) + \frac{(1-\iota_p)}{(1+\beta\gamma s\iota_p)} (\varepsilon_t^p - \beta\gamma s E_t \varepsilon_{t+1}^p). \end{aligned} \quad (\text{B12})$$

Real marginal cost:

$$\widehat{mc}_t = (1 - \alpha) \widehat{w}_t + \alpha \widehat{r}_t^k - \varepsilon_t^a. \quad (\text{B13})$$

Consumption equation featuring habits and non-separability between consumption and labor in the utility function:

$$\widehat{c}_t = \frac{h/(1+\gamma)}{1+h/(1+\gamma)} \widehat{c}_{t-1} + \frac{1}{1+h/(1+\gamma)} E_t \widehat{c}_{t+1} + \frac{(\sigma_c - 1)w\theta_w / ((\theta_w - 1)c)}{\sigma_c(1+h/(1+\gamma))} \left(\widehat{l}_t - E_t \widehat{l}_{t+1} \right) - \frac{1-h/(1+\gamma)}{\sigma_c(1+h/(1+\gamma))} (r_t - r + \varepsilon_t^b). \quad (\text{B14})$$

Taylor-type monetary policy rule:

$$R_t - R = \mu_R (R_{t-1} - R) + (1 - \mu_R) \left[\mu_\pi (\pi_t - \pi) + \mu_y (\widehat{y}_t - \widehat{y}_t^p) \right] + \mu_{dy} \left[(\widehat{y}_t - \widehat{y}_t^p) - (\widehat{y}_{t-1} - \widehat{y}_{t-1}^p) \right] + \varepsilon_t^R. \quad (\text{B15})$$

Goods market equilibrium:

$$\widehat{y}_t = \frac{c}{y} \widehat{c}_t + \frac{i}{y} \widehat{i}_t + \frac{r^k k}{y} \widehat{u}_t + \frac{\varepsilon^g}{y} \varepsilon_t^g + \frac{(\delta_n / (1 - \delta_n)) e c}{y} (\widehat{n}_{t+1}^e + \widehat{e}c_t). \quad (\text{B16})$$

Production technology for the average-productivity establishment:

$$\widehat{y}_t = \alpha \widehat{k}_t + (1 - \alpha) \widehat{l}_t + \varepsilon_t^a. \quad (\text{B17})$$

Fisher equation:

$$r_t = R_t - E_t \pi_{t+1}^c. \quad (\text{B18})$$

Wage inflation equation with Calvo (1983) sticky wages and indexation:

$$\begin{aligned} \pi_t^w - \pi^w = & \iota_w (\pi_{t-1}^c - \pi^c) + \beta E_t (\pi_{t+1}^w - \pi^w) - \beta \iota_w (\pi_t^c - \pi^c) \\ & + \frac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w} (\widehat{mrs}_t - \widehat{w}_t) + (1 - \iota_w) (\varepsilon_t^W - \beta E_t \varepsilon_{t+1}^W). \end{aligned} \quad (\text{B19})$$

where the log-linearized household marginal rate of substitution is,

$$\widehat{mrs}_t = \sigma_l \widehat{l}_t + \left(\frac{1}{1-h/(1+\gamma)} \widehat{c}_t - \frac{h/(1+\gamma)}{1-h/(1+\gamma)} \widehat{c}_{t-1} \right), \quad (\text{B20})$$

and the real wage dynamics are determined by the log-linear expression implied by its definition ($w_t = W_t/P_t^c$),

$$\widehat{w}_t = \widehat{w}_{t-1} + \pi_t^w - \pi_t^c. \quad (\text{B21})$$

Labor market equilibrium condition:

$$\widehat{l}_t = \widehat{n}_t + \widehat{\bar{l}}_t. \quad (\text{B22})$$

Capital market equilibrium condition:

$$\widehat{k}_t^s = \widehat{n}_t + \widehat{\bar{k}}_t. \quad (\text{B23})$$

As in Smets and Wouters (2007), the log-linearized investment equation is,

$$\widehat{i}_t = i_1 \widehat{i}_{t-1} + (1 - i_1) E_t \widehat{i}_{t+1} + i_2 \widehat{q}_t + \varepsilon_t^i, \quad (\text{B24})$$

where $i_1 = \frac{1}{1 + \beta(1 + \gamma)^{-\sigma_c}}$, and $i_2 = \frac{i_1}{(1 + \gamma)^2 \varphi_k}$, and the value of capital goods (Tobin's q) is given, in log-linear terms by the arbitrage condition,

$$\widehat{q}_t = q_1 E_t \widehat{q}_{t+1} + (1 - q_1) E_t \widehat{r}_{t+1}^k - (r_t - r + \varepsilon_t^b), \quad (\text{B25})$$

where $q_1 = \frac{(1 - \delta_k)}{(r^k + 1 - \delta_k)}$. The equilibrium rental rate of capital can be found in the input demand equations of the representative firm

$$\widehat{r}_t^k = \widehat{w}_t - \left(\widehat{\bar{k}}_t - \widehat{\bar{l}}_t \right). \quad (\text{B26})$$

Also, following Smets and Wouters (2007), the loglinear expression for capital accumulation is,

$$\widehat{k}_t = k_1 \widehat{k}_{t-1} + (1 - k_1) \widehat{i}_t + k_2 \varepsilon_t^i, \quad (\text{B27})$$

where $k_1 = \frac{1 - \delta_k}{1 + \gamma}$ and $k_2 = (1 - k_1) / i_2$.

The supply of capital can be adjusted in the intensive margin (utilization rate) as well as the extensive margin,

$$\widehat{k}_t^s = \widehat{u}_t + \widehat{k}_{t-1}, \quad (\text{B28})$$

and the log-linearized variable capital utilization rate is,

$$\widehat{u}_t = \left(\frac{1 - \sigma_a}{\sigma_a} \right) \widehat{r}_t^k. \quad (\text{B29})$$

Entry congestion cost

$$\widehat{e}c_t = \varsigma (\widehat{n}_{t+1}^e - \widehat{n}_t). \quad (\text{B30})$$

Entry rate dynamics (semi-loglinear approximation)

$$e_t - e = \frac{e}{\varsigma} \left(\frac{v}{ec} \widehat{v}_{t-1} - \frac{f^e}{ec} \varepsilon_{t-1}^e \right) + (\widehat{n}_t^s - \widehat{n}_{t-1}). \quad (\text{B31})$$

Exit rate dynamics (semi-loglinear approximation)

$$x_t - x = \kappa (1 - x) \widehat{z}_t^{cr} \quad (\text{B32})$$

The potential (natural-rate) block is obtained repeating all the equations (B1)-(B32) with p superscript to denote the values reached under no rigidity on both price and wage adjustments, with the exceptions of the New Keynesian Phillips curve (B12) that is replaced by the constant price mark-up condition,

$$\widehat{\rho}_t^p = \widehat{mc}_t^p, \quad (\text{B12}^p)$$

and the wage inflation curve (B19) that is replaced by the constant wage mark-up condition,

$$\widehat{mrs}_t^p = \widehat{w}_t^p. \quad (\text{B19}^p)$$

Endogenous variables (64):

The following 64 variables: \widehat{n}_{t+1} , \widehat{n}_{t+1}^e , \widehat{n}_t^x , \widehat{n}_t^s , \widehat{ec}_t , \widehat{z}_t^{cr} , \widehat{lv}_t , \widehat{v}_t , \widehat{d}_t , $\widehat{\rho}_t$, \widehat{y}_t , \widehat{c}_t , \widehat{i}_t , \widehat{u}_t , \widehat{q}_t , \widehat{k}_t , \widehat{k}_t^s , \widehat{l}_t , $\widehat{\widehat{y}}_t$, $\widehat{\widehat{l}}_t$, $\widehat{\widehat{k}}_t$, $\widehat{\widehat{mc}}_t$, $r_t - r$, $R_t - R$, $\pi_t - \pi$, $\pi_t^c - \pi^c$, $\pi_t^w - \pi^w$, \widehat{r}_t^k , \widehat{w}_t , \widehat{mrs}_t , e_t , x_t and the same set with p superscript to bring the variables corresponding to the potential block.

Exogenous variables (9):

- technology shock: $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ with $\eta_t^a \sim N(0, \sigma_{\eta^a}^2)$
- risk-premium shock: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ with $\eta_t^b \sim N(0, \sigma_{\eta^b}^2)$
- monetary policy shock: $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$ with $\eta_t^R \sim N(0, \sigma_{\eta^R}^2)$
- fiscal policy shock: $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \rho_{ga} \eta_t^a + \eta_t^g$ with $\eta_t^g \sim N(0, \sigma_{\eta^g}^2)$
- investment shock: $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ with $\eta_t^i \sim N(0, \sigma_{\eta^i}^2)$
- price-push shock: $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p - \mu_p \eta_{t-1}^p + \eta_t^p$ with $\eta_t^p \sim N(0, \sigma_{\eta^p}^2)$
- wage-push shock: $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w - \mu_w \eta_{t-1}^w + \eta_t^w$ with $\eta_t^w \sim N(0, \sigma_{\eta^w}^2)$
- entry cost shock: $\varepsilon_t^e = \rho_e \varepsilon_{t-1}^e + \eta_t^e$ with $\eta_t^e \sim N(0, \sigma_{\eta^e}^2)$
- liquidation value shock: $\varepsilon_t^x = \rho_x \varepsilon_{t-1}^x + \eta_t^x$ with $\eta_t^x \sim N(0, \sigma_{\eta^x}^2)$

Set of non-linear equations that define the detrended steady state (long-run equilibrium)

There are 21 endogenous variables: n , n^e , n^x , n^s , r , r^k , v , d , \widetilde{mc} , $\widetilde{\rho}$, \widetilde{y} , \widetilde{k} , \widetilde{l} , y , c , i , w , ec , lv , z^{cr} , and \widetilde{z} . The non-linear steady-state system to solve is;

$$\widetilde{z} = z_{\min} \left(\frac{\kappa}{\kappa - (\theta_p - 1)} \right)^{\frac{1}{\theta_p - 1}}, \quad (\text{SSB1})$$

$$\frac{n^e}{n} = \frac{1 - \left(\frac{z_{\min}}{z^{cr}} \right)^\kappa}{\left(\frac{z_{\min}}{z^{cr}} \right)^\kappa}, \quad (\text{SSB2})$$

$$\frac{n^s}{n} = \left(\frac{z_{\min}}{z^{cr}} \right)^\kappa, \quad (\text{SSB3})$$

$$\frac{n^x}{n} = 1 - \left(\frac{z_{\min}}{z^{cr}}\right)^\kappa, \quad (\text{SSB4})$$

$$r = \beta^{-1} (1 + \gamma)^{\sigma_c} - 1, \quad (\text{SSB5})$$

$$r^k = \beta^{-1} (1 + \gamma)^{\sigma_c} + \delta_k - 1, \quad (\text{SSB6})$$

$$v = \frac{\beta(1+\gamma)^{1-\sigma_c} \left(\frac{n^s}{n} d + \frac{n^x}{n} lv\right)}{1 - \beta(1+\gamma)^{1-\sigma_c} \frac{n^s}{n}}, \quad (\text{SSB7})$$

$$lv = \frac{\beta(1+\gamma)^{1-\sigma_c} \left(\frac{n^s}{n}\right)}{1 - \beta(1+\gamma)^{1-\sigma_c} \left(\frac{n^s}{n}\right)} \left(\frac{1 - \frac{\tilde{z}}{z^{cr}} \tilde{m}c}{1 - \tilde{m}c}\right) \frac{d}{n}, \quad (\text{SSB8})$$

$$f^e + ec = v \quad (\text{SSB9})$$

$$lv = (1 - \tau) f_e, \quad (\text{SSB10})$$

$$y = c + i + \varepsilon^g + (ec) n^e, \quad (\text{SSB11})$$

$$ec = \Theta \left(\frac{n^e}{n}\right)^\varsigma, \quad (\text{SSB12})$$

$$y = n \tilde{\rho} \tilde{y}, \quad (\text{SSB13})$$

$$\tilde{\rho} = n^{\frac{\theta_p}{\theta_p - 1}}, \quad (\text{SSB14})$$

$$d = (\tilde{\rho})^{-\theta_p} y (\tilde{\rho} - \tilde{m}c), \quad (\text{SSB15})$$

$$\tilde{y} = \tilde{z} \left(\tilde{k}\right)^\alpha \left(\tilde{l}\right)^{1-\alpha}, \quad (\text{SSB16})$$

$$\tilde{m}c = \frac{\theta_p - 1}{\theta_p} \tilde{\rho}, \quad (\text{SSB17})$$

$$\frac{1}{\tilde{z}} \left(\frac{w}{1 - \alpha}\right)^{1-\alpha} \left(\frac{r^k}{\alpha}\right)^\alpha = \tilde{m}c, \quad (\text{SSB18})$$

$$w = \frac{\theta_w}{(\theta_w - 1)} (c - h(1 + \gamma)^{-1} c) \left(n \tilde{l}\right)^{\sigma_l}, \quad (\text{SSB19})$$

$$\tilde{k} = \left(\frac{\alpha \tilde{m}c \tilde{z}}{r^k}\right)^{\frac{1}{1-\alpha}}, \quad (\text{SSB20})$$

$$i = (\gamma + \delta_k) n \tilde{k}. \quad (\text{SSB21})$$

C. Average productivity.

The probability density function, $g(z)$, and the cumulative distribution function, $G(z)$, of the Pareto distribution that delivers firm-specific productivities are, respectively,

$$g(z) = \begin{cases} \frac{\kappa(z_{\min})^\kappa}{z^{\kappa+1}}, & \text{if } z \geq z_{\min} \\ 0, & \text{if } z < z_{\min} \end{cases}$$

$$G(z) = \begin{cases} \int_{z_{\min}}^z g(z) dz = 1 - \left(\frac{z_{\min}}{z}\right)^\kappa, & \text{if } z \geq z_{\min} \\ 0, & \text{if } z < z_{\min} \end{cases}.$$

The average productivity across all firms (with CES aggregation *à la* Dixit-Stiglitz) is

$$\tilde{z}_t = \left[(n_t^x/n_{t-1}) (\tilde{z}_t^x)^{\theta_p-1} + (n_t^s/n_{t-1}) (\tilde{z}_t^s)^{\theta_p-1} \right]^{1/(\theta_p-1)}, \quad (\text{C1})$$

where \tilde{z}_t^x is the average productivity across exiting firms, and \tilde{z}_t^s is the average productivity across surviving firms.

Following Hamano and Zanetti (2017), \tilde{z}_t^s is obtained from the Dixit-Stiglitz aggregation scheme bounded in the open interval between critical productivity z_t^{cr} and $+\infty$

$$\tilde{z}_t^s = \left[\frac{1}{1 - G(z_t^{cr})} \int_{z_t^{cr}}^{\infty} z^{\theta_p-1} g(z) dz \right]^{\frac{1}{\theta_p-1}},$$

where using $g(z)$ defined above yields

$$\tilde{z}_t^s = \left[\frac{1}{1 - G(z_t^{cr})} \int_{z_t^{cr}}^{\infty} \kappa (z_{\min})^\kappa z^{\theta_p-1-\kappa-1} dz \right]^{\frac{1}{\theta_p-1}}.$$

The critical productivity z_t^{cr} determines the split-up of firms between survival and exit in the cumulative distribution function, which identifies the survival rate as $1 - G(z_t^{cr}) = \left(\frac{z_{\min}}{z_t^{cr}}\right)^\kappa$. This can be introduced in the expression of \tilde{z}_t^s to obtain

$$\tilde{z}_t^s = \left[\left(\frac{z_{\min}}{z_t^{cr}}\right)^{-\kappa} \int_{z_t^{cr}}^{\infty} \kappa (z_{\min})^\kappa z^{\theta_p-1-\kappa-1} dz \right]^{\frac{1}{\theta_p-1}}.$$

Taking the fixed elements outside the integral, we have

$$\tilde{z}_t^s = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(\frac{z_{\min}}{z_t^{cr}}\right)^{-\kappa} \int_{z_t^{cr}}^{\infty} z^{\theta_p-1-\kappa-1} dz \right]^{\frac{1}{\theta_p-1}}.$$

The rule for the integral of an exponential function implies

$$\tilde{z}_t^s = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(\frac{z_{\min}}{z_t^{cr}}\right)^{-\kappa} \frac{z^{\theta_p-1-\kappa}}{\theta_p-1-\kappa} \Big|_{z_t^{cr}}^{\infty} \right]^{\frac{1}{\theta_p-1}},$$

which leads to

$$\tilde{z}_t^s = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(1 - \left(\frac{z_{\min}}{z_t^{cr}} \right)^\kappa \right)^{-1} \left(\frac{0 - (z_t^{cr})^{\theta_p-1-\kappa}}{\theta_p-1-\kappa} \right) \right]^{\frac{1}{\theta_p-1}},$$

or, alternatively

$$\tilde{z}_t^s = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(\frac{z_{\min}}{z_t^{cr}} \right)^{-\kappa} \left(\frac{(z_t^{cr})^{\theta_p-1-\kappa}}{\kappa - (\theta_p-1)} \right) \right]^{\frac{1}{\theta_p-1}},$$

which simplifies initially to

$$\tilde{z}_t^s = \kappa^{1/(\theta_p-1)} \left[\left(\frac{(z_t^{cr})^{\theta_p-1}}{\kappa - (\theta_p-1)} \right) \right]^{\frac{1}{\theta_p-1}},$$

and finally to

$$\tilde{z}_t^s = z_t^{cr} \left(\frac{\kappa}{\kappa - (\theta_p-1)} \right)^{\frac{1}{\theta_p-1}}. \quad (\text{C2})$$

Analogously to the case of surviving firms, the average productivity for the set of firms that decide to exit in period t is bounded between minimum productivity, z_{\min} , and the time-varying cut-off productivity, z_t^{cr} , as follows

$$\tilde{z}_t^x = \left[\frac{1}{G(z_t^{cr})} \int_{z_{\min}}^{z_t^{cr}} z^{\theta_p-1} g(z) dz \right]^{\frac{1}{\theta_p-1}}.$$

Using the pdf specification $g(z)$ gives

$$\tilde{z}_t^x = \left[\frac{1}{G(z_t^{cr})} \int_{z_{\min}}^{z_t^{cr}} \kappa (z_{\min})^\kappa z^{\theta_p-1-\kappa-1} dz \right]^{\frac{1}{\theta_p-1}}.$$

The critical productivity z_t^{cr} determines the split-up of firms between survival and exit in the cumulative distribution function, which identifies the exit rate as $G(z_t^{cr}) = 1 - \left(\frac{z_{\min}}{z_t^{cr}} \right)^{-\kappa}$. This can be substituted in the expression of \tilde{z}_t^x to obtain

$$\tilde{z}_t^x = \left[\left(1 - \left(\frac{z_{\min}}{z_t^{cr}} \right)^{-\kappa} \right)^{-1} \int_{z_{\min}}^{z_t^{cr}} \kappa (z_{\min})^\kappa z^{\theta_p-1-\kappa-1} dz \right]^{\frac{1}{\theta_p-1}}.$$

Taking the fixed elements outside the integral, we have

$$\tilde{z}_t^x = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(1 - \left(\frac{z_{\min}}{z_t^{cr}} \right)^{-\kappa} \right)^{-1} \int_{z_{\min}}^{z_t^{cr}} z^{\theta_p-1-\kappa-1} dz \right]^{\frac{1}{\theta_p-1}}.$$

The rule for the integral of an exponential function implies

$$\tilde{z}_t^x = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(1 - \left(\frac{z_{\min}}{z_t^{cr}} \right)^{-\kappa} \right)^{-1} \frac{z^{\theta_p-1-\kappa}}{\theta_p-1-\kappa} \Big|_{z_{\min}}^{z_t^{cr}} \right]^{\frac{1}{\theta_p-1}}.$$

which leads to

$$\tilde{z}_t^x = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(1 - \left(\frac{z_{\min}}{z_t^{cr}} \right)^{-\kappa} \right)^{-1} \left(\frac{(z_t^{cr})^{\theta_p-1-\kappa} - (z_{\min})^{\theta_p-1-\kappa}}{\theta_p-1-\kappa} \right) \right]^{\frac{1}{\theta_p-1}}.$$

or, alternatively

$$\tilde{z}_t^x = (\kappa z_{\min}^\kappa)^{1/(\theta_p-1)} \left[\left(1 - \left(\frac{z_{\min}}{z_t^{cr}} \right)^{-\kappa} \right)^{-1} \left(\frac{(z_{\min})^{\theta_p-1-\kappa} - (z_t^{cr})^{\theta_p-1-\kappa}}{\kappa - (\theta_p-1)} \right) \right]^{\frac{1}{\theta_p-1}}. \quad (\text{C3})$$

Using both (C2) and (C3), respectively for \tilde{z}_t^s and \tilde{z}_t^x , in the average productivity expression (C1), it is reached

$$\tilde{z}_t = \left[\begin{aligned} & (n_t^x/n_{t-1}) \left(\kappa z_{\min}^\kappa \left[\left(1 - (z_{\min}/z_t^{cr})^\kappa \right)^{-1} \left(\frac{(z_{\min})^{\theta_p-1-\kappa} - (z_t^{cr})^{\theta_p-1-\kappa}}{\kappa - (\theta_p-1)} \right) \right] \right) \\ & + (n_t^s/n_{t-1}) (z_t^{cr})^{\theta_p-1} \left(\frac{\kappa}{\kappa - (\theta_p-1)} \right) \end{aligned} \right]^{1/(\theta_p-1)}.$$

Survival and exit rates as functions of critical productivity, $n_t^s/n_{t-1} = (z_{\min}/z_t^{cr})^\kappa$ and $n_t^x/n_{t-1} = 1 - (z_{\min}/z_t^{cr})^\kappa$, can be introduced to obtain

$$\tilde{z}_t = \left[\begin{aligned} & \left(1 - (z_{\min}/z_t^{cr})^\kappa \right) \left(\kappa z_{\min}^\kappa \left[\left(1 - (z_{\min}/z_t^{cr})^\kappa \right)^{-1} \left(\frac{(z_{\min})^{\theta_p-1-\kappa} - (z_t^{cr})^{\theta_p-1-\kappa}}{\kappa - (\theta_p-1)} \right) \right] \right) \\ & + (z_{\min}/z_t^{cr})^\kappa (z_t^{cr})^{\theta_p-1} \left(\frac{\kappa}{\kappa - (\theta_p-1)} \right) \end{aligned} \right]^{1/(\theta_p-1)},$$

which simplifies massively to

$$\tilde{z}_t = z_{\min}^\kappa \left(\frac{\kappa}{\kappa - (\theta_p-1)} \right)^{1/(\theta_p-1)}.$$

D. Data and measurement equations

The following table summarizes the data definitions and the measurement equations used for the model estimation:

Series definition	U.S. Data	Model measurement equation
Quarterly change in per-capita Real GDP, Y_t/L_t	$100\log\left(\frac{Y_t/L_t}{Y_{t-1}/L_{t-1}}\right)$	$\gamma + \hat{y}_t - \hat{y}_{t-1}$
Quarterly change in per-capita Real Personal Consumption Expenditures, C_t/L_t	$100\log\left(\frac{C_t/L_t}{C_{t-1}/L_{t-1}}\right)$	$\gamma + \hat{c}_t - \hat{c}_{t-1}$
Quarterly change in per-capita Real Fixed Private Investment, I_t/L_t	$100\log\left(\frac{I_t/L_t}{I_{t-1}/L_{t-1}}\right)$	$\gamma + \hat{i}_t - \hat{i}_{t-1}$
Quarterly change in real compensation per Hour in nonfarm business sector, W_t/\tilde{P}_t	$100\log\left(\frac{W_t/\tilde{P}_t}{W_{t-1}/\tilde{P}_{t-1}}\right)$	$\gamma + \hat{w}_t - \hat{w}_{t-1}$
Hours per worker (in natural logarithm) in nonfarm business sector, $h_t EMP_t/LF_t$	$\log(h_t EMP_t/LF_t)$	$l + \hat{l}_t$
Quarterly change in GDP Price Deflator, \tilde{P}_t	$100\log\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}}\right)$	$\pi + (\pi_t - \pi)$
Quarterly shadow Federal Funds Rate, R_t^{QE}	$R_t^{QE}/4$	$r + \pi + (R_t - R)$
Establishment entry rate (effective), e_t	$100\left(\frac{N_t^e}{N_t} - 1\right)$	$e + e(\hat{n}_t^e - \hat{n}_{t-1} + \hat{n}_t^s - \hat{n}_{t-1})$
Establishment exit rate, x_t	$100\left(\frac{N_t^x}{N_t} - 1\right)$	$x + x(\hat{n}_t^x - \hat{n}_{t-1})$

Actual data series used as observables in the estimation are plotted within the next Figure:

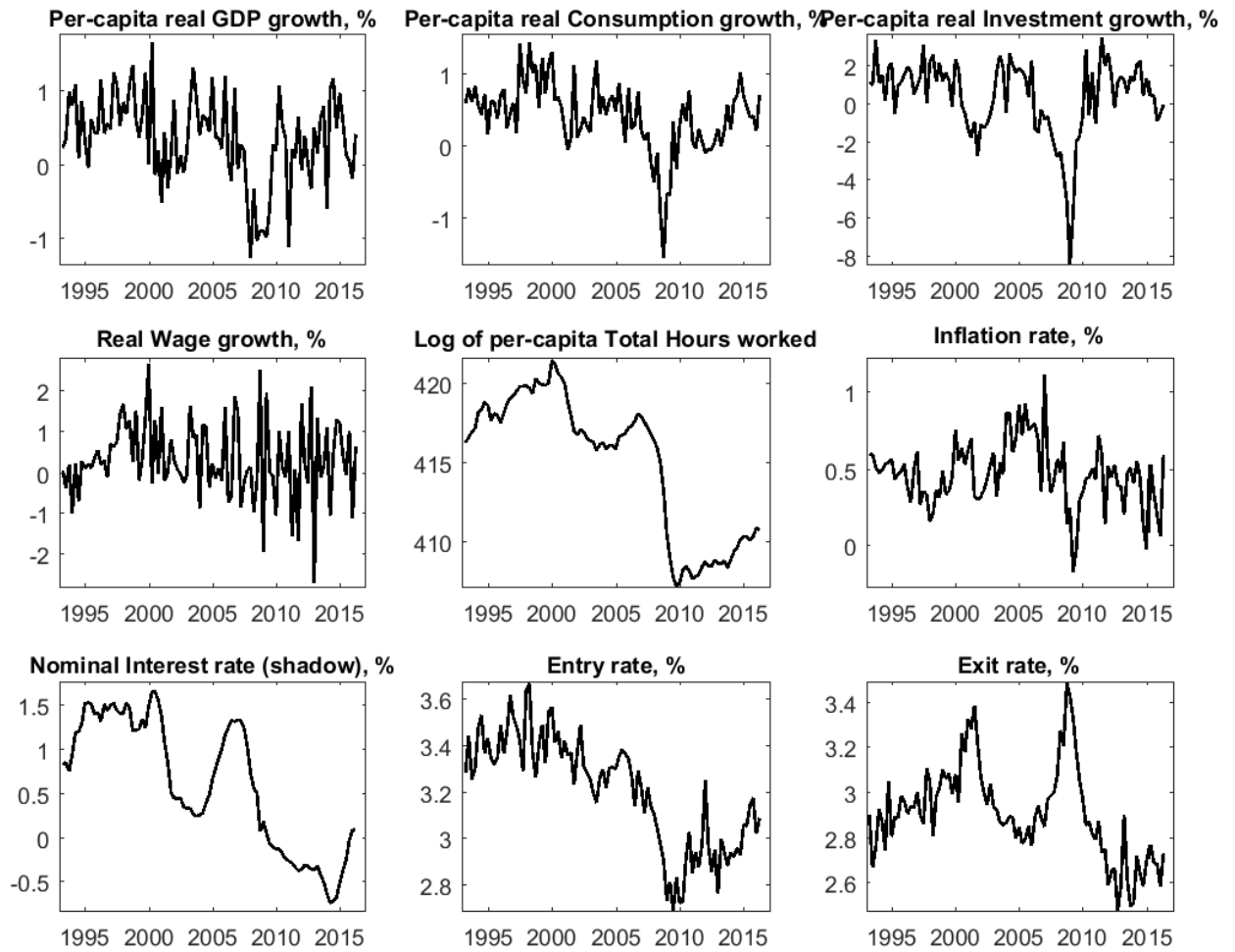


Figure 1: Observable series from the US economy (1993:2-2016:2).

E. The loglinearized equation for short-run fluctuations of critical productivity, z^{cr}

The exit condition at the margin is defined in the text as follows,

$$E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) d_{t+j}^{cr}(\omega) = l v_t, \quad (\text{E1})$$

which can be rewritten in log-linear terms to read, Using the Dixit-Stiglitz demand constraints, and the first order conditions of labor demand and capital demand, the left-hand side of (E1) is

$$E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \left(\left(\frac{P_{t+j}(\omega)}{P_{t+j}^c} \right)^{-\theta_p} y_{t+j} \left[\frac{P_{t+j}(\omega)}{P_{t+j}^c} - m c_{t+j}^{cr}(\omega) \right] \right),$$

where $m c_{t+j}^{cr}(\omega)$ is computed at the critical productivity z_t^{cr} fixed in period t for all future periods because firm-level productivity is time invariant. Relative prices, $\tilde{\rho}_{t+j} = \tilde{P}_{t+j}/P_{t+j}^c$ can be introduced to obtain

$$E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \left(\left(\frac{P_{t+j}(\omega) \tilde{\rho}_{t+j}}{\tilde{P}_{t+j}} \right)^{-\theta_p} y_{t+j} \left[\frac{P_{t+j}(\omega) \tilde{\rho}_{t+j}}{\tilde{P}_{t+j}} - m c_{t+j}^{cr}(\omega) \right] \right). \quad (\text{E2})$$

The relationship between the firm-specific price and the price at the average productivity is $P_{t+j}(\omega) = \frac{\tilde{z}}{z(\omega)} \tilde{P}_{t+j}$ for any $t+j$ period.⁴ Using this result in (E2) yields

$$E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \left(\left(\frac{\tilde{z}}{z(\omega)} \tilde{\rho}_{t+j} \right)^{-\theta_p} y_{t+j} \left[\frac{\tilde{z}}{z(\omega)} \tilde{\rho}_{t+j} - m c_{t+j}^{cr}(\omega) \right] \right). \quad (\text{E3})$$

Meanwhile, the average real marginal cost of any $t+j$ period is defined at the steady-state average productivity, \tilde{z} , which implies, $m c_{t+j}^{cr}(\omega) = \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}(\omega)}$, and once inserted in (E3) gives,

$$E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j}(\omega) \left(\left(\frac{\tilde{z}}{z(\omega)} \tilde{\rho}_{t+j} \right)^{-\theta_p} y_{t+j} \left[\frac{\tilde{z}}{z(\omega)} \tilde{\rho}_{t+j} - \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}(\omega)} \right] \right). \quad (\text{E4})$$

The loglinear approximation to (E1) is

$$(1 - \beta\gamma s) E_t \sum_{j=1}^{\infty} (\beta\gamma s)^j \left(\hat{\beta}_{t+j} + \hat{s}_{t+j}(\omega) + \hat{d}_{t+j}^{cr} \right) = \hat{l} v_t \quad (\text{E5})$$

Applying log-linearizing techniques to (E4) results in the following linear expression for the expected stream of dividends

$$E_t \sum_{j=1}^{\infty} (\beta s)^j \hat{d}_{t+j}^{cr} = \frac{\beta\gamma s}{1-\beta\gamma s} \frac{\tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}}{\tilde{\rho} - \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}} \hat{z}_t^{cr}(\omega) + E_t \sum_{j=1}^{\infty} (\beta\gamma s)^j \left(\hat{y}_{t+j} + \left(\frac{\tilde{\rho}}{\tilde{\rho} - \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}} - \theta_p \right) \hat{\rho}_{t+j} - \left(\frac{\tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}}{\tilde{\rho} - \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}} \right) \hat{\tilde{m} c}_{t+j} \right). \quad (\text{E6})$$

Inserting (E6) in (E5) yields

$$\frac{\beta\gamma s \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}}{\tilde{\rho} - \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}} \hat{z}_t^{cr}(\omega) + (1 - \beta\gamma s) E_t \sum_{j=1}^{\infty} (\beta\gamma s)^j \left(\hat{\beta}_{t+j} + \hat{s}_{t+j}(\omega) + \hat{y}_{t+j} + \left(\frac{\tilde{\rho}}{\tilde{\rho} - \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}} - \theta_p \right) \hat{\rho}_{t+j} - \left(\frac{\tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}}{\tilde{\rho} - \tilde{m} c_{t+j} \frac{\tilde{z}}{z_t^{cr}}} \right) \hat{\tilde{m} c}_{t+j} \right) = \hat{l} v_t$$

⁴We will prove this property for period t in subsection F of this appendix.

and solving for $\widehat{z}_t^{cr}(\omega)$, it is obtained

$$\widehat{z}_t^{cr}(\omega) = \frac{(\tilde{\rho} - \widehat{m}c_{z^{cr}}^{\tilde{z}})}{\beta\gamma s \widehat{m}c_{z^{cr}}^{\tilde{z}}} \widehat{l}v_t - \frac{(1-\beta\gamma s)(\tilde{\rho} - \widehat{m}c_{z^{cr}}^{\tilde{z}})}{\beta\gamma s(\widehat{m}c_{z^{cr}}^{\tilde{z}})} E_t \sum_{j=1}^{\infty} (\beta\gamma s)^j \begin{pmatrix} \left(\frac{\widehat{m}c_{z^{cr}}^{\tilde{z}}}{\tilde{\rho} - \widehat{m}c_{z^{cr}}^{\tilde{z}}}\right) \widehat{m}c_{t+j} - \widehat{\beta}_{t+j} \\ -\widehat{s}_{t+j}(\omega) - \widehat{y}_{t+j} - \left(\frac{\tilde{\rho}}{\tilde{\rho} - \widehat{m}c_{z^{cr}}^{\tilde{z}}} - \theta_p\right) \widehat{\rho}_{t+j} \end{pmatrix}. \quad (\text{E7})$$

Taking (E7) one period ahead and computing $\widehat{z}_t^{cr}(\omega) - \beta\gamma s E_t \widehat{z}_{t+1}^{cr}(\omega)$ gives

$$\begin{aligned} \widehat{z}_t^{cr}(\omega) &= \frac{(\tilde{\rho} - \Omega)}{\beta s \Omega} \left(\widehat{l}v_t - \beta s E_t \widehat{l}v_{t+1} \right) + \beta\gamma s E_t \widehat{z}_{t+1}^{cr}(\omega) \\ &+ (1 - \beta\gamma s) E_t \left(\widehat{m}c_{t+1} - \frac{(\tilde{\rho} - \Omega)}{\Omega} \left(\widehat{y}_{t+1} + \widehat{\beta}_{t+1} + \widehat{s}_{t+1}(\omega) \right) - \frac{\tilde{\rho} - (\tilde{\rho} - \Omega)\theta_p}{\Omega} \widehat{\rho}_{t+1} \right), \end{aligned} \quad (\text{E8})$$

with $\Omega = \widehat{m}c_{z^{cr}}^{\tilde{z}}$ and where the stochastic discount factor and the expected survival rate in loglinear terms are

$$E_t \widehat{\beta}_{t+1} = -(R_t - E_t \pi_{t+1}^c + \varepsilon_t^b) \quad (\text{E9})$$

$$E_t \widehat{s}_{t+1}(\omega) = E_t \widehat{n}_{t+1}^a(\omega) - \widehat{n}_t(\omega) = -\kappa E_t \widehat{z}_{t+1}^{cr}(\omega), \quad (\text{E10})$$

recalling the inverse relation between the survival rate and the critical productivity, $n_{t+1}^a(\omega)/n_t(\omega) = (z_{\min}/z_t^{cr})^\kappa$. Plugging both (E9) and (E10) in (E8), we get

$$\begin{aligned} \widehat{z}_t^{cr}(\omega) &= \frac{(\tilde{\rho} - \Omega)}{\beta s \Omega} \left(\widehat{l}v_t - \beta\gamma s E_t \widehat{l}v_{t+1} \right) + \left(\beta\gamma s + \frac{\kappa(1-\beta\gamma s)(\tilde{\rho} - \Omega)}{\Omega} \right) E_t \widehat{z}_{t+1}^{cr}(\omega) \\ &+ (1 - \beta\gamma s) E_t \left(\widehat{m}c_{t+1} - \frac{(\tilde{\rho} - \Omega)}{\Omega} \widehat{y}_{t+1} + \frac{(\tilde{\rho} - \Omega)}{\Omega} (R_t - E_t \pi_{t+1}^c + \varepsilon_t^b) - \frac{\tilde{\rho} - (\tilde{\rho} - \Omega)\theta_p}{\Omega} \widehat{\rho}_{t+1} \right). \end{aligned} \quad (\text{E11})$$

Since $\widehat{z}_t^{cr}(\omega)$ depends in (E11) exclusively in current and expected future economy-wide variables, the average critical productivity, $\widehat{z}_t^{cr} = \int_0^{n_t} \widehat{z}_t^{cr}(\omega) d\omega$ will have log fluctuations from steady state of identical magnitude to the firm-specific critical productivity $\widehat{z}_t^{cr}(\omega)$

$$\widehat{z}_t^{cr} = \widehat{z}_t^{cr}(\omega),$$

and the dynamics of the aggregate exit rate would be as follows

$$\widehat{n}_t^x - \widehat{n}_{t-1} = \kappa \left(\frac{1 - \delta_n}{\delta_n} \right) \widehat{z}_t^{cr}.$$

F. Aggregation

Producer Price Index (PPI) and Consumer Price Index (CPI).

The average of firm-specific prices $P_t(\omega)$ can be computed using the Dixit-Stiglitz weighted average of the outcome of its current and past Calvo-type lotteries

$$\tilde{P}_t(\omega) = \left[\begin{aligned} &(1 - \xi_p)P_t^*(\omega)^{1-\theta_p} + (1 - \xi_p)\xi_p \left(\Pi_{t-1,t}^P P_{t-1}^*(\omega) \right)^{1-\theta_p} \\ &+ (1 - \xi_p)\xi_p^2 \left(\Pi_{t-2,t}^P P_{t-2}^*(\omega) \right)^{1-\theta_p} + \dots \end{aligned} \right]^{1/(1-\theta_p)} \quad (\text{F1})$$

where $P_{t-j}^*(\omega)$ is the optimal price set j periods ago and $\Pi_{t-j,t}^P$ is the price indexation factor applied from period $t-j$ to period t . Recalling the optimal pricing of the representative establishment (see section A of this Appendix), we obtained a relative optimal price $P_t^*(\omega) = \frac{\tilde{z}}{z(\omega)} \tilde{P}_t^*$ determined by relative firm-level productivities, which can be generalized for any $t-j$ period as follows

$$P_{t-j}^*(\omega) = \frac{\tilde{z}}{z(\omega)} \tilde{P}_{t-j}^*. \quad (\text{F2})$$

Inserting (F2) for $j = 0, 1, 2, \dots$, in (F1) yields

$$\tilde{P}_t(\omega) = \left[\begin{aligned} &(1 - \xi_p) \left(\frac{\tilde{z}}{z(\omega)} \tilde{P}_t^* \right)^{1-\theta_p} + (1 - \xi_p)\xi_p \left(\Pi_{t-1,t}^P \frac{\tilde{z}}{z(\omega)} \tilde{P}_{t-1}^* \right)^{1-\theta_p} \\ &+ (1 - \xi_p)\xi_p^2 \left(\Pi_{t-2,t}^P \frac{\tilde{z}}{z(\omega)} \tilde{P}_{t-2}^* \right)^{1-\theta_p} + \dots \end{aligned} \right]^{1/(1-\theta_p)},$$

where $\frac{\tilde{z}}{z(\omega)}$ can be extracted from the bracketed term to reach

$$\tilde{P}_t(\omega) = \frac{\tilde{z}}{z(\omega)} \left[\begin{aligned} &(1 - \xi_p) \left(\tilde{P}_t^* \right)^{1-\theta_p} + (1 - \xi_p)\xi_p \left(\Pi_{t-1,t}^P \tilde{P}_{t-1}^* \right)^{1-\theta_p} \\ &+ (1 - \xi_p)\xi_p^2 \left(\Pi_{t-2,t}^P \tilde{P}_{t-2}^* \right)^{1-\theta_p} + \dots \end{aligned} \right]^{1/(1-\theta_p)}. \quad (\text{F3})$$

The PPI is the average price set by establishments that operate with the average productivity, that is computed through the Dixit-stiglitz aggregator as follows

$$\tilde{P}_t = \left[(1 - \xi_p) \left(\tilde{P}_t^* \right)^{1-\theta_p} + (1 - \xi_p)\xi_p \left(\Pi_{t-1,t}^P \tilde{P}_{t-1}^* \right)^{1-\theta_p} + (1 - \xi_p)\xi_p^2 \left(\Pi_{t-2,t}^P \tilde{P}_{t-2}^* \right)^{1-\theta_p} + \dots \right]^{1/(1-\theta_p)}, \quad (\text{F4})$$

and which can be inserted in (F3) to yield

$$\tilde{P}_t(\omega) = \frac{\tilde{z}}{z(\omega)} \tilde{P}_t, \quad (\text{F5})$$

implying implies the same proportional relationship for average prices as the one we found in (A11) for optimal prices. Next, we will also find a relationship between the PPI and the CPI. The Dixit-Stiglitz aggregator for the CPI is

$$P_t^c = \left[\int_0^{n_t} P_t^{1-\theta_p}(\omega) d\omega \right]^{\frac{1}{1-\theta_p}},$$

where inserting (F4) for the average price of firms with productivity ω , we have

$$P_t^c = \left[\int_0^{n_t} \left(\frac{\tilde{z}}{z(\omega)} \tilde{P}_t \right)^{1-\theta_p} d\omega \right]^{\frac{1}{1-\theta_p}}. \quad (\text{F6})$$

The elements that are not firm specific can be moved outside the integral in (F5) to reach

$$P_t^c = \left[\tilde{z}^{1-\theta_p} \tilde{P}_t^{1-\theta_p} \int_0^{n_t} z(\omega)^{\theta_p-1} d\omega \right]^{1/(1-\theta_p)}. \quad (\text{F7})$$

The average productivity observed in period t is

$$\tilde{z}_t = \left[n_t^{-1} \int_0^{n_t} z(\omega)^{\theta_p-1} d\omega \right]^{1/(\theta_p-1)},$$

which implies

$$(\tilde{z}_t)^{\theta_p-1} n_t = \int_0^{n_t} z(\omega)^{\theta_p-1} d\omega \quad (\text{F8})$$

Combining (F8) and (F7) gives

$$P_t^c = \tilde{z} \tilde{P}_t \left[(\tilde{z}_t)^{\theta_p-1} n_t \right]^{1/(1-\theta_p)},$$

which simplifies to the following expression for the consumer price index

$$P_t^c = \frac{\tilde{z}}{\tilde{z}_t} \tilde{P}_t n_t^{1/(1-\theta_p)},$$

and using the property of constant average productivity, $\tilde{z}_t = \tilde{z}$, we have

$$P_t^c = \tilde{P}_t n_t^{1/(1-\theta_p)}.$$

Aggregate labor demand, $\int_0^{n_t} l_t(\omega) d\omega$.

Firm-level labor demand is consistent with the first order condition of the firm

$$w_t = mc_t(\omega) \frac{(1-\alpha) y_t(\omega)}{l_t(\omega)},$$

that brings the amount of firm-specific labor demand

$$l_t(\omega) = mc_t(\omega) \frac{(1-\alpha) y_t(\omega)}{w_t}. \quad (\text{F9})$$

For the firm that produces using the steady-state average productivity \tilde{z} , the amount of labor demand is

$$\tilde{l}_t = \tilde{m}c_t \frac{(1-\alpha) \tilde{y}_t}{w_t}. \quad (\text{F10})$$

Making the ratio between (F9) and (F10) yields

$$l_t(\omega) = \frac{mc_t(\omega)}{\tilde{m}c_t} \frac{y_t(\omega)}{\tilde{y}_t} \tilde{l}_t. \quad (\text{F11})$$

The definition of the real marginal cost implies $mc_t(\omega) = \frac{\tilde{z}}{z(\omega)} \tilde{m}c_t$ whereas the Dixit-Stiglitz demand constraint brings $y_t(\omega) = \left(\frac{P_t(\omega)}{\tilde{P}_t}\right)^{-\theta_p} \tilde{y}_t$ which can be jointly used in (F11) to obtain

$$l_t(\omega) = \frac{\tilde{z}}{z(\omega)} \left(\frac{P_t(\omega)}{\tilde{P}_t}\right)^{-\theta_p} \tilde{l}_t. \quad (\text{F12})$$

Next, the average price with average productivity and any firm-specific price are proportional to their relative productivities (as jointly implied by F3 and F4)

$$\tilde{P}_t = \frac{z(\omega)}{\tilde{z}} P_t(\omega),$$

that we plug in (F12) to reach

$$l_t(\omega) = \left(\frac{\tilde{z}}{z(\omega)}\right)^{1-\theta_p} \tilde{l}_t. \quad (\text{F13})$$

The aggregate labor demand consistent with (F13) is

$$\int_0^{n_t} l_t(\omega) d\omega = \int_0^{n_t} \left(\frac{\tilde{z}}{z(\omega)}\right)^{1-\theta_p} \tilde{l}_t d\omega,$$

that is equivalent to

$$\int_0^{n_t} l_t(\omega) d\omega = \int_0^{n_t} \tilde{z}^{1-\theta_p} \tilde{l}_t z(\omega)^{\theta_p-1} d\omega,$$

and moving outside the integral terms

$$\int_0^{n_t} l_t(\omega) d\omega = \tilde{z}^{1-\theta_p} \tilde{l}_t \int_0^{n_t} z(\omega)^{\theta_p-1} d\omega. \quad (\text{F14})$$

Recalling the definition of average firm-level productivity in period t

$$\tilde{z}_t = \left[n_t^{-1} \int_0^{n_t} z(\omega)^{\theta_p-1} d\omega \right]^{1/(\theta_p-1)},$$

and using it in (F14) yields

$$\int_0^{n_t} l_t(\omega) d\omega = \tilde{z}^{1-\theta_p} \tilde{l}_t (\tilde{z}_t)^{\theta_p-1} n_t,$$

or, alternatively,

$$\int_0^{n_t} l_t(\omega) d\omega = \left(\frac{\tilde{z}_t}{\tilde{z}}\right)^{\theta_p-1} n_t \tilde{l}_t.$$

Using the property of constant average productivity, $\tilde{z}_t = \tilde{z}$, we have

$$\int_0^{n_t} l_t(\omega) d\omega = n_t \tilde{l}_t.$$

Aggregate demand for capital, $\int_0^{n_t} k_t(\omega) d\omega$.

From the first order conditions of the representative firm, the capital demand is

$$k_t(\omega) = mc_t(\omega) \frac{\alpha y_t(\omega)}{r_t^k}.$$

Using analogous steps to those taken for the aggregate labor demand, firm-specific capital demand is related to the demand under average productivity as follows

$$k_t(\omega) = \left(\frac{\tilde{z}}{z(\omega)} \right)^{1-\theta_p} \tilde{k}_t, \quad (\text{F15})$$

and the aggregate capital demand becomes

$$\int_0^{n_t} k_t(\omega) d\omega = \left(\frac{\tilde{z}_t}{\tilde{z}} \right)^{\theta_p-1} n_t \tilde{k}_t.$$

Using the property of constant average productivity, $\tilde{z}_t = \tilde{z}$, we have

$$\int_0^{n_t} k_t(\omega) d\omega = n_t \tilde{k}_t.$$

$$\text{Aggregate output, } y_t = \left[\int_0^{n_t} y_t(\omega)^{\frac{\theta_p-1}{\theta_p}} d\omega \right]^{\frac{\theta_p}{\theta_p-1}}$$

Aggregate output is obtained as the Dixit-Stiglitz consumption bundle for a variable number of varieties n_t

$$y_t = \left[\int_0^{n_t} y_t(\omega)^{\frac{\theta_p-1}{\theta_p}} d\omega \right]^{\frac{\theta_p}{\theta_p-1}},$$

where using the Cobb-Douglas production function (A7)

$$y_t = \left[\int_0^{n_t} \left(e^{\varepsilon_t^a} z(\omega) k_t^\alpha(\omega) (e^{\gamma t} l_t(\omega))^{1-\alpha} \right)^{\frac{\theta_p-1}{\theta_p}} d\omega \right]^{\frac{\theta_p}{\theta_p-1}},$$

and also the amounts of establishment-level demands for labor, (F13), and capital, (F15), it is obtained

$$y_t = \left[\int_0^{n_t} \left(e^{\varepsilon_t^a} z(\omega) \left(\frac{\tilde{z}}{z(\omega)} \right)^{1-\theta_p} (\tilde{l}_t)^\alpha (e^{\gamma t} \tilde{l}_t)^{1-\alpha} \right)^{\frac{\theta_p-1}{\theta_p}} d\omega \right]^{\frac{\theta_p}{\theta_p-1}} \quad (\text{F16})$$

The definition of output produced at the establishment with average productivity \tilde{z} , i.e. $\tilde{y}_t = e^{\varepsilon_t^a} \tilde{z} \tilde{k}_t^\alpha (e^{\gamma t} \tilde{l}_t)^{1-\alpha}$, can be inserted in (F16) to yield

$$y_t = \left[\int_0^{n_t} \left(\left(\frac{\tilde{z}}{z(\omega)} \right)^{-\theta_p} \tilde{y}_t \right)^{\frac{\theta_p-1}{\theta_p}} d\omega \right]^{\frac{\theta_p}{\theta_p-1}},$$

where taking elements outside the integral, we get

$$y_t = \tilde{y}_t \tilde{z}^{-\theta_p} \left[\int_0^{n_t} z(\omega)^{\theta_p-1} d\omega \right]^{\frac{\theta_p}{\theta_p-1}} \quad (\text{F17})$$

Plugging the definition of the average productivity in (F17), $\tilde{z}_t = \left[n_t^{-1} \int_0^{n_t} z(\omega)^{\theta_p-1} d\omega \right]^{1/(\theta_p-1)}$, gives

$$y_t = \tilde{y}_t \tilde{z}^{-\theta_p} \tilde{z}_t^{\theta_p} n_t^{\frac{\theta_p}{\theta_p-1}},$$

where using the property of constant average productivity $\tilde{z}_t = \tilde{z}$ results in the simpler expression

$$y_t = \tilde{y}_t n_t^{\frac{\theta_p}{\theta_p-1}} \quad (\text{F18})$$

Finally, since the relative price is connected to the number of varieties as follows

$$\frac{\tilde{P}_t}{P_t^c} = n_t^{-1/(1-\theta_p)}$$

and (F18) can be rewritten in a way that displays $n_t^{-1/(1-\theta_p)}$

$$y_t = \tilde{y}_t n_t n_t^{-1/(1-\theta_p)},$$

then we can obtain an expression that shows how aggregate output, y_t , can be decomposed between the intensive margin (output per establishment with average productivity, $\frac{\tilde{P}_t}{P_t^c} \tilde{y}_t$) and the extensive margin (number of establishments, n_t)

$$y_t = n_t \left(\frac{\tilde{P}_t}{P_t^c} \tilde{y}_t \right).$$

G. The overall resources constraint

The household budget constraint is,

$$\frac{W_t(i)}{P_t^c} l_t(i) + r_t^k u_t(i) k_{t-1}(i) + \left[\frac{n_t^s}{n_{t-1}} (d_t + v_t) + \frac{n_t^x}{n_{t-1}} l v_t \right] (x_{t-1}(i) + n_t^e(i)) - t_t(i) = c_t(i) + i_t(i) + a(u_t(i)) k_{t-1}(i) + v_t x_t(i) + \frac{b_t(i)}{\exp(\varepsilon_t^b)(1+r_t)} - b_{t-1}(i) + (\exp(\varepsilon_t^e) f^e + e c_t) n_{t+1}^e(i).$$

First, plugging the labor demand constraint, $l_t(i) = (W_t(i)/W_t)^{-\theta_w} l_t$, and the definition of the real wage, $w_t = W_t/P_t^c$, it is obtained

$$w_t (W_t(i)/W_t)^{1-\theta_w} l_t + r_t^k u_t(i) k_{t-1}(i) + \left[\frac{n_t^s}{n_{t-1}} (d_t + v_t) + \frac{n_t^x}{n_{t-1}} l v_t \right] (x_{t-1}(i) + n_t^E(i)) - t_t = c_t(i) + i_t(i) + a(u_t(i)) k_{t-1}(i) + v_t x_t(i) + \frac{b_t(i)}{\exp(\varepsilon_t^b)(1+r_t)} - b_{t-1}(i) + (\exp(\varepsilon_t^e) f^e + e c_t) n_{t+1}^e(i).$$

The aggregation across households implies

$$w_t l_t + r_t^k u_t k_{t-1} + \left[\frac{n_t^s}{n_{t-1}} (d_t + v_t) + \frac{n_t^x}{n_{t-1}} l v_t \right] (x_{t-1} + n_t^e) - t_t = c_t + i_t + a(u_t) k_{t-1} + v_t x_t + \frac{b_t(i)}{\exp(\varepsilon_t^b)(1+r_t)} - b_{t-1} + (\exp(\varepsilon_t^e) f^e + e c_t) n_{t+1}^e,$$

where we used aggregation schemes for nominal wages, capital utilization, the stock of capital, equity shares and bonds. Introducing the equilibrium condition for the portfolio shares, $x_{t-1} = n_{t-1}$ and $x_t = n_t$, it is obtained,

$$w_t l_t + r_t^k u_t k_{t-1} + \left[\frac{n_t^s}{n_{t-1}} (d_t + v_t) + \frac{n_t^x}{n_{t-1}} l v_t \right] (n_{t-1} + n_t^e) - t_t = c_t + i_t + a(u_t) k_{t-1} + v_t n_t + \frac{b_t}{\exp(\varepsilon_t^b)(1+r_t)} - b_{t-1} + (\exp(\varepsilon_t^e) f^e + e c_t) n_{t+1}^e,$$

The law of motion for the number of varieties, $n_t = (n_t^s/n_{t-1})(n_{t-1} + n_t^e)$, serves to cancel the equity term $v_t n_t$ in order to yield

$$w_t l_t + r_t^k u_t k_{t-1} + n_t d_t + \frac{n_t^x}{n_t^s} n_t l v_t - t_t = c_t + i_t + a(u_t) k_{t-1} + \frac{b_t}{\exp(\varepsilon_t^b)(1+r_t)} - b_{t-1} + (\exp(\varepsilon_t^e) f^e + e c_t) n_{t+1}^e,$$

where replacing the tax variable for the expression implied by the government constraint, $\varepsilon_t^g =$

$$t_t + \exp(\varepsilon_t^e) f^e n_{t+1}^e - \exp(\varepsilon_t^x) (1 - \tau) f^e (n_t^x + (n_t^x/n_{t-1}) n_t^e) + \frac{b_t}{\exp(\varepsilon_t^b)(1+r_t)} - b_{t-1},$$

$$w_t l_t + r_t^k u_t k_{t-1} + n_t d_t + \frac{n_t^x}{n_t^s} n_t l v_t = \varepsilon_t^g + (1 - \tau) f^e (n_t^x + (n_t^x/n_{t-1}) n_t^e) + c_t + i_t + a(u_t) k_{t-1} + e c_t n_{t+1}^e.$$

Recalling the expression to obtain the liquidation value, $l v_t = \exp(\varepsilon_t^x) (1 - \tau) f^e$, and using $\frac{n_t^x}{n_t^s} n_t = (n_t^x + (n_t^x/n_{t-1}) n_t^e)$, we reach,

$$w_t l_t + r_t^k u_t k_{t-1} + n_t d_t = \varepsilon_t^g + c_t + i_t + a(u_t) k_{t-1} + e c_t n_{t+1}^e,$$

Next, introducing the input markets equilibria, $l_t = n_t \tilde{l}_t$, and, $u_t k_{t-1} = n_t \tilde{k}_t$, yields,

$$w_t n_t \tilde{l}_t + r_t^k n_t \tilde{k}_t + n_t d_t = c_t + i_t + a(u_t) k_{t-1} + \varepsilon_t^g + e c_t n_{t+1}^E.$$

The average dividend of firms that produce single goods, $d_t = \tilde{\rho}_t \tilde{y}_t - w_t \tilde{l}_t - r_t^k \tilde{k}_t$, can be substituted in the previous expression to obtain,

$$n_t \tilde{\rho}_t \tilde{y}_t = c_t + i_t + a(u_t) k_{t-1} + \varepsilon_t^g + e c_t n_{t+1}^e.$$

Recalling the relation between aggregate output and firm-level output, $y_t = n_t \tilde{\rho}_t \tilde{y}_t$, we have

$$y_t = c_t + i_t + a(u_t) k_{t-1} + \varepsilon_t^g + e c_t n_{t+1}^e,$$

and in a log-linear approximation around the detrended steady-state

$$\hat{y}_t = \frac{c}{y} \hat{c}_t + \frac{i}{y} \hat{i}_t + \frac{r^k k}{y} \hat{u}_t + \frac{\varepsilon^g}{y} \varepsilon_t^g + \frac{(\delta_n / (1 - \delta_n)) e c}{y} (\hat{n}_{t+1}^e + \hat{e} c_t).$$

H. Estimated shock decomposition for US data

Using the "shock_decomposition" routine of Dynare, we have obtained and plotted the quarter-to-quarter estimated shock decomposition of the rate of growth of US real Gross Domestic Product (GDP) per capita, the rate of growth of the US Total Private Establishments (TPE), the US establishment entry (births) rate, and the US establishment exit (deaths) rate.

Next, Figures 2-5 display the results with the following legend labeling: e_x is the contribution of the liquidation value shock, e_e is the contribution of the entry cost shock, e_W is the contribution of the wage-push indexation shock, e_P is the contribution of the price-push indexation shock, e_g is the contribution of the fiscal/net exports spending shock, e_i is the contribution of the adjustment cost of investment shock, e_R is the contribution of the Taylor-type monetary policy rule shock, e_b is the contribution of the risk-premium shock, and e_a is the contribution of the technology shock. The "initial values" share reports the contribution that is not explained by any of the nine exogenous variables.

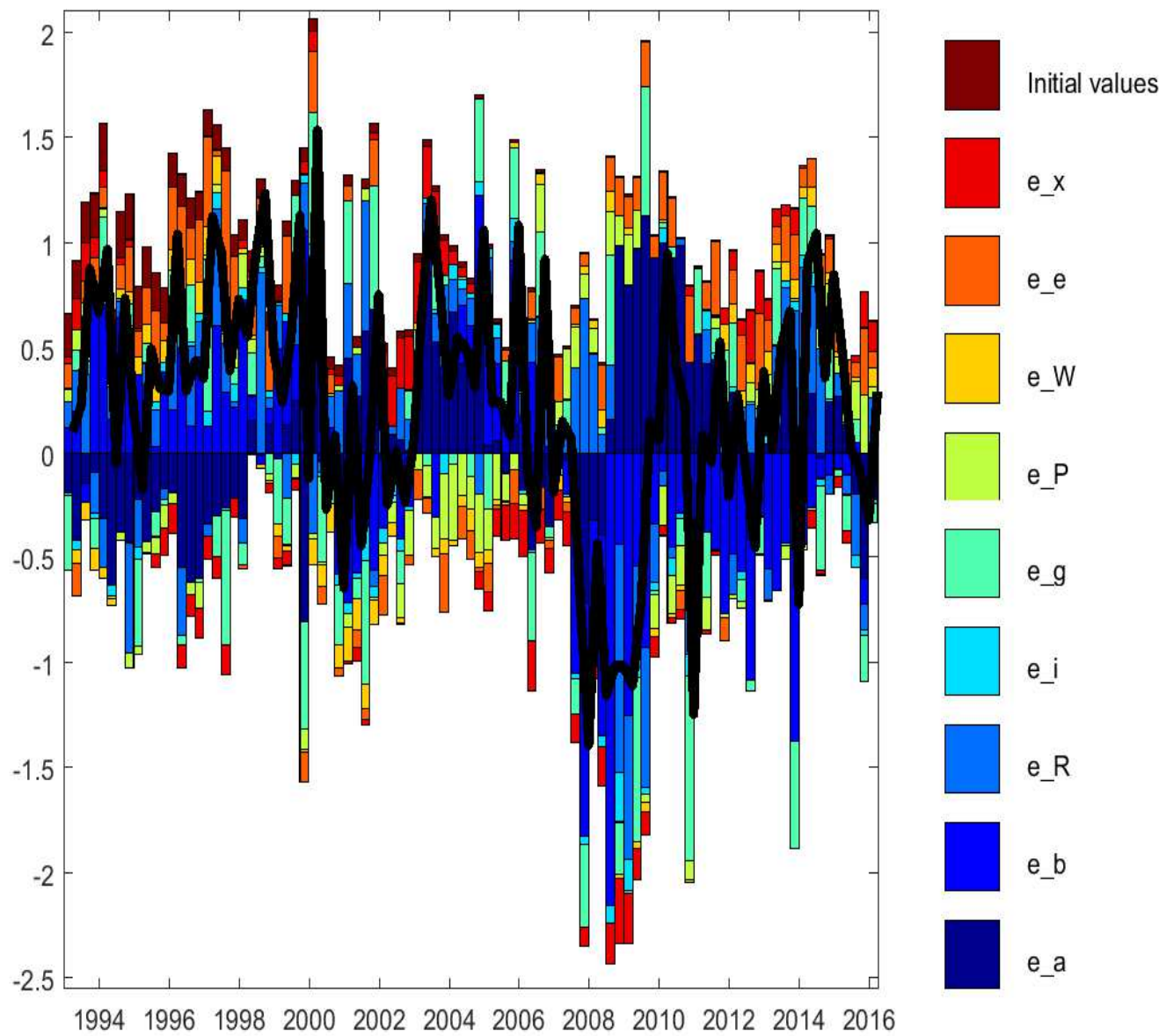


Figure 2: Shock decomposition: quarterly growth rate of US real GDP per capita (1993:2 to 2016:2).

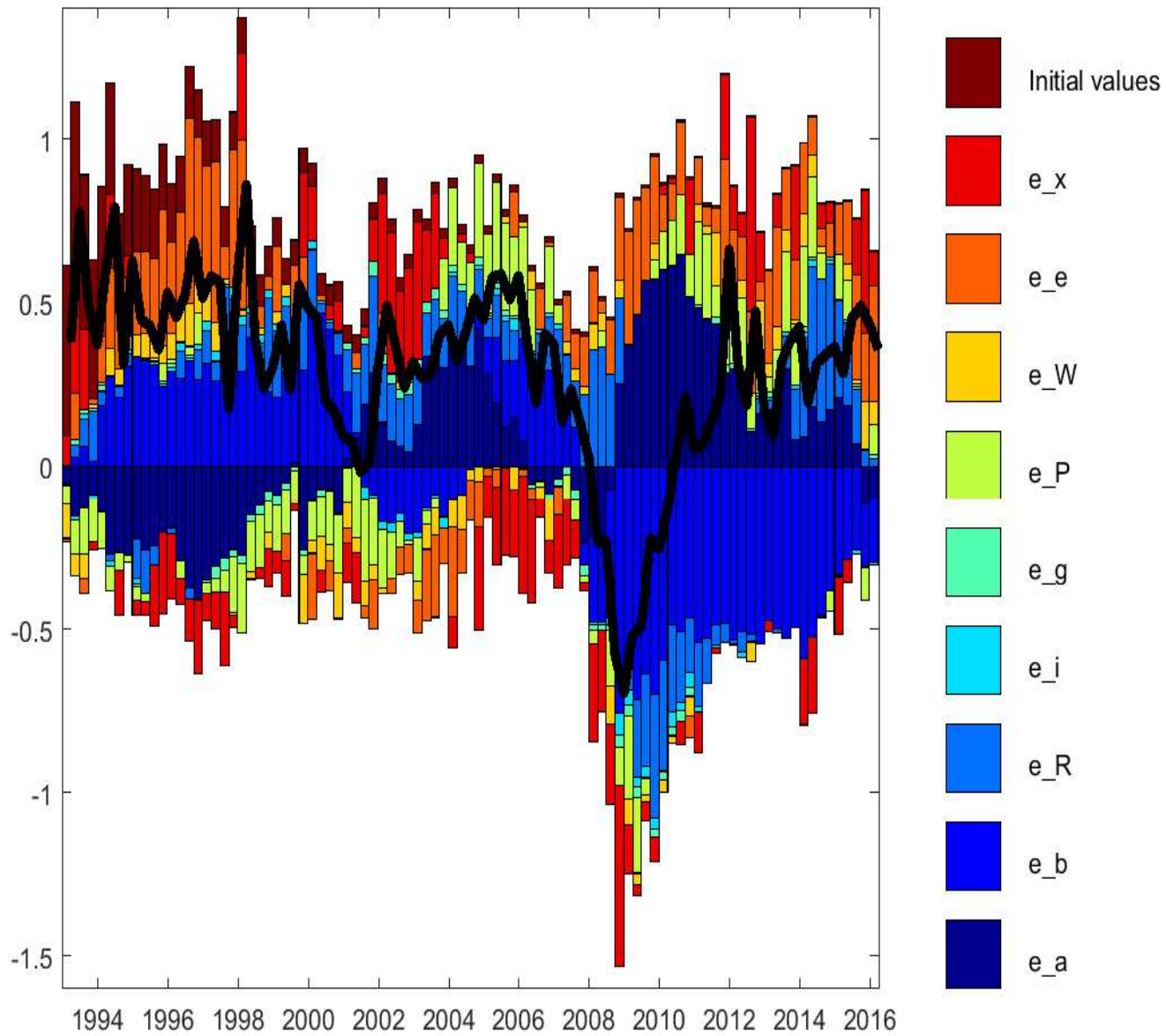


Figure 3: Shock decomposition: quarterly growth rate of US Total Private Establishments per capita (1993:2 to 2016:2).

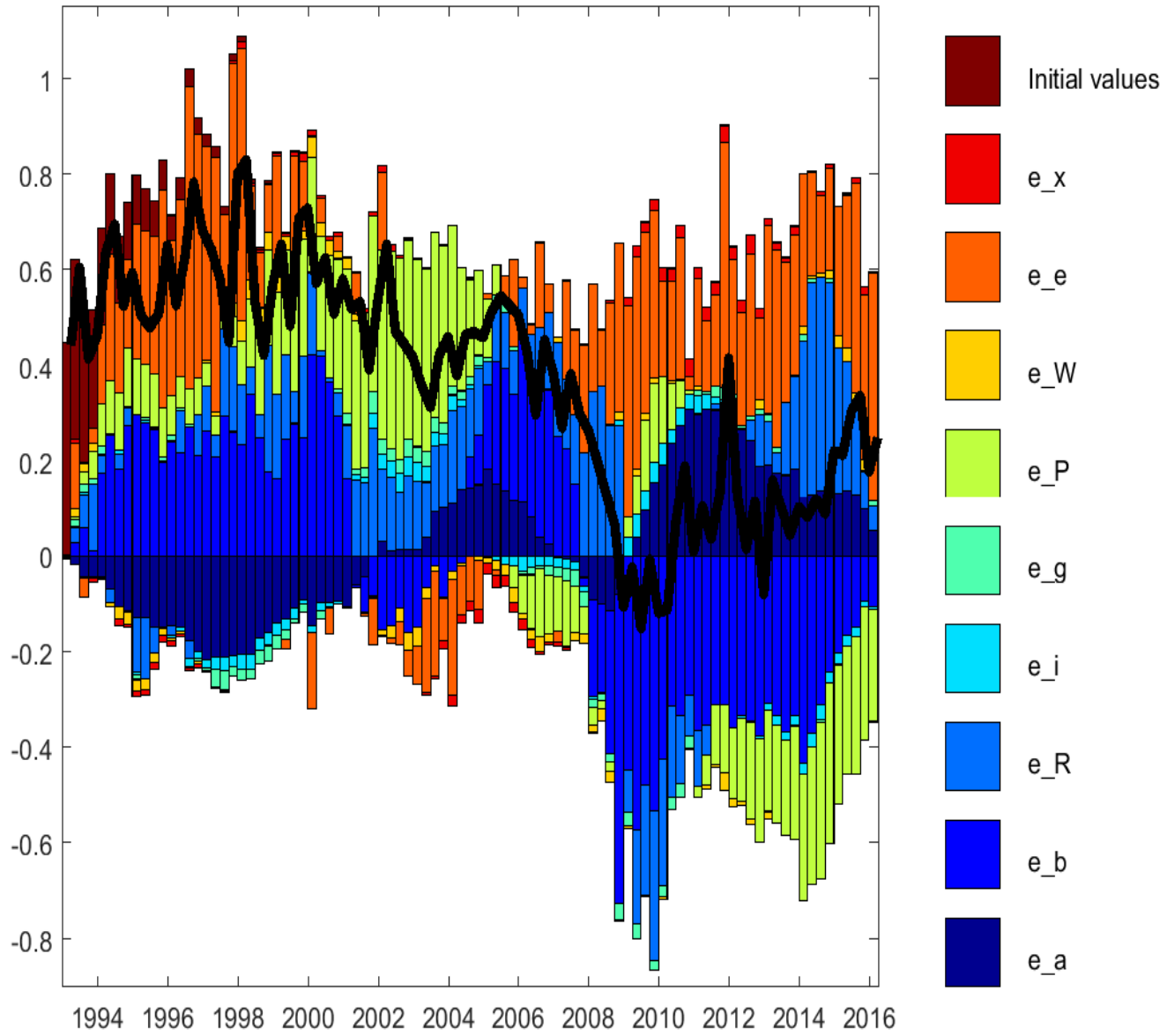


Figure 4: Shock decomposition: rate of US establishment entry (1993:2 to 2016:2).

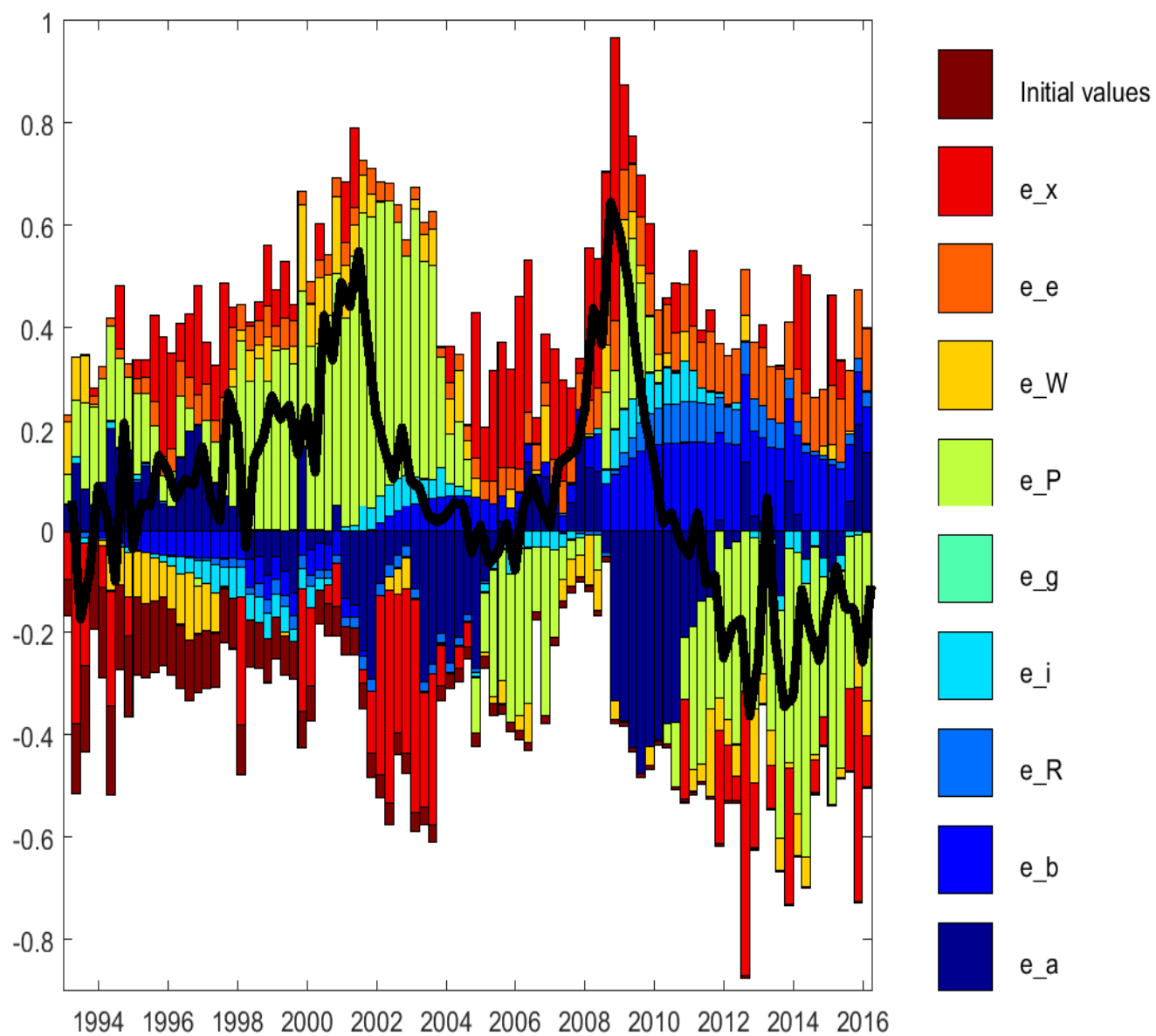


Figure 5: Shock decomposition: rate of US establishment exit (1993:2 to 2016:2).

I. The sources of fluctuations in the Great Recession

The next two Tables collect the estimates of the structural parameters of the model for a sample period that corresponds to the Great Recession (2007:1-2016:2):

Estimation of the structural parameters in the Great Recession

	Priors			Posteriors	
	Distr	Mean	Std D.	Mean	90% HPD interval
h : consumption habits	Beta	0.70	0.15	0.57	[0.46, 0.69]
σ_c risk aversion	Normal	1.50	0.25	0.92	[0.81, 1.02]
σ_l : inverse Frisch elasticity	Normal	2.00	0.50	1.59	[0.71, 2.45]
ξ_p : Calvo price rigidity	Beta	0.50	0.15	0.80	[0.75, 0.84]
ξ_w : Calvo wage rigidity	Beta	0.50	0.15	0.96	[0.94, 0.98]
l_p : price indexation	Beta	0.50	0.15	0.38	[0.18, 0.56]
l_w : wage indexation	Beta	0.50	0.15	0.26	[0.11, 0.40]
φ_k : capital adj. cost elasticity	Normal	4.00	1.50	2.76	[0.81, 4.74]
σ_a capital utilization cost elasticity	Beta	0.50	0.15	0.79	[0.64, 0.93]
ζ : entry cost elasticity	Normal	2.00	0.50	2.41	[1.69, 3.18]
X : steady-state exit rate	Gamma	0.0292	0.0025	0.0296	[0.0280, 0.0311]
κ : exit shape	Normal	5.00	1.50	3.36	[2.77, 3.95]
α : capital share in production	Beta	0.36	0.10	0.14	[0.10, 0.17]
θ_p : Dixit-Stigitz elasticity	Normal	3.80	1.00	2.51	[2.22, 2.82]
μ_π : inflation in Taylor rule	Normal	1.50	0.25	1.53	[1.22, 1.84]
μ_y : output gap in Taylor rule	Normal	0.12	0.05	0.09	[0.05, 0.13]
$\mu_{\Delta y}$: output gap change in Taylor rule	Normal	0.12	0.05	0.14	[0.08, 0.19]
μ_R : inertia in Taylor rule	Beta	0.75	0.15	0.80	[0.72, 0.89]
γ : steady-state technology growth, %	Normal	0.35	0.10	0.07	[0.02, 0.12]
π : steady-state rate of inflation, %	Normal	0.45	0.10	0.48	[0.37, 0.60]
l : steady-state log of hours	Normal	415.0	5.00	411.4	[410.1, 412.9]

Estimation of the exogenous processes in the Great Recession

	Priors			Posteriors	
	Distr	Mean	Std D.	Mean	90% HPD interval
σ_{η^a} : Std. dev. of technology innov.	Invgamma	0.10	2.00	0.78	[0.62, 0.95]
σ_{η^b} : Std dev of risk-premium innov.	Invgamma	0.10	2.00	0.19	[0.13, 0.24]
σ_{η^R} : Std dev of monetary innov.	Invgamma	0.10	2.00	0.14	[0.11, 0.17]
σ_{η^g} : Std dev of fiscal innov.	Invgamma	0.10	2.00	1.93	[1.55, 2.30]
σ_{η^i} : Std dev of investment innov.	Invgamma	0.10	2.00	0.35	[0.23, 0.47]
σ_{η^p} : Std dev of price-push innov.	Invgamma	0.10	2.00	0.49	[0.29, 0.69]
σ_{η^w} : Std of wage-push innov.	Invgamma	0.10	2.00	1.30	[0.91, 1.66]
σ_{η^e} : Std dev of entry cost innov.	Invgamma	0.10	2.00	0.47	[0.27, 0.66]
σ_{η^x} : Std dev of liquidation innov.	Invgamma	0.10	2.00	0.47	[0.21, 0.73]
ρ_a : Autocorr. of technology shock	Beta	0.50	0.20	0.74	[0.65, 0.84]
ρ_b : Autocorr. of risk-premium shock	Beta	0.50	0.20	0.95	[0.92, 0.98]
ρ_R : Autocorr. of monetary shock	Beta	0.50	0.20	0.49	[0.30, 0.66]
ρ_g : Autocorr. of fiscal shock	Beta	0.50	0.20	0.55	[0.36, 0.75]
ρ_i : Autocorr. of investment shock	Beta	0.50	0.20	0.69	[0.49, 0.87]
ρ_p : Autocorr. of price-push shock	Beta	0.50	0.20	0.44	[0.17, 0.74]
ρ_w : Autocorr. of wage-push shock	Beta	0.50	0.20	0.22	[0.06, 0.35]
ρ_e : Autocorr. of entry cost shock	Beta	0.50	0.20	0.52	[0.29, 0.78]
ρ_x : Autocorr. of liquidation shock	Beta	0.50	0.20	0.59	[0.38, 0.79]
μ_p : MA(1) of price-push shock	Beta	0.50	0.20	0.53	[0.28, 0.79]
μ_w : MA(1) of wage-push shock	Beta	0.50	0.20	0.92	[0.85, 0.99]
ρ_{ga} : cross effect tech.-fiscal/NX	Beta	0.50	0.20	0.66	[0.39, 0.95]

We will examine here the origins of the aggregate fluctuations in the shock decomposition of the estimated model. The sample period we look at begins in 2007:1 and ends at the end of the sample period, 2016:2, to contain the financial crisis of 2008 and the years afterwards that belong to the so-called Great Recession period.⁵

Figure 6 displays the partial quarterly contribution of each estimated shock of the model to the actual fluctuations of US real GDP growth during the Great Recession. Technology shocks are

⁵There is no reference to the role of entry and exit for business cycle fluctuations during the subsample period that belongs to the Great Moderation era (1993:2-2006:4) because it was found to be rather poor. The Bayesian estimation of the model over this subsample period was not very successful in replicating second-moment statistics, probably because of the little influence of net business formation for aggregate fluctuations (documented in Section 2 of the paper).

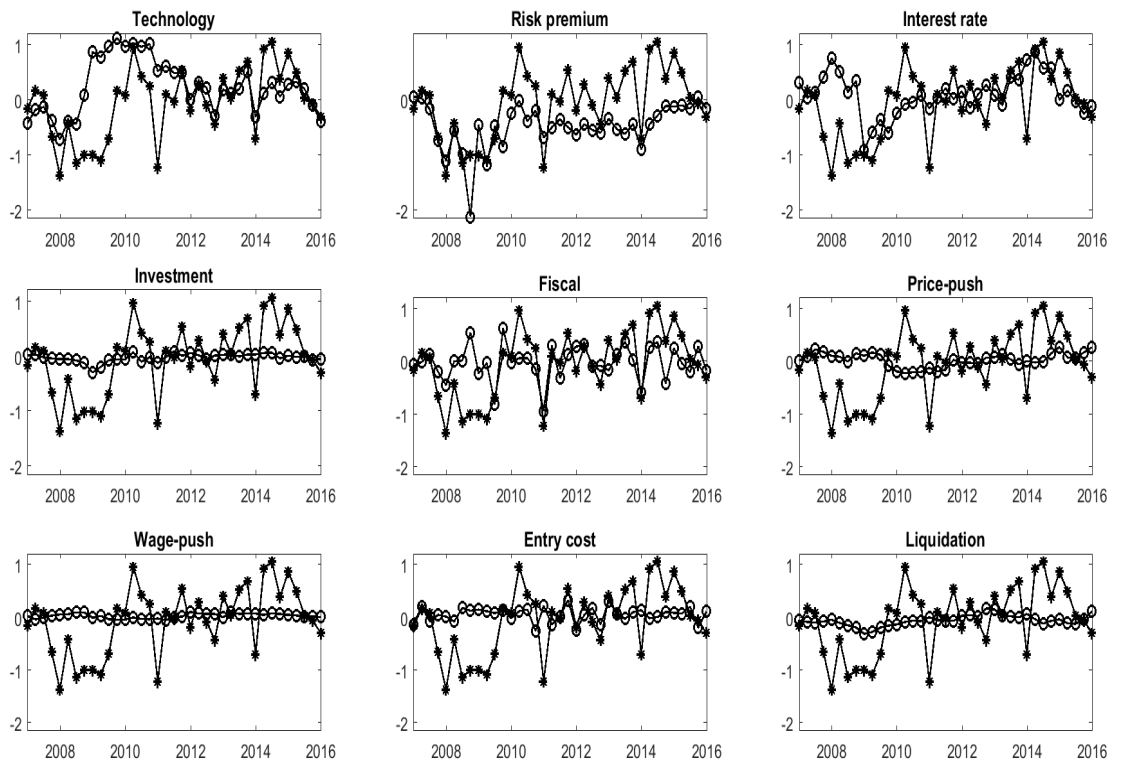


Figure 6: Sources of US real GDP growth variability (lines marked with *) during the Great Recession.

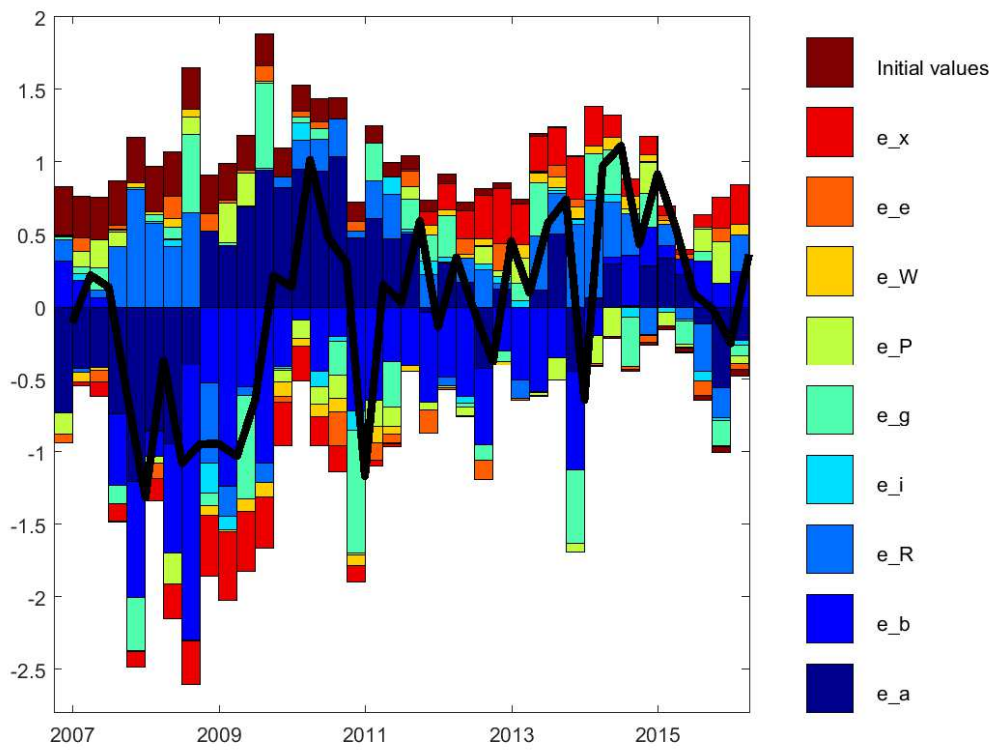


Figure 7: US real GDP during the Gret Recession period (2007:1-2016:2). Quarterly shock decomposition.

really important for the recovery path after the financial crisis. In 2009-2011, Figures 6-7 show that technology innovations contribute at around 1% positive for US economic growth and numbers remain on the positive side until 2012. It could be argued that the enormous business destruction that took place during the financial crisis of 2008 (more than 100,000 establishments closed in net terms) led to economy-wide technological innovations a few quarters later. This can be considered a Schumpeterian interpretation (creative destruction), supported by the estimation results showing technology shocks help the building up of the recovery path. In the second cell of Figure 6, we can see the severity of the adverse risk-premium shock during the financial crisis. In the third quarter of 2008 (Lehman Brothers' bankruptcy) the contractionary effects of the risk premium shock had an estimated impact of a 2.2% reduction of real GDP. The risk premium shock is still contractionary until 2010, though its size and effect on US growth is diminishing over time. Interest-rate shocks show the role of unconventional monetary policy during the Great Recession. Initially, the Fed intervention to cut interest rates to 0% provided some stimulus in 2008, with an average quarterly contribution to real GDP growth of +0.45%. Later, the massive asset purchase program of the Fed (QE policies), give a second wave of monetary stimulus in 2013-2015.⁶ In 2014, the year in which the Fed'd balance sheet reached its highest value (around 4 trillion dollars) the average quarterly effect monetary shocks on real GDP growth is +0.63%.⁷ Fiscal policy turns influential for US real GDP in several punctual quarters.⁸ In particular, there is a contractionary fiscal shock in 2011:1 that has a negative impact of -0.87% on real GDP growth. Other adverse fiscal shocks are displayed in Figures 6-7 corresponding to the fiscal cliff turbulences occurred in 2012-2014. All the remaining shocks play a minor role on explaining US real GDP during the Great Recession. We could just mention the price shocks in 2010-2011 as a consequence of the increase in the cost of energy (oil price jumped over \$90 a barrel), which are found to have a negative impact on US growth of around -0.2% per quarter.

The following Table provides the mean contribution and the standard deviation of the innovations from the nine shocks of the model, comparing across the full sample period and the Great Recession:

⁶The observed series of nominal interest rate may capture the effects of the QE policies because we have used the Wu-Xia (2016) series of shadow interest rates which include negative observation.

⁷Actually, the expansionary effects of monetary shocks estimated in the four quarters of 2014 (in terms of growth of real GDP per capita) are +0.68% (Q1), +0.81% (Q2), +0.51% (Q3) and +0.50% (Q4).

⁸As discussed in Smets and Wouters (2007), the fiscal shock may also capture changes in external demand (net exports) that are not considered in the closed-economy setup of the model.

Shock decomposition for US real GDP growth, $\Delta\hat{y}_t$

	Full sample, 1993-2016		Great Recession, 2007-16	
	Mean	Std dev	Mean	Std dev
Technology, η^a	0.07	0.45	0.20	0.50
Risk-premium, η^b	-0.08	0.52	-0.46	0.45
Interest rate, η^R	0.09	0.31	0.06	0.39
Investment, η^i	0.00	0.06	-0.01	0.06
Fiscal/NX, η^g	0.00	0.30	-0.02	0.32
Price-push, η^p	-0.01	0.13	0.05	0.14
Wage-push, η^w	0.01	0.07	0.03	0.05
Entry cost, η^e	0.10	0.17	0.13	0.12
Liquidation, η^x	-0.01	0.13	-0.03	0.13

The sources of economic growth during the Great Recession are the technology shock (+0.20% per quarter) and, in a weaker extent, the entry cost shock (+0.13% per quarter), the interest rate shock (+0.06%), and the price-push shock (+0.05% per quarter). Meanwhile, the recession is mostly justified on the demand-side risk premium shocks with a negative effect of -0.46% per quarter on US real GDP growth. Fiscal shocks are quite volatile (with continuous ups and downs) because its standard deviation is almost as high as that of the interest rate shocks, but the overall effect is quantitatively small. Finally, entry cost shocks and liquidation shocks have opposite sign effects: the exogenous component of entry favours economic growth (+0.13% per quarter) while the exit shock reduces it at -0.03% per quarter.