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A PORTFOLIO-CHOICE MODEL TO ANALYZE THE RECENT GROSS CAPITAL FLOWS BETWEEN CANADA AND THE US

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A portfolio-choice model to analyze the recent gross capital flows between Canada and the US*

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Abstract

We calibrate a two-country New Keynesian model with endogenous portfolio choice and valuation effects to discuss the determinants of the increase in Canadian Net Foreign Assets with the US observed after 2012. Furthermore, we discuss the shocks that may explain the "reversed two-way" capital flows pattern recently characterizing the Canada-US asset trading: Canada has a negative position on bond holdings owned by US investors while a positive balance emerges on its equity holdings from US firms. The combination of a global technology shock, the US fiscal contraction, an adverse wage-push shock in the US and the greater monetary stimulus in the US than in Canada (QE) provide insights to describe the recent capital flows between Canada and the US. Both the QE and the negative wage-push shock in the US play a crucial role as explanatory factors through substantial valuation effects.

Keywords: US-Canada capital flows, portfolio choice model, business cycles.

JEL codes: E44, F41, F44, E12.

1 Introduction

During the last few decades, both gross capital and trade flows across countries have reached unprecedented levels. Canada and the US are known to have the world's largest bilateral trade relationship.[1]

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[1] Policymakers of OCDE countries (especially, the US, the UK and EU countries) and emerging countries (especially, China) are nowadays merged into controversial debates on the global economy's uncertain trajectory, increasingly protectionist trade policies and a need for strengthened global institutions. Thus, Canada and the US economies provide natural

This link is generally characterized by a high Canadian net trade surplus, which reached a peak value of 7.8 billions of US dollars in 2008. After the 2008 financial crisis, trade surplus in Canada has been gradually reduced and it has punctually become even negative in 2016. Regarding financial transactions, Canadian overall Net Foreign Asset (NFA) position has been traditionally negative until 2015, when it turned positive for the first time in decades (see Figure 1). The evolution of Canadian NFA is vastly dominated by the capital flows with the US because it accounts for the largest share in total position. Moreover, the negative US position outweights all positive positions with the rest of the world. As net exports have declined while the variations in NFA have been typically positive, there have been significant valuation effects on Canadian assets and liabilities that explain the upwards trend in the NFA position of Canada with the US.

Both net Foreign Direct Investment (FDI) and Portfolio Equity (PE) account for the largest shares in the contributions for Canadian NFA. Their overall sizes have increased dramatically since 1990.[2] We have built a quarterly time series that results from adding up net FDI and net PE of Canada with respect to the US, which it has shown an upwards trend to reached almost 1.25 times Canadian GDP in 2017 (see Figure 1). Meanwhile, the quarterly balance of Canadian net Portfolio Bonds (PB) with the US has been always negative, since Canada bond liabilities are way larger than Canada bond assets held in the US. Thus, the Canada-US foreign asset relationships present a "reversed two-way" capital flows pattern. Generally, a "two-way pattern" refers to the case in which emerging economies net bonds positions are positive and their net equity positions are negative with respect to advanced economies. After 2011, the gap between net equity and net bond position starts increasing in Canada, as Figure 1 clearly shows. This means that US investors buy equity from emerging countries while they sell US-equity to other advanced countries, such as Canada. In addition, US investors sell domestic US-bonds to emerging countries while they buy bonds from other advanced countries, such as Canada.[3]

With the motivation of studying these empirical findings, we propose a two-economy optimizing model with endogenous gross trade and financial assets. The model is going to be calibrated to reproduce fluctuations macroeconomic variables from Canada and the US over the period between 1990 and 2018. Our approach is based on the open-economy New Keynesian literature related to seminal papers such as Smets and Wouters (2002) and Galí and Monacelli (2005), and its extension to

examples that are vital to understand how economic integration thorough trade and foreign capital affects macroeconomic aggregates, financial stability and business cycle synchronization.

[2] Canadian holdings of FDI and PE in US firms have increased from representing 50% and 20% of Canadian GDP, respectively, in 1990 to almost 180% and 200%, respectively, in 2018.

[3] We left out for future research the analysis of "reverse" international capital flows between US and other advanced countries.

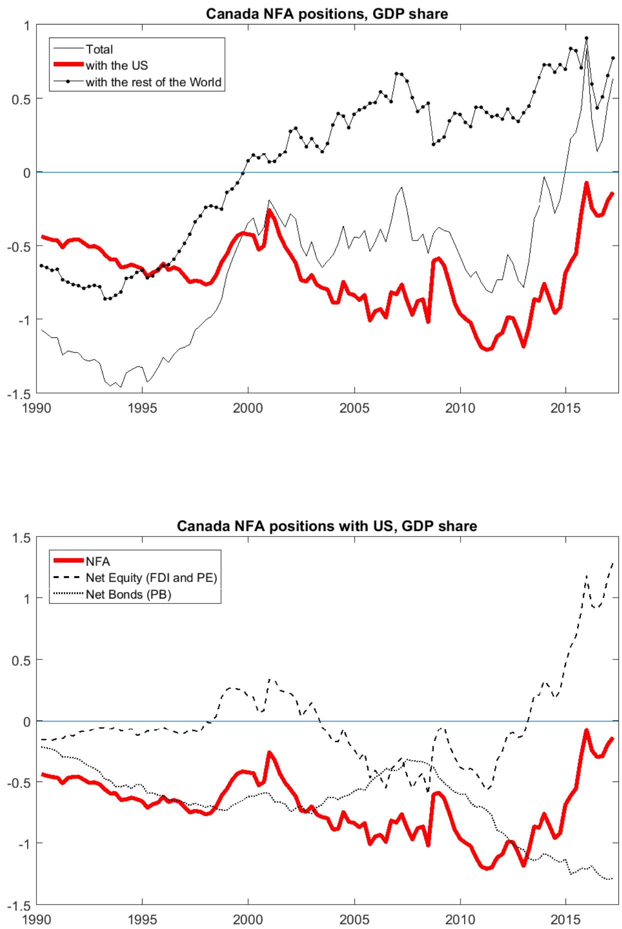

Figure 1: Net Foreign Asset positions for Canada (1990-2018)

incorporate endogenous portfolio choice initiated by Devereux and Sutherland (2009, 2010 and 2011) and Tille *et al.* (2008 and 2010). In general, models that incorporate portfolio choice into two-country general equilibrium frameworks, are highly theoretical and small in size. Hence, they do not contain enough elements, such as nominal and real frictions both at the local and at the international level, to provide realistic business cycle fluctuations for macroeconomic variables that are related in an open economy environment. Another important advantage of our model is that variations in the NFA position are pinned down not only by current account fluctuations but also due to valuation effects, which we have found to be key in explaining the increase in Canadian NFA.

Therefore, the first contribution of our paper on the modeling side is to introduce a wide set of nominal and real frictions into a medium-scale fully-fledged two-country New Keynesian model with endogenous gross trade and portfolio variables. This paper builds on Del Villar (2018), who provides a two-country New Keynesian model with portfolio choice of equity and bonds, nominal price rigidities following Calvo (1983)'s fixed probabilities, home good bias, and incomplete financial markets to study the factors behind heterogenous capital flows across emerging and advanced countries. We extend her benchmark model by including nominal wage rigidities *á la* Calvo (1983), price and wages indexation rules, consumption habits on household preferences, and additional exogenous processes to account for the business cycle synchronization across Canada and the US.

The second contribution of our paper is to provide a quantitative analysis on the overall model performance and on its ability to explain the highlighted empirical facts for the recent evolution of the Canada NFA position. This paper is the first that calibrates the parameters of the model with actual data from Canada and the US. Furthermore, we demonstrate that the proposed model performs reasonably well in explaining business cycle statistical moments of Canada, the US and their degree of cyclical synchronization.[4] Besides, our model allows for valuation effects channel through portfolio choices that significantly affect the dynamics of the NFA balances.

The organization of the rest of the paper is as follows. Section 2 reviews selection of related literature. Section 3 provides an overview of the two-country New Keynesian model describing the optimizing programs of domestic households and firms. Section 4 brings the parameter calibration. Section 5 presents the business cycle analysis of impulse response functions to evaluate the propagation channels of the shocks of the model and their international effects. Section 6 searches for the determinants of the increase in the NFA position of Canada with the US and their reversed two-way

[4] Schmitt-Grohé (1998) highlighted that the positive responses of Canadian output, employment, investment, exports, imports and terms of trade to positive shocks in the US cannot be explained using the standard international business cycle model.

capital flows. Section 7 concludes with a summary of the main results of the paper.

2 Selective literature review

Since the late 1990s, there have been additional elements included to the New Keynesian model to introduce the behavior of the exchange rate and the international transmission of shocks in open economies. After the the seminal paper by Obstfeld and Rogoff (1995), this framework coined the name New Open-Economy Macroeconomics (NOEM). These models generally focus the attention on net foreign assets and current account dynamics and not on the gross asset flows.[5] The main extensions derived in the literature aim at characterizing the price of exported and imported goods by implementing producer (or local) currency pricing, homogenous pricing (or pricing to market) and sticky (or flexible) prices. Other extensions focus the attention on the international risk sharing properties to discuss optimal monetary policy and economic policy coordination. Nonetheless, there is a growing empirical literature that studies the increasing significance of the valuation effects channel created by large and heterogenous gross capital flow movements across countries (Gourichas and Rey, 2007 and 2014). Still standard NOEM models abstract from gross capital flows which ultimately help to understand NFA dynamics and the international transmission of shocks through financial markets. There are just a few general equilibrium models which incorporate endogenous portfolio choice in an open economy framework because, until recently, there was no suitable computable method to solve portfolio choice in the context of DSGE models. In this regard, both Devereux and Sutherland (2008, 2010 and 2011), and Tille *et al.* (2008 and 2010) have developed novel methods to facilitate portfolio model solution within general equilibrium frameworks.[6]

The bulk of research within the NOEM literature has been highly theoretical and based on small-stylized models such as Ghironi (1999) or Justiniano and Preston (2004). The small size of these models at the local and the international dimensions of the economy, does not permit an empirical test of the main implications of these models for a relatively wide range of macro-aggregates. Nowadays, many Central Bankers in industrialized economies use extensions to the NOEM model that include more realistic features to facilitate the empirical check, such as nominal rigidities, capital accumulation with adjustment cost and traded and non-traded sectors, although most of them still lack of endogenous portfolio choice.[7]

[5]Other important contributions to the NOEM literature are Corsetti and Peseti (2001), Kollmann (2002) and Galí and Monacelli (2005).

[6]See Coeurdacier and Rey (2013) for an extended literature revision on Open Economy Financial Macroeconomics. Also, for portfolio choice studies see Engel and Matsumoto (2009), and Coeurdacier *et al.* (2010 and 2013).

[7]Examples of two (or multi-country) models at monetary and financial institutions are Laxton and Pesenti (GEM-

Finally, our model is also fed with the vast literature on cross-country business cycle synchronization. Generally, output and other macroeconomic variables are found to be positively correlated across industrialized countries. In fact, we find for our sample period that the correlation for US and Canada real output growth per capita is 0.56. Schmitt-Grohe (1998) suggest that the majority of international real-business-cycle models cannot account for the pro-cyclicality between the Canadian economy and innovations to US output. In general, she concludes that trade alone does not explain the well-evidenced cross-country co-movements of the macroeconomic variables at business cycle frequency.

Our model addresses some of the limitations of previous theoretical and empirical literature by providing a medium-scale fully-fledged open economy New Keynesian model with endogenous gross trade and gross capital flows, to empirically analyze Canada and US bilateral economic relationships during the last few decades.

3 Two-country New Keynesian model

The model incorporates two economies that are referred as either the home or the foreign economy.[8] Free international trading among them takes place in markets for consumption goods and financial assets. There are domestic markets for labor services. Financial assets are of two types: equity issued by either domestic or foreign firms with a variable return determined by the dividend and bonds issued by either the domestic or the foreign governments that yield a risk-free interest rate. There is a flexible exchange rate toe equalize purchasing power and central banks operate their monetary policy by implementing a Taylor (1993)-type monetary policy rule. Both sticky prices and sticky wages introduce nominal rigidities to capture real effects of demand-side shocks.

3.1 Households

Let each economy contain a continuum of households indexed by j $\in [0, 1]$. The preferences of a representative infinitely-lived j household at time t are expressed in an intertemporal utility function whose arguments are a consumption index $c_t(j)$ and labour hours worked, $n_t(j)$. External consumption habits are determined by the parameter $0 < h < 1$ which measures the influence of lagged aggregate

Global Economy Model at the IMF, 2003), Erceg, Guerrieri and Gust (SIGMA at the Federal Reserve Board, 2003), Benigno and Thoenissen (Bank of England, 2003), Murchison, Rennison and Zhu (Bank of Canada, 2004), Adolfson *et al.* (Riksbank, 2005), and Kortelainen (Bank of Finland, 2002).

[8] The foreign economy will not be explicitly displayed here because it is structurally identical to the one presented in this section. See the Appendix for a complete description of all the model variables, parameters and equations.

consumption, c_{t-1}, on smoothing household-level consumption. The instantaneous utility function in period t takes the following form

$$U(c_t(j), n_t(j)) = \frac{(c_t(j) - hc_{t-1})^{1-\sigma}}{1-\sigma} - \psi \frac{n_t(j)^{1+\gamma}}{1+\gamma} \tag{1}$$

where $\sigma > 0$ is the risk aversion parameter, $\gamma > 0$ is the inverse of Frisch labor supply elasticity, and $\psi > 0$ is a scale parameter that weighs labor disutility with respect to total utility.

Following Schmitt-Grohe and Uribe (2003), β_t is an endogenous discount factor to ensure a stationary wealth distribution in the linearized approximated dynamic model.[9] In particular, the discount factor is a function of aggregate consumption determined as follows

$$\beta_{t+1} = \beta_t(1 + c_t)^{-\varsigma} \tag{2}$$

where $\varsigma > 0$ is a discount rate parameter. Due to identical preferences and symmetric equilibrium, household-level and aggregate consumption are equal, $c_t(j) = c_t$. The consumption bundle, $c_t(j)$, is represented by a Dixit and Stiglitz (1977)'s consumption index composed by baskets of home consumption goods, $c_{H,t}$, i.e. produced by home (H) firms, and foreign consumption goods, $c_{F,t}$, i.e. produced by foreign (F) firms and purchased (imported) by domestic households

$$c_t \equiv \left[(1-\alpha)^{\frac{1}{\theta}} (c_{H,t})^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} (c_{F,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where $\theta > 1$ denotes the elasticity of substitution between domestic and foreign goods from the viewpoint of a domestic household and $0 < \alpha < 1$ is inversely related to the degree of home bias in preferences.[10] For simplicity, we assume identical Dixit-Stiglitz aggregation schemes for $c_{H,t}$ and $c_{F,t}$

$$c_{H,t} \equiv \left(\int_0^1 c_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}, \; c_{F,t} \equiv \left(\int_0^1 c_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

with $\epsilon > 1$ denoting the elasticity of substitution between goods produced within the same economy. The optimal choices of domestic and imported goods imply the standard demand functions

$$c_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} c_t, \quad c_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} c_t, \tag{4}$$

where $P_{H,t}$ is the price for domestically produced goods expressed in domestic currency (Producer Pride Index, PPI) and $P_{F,t}$ is the price for foreign produced goods expressed also in domestic currency. The

[9] They propose five different ways to induce stationarity in an open economy model. We choose the endogenous discount factor for simplicity.

[10] If price indices for domestic and foreign goods are equal (as assumed in the steady state equilibrium), the model parameter α corresponds to the share of domestic consumption allocated to imported goods.

consumption-based price aggregation that corresponds to the Dixit-Stiglitz scheme gives the following Consumer Price Index (CPI)

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\theta} + \alpha P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{5}$$

and the rate of inflation from the CPI in period t therefore is $\pi_{t-1}^{CPI} = P_t/P_{t-1} - 1$.

The model allows for household heterogeneity on the labor services provided to firms and sticky wages, following Erceg *et al.* (2000). Thus, the representative j-type household faces the following Dixit-Stiglitz labour demand constraint that determines the amount of specific labour supply inversely depending on the relative wage.

$$n_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta_w} n_t \tag{6}$$

where $W_t(j)$ is the nominal wage earned by the type of labor supplied by household j, W_t and n_t are the Dixit-Stiglitz aggregate nominal wage and labor, respectively, and $\theta_w > 1$ is the elasticity of substitution between differentiated labor services.

Income is obtained from selling labour services in the market and from last-period portfolio holdings payments. There is a lump-sum tax charged by the government to the household. The uses of household income are purchases of consumption goods, net purchases of equity shares from home and foreign incumbents, and net purchases of domestic and foreign government bonds held during next period. The budget constraint imposed in period t for a representative household expressed in nominal terms

$$W_t(j)n_t(j) + (D_t + V_t)\, S_{H,t-1} + e_t\,(D_t^* + V_t^*)\, S_{F,t-1} + B_{H,t-1} + e_t B_{F,t-1} - Tax_t =$$
$$P_t c_t + V_t S_{H,t} + e_t V_t^* S_{F,t} + (1+R_t)^{-1} B_{H,t} + (1+R_t^*)^{-1} e_t B_{F,t},$$

where V_t refers to domestic equity value, V_t^* to foreign equity value, D_t and D_t^* refer, respectively, to domestic and foreign firm dividends, $S_{H,t}$ refers to the share of domestic equity held by domestic households and $S_{F,t}$ refers to that of foreign equity. $B_{H,t}$ and $B_{F,t}$ are the amount of domestic and foreign government bonds purchased by the domestic household in period t to be reimbursed in $t+1$. R_t refers to nominal interest rate set by the central bank in the domestic economy, R_t^* that of the foreign economy and e_t is the nominal exchange rate expressed in foreign currency. Using (6) in the labor income and introducing the aggregate nominal wage gives

$$\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} W_t n_t + (D_t + V_t)\, S_{H,t} + e_t\,(D_t^* + V_t^*)\, S_{F,t} + B_{H,t} + e_t B_{F,t} - Tax_t =$$
$$P_t c_t + V_t S_{H,t+1} + e_t V_t^* S_{F,t+1} + (1+R_t)^{-1} B_{H,t+1} + (1+R_t^*)^{-1} e_t B_{F,t+1}$$

Dividing both sides by the CPI, P_t, introducing the real exchange rate, $q_t = \frac{e_t P_t^*}{P_t}$, and the aggregate real wage, $w_t = \frac{W_t}{P_t}$, in the previous expression brings the budget constraint in real terms

$$\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + \frac{1}{P_t}\left(D_t + V_t\right) S_{H,t} + \frac{1}{P_t^*} q_t \left(D_t^* + V_t^*\right) S_{F,t} + \frac{B_{H,t}}{P_t} + q_t \frac{B_{F,t}}{P_t^*} - \frac{Tax_t}{P_t} = \quad (7)$$
$$c_t + \frac{1}{P_t} V_t S_{H,t+1} + \frac{1}{P_t^*} q_t V_t^* S_{F,t+1} + \left(1+R_t\right)^{-1} \mathbb{E}_t \frac{P_{t+1}}{P_{t+1}} \frac{B_{H,t+1}}{P_t} + \left(1+R_t^*\right)^{-1} e_t \frac{P_t^*}{P_t^*} \mathbb{E}_t \frac{P_{t+1}^*}{P_{t+1}^*} \frac{B_{F,t+1}}{P_t}$$

measured in terms of domestic bundles of consumption goods. Using the definition of the expected CPI inflation, $\mathbb{E}_t \frac{P_{t+1}}{P_t} = \mathbb{E}_t \left(1 + \pi_{t+1}^{CPI}\right)$, with the rational expectation operator evaluated in period t, $\mathbb{E}_t$, and the Fisher relation that introduces the *ex ante* real interest rate, $1 + r_t = \frac{1+R_t}{\mathbb{E}_t\left(1+\pi_{t+1}^{CPI}\right)}$, for both the domestic and foreign economies, transforms (7) as follows

$$\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + \frac{1}{P_t}\left(D_t + V_t\right) S_{H,t} + \frac{1}{P_t^*} q_t \left(D_t^* + V_t^*\right) S_{F,t} + \frac{B_{H,t}}{P_t} + q_t \frac{B_{F,t}}{P_t^*} - \frac{Tax_t}{P_t} =$$
$$c_t + \frac{1}{P_t} V_t S_{H,t+1} + \frac{1}{P_t^*} q_t V_t^* S_{F,t+1} + \left(1+r_t\right)^{-1} \frac{B_{H,t+1}}{P_{t+1}} + \left(1+r_t^*\right)^{-1} q_t \frac{B_{F,t+1}}{P_{t+1}^*} \quad (8)$$

To simplify notation, we suggest taking variables on lower-case letters to refer to the real value of those variables in upper case letters measured as units of the domestic consumption bundle. For example, $d_t = \frac{D_t}{P_t}$. Applying such notation rule through the expression (8) yields

$$\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + \left(d_t + v_t\right) S_{H,t} + q_t \left(d_t^* + v_t^*\right) S_{F,t} + b_{H,t} + q_t b_{F,t} - tax_t =$$
$$c_t + v_t S_{H,t+1} + q_t v_t^* S_{F,t+1} + \left(1+r_t\right)^{-1} b_{H,t+1} + \left(1+r_t^*\right)^{-1} q_t b_{F,t+1} \quad (9)$$

The representative household will maximize intertemporal utility subject to budget constraints as the one in period t, (9), and labor supply constraints as the one in period t, (6). Hence, the household will compute first order conditions to determine the optimal choices of consumption, c_t, the specific nominal wage, $W_t(j)$, the ownership shares of both domestic equity, $S_{H,t+1}$, and foreign equity, $S_{F,t+1}$, the purchases of both domestic bonds, $b_{H,t+1}$, and foreign bonds, $b_{F,t+1}$.

Sticky wages are introduced also as in Erceg *et al.* (2000), assuming that there is only a proportion $1 - \eta_w$ of households who can set optimally the nominal wages according to the Calvo (1983)-type fixed probability scheme. The remaining η_w share of the households will have to follow a weighted-indexation rule on lagged CPI inflation and steady-state CPI inflation. For an adjustment to take place in period t, the indexation factor is

$$(1 + idx_t^w) = (1 + \pi_{t-1}^{CPI})^{\kappa_w} (1 + \pi + \varepsilon_t^W)^{1-\kappa_w}$$

which includes the weight parameter $0 < \kappa_w < 1$ and a wage-push AR(1) shock, $\varepsilon_t^W = \rho_W \varepsilon_{t-1}^W + u_t^W$ with white-noise innovations $u_t^W \sim N(0, \sigma_{u^W})$. The first order condition on the optimal choice of the nominal wage brings a relative wage as a mark-up on the ratio between the marginal rate of substitution and the real wage

$$\widetilde{W}_t(j) = \frac{W_t(j)}{W_t} = \left(\frac{\theta_w}{\theta_w - 1}\right) \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \eta_w^k \frac{\beta_{t+k+1}}{\beta_{t+k}} \xi_{t+k} n_{t+k} \Pi_{s=1}^k \left(\frac{1+\pi_{t+s}^w}{1+idx_{t+s}^w}\right)^{\theta_w}}{\mathbb{E}_t \sum_{k=0}^{\infty} \eta_w^k \frac{\beta_{t+k+1}}{\beta_{t+k}} \lambda_{t+k} w_{t+k} n_t \Pi_{s=1}^k \left(\frac{1+\pi_{t+s}^w}{1+idx_{t+s}^w}\right)^{\theta_w - 1}}$$

where π^w gives the quarterly rate of nominal wage inflation and both λ and ξ, respectively, denote the marginal utility of consumption and the marginal disutility of labour for the households that can set the optimal wage in period t. From the Dixit-Stiglitz aggregator of nominal wages,

$$W_t = \left[(1 - \eta_w) W_t(j)^{1-\theta_w} + \eta_w \left((1 + idx_t^w) W_{t-1}\right)^{1-\theta_w}\right]^{1/(1-\theta_w)},$$

we can obtain the following expression for the relative wage

$$\widetilde{W}_t(j)^{(\theta_w - 1)} = (1 - \eta_w) + \eta_w \left[(1 + \pi_{t-1}^{CPI})^{\kappa_w} (1 + \pi_{ss}^{CPI} + \varepsilon_t^W)^{1-\kappa_w}\right]^{(1-\theta_w)} (1 + \pi_t^w)^{\theta_w - 1} \widetilde{W}_t(j)^{(\theta_w - 1)}$$

3.2 Firms

There is a continuum of producers of differentiated consumption goods that operate under monopolistic competition and seek to maximize their profits. They are indexed in the unit interval, so as to have the representative i firm where $i \in [0, 1]$. In this setup, there is no physical capital and the amount of output produce depends on labor employed and a technology shock. Let us denote $P_{H,t}(i)$ as the price set by the representative domestic (home) firm i in period t, and $P_{H,t}$ as the home producer price index. Firm dividend obtained by the representative firm is

$$d_t(i) = \frac{P_{H,t}(i)}{P_{H,t}} y_t(i) - \frac{P_t}{P_{H,t}} \frac{W_t}{P_t} n_t(i)$$

which faces a Dixit-Stiglitz demand constraint

$$y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} y_t$$

that implies

$$d_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon} y_t - \frac{P_t}{P_{H,t}} \frac{W_t}{P_t} n_t(i)$$

It should be noticed that firm dividend is measured in real terms as bundles of the consumption goods produced by all domestic firms. This is a different unit of measure from the bundle of all consumption goods for domestic households (which would also include imported goods produced by foreign firms). Therefore, it is convinient to introduce the relative prices as the ratio of the price index of domestically produced goods (Producer Price Index, PPI), denoted as $P_{H,t}$, and price of bundles of all consumption goods (CPI)

$$RP_t \equiv \frac{P_{H,t}}{P_t}; \tag{10}$$

which for the foreign economy would be $RP_t^* \equiv P_{F,t}^*/P_t^*$. The aggregation across the continuum of firms and the relative price definition, (10), result in the following real aggregate dividend, in terms of the household consumption bundle,

$$d_t \equiv \frac{P_{H,t}}{P_t}\int_0^1 d_t(i)di = RP_t\left(\int_0^1\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon}di\right)y_t - w_t\int_0^1 n_t(i)di$$

The production technology is linear on labor

$$y_t(i) = e^{\varepsilon_t^A}An_t(i)$$

and incorporates, through an exponential function, an AR(1) productivity shock, $\varepsilon_t^A = \rho_A\varepsilon_{t-1}^A + u_t^A$ with white-noise innovations $u_t^A \sim N(0,\sigma_{u^A})$. Using this production function and the Dixit-Stiglitz demand constraint in the labor cots of the aggregate dividend gives

$$d_t = y_t\left[RP_t\left(\int_0^1\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon}di\right) - \left(\frac{w_t}{e^{\varepsilon_t^A}A}\right)\int_0^1\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon}di\right] \tag{11}$$

Following Schmitt-Grohe and Uribe (2006), we define the price dispersion indicators

$$PD_t \equiv \int_0^1\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon}di \quad ; \quad PDD_t \equiv \int_0^1\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon}di$$

which can be plugged in (11) to reach the final expression for the aggregate dividend (measured in terms of domestically-produced bundles of consumption goods)

$$d_t = y_t\left[RP_tPD_t - \left(\frac{w_t}{e^{\varepsilon_t^A}A}\right)PDD_t\right] \tag{12}$$

Each firm sets the price of a unique differentiate good and earns some monopoly profit. We include price stickiness following Calvo (1983)-type rigidity for price adjustments. In this way, a fraction of $(1-\eta_p)$ randomly selected firms set optimal prices each period, with an individual firm's probability of re-setting in any given period being completely independent of the time elapsed since it last re-optimized its price. For cases when the firm cannot setthe optimal price, the price indexation factor takes this specific form

$$(1 + idx_{t+k}^p) = (1 + \pi_{t+k-1}^{PPI})^{\kappa_p}(1 + \pi + \varepsilon_{t+k}^P)^{1-\kappa_p}$$

that combines adjustments to lagged producer price inflation, $\pi_t^{PPI} = P_{H,t}/P_{H,t-1} - 1$, the steady-state price inflation, π, and the AR(1) price-push shock, $\varepsilon_t^p = \rho_p\varepsilon_{t-1}^p + u_t^p$ with white-noise innovations $u_t^p \sim N(0,\sigma_{u^p})$.

Assuming that the Calvo signal allows optimal pricing, the representative firm would choose $P_{H,t}(i)$ by maximizing the following intertemporal profit function

$$\sum_{k=0}^{\infty}\mathbb{E}_t\left[\frac{\Theta_{t+k+1}}{\Theta_{t+k}}\eta_p^k y_{t+k}\left(\int_0^1\left(\frac{RP_{t+k}(1+idx_{t+k}^p)P_{H,t}(i)}{P_{H,t+k}}\right)^{1-\epsilon}di-\left(\frac{w_{t+k}}{e^{\varepsilon_{t+k}^A}A}\right)\int_0^1\left(\frac{(1+idx_{t+k}^p)P_{H,t}(i)}{P_{H,t+k}}\right)^{-\epsilon}di\right)\right]$$

where $\frac{\Theta_{t+k+1}}{\Theta_{t+k}}$ is the stochastic discount factor between period $t+k$ and period $t+k+1$.[11] The (relative) optimal price obtained from the firm's maximization problem is

$$\widetilde{P_{H,t}}(i)=\frac{P_{H,t}(i)}{P_{H,t}}=\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\frac{\Theta_{t+k+1}}{\Theta_{t+k}}\eta_p^k\left(mc_{t+k}\right)y_{t+k}\Pi_{s=1}^k\left(\frac{1+\pi_{t+s}^p}{1+idx_{t+s}^p}\right)^{\epsilon}}{\mathbb{E}_t\sum_{k=0}^{\infty}\frac{\Theta_{t+k+1}}{\Theta_{t+k}}\eta_p^k y_{t+k}\Pi_{s=1}^k\left(\frac{1+\pi_{t+s}^p}{1+idx_{t+s}^p}\right)^{\epsilon-1}} \tag{13}$$

where $0<\eta_p<1$ is the Calvo probability and $mc_{t+k}=\frac{w_{t+k}}{RP_{t+k}\left(e^{\varepsilon_{t+k}^A}A\right)}$ is the real marginal cost in period $t+j$. From the Dixit-Stiglitz aggregator, we define the PPI of domestic firms,

$$P_{H,t}=\left[\left(1-\eta_p\right)P_{H,t}(i)^{1-\epsilon}+\eta_p\left(\left(1+idx_t^p\right)P_{H,t-1}\right)^{1-\epsilon}\right]^{1/(1-\epsilon)},$$

which implies this dynamic equation for the relative price of the representative firm

$$\widetilde{P_{H,t}}(i)^{(\epsilon-1)}=(1-\eta_p)+\eta_p\left[(1+\pi_{t-1}^{PPI})^{\kappa_p}(1+\pi+\varepsilon_t^P)^{1-\kappa_p}\right]^{(1-\epsilon)}(1+\pi_t^{PPI})^{\epsilon-1}\widetilde{P_{H,t}}(i)^{(\epsilon-1)} \tag{14}$$

3.3 Central bank

Nominal interest rate (R_t) is determined through a reaction function describing Taylor (1993)-type monetary policy decisions made by the the central bank

$$1+R_t=\left((1+r)\left(1+\pi\right)^{(1-\mu_\pi)}\right)^{(1-\mu_R)}(1+R_{t-1})^{\mu_R}\left(1+\pi_t^{CPI}\right)^{(1-\mu_R)\mu_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{(1-\mu_R)\mu_y}e^{\varepsilon_t^R} \tag{15}$$

where $\mu_\pi>1$ is the policy coefficient for responses to CPI inflation, $\mu_y>0$ is the policy coefficient for response to output growth, $0<\mu_R<1$ is the smoothing coefficient for gradual adjustments of the nominal interest rate and there is also an AR(1) monetary policy shock $\varepsilon_t^R=\rho_R\varepsilon_{t-1}^R+u_t^R$ with white-noise innovations $u_t^R\sim N(0,\sigma_{u^R})$. The constant term serves to pin down the steady-state rate relationship between nominal interest rate, real interest rate and inflation, $1+R=(1+r)(1+\pi)$.

A similar rule, with specific policy coefficients, is assumed for the monetary policy of the foreign economy.

[11] The domestic firm discount factor is not the household's inter-temporal marginal rate of substitution since the firm is owned by domestic and foreign agents, thus a weighted combination of the home and foreign discount factors is utilized as in Devereux and Sutherland (2010)

3.4 Government

Finally, lump-sum transfers of the government are financed by selling bonds to local and foreign households. In turn, the government budget constraint becomes in aggregate output terms. Note that r_t refers to real interest rate.

$$g_t = tax_t + \frac{b_{H,t+1}}{1+r_t} - b_{H,t} + \frac{b^*_{H,t+1}}{1+r_t} - b^*_{H,t} \tag{16}$$

where public spending g_t is exogenously determined. Particularly, deviations from the steady-state level of government purchases are determined as follows

$$g_t = e^{\varepsilon_t^g} g_{ss}$$

and the exogenous component ε_t^g is generated by an AR(1) time series $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \rho_{gA} u_t^A + u_t^g$ with white-noise innovations $u_t^g \sim N(0, \sigma_{u^g})$ and, following Smets and Wouters (2007), a cross effect coming from the innovations of the technology shock.[12]

3.5 Equilibrium conditions and the balance of payments

For a representative j good produced in the domestic economy, the market clearing condition is

$$y_t(i) = c_{H,t}(i) + c^*_{H,t}(i) + g_t(i)$$

The optimal choices of the home differentiated consumption good, decided by domestic and foreign households are, respectively,

$$c_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} c_{H,t} \text{ and } c^*_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon^*} c^*_{H,t}$$

Meanwhile, the government purchases the same amount of all domestic and foreign goods. Implementing the aggregation across domestically-produced goods, we obtain the aggregate goods market clearing condition

$$PD_t y_t = PD_t c_{H,t} + PD_t^* c^*_{H,t} + g_t \tag{17}$$

where $c_{H,t}$ corresponds to domestic demand for domestic goods and $c^*_{H,t}$ foreign demand for domestic goods (domestic exports), g_t is the amount of government purchases of domestic bundles, PD_t is the

[12] A innovation to the technology shock would result in a higher autonomous spending through either capital accumulation or net exports from the rest of the world excluding the foreign economy. Both elements are ignored in the model setup and could be captured by the cross correlation between technology innovations and variations in the exogenous component of aggregate spending.

price dispersion defined above, and PD_t^* is the following price dispersion indicator that takes the foreign elasticity of substitution

$$PD_t^* \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\epsilon^*} di$$

Asset markets clear at all times according to the following equilibrium conditions

$$S_{H,t} + S_{H,t}^* = S_{F,t} + S_{F,t}^* = 1; \tag{18}$$

$$b_{H,t} + b_{H,t}^* = b_{F,t} + b_{F,t}^* = 0 \tag{19}$$

Noticing that $S_{H,t}^*$ refers to the foreign share of domestic equity, and $S_{F,t}^*$ would refer to the foreign share of foreign equity. The same applies to $b_{F,t}^*$ and $b_{H,t}^*$.

In the labor market, the equilibrium condition is

$$\int_0^1 n_t(i)di = n_t$$

which combined with the production function, $y_t(i) = e^{\varepsilon_t^A} n_t(i)$, and the Dixit-Stiglitz demand constraint, $y_t(i) = (P_{H,t}(i)/P_{H,t})^{-\epsilon} y_t$, result in the following aggregate production function

$$y_t \int_0^1 (P_{H,t}(i)/P_{H,t})^{-\epsilon} di = e^{\varepsilon_t^A} n_t$$

or

$$PDD_t y_t = e^{\varepsilon_t^A} n_t$$

Finally, let us discuss the key ingredients of the balance of payments for the domestic economy: the net exports from trading in the goods market and the net foreign assets position from trading in the financial markets. Exports for domestic firms are decided by foreign households. Recalling the choice of foreign goods, (4), and applying it to the decision of foreign households, exports of domestic firms would be

$$ex_t = \alpha^* \left(\frac{P_{H,t}}{e_t P_t^*}\right)^{-\theta^*} c_t^*$$

where using the definition of the real exchange rate, $q_t = e_t P_t^*/P_t$, and relative prices, (10), we get

$$ex_t = \alpha^* \left(\frac{RP_t}{q_t}\right)^{-\theta^*} c_t^*$$

Imports are decided by domestic households as an inverse function of its relative price, (see $c_{F,t}$ in (4)), which means

$$im_t = \alpha \left(RP_t^* q_t\right)^{-\theta} c_t,$$

The trade balance of consumption goods determines net exports

$$NX_t = ex_t - ex_t$$

For the trading of finantial assets, we define the net foreign asset holdings in period t from the joint contribution of equity and bonds,

$$NFA_t \equiv [\alpha_{EF,t} + \alpha_{BF,t} - \alpha^*_{EH,t} - \alpha^*_{BH,t}]$$

where $\alpha_{EF,t} = q_t v^*_t S_{F,t+1}$ is the foreign equity holdings of the domestic households (expressed in domestic bundles through the real exchange rate), $\alpha_{BF,t} = (1 + r^*_t)^{-1} q_t b_{F,t+1}$ is the amount of foreign bond holdings of the domestic households (also expressed in domestic bundles), $\alpha^*_{EH,t} = v_t S^*_{H,t+1}$ is the amount of domestic equity purchased by foreign households and $\alpha^*_{BH,t} = (1 + r_t)^{-1} b^*_{H,t+1}$ is the amount of domestic bonds owned by foreign households. The reference asset will be domestic equity and the asset holdings of domestic households at the end of period $t-1$ are allocated in the column vector

$$\alpha_{t-1} = \begin{bmatrix} \alpha_{EF,t-1} \\ \alpha_{BH,t-1} \\ \alpha_{BF,t-1} \end{bmatrix} = \begin{bmatrix} q_{t-1} v^*_{t-1} S_{F,t} \\ (1 + r_{t-1})^{-1} b_{H,t} \\ \left(1 + r^*_{t-1}\right)^{-1} q_{t-1} b_{F,t} \end{bmatrix}$$

Moreover, the return differentials with respect to domestic equity used to determine the valuation effects and the NFA position are

$$r_{x,t-1} = \begin{bmatrix} \frac{q_t}{q_{t-1}} (1 + r_{EF,t-1}) - 1 - r_{EH,t-1} \\ r_{BH,t-1} - r_{EH,t-1} \\ \frac{q_t}{q_{t-1}} (1 + r_{BF,t-1}) - 1 - r_{EH,t-1} \end{bmatrix} = \begin{bmatrix} \frac{q_t}{q_{t-1}} \left(\frac{d^*_t + v^*_t}{v^*_{t-1}}\right) - \left(\frac{d_t + v_t}{v_{t-1}}\right) \\ r_{t-1} - \left(\left(\frac{d_t + v_t}{v_{t-1}}\right) - 1\right) \\ \frac{q_t}{q_{t-1}} \left(1 + r^*_{t-1}\right) - \left(\frac{d_t + v_t}{v_{t-1}}\right) \end{bmatrix}$$

which includes the differential, with respect to the domestic equity return, of returns from foreign equity (first row), domestic bonds (second row) and foreign bonds (third row). As carefully proved in the technical appendix, we could combine the household budget constraint, the government budget constraint, equilibrium conditions of the asset markets and the definitions of $r_{x,t-1}$ and α_{t-1} to obtain the following dynamic equation for the NFA position of the domestic economy

$$NFA_t - \left(\frac{d_t + v_t}{v_{t-1}}\right) NFA_{t-1} = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t - g_t - c_t + r'_{x,t}\alpha_{t-1}$$

Moreover, the valuation effects can be defined from the return differentials and the gain in the market value of lagged NFA

$$VAL_t = r'_{x,t}\alpha_{t-1} + \left(\frac{v_t}{v_{t-1}} - 1\right) NFA_{t-1}$$

in a way to extracted from the expression of NFA above to have the link between the change in NFA and valuation effects

$$\triangle NFA_t = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t + \left(\frac{d_t}{v_{t-1}}\right) NFA_{t-1} - (g_t + c_t) + VAL_t$$

The variation of NFA can be explained from two sources:

i) the difference between total domestic income, $w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t + \left(\frac{d_t}{v_{t-1}}\right) NFA_{t-1}$, and total domestic expenditures both from the public and the private sectors, $g_t + c_t$ which in national accounting terms would proxy for net exports, and

ii) the valuation effects that results from changes in the value of net asset holdings expressed in domestic currency.

The complete set of dynamic equations of the model and can be reviewed in the technical appendix. The model can be solved numerically using the *stoch_ simul* routine in Dynare. Even though the model is introduced in its original (non-linear) form, the numerical methods taken for obtaining the solution form use first-order approximations. The calibration of the model parameters is discussed next.

4 Calibration

The two-economy model is calibrated to represent Canada as the home economy and the US as the foreign economy. One time period in the model corresponds to one quarter, which is consistent with the short-run analysis of business cycle fluctuations. A symmetric calibration is initially assumed for both economies due to their high degree of economic integration and the similarities in the institutional framework and market regulation. Nevertheless, some of the parameters take country-specific values in order to accommodate differences in economic outcomes. The selection criteria for the asymmetric parameter calibration is twofold. First, we calibrate a subset of parameters to match long-run data properties individually observed in either US or Canada data. Second, the parameters that shape the stochastic elements of the model are specified at values that fairly replicate some of the business cycle patterns documented from either the US or Canada. In particular, we will look at second-moment statistics of real GDP growth, price inflation, wage inflation and the nominal interest rate. The data considered correspond to quarterly observations for the period 1990:1 to 2018:2, expressed in per capita terms of the population older than 16 years and seasonally adjusted. The values for the baseline model calibration are reported in Tables 1 and 2.

Table 1. Parameter calibration. Non-stochastic elements.

	Canada	US
Elasticity of consumption marginal utility	$\sigma = 1.39$	$\sigma^* = 1.39$
Consumption habits	$h = 0.71$	$h^* = 0.71$
Elasticity of hours marginal disutility	$\gamma = 1.92$	$\gamma^* = 1.92$
Weight of hours disutility	$\psi = 6.78$	$\psi^* = 6.78$
Discount rate parameter	$\varsigma = 0.0075$	$\varsigma^* = 0.0059$
Labor productivity	$A = 1.0$	$A^* = 1.26$
Elasticity of substitution across domestic goods	$\epsilon = 6.0$	$\epsilon^* = 6.0$
Elasticity of substitution across labor services	$\theta_w = 3.0$	$\theta_w^* = 3.0$
Elasticity of substitution between domestic and foreign goods	$\theta = 1.5$	$\theta^* = 1.5$
Home good bias	$\alpha = 0.36$	$\alpha^* = 0.26$
Steady-state ratio of autonomous spending over GDP	$g_{ss}/y_{ss} = 0.41$	$g_{ss}^*/y_{ss}^* = 0.38$
Calvo probability for price stickiness	$\eta_p = 2/3$	$\eta_p^* = 0.85$
Calvo probability for wage stickiness	$\eta_w = 0.75$	$\eta_w^* = 0.5$
Weight of price indexation on lagged inflation	$\kappa_p = 0.2$	$\kappa_p^* = 0.2$
Weight of wage indexation on lagged inflation	$\kappa_w = 0.2$	$\kappa_w^* = 0.2$
Inflation coefficient in monetary policy rule	$\mu_\pi = 1.5$	$\mu_\pi^* = 1.5$
Output coefficient in monetary policy rule	$\mu_y = 0.5/4$	$\mu_y^* = 0.5/4$
Smoothing coefficient in monetary policy rule	$\mu_R = 0.9$	$\mu_R^* = 0.9$

The parameters that characterize household preferences have been set identically for both economies. Thus, we use the values of the elasticities on consumption marginal utility, σ, and hours marginal disutility, γ, found for the US in the estimated DSGE model of Smets and Wouters (2007) for both the US and Canada. Likewise, the consumption habits parameter, h, takes the value reported by Smets and Wouters (2007). The weight of the hours disutility contribution to the overall utility takes the value that results in a normalized labor $n = 1$ in the steady state solution of the model for Canada.

The discount factor parameter ς has a substantial impact on the steady-state real interest rate, r. Thus, it also plays a crucial role on the steady state value of firm equity ($\nu = d/r$, where v is equity and d is firm dividend in steady state) and the net foreign asset position. Since the historical average of net foreign assets of Canada with respect to the US is negative, we have decided to calibrate ς at different values for Canada and the US. Our target has been the average net foreign assets over GDP for Canada in its trading with the US from 1990 to 2018. We make it correspond to the steady-state ratio NFA/y in the model. After the calibration, $\varsigma = 0.0075$ and $\varsigma^* = 0.0059$, the model delivers $NFA/y = -0.65$ close enough to the mean value of -0.63 observed in the data displayed in Figure 1. The steady-state real interest rate in the model is common for both economies at $r = r^* = 0.0033$, an annualized rate of 1.32%.

The production technology ignores capital accumulation and a linear function relates employment to output produced. Since the average per capita real GDP has been 18% higher in the US than Canada, we have decided to set labor productivity A as a country-specific value that matches the average Canada/US ratio in the steady-state solution of the model. Setting $A = 1.0$ and $A^* = 1.26$, the steady-state solution of the model implies $y/y^* = 0.85$, which is pricesely the inverse value of 1.18.

The internal elasticities of substitution in the goods market (demand for consumption goods, ϵ) and in the labor market (supply for labor services, θ_w) are set are standard values from the DSGE literature for both the US and Canada. Thus, we fix $\epsilon = \epsilon^* = 6.0$ to imply a 20% steady-state mark-up of prices over the marginal cost, and $\theta_w = \theta_w^* = 3$ to have a wedge between the real wage and the marginal rate of substitution of 50%. It is known that the estimation of the intra-temporal elasticity of substitution between domestic and foreign goods, θ, is quite controversial (Justiniano and Preston, 2004; Adolfson *et al.,* 2004). Most of the existing theoretical papers use a value of 1.5, as suggested by Backus *et al.* (1992). We follow this criterion for both Canada and the US and give $\theta = \theta^* = 1.5$.

The home goods bias parameter is set at $\alpha = 0.36$ in Canada and at a lower value $\alpha^* = 0.26$ in the US. It should be noticed that a lower α implies a stronger preference for domestic good relative to foreign goods. We have introduced this asymmetry to render a positive net exports for Canada with the US. The data show a mean value of Canadian net exports to the US equivalent to 3% of Canadian GDP over the sample period 1990-2018. Our steady-state solution gives $nx/y = 0.03$. The mean value of Canadian exports to the US and Canadian imports from the US over Canadian GDP are also replicated in the steady state solution of the model with $ex/y = 0.22$ and $im/y = 0.19$.

The steady state share of government expenditures over GDP is calibrated at $g/y = 0.41$ in Canada and, slightly lower in the US, $g^*/y^* = 0.38$. This choice is based on the fact that private consumption represents a higher share over GDP in the US than in Canada (67% versus 56% on average from 1990 to 2018). Having a lower share of autonomous spending raises the share from purchases of consumption goods.[13] The steady-state solution of the model brings $c/y = 0.56$ and $c^*/y^* = 0.65$ which provides a good matching to the actual ratios observed in Canada and the US.

[13] Our model abstracts from capital accumulation and net exports with the rest of the world. Thus, the government share parameter is set to a higher value than its empirical counterpart for Canada and the US, since it captures all factors determining output that are model-exogenous, including investment and net exports.

The monetary policy parameters are set at identical values for the US and Canada. Both the inflation and output coefficients are the ones recommended in the original Taylor (1993) rule, $\mu_\pi = \mu_\pi^* = 1.5$ and $\mu_y = \mu_y^* = 0.5/4$, whereas the smoothing parameter is at $\mu_R = \mu_R^* = 0.9$ to reproduce the long inertia and slow adjustments of policy rates set by the Fed and the Bank of Canada.

The remaining parameters have been calibrated looking at the characteristics of the quarterly fluctuations observed in the data. In particular, we have paid attention to the series of (per capita) real GDP growth, producer price inflation (from the GDP implicit price deflator), nominal wage inflation and the nominal interest rates displayed in Figure 2 and with second-moment statistics reported in Table 3.[14] Price inflation less volatility and more persistence in the US than in Canada. Thus, we have assumed more price rigidities in the US and set a Calvo probability for the foreign economy at $\eta_p^* = 0.85$, whereas the domestic economy (Canada) takes a substantial lower value $\eta_p = 2/3$. No autocorrelation is assumed for the price-push shocks of both economies to keep the inflation autocorrelation low. The lagged inflation component of the price indexation rule is fixed at a small value for both economies $\kappa_p = \kappa_p^* = 0.2$ also to avoid excessive inflation inertia in the model. On the comparison shown at Table 3, it can be observed that model simulations provide a good fit of Canadian inflation volatility (standard deviation) but US inflation volatility is significantly lower in the data (both from PPI and CPI inflation). Moreover, the model overestimates the inflation inertia, as it embeds a price rigidity structure that inherently results in high autocorrelation for inflation. Similar comments can be mentioned for the US inflation.

Regarding wage inflation, we introduce asymmetric behavior on wage setting. The Calvo probability for wage stickiness is set at the standard value $\eta_w = 0.75$ in Canada and at a lower value in the US, $\eta_w^* = 0.5$. As Table 3 reports, the model brings more persistence and less volatility of wage inflation in Canada than in the US which provides a reasonable fit to the behavior observed in the data.[15] Wage indexation on lagged inflation is weak both in Canada ($\kappa_w = 0.2$) and in the US ($\kappa_w^* = 0.2$) to generate low wage inflation inertia but still accommodate for the effects of wage-push shocks entering the wage indexation rule.

Regarding the calibration of the generating processes for the exogenous variables (see Table 2), the technology shock and the monetary policy shock are common for both economies (global shocks). This helps to obtain a business cycle synchronization and it gives a positive correlation between output

[14] The time series of the US nominal interest rate correspond to the shadow interest rate elaborated by Wu and Xia (2016) to bring the effects of uncomventional monetary policy in the quarters of the zero lower bound constraint. A detailed description of the time series taken from the data is available in the technical appendix.

[15] Our sticky-wage model cannot replicate the negative autocorrelation of US wage inflation, which comes explained by the erratic fluctuations observed after 2008 (see Figure 3).

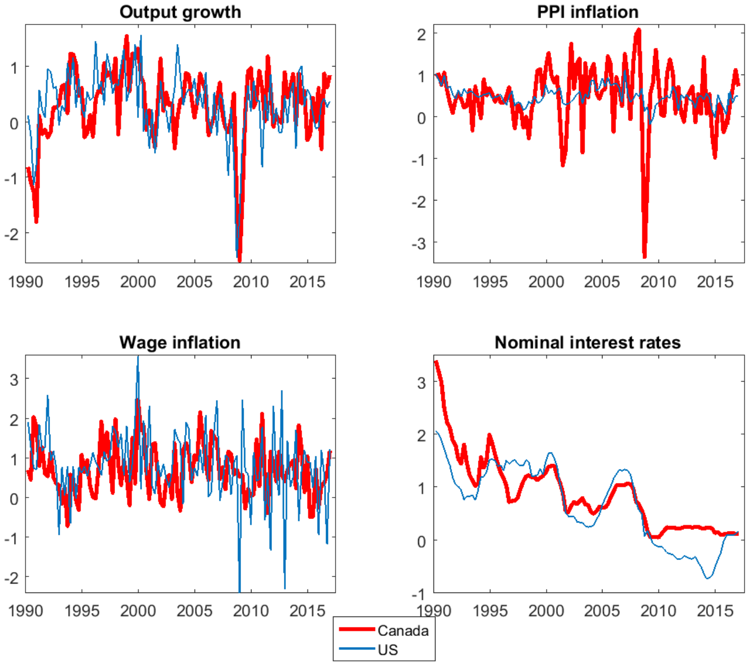

Figure 2: Quarterly macroeconomic fluctuations in Canada and the US, 1990-2018

growth of both the domestic economy (Canada) and the foreign economy (US) in the model at 0.52, close to the number found in the data, 0.56. The coefficient of autocorrelation of the technology shock is the usual value, 0.95 and the autocorrelation of the monetary policy shock is low at 0.35, close to the estimated number reported by Smets and Wouters (2007). The standard deviation of the innovations to technology and monetary shocks are decided to match the variability of output growth observed in the data and also to obtain reasonable shares of the impact of these shocks on the long-run output growth variance decomposition.[16]

[16] In the calibrated model, technology shocks explain 40% of output growth variability for Canada and 14% for the US. Monetary policy shocks take a share of 23% of Canadian output growth variance decomposition and 24% of that of the US output growth.

Table 2. Parameter calibration. Stochastic elements.

	Canada	US
Technology shock, standard deviation of the innovation, %	$\sigma_{u^A} = 0.89$	
Technology shock, coefficient of autocorrelation	$\rho_A = 0.95$	
Monetary policy shock, standard deviation of the innovation, %	$\sigma_{u^R} = 0.17$	
Monetary policy shock, coefficient of autocorrelation	$\rho_R = 0.35$	
Public spending shock, standard deviation of the innovation, %	$\sigma_{u^g} = 1.05$	$\sigma^*_{u^g} = 1.41$
Public spending shock, coefficient of autocorrelation	$\rho_g = 0.9$	$\rho^*_g = 0.9$
Public spending shock, cross correlation with tech. innovations	$\rho_{gA} = 0.6$	$\rho^*_{gA} = 0.6$
Price-push shock, standard deviation of the innovation, %	$\sigma_{u^p} = 0.3$	$\sigma^*_{u^p} = 0.09$
Price-push shock, coefficient of autocorrelation	$\rho_p = 0.0$	$\rho^*_p = 0.0$
Wage-push shock, standard deviation of the innovation, %	$\sigma_{u^W} = 0.67$	$\sigma^*_{u^W} = 1.10$
Wage-push shock, coefficient of autocorrelation	$\rho_W = 0.0$	$\rho^*_W = 0.0$

The public spending shock features long inertia for both Canada and the US (coefficient of autocorrelation at 0.9 in both cases), and a higher volatility on the innovations for the US in order to match the standard deviation of output growth observed in the data. The price-push shocks have no serial correlation to prevent the price inflation autocorrelation from rising. The standard deviation of the price-push innovations are calibrated at the values that give a good fit to the price inflation volatility observed in Canadian data. The standard deviation of US price shocks is just 30% of that set for Canada in order to replicate the relative observed variability of Canada and US inflation. As the wage-push shocks, they are not serially correlated because of the lack of persistence of wage inflation in the data and the innovations volatility is greater in the US also to approximately match the observed relative standard deviations.

Table 3. US-Canada descriptive statistics and their model-based values with baseline calibration

	Canada		US	
	Data	Model	Data	Model
Second-moment statistics				
Standard deviation of GDP rate of growth, %	0.68	0.67	0.60	0.59
Standard deviation of PPI inflation, %	0.74	0.61	0.22	0.53
Standard deviation of CPI inflation, %	0.64	0.56	0.62	0.54
Standard deviation of wage inflation, %	0.66	0.65	0.94	1.05
Standard deviation of the interest rate, %	0.94	0.30	0.69	0.31
Cross correlation across output growth	0.56	0.52	0.56	0.52
Cross correlation between PPI inflation and output growth	0.26	-0.41	-0.03	-0.06
Cross correlation between CPI inflation and output growth	-0.26	-0.34	0.16	-0.11
Cross correlation between wage inflation and output growth	-0.01	-0.02	-0.09	0.14
Cross correlation between the interest rate and output growth	-0.24	-0.23	0.11	-0.16
Autoorrelation of output growth	0.56	0.33	0.40	0.14
Autocorrelation of PPI inflation	0.36	0.74	0.52	0.89
Autocorrelation of CPI inflation	0.25	0.79	0.20	0.87
Autocorrelation of wage inflation	0.19	0.35	-0.23	0.34
Autocorrelation of interest rates	0.98	0.88	0.98	0.89

Finally, let us discuss the calibration of the asset holdings that enter the NFA equation to account for the valuation effects. The standard solution method, based on first-order approximations, cannot pin down a unique solution path for each portfolio asset holdings (e.g., $\alpha_{EH,t}$, $\alpha_{EF,t}$, $\alpha_{BH,t}$ and $\alpha_{BF,t}$ for the domestic economy).[17] The focus of this paper is to understand the factors behind a large increase in the Canadian NFA, given the specific position of Canada in each financial asset with the US. In fact, in the data (taking the period 1990-2018), the average US bond holdings owned by Canadian investors represent 17% of Canadian GDP, the average Canadian bond holdings owned by US investors is 83% of Canadian GDP and US equity holdings owned by Canadian investors account for 145% of Canadian GDP. These facts are used to fix the values of asset holdings as the following proportions to steady-state domestic output: $\alpha_{EF} = 1.45 y_{ss}$, $\alpha_{BH} = -0.83 y_{ss}$, and $\alpha_{BF} = 0.17 y_{ss}$.

[17] See the Appendix for a more detailed explanation.

5 Impulse-response functions

Next, we are going to discuss the propagation channels from changes in the exogenous variables to their effects over the endogenous variables. The calibrated model incorporates eight shocks and, due to space restrictions, we will analyze here the dynamic effects of five of them: a technology shock (Figure 3), a monetary policy shock (Figure 4), a public spending shock (Figure 5), a price-push shock (Figure 6), and a wage-push shock (Figure 7). In the case of the technology and monetary shock, the exogenous variation would be simultaneously affecting both the home (Canada) and foreign (US) economies as examples of global shocks. The other three cases (fiscal, price and wage shocks) represent innovations that initially enter the home (Canada) economies. The specific shocks to the foreign economy (US) are not discussed here but the effects for the domestic economy (Canada) can somehow be anticipated by the responses of the foreign economy to domestic shocks in Figures 5, 6 and 7.

Figure 3 plots the responses to a one standard deviation technology shock, which would increase labor productivity by 0.89% simultaneously in the home economy (Canada) and the foreign economy (US). The fraction of firms that receive the Calvo signal to set the optimal prices will decide a lower price and they will observe a higher demand for consumption goods. As more fractions adjust optimally the price in Canada than in the US, the PPI inflation rate falls further in Canada. The rates of CPI inflation report more similar declines across the two economies because they incorporate the prices of foreign goods (imports). The central banks will reduce the nominal interest rate in reaction to lower CPI inflation falls (and despite the output growth). The real interest rate increases as the expected inflation slides down below the steady state rate. Exports and imports increase for both economies because international trading rises with higher household income and the taste for both domestic and foreign goods. Canadian net exports turn positive: taking advantage of the real exchange rate depreciating due to lower inflation in Canada than in the US. Wage inflation falls in both cases because of the decline in the marginal rate of substitution between hours and consumption. The fraction of households that can set their optimal wage will prefer a lower wage that increases their labor supply. As there are more firms setting a lower wage in the US than Canada (lower Calvo probability assumed in the calibration), wages fall more sharply in Canada. In turn, the real wage will be higher in Canada than in the US, while firm dividends will be higher in the US than in Canada.

The effects of a global monetary policy shock are displayed in Figure 4. An interest-rate shock that identically enters the monetary policy rule (15) for the home economy (Canada) and the analogous rule for the foreign economy (US) can represent the scenario of a higher cost of borrowing that emerged

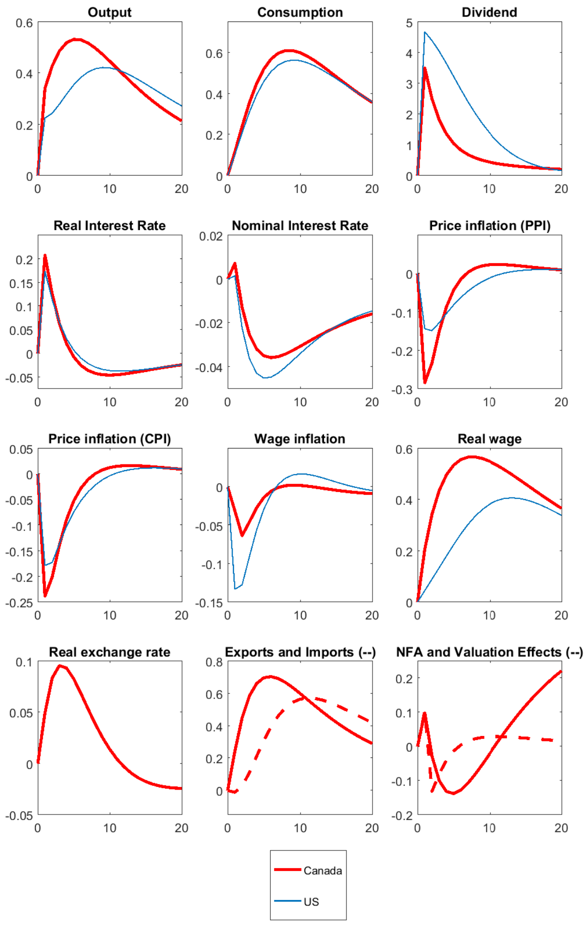

Figure 3: Impulse-response functions following a (global) technology shock

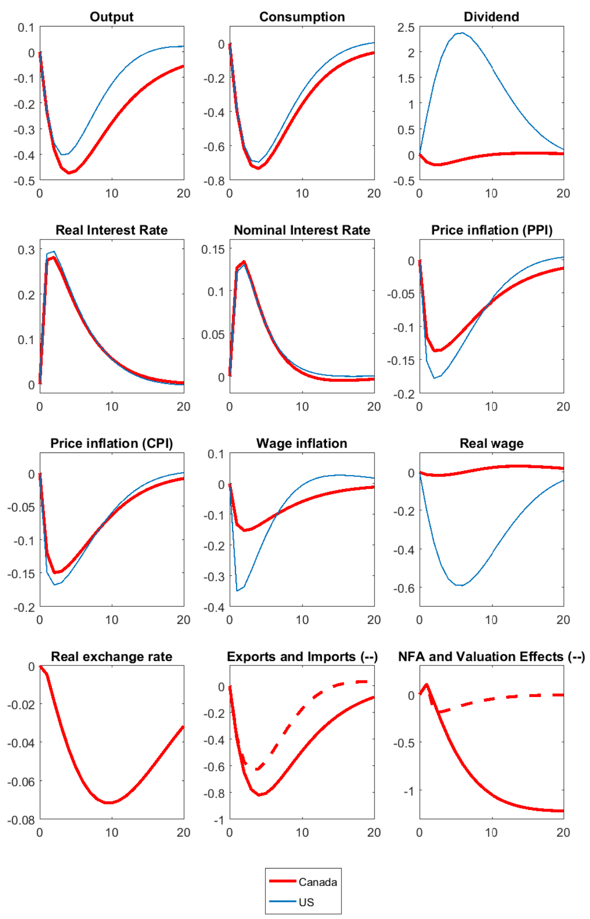

Figure 4: Impulse-response functions following a (global) interest rate shock

in the advance economies during the financial crisis of 2008. The shock results in a an increase of the nominal interest rate of 12.5 basis points (0.5% increase of the annualized nominal interest rate). The real interest rate reports a larger increase (close to 30 basis points) due to the expected deflation that the contractionary shock generates. As household demand for consumption goods falls, firms demand less labor and cut production downwards. Thus, the real wage falls and the fraction of firms that can optimize on price setting would decide to charge a lower price. Nominal interest rates, consumption and real interest rates report similar responses for both Canada and the US. However, wage inflation and the real wage would have a more severe adjustment downwards in the US than in Canada. With a lower Calvo probability for wage rigidity, there will be a larger fraction of US households that would set a lower wage as they wish to supply more labor in response to a decrease in the marginal disutility from labor. The differences in the response of wage inflation makes the real wage fall significantly in the US whereas it barely moves down in Canada. The dividends of US firms become higher taking advantage of the lower cost of production, whereas Canadian firms see initially small profit reduction.

Firms price inflation (PPI) report similar declining patterns in US and Canada, with a more severe fall in the US due to the larger reduction in the marginal cost of production (and despite having less firms adjusting prices optimally). In turn, CPI inflation gets reduced slightly further in the US than in Canada. Regarding international trade, both Canadian exports and imports fall with the global recession. Moreover, Canadian net exports are negative because exports fall deeper than the reduction of imports. The reason for the negative current account effect in Canada is its real exchange rate appreciation (lower value) which comes along from the higher relative CPI prices in Canada. As a consequence, the US takes advantage of the external Canadian demand to reduce the negative impact of the monetary shock. The recession is milder in the US than in Canada as it is showed in the comparison of the output responses displayed in Figure 4.

The effects of a country-specific public spending shock in Canada can be seen in Figure 5. As the autonomous component of aggregate demand rises firms increase their sales and demand more labor. The cost of production increases in Canada and the fraction of firms that can set the optimal price will move it upwards. The rate of PPI inflation rises. The increase of home prices make Canadian households substitute domestic goods for US goods. Subsequently, the CPI inflation reports an initial drop at the time of the shock that is quickly corrected with the effect of higher prices of domestic goods.

The public spending shock has crowding-out effects on both domestic consumption and net exports. As the real interest rate increases, consumption reports a fall with a trough value observed five quarters after the shock of size equivalent to 1/6 of the impact on aggregate output. Meanwhile, the real

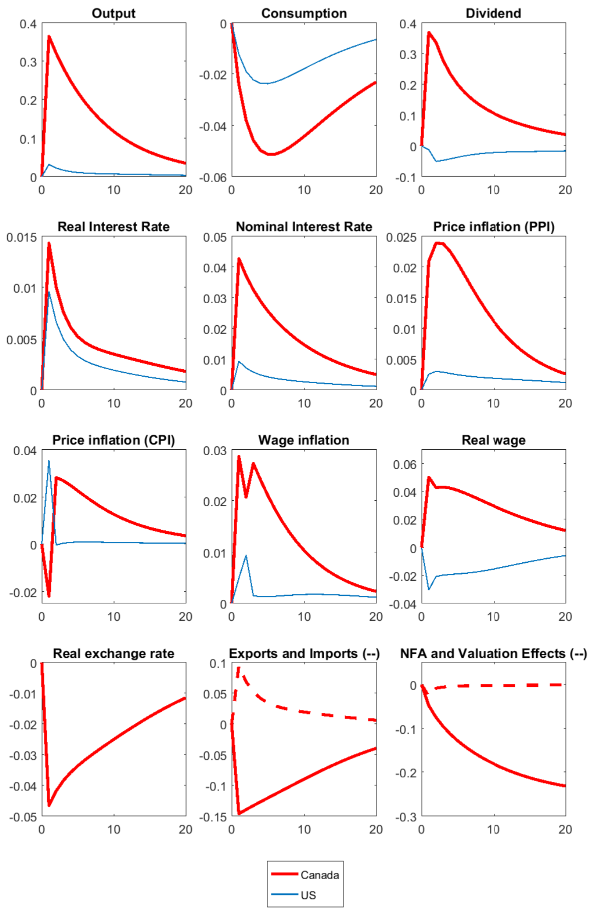

Figure 5: Impulse-response functions following a Canadian spending shock

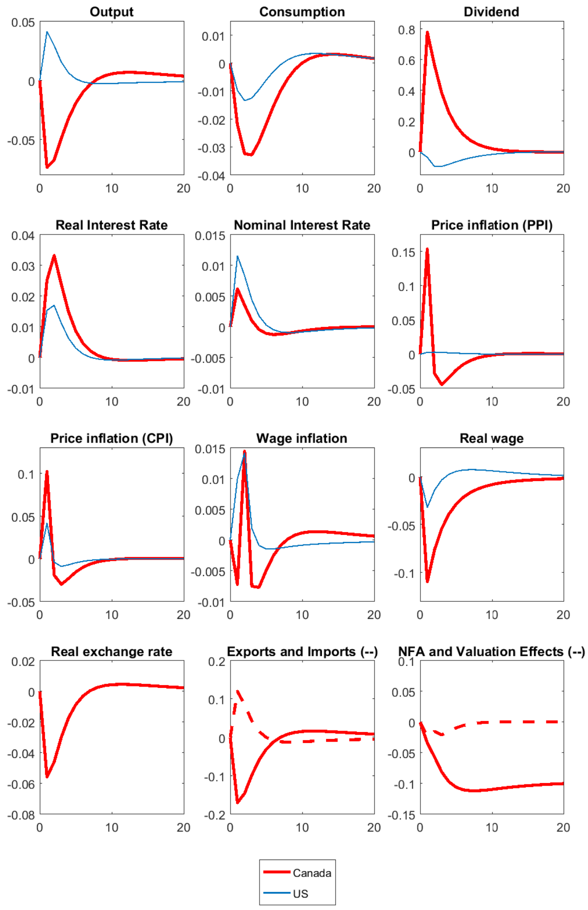

Figure 6: Impulse-response functions following a Canadian price-push shock

exchange rate appreciates and net exports are negative jointly caused by the fall in exports and the increase of imports.

The effects transmitted to the foreign (US) economy are mildly expansionary through external demand. Canadian imports are US exports that increase the demand for US goods. In turn, the responses observed in the US are higher output produced by firms, some price and wage inflation, higher nominal and real interest rates, and some crowding-out effect on consumption. All the responses have a significantly smaller size than the ones found for Canada.

Figure 6 provides the responses of a price-push shock that only hits the home economy. The fraction of Canadian firms that cannot set the optimal price will charge a higher selling price as they apply the indexation rule. The effects of the inflation shock on aggregate output are of reversed sign between the home economy (Canadian output falls) and the foreign economy (US output rises). As prices of Canadian producers increase the two endogenous components of aggregate demand (consumption and

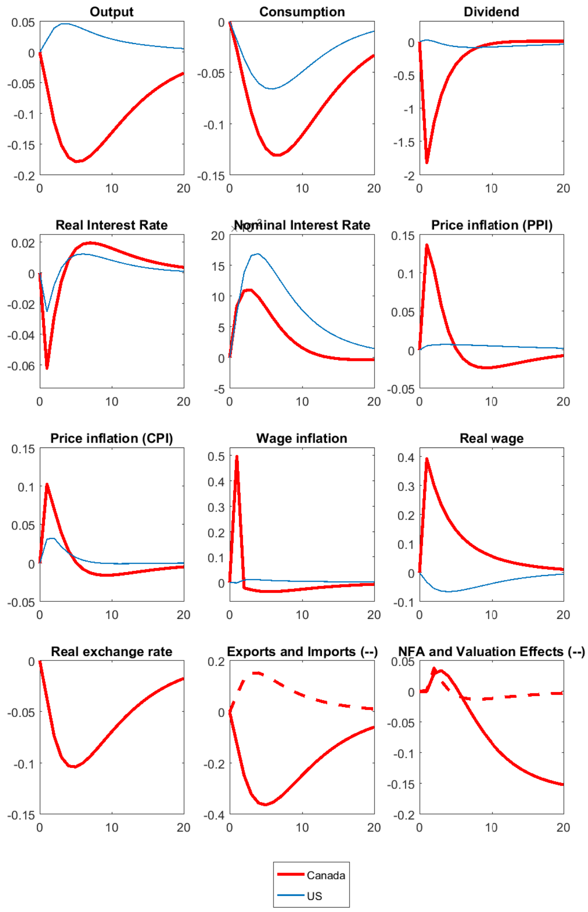

Figure 7: Impulse-response functions following a Canadian wage-push shock

net exports) react with falls. The drop in domestic consumption is due to the preference for future consumption (savings) as households see the higher interest rates set by the central bank in reaction to the inflationary episode. Meanwhile, the real exchange rate appreciates with higher producer prices in Canada than in the US and net exports move downwards. Since households of both economies substitute between domestic and foreign goods, Canadian exports fall and their imported goods from the US increase.

The effect on wage inflation is very little. Following the impact of less labor employed in their marginal disutility, Canadian households decide to set lower wages while US households prefer higher wages. In both economies, the real wage falls because of higher inflation, especially in Canada where the impact of the inflation shock is primarily received. Firm dividends in Canada increase substantially because of higher firm revenues (higher selling prices) and lower costs of production (lower real wage).

Finally, the propagation of a Canada wage-push shock can be examined in Figure 7. Those Canadian households who cannot decide the optimal nominal wage adjust it upwards in the implementation of the wage indexation rule. Wage inflation rises and both the real wage and the cost of production move up. The pool of Canadian firms that can revise the price optimally respond to the higher cost of production charging higher prices. Producer inflation rises and the central bank announces a higher nominal interest rate as prescribed by the Taylor (1993)-type rule (15). The Canadian real wage goes up and firm dividends suffer a significant decline as a consequence of the larger cost of labor.

The real exchange rate appreciates due to the relative increase of Canadian CPI, and consumers substitute Canadian goods for US goods. As exports fall and imports rise, Canadian net exports move down and the aggregate demand drops. Moreover, consumption of domestic goods describes a u-shape downwards pattern in reaction to higher expected real interest rates (from the initial decline). The overall effect on output is also characterized by a u-shape plot. As for the foreign (US) economy, there is some price inflation and output growth caused by the Canadian wage shock. The boost of external demand (imports of Canadian households from US firms increase) explains that US output rises. As for inflation, there is a pass-through effect from the price of Canadian imported goods to the US price index (CPI). This effect eventually rises the nominal and real interest rates, which bring a fall of US consumption in a similar pattern to that of Canadian consumption.

6 The reversed two-way capital flows

This section takes the model to discuss the determinants, specifically, of the recent evolution in the Net Foreign Asset (NFA) position of Canada with respect to the US. As documented above (see Figure 1), there have been "reversed two-way" capital flows between Canada and the US from 2012 to 2018:

- FDI and the net Portfolio Equity position of Canada with respect of the US describes an upwards trend that moves the initial -50% of GDP in 2012 to more than 100% of GDP in 2018.

- The net Portfolio Bonds position remains on the negative sign and the unbalance keeps growing over the 2012-2018 period to reach the size of Canadian GDP

Since the quantitative implications for NFA are larger in the upwards move of net equity holdings than the downwards move of net bond holdings, the NFA position of Canada with respect to the US switches from being markedly negative upwards to close to the zero level. These capital flows took place along a significant reduction of Canadian net exports with the US, which started after the financial crisis of 2008.[18] Therefore, valuation effects should explain the variations observed in the

[18] Betwwen 1998 and 2007, Canadian net exports with the US represented on average 5.2% of Canadian GDP. From 2008 to 2017, this number has fallen to 1.7%. In the second quarter of 2016, there was even a current account deficit for Canada with the US equivalent to -0.28% of its GDP.

position of the Canadian NFA with the US.

As the model cannot directly show fluctuations of gross capital flows due to the identification issues on asset holdings, we will analyze the return differentials for the international multi-asset portfolio to discuss the determinants of the reversed two-way phenomenon. Hence, let us introduce the *effective* rates of return obtained in the four assets available for the home/foreign households. These will be different from the *ex ante* expected returns that show up in the first order conditions. Obviously, the no arbitrage condition requires that *ex ante* all the returns get equalized in the portfolio choice. The evolution *ex post* may determine strategic decisions on how to reallocate asset holdings. Such effective returns on Canadian bonds, US bonds, Canadian equity and US equity (all of them expressed in terms of Canadian currency and evaluated in percentage annualized terms) are

$$
\begin{aligned}
r_{B,t}^{CAN} &= 400\left[(1+r_{t-1})\frac{1+E_{t-1}\pi_t^{CPI}}{1+\pi_t^{CPI}}-1\right] \\
r_{B,t}^{US} &= 400\left[(1+r_{t-1}^{*})\frac{1+E_{t-1}\pi_t^{CPI,*}}{1+\pi_t^{CPI,*}}\frac{q_t}{q_{t-1}}-1\right] \\
r_{E,t}^{CAN} &= 400\left[\left(\frac{d_t+v_t}{v_{t-1}}\right)-1\right], \text{ and} \\
r_{E,t}^{US} &= 400\left[\left(\frac{d_t^{*}+v_t^{*}}{v_{t-1}^{*}}\right)\frac{q_t}{q_{t-1}}-1\right].
\end{aligned}
$$

The differences between *ex ante* and *ex post* returns emerge from the wedge created between expected inflation and actual inflation and from the evolution of the real exchange rate that would imply some gains or losses when conducting international transactions. Our conjecture to explain the portfolio asset substitutions that would bring reversed two-way capital flows is

$$
\begin{aligned}
r_{B,t}^{CAN} &> r_{B,t}^{US}, \text{ and} \\
r_{E,t}^{US} &> r_{E,t}^{CAN},
\end{aligned} \tag{20}
$$

together with an increase in the overall NFA position of Canada with the US

$$
\triangle NFA_t = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t + \left(\frac{d_t}{v_{t-1}}\right) NFA_{t-1} - (g_t + c_t) + VAL_t > 0 \tag{21}
$$

where the valuation effects, VAL_t, are determined by return differentials and gains/losses from the lagged NFA position

$$
VAL_t = r_{x,t}'\alpha_{t-1} + \left(\frac{v_t}{v_{t-1}} - 1\right) NFA_{t-1} \tag{22}
$$

Figure 8 provides the plots of the returns of r_B^{CAN}, r_B^{US}, r_E^{CAN} and r_E^{US} following each one of the eight shocks of the model, and Table 4a reports the values obtained in the responses of these returns. Meanwhile, the responses of both NFA and valuation effects for the home economy (Canada) had been plotted in the corresponding last boxes of Figures 3 through 7 and numerically documented, for all shocks, in Table 4b.[19]

The global technology shock brings an economic expansion and pushes up the annualized returns on both equity (around 3.5%) and bonds (around 2.25%) above the steady-state real rate of return (1.32%). In the comparison across asset types, Figure 8 displays and Table 4a reports that equity investments turn more profitable than purchasing bonds both in the US and Canada. Such equity premium is 1.06% in Canada and 1.34% in the US. The return differentials across countries are barely noticeable at first eyesight on Figure 8. As Table 4a documents, numbers are small, slightly favorable to Canadian bonds (4 basis points) and US equity (23 basis points), but their order of magnitude would not probably justify the massive capital flows observed in the data. The NFA position plotted in Figure 3 indicates that there are initially net losses on Canadian asset holdings which are reversed towards net gains five quarters after the shock and even move onto the positive side from the tenth quarter after the shock onwards. Valuation effects are positive at the quarter of the shock because the bond returns fall below the equity return and Canadian bonds are owned by US citizens. One quarter after the shock, the valuation effects turn negative because the return differentials vanish and the market value of Canadian foreign debt with the US rises. After four quarters, Canadian NFA starts rising because valuation effects disappear and there are positive net exports. The response of Canadian NFA twelve quarters after the shock is positive (+0.067) as Table 4b reports.

Hence, it could be argued that a positive global shock brings the conditions for two-reversed capital flows with two limitations: the return differentials are quantitatively small and the positive response of NFA is found with lag of approximately 2.5 years.

As displayed in Figure 8, the global monetary (interest-rate) shock, that simultaneously occurs in Canada and the US, has a positive impact on the return of bonds and a severe and negative effect on the equity return. Both firm dividends and equity value fall as interest rates increase. The rates of return are similar for Canada and the US with little differences in bond premia (see Table 4a). The interest rate of the Canadian bond rises to 1.81% but falls 9.8 basis points below that of the US bond. Equity returns turn clearly negative in both economies (near -6%), and the fall of Canadian equity is 34 basis points deeper than US equity. The equity return differentials may explain the Canadian purchases of US equity observed in the data, but the US purchases of Canadian bonds is not supported

[19] It should be noticed that the valuation effects are obtained in the calibrated model for fixed asset holdings at $\alpha_{EF} = 1.45y$, $\alpha_{BH} = -0.83y$, and $\alpha_{BF} = 0.17y$, which implies a negative effects from the excess return of home (Canada) bonds. As Canadian bonds are owned by US households, an increase in the interest rate of Canadian bonds would bring negative valuation effects for the Canadian NFA position.

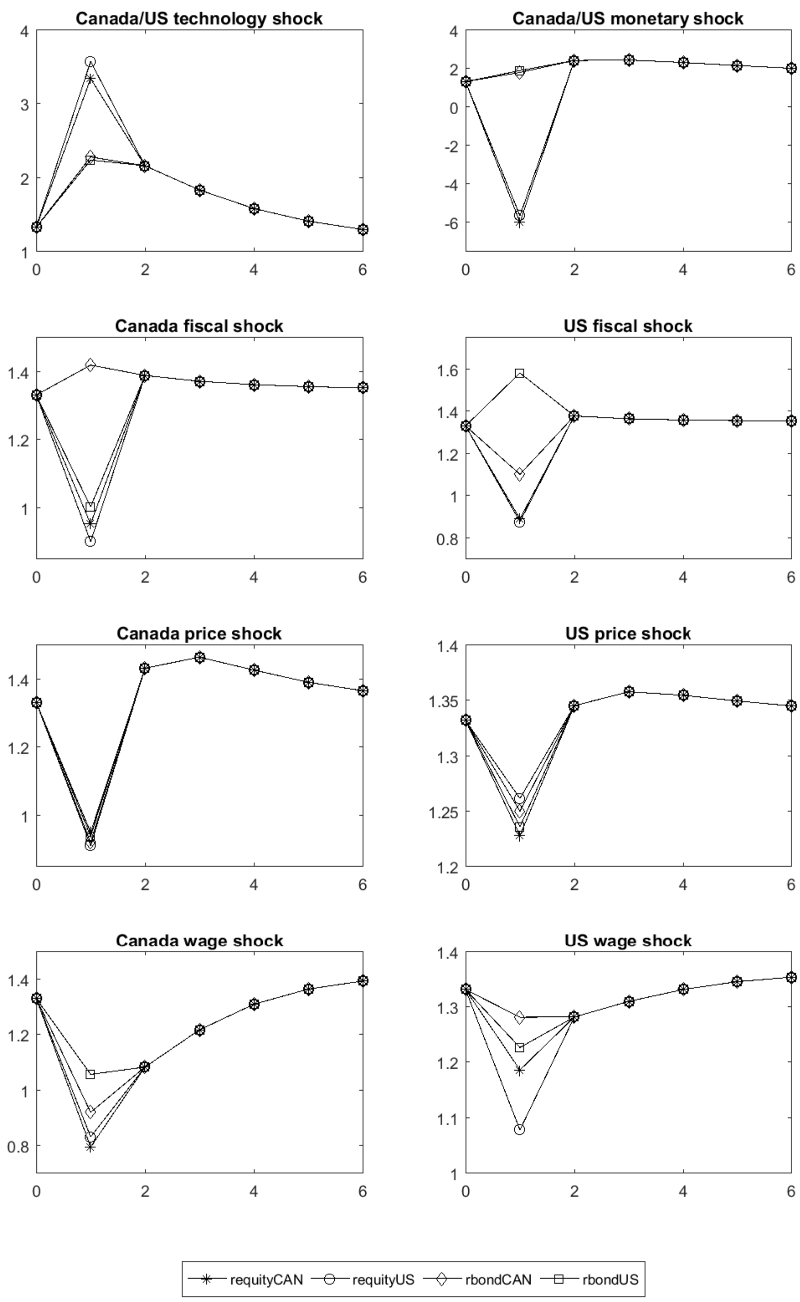

Figure 8: Responses of the rates of return, % annualized

by the return differential favorable to the US bond. Thus, we cannot conclude that a global monetary shock explains the reversed two way capital flows observed from 2012 to 2018.

The valuation effects after a global interest-rate shock are displayed in Figure 4 and reported in Table 4b. The immediate effect is positive because the market value of the existing debt with the US falls (and despite the higher cost of the interest service of Canadian debt owned by US households). One quarter after the shock NFA begins to fall substantially and continuously over the next fifteen quarters of so. The reasons for this net borrowing to the US is the trade deficit (negative Canadian net exports) and the negative valuation effects caused by the recovery of the market value of US debt. Twelve quarters after the shock, the Canadian NFA position accumulates a decline of -1.15% units relative to the steady-state level of output.

Table 4a. Responses of asset returns to shocks, % annualized

	Canada		US		Differentials	
$r_{ss} = 1.32\%$	Bonds, r_B^{CAN}	Equity, r_E^{CAN}	Bonds, r_B^{US}	Equity, r_E^{US}	$r_B^{CAN} - r_B^{US}$	$r_E^{US} - r_E^{CAN}$
Canada/US tech. shock	2.283	3.345	2.242	3.578	0.041	0.233
Canada/US mon. shock	1.810	-5.973	1.908	-5.629	-0.098	0.344
Canada fiscal shock	1.420	0.954	1.004	0.902	0.416	-0.052
US fiscal shock	1.103	0.891	1.583	0.878	-0.480	-0.013
Canada price shock	0.920	0.951	0.941	0.912	-0.021	-0.039
US price shock	1.250	1.228	1.236	1.262	0.014	0.032
Canada wage shock	0.921	0.795	1.058	0.832	-0.137	0.037
US wage shock	1.282	1.187	1.227	1.079	0.055	-0.108

Table 4b. Responses of Canadian NFA to shocks, % of output

	Quarter of the shock	4 quarters after the shock	12 quarters after the shock
Canada/US tech. shock	0.099	-0.138	0.067
Canada/US monetary shock	0.107	-0.601	-1.148
Canada fiscal shock	-0.048	-0.128	-0.201
US fiscal shock	0.014	0.013	0.004
Canada price shock	-0.034	-0.106	-0.105
US price shock	0.019	0.011	0.003
Canada wage shock	-0.000	0.001	-0.118
US wage shock	-0.043	-0.012	-0.034

A specific public spending shock in Canada results in a interest rate spread for the Canadian bond respect to the US bond (see Figure 8). In annualized percent terms, the gap $r_B^{CAN} - r_B^{US}$ is close to 0.42%. This might explain net purchases of Canadian bonds out of sales of US bonds. This is one way of the capital flows documented in this paper. The other way –net purchases of US equity– cannot be anticipated by the return differential of this Canadian public spending shock. Equity returns fall both in Canada and in the US, with a little gap favorable to Canadian equity (3.9 basis points). The extra payoffs of Canadian bond returns to US owners explains the valuation effects that make Canada NFA move downwards at the time of the shock. In the following quarters, the Canadian net borrowing position continues because of its current account deficit with the US. The persistence of such negative next exports accumulates an overall negative effect of -0.2% of steady-state output twelve quarters after the shock.

If the public spending (fiscal) shock takes place in the US economy, the results bring return differentials favorable to the US bond and the Canadian equity (see Table 4a). Therefore, if the shock took a negative realization (fiscal contraction), the return differentials would satisfy (20) for the empirical test of the Canada/US capital flows. As displayed in Figure 8, the gap between bond returns is significant (0.48%), but the distance between equity return is tiny (as Table 4a informs it is just 1.3 basis points). So, although we could argue that (20) holds for a US fiscal contraction the Canada/US equity flows that generates would not be large because of the small return differential. The response of the Canadian NFA is quantitatively small (see numbers in Table 4b) and of positive sign due to the lending capacity that emerges from the increase of Canadian net exports with the US. Thus, in the case of a US fiscal contraction shock the Canada NFA position would take the opposite downwards direction which cannot meet the movement observed during the reversed two-way capital flows, implied by (21).

An inflation shock introduced in the price indexation scheme of Canadian firms reduces the rates of return of the four available assets. As Figure 8 shows and Table 4a informs, there is a simultaneous cut of about 0.4% in the return of bonds and equity, both from Canada and the US. In Canada, there is a little domestic equity premium over the domestic bond, whereas the opposite occurs in the US. Anyway, the shock does not provide significant interest rate differentials neither across assets nor across economies. It could be mentioned that Canadian equity yields a lower return than US equity as firms from Canada suffer the adverse effects of the shock originated in Canada more than US firms. Meanwhile, the Canadian bond provides 2 basis points of higher return than the US bond. If the shock were deflationary (negative realization), the simulated return differentials would satisfy (20). When we re-examine the response of Canada NFA position (Figure 6), the negative net exports motivate a fall over four quarters after the shock (valuation effects are small in this case). Accordingly, if the shock were deflationary on Canadian goods, the current account would register positive values and the NFA position had increased. We could, thus, argue that a deflationary price push-shock in Canada could explain the reversed two-way capital flows. However, there are two limitations to this argument. First, the quantitative effects found, in terms of either return differentials or NFA variations, are small. And secondly, actual data on Canada and US producer price inflation between 2012 and 2018 do not show any signal of price shocks in Canada relative to the US (see Figure 2 for the plots of the inflation rates and Table 5 for their average values).

If the price-push shock hits the US economy, the four asset returns also drop, but they do it at a lower extent to what they did after a Canada price shock because the size of shock is smaller.[20] The effects are symmetrically reversed from the Canadian price-push shock. Hence, the return differentials are favorable to Canadian bonds and US equity (see Table 4a). But these numbers are really little. 1.4 basis points and 3.2 points, respectively, which does not justify the massive capital flows found in the data. Besides, we have already mentioned that the rates of inflation in Canada and the US have not been significantly different over the period. The Canadian NFA improves due to its increase in net exports and the lack of significant valuation effects. Numbers reported in Table 4b are again very small and there should be a huge US price-push shock (e.g., 100 times its calibrated standard deviation) to explain an increase in NFA of similar size to the one observed in the data.

Moving to wage-push shocks, Figure 8 shows a more apparent differentiation of asset returns. As expected, a wage-push shock reduces the rates of return on all the assets. The effects are larger after a Canadian wage-push shock because of the higher wage stickiness calibrated for the Canadian (home) economy. As documented in Table 4a, equity returns fall deeper than bond returns and home assets report a larger effect than foreign assets. Regarding the return differentials for testing (20), Table 4a reports $r_B^{CAN} - r_B^{US} < 0$ and $r_E^{US} - r_E^{CAN} > 0$ after a Canada wage-push shock and switched signs after a US wage-push shock. Therefore, neither shock satisfies (20). The effects of wage-push shocks on the Canadian NFA position are rather small (see last two rows of Table 4b). Figure 7 plots the response of Canadian NFA to a Canada wage-push shock. The fall in net exports brings borrowing needs and explains a decreasing pattern in the NFA position of Canada with the US. Valuation effects are little. When the wage-push shock hits the (foreign) US economy, the Canadian net exports rise but there are some negative valuation effects that dominate. Canada NFA slides downwards to the negative values reported in Table 4b. The negative valuation effects come from the extra interest-rate payments of

[20] It may be recalled that the shocks have a size equivalent to one calibrated standard deviation and the baseline calibration assumes that the price-push shock is less volatile in the US than in Canada to be consistent with the empirical evidence on their relative inflation volatility.

Canadian bonds owned by US households and also in the lower return of US equity held by Canadian households (see last row of Table 4a).

Summarizing, we can say that only price-push shocks can be explanatory factors to the reversed two-way capital flows between the US and Canada that characterized the rise of the Canada NFA position with the US from 2012 to 2018. All the remaining shocks fail to jointly satisfy (20) and (21). A combination of shocks may still explain the reversed two-way capital flows. Thus, we twist our strategy to define the sources of exogenous variability from a prospective look at the data. The type and size of the shocks are going to be specified from the economic facts that characterized the US and Canada in the period between 2012 and 2018. This opens the next subsection.

Table 5. Mean values of selected variables (quarterly, %)

	Canada	US	Difference
Full sample, 1990-2018			
Output growth	0.30	0.32	-0.02
Price inflation (PPI)	0.49	0.50	-0.01
Wage inflation	0.72	0.82	-0.10
Nominal interest rate	0.88	0.68	0.20
Subsample of increasing Canadian NFA, 2012-2018			
Output growth	0.37	0.25	0.12
Price inflation (PPI)	0.36	0.40	-0.04
Wage inflation	0.63	0.53	0.10
Nominal interest rate	0.19	-0.29	-0.48

Replicating the scenario of 2012

The price-push shocks emerge as the candidates to explain the excess returns and NFA sign observed in the US/Canada capital flows. Are they found in the data? Looking at Figure 2, no substantial change is found in the fluctuations of either US or Canada producer price inflation. Table 5 reports the average values and, again, numbers are quite homogeneous across samples. Both US and Canada had some disinflation in the 2012-2018 period compared to the full sample, but the extent of the disinflation is similar and the difference between Canada and US price inflation remains small. Since individual shocks cannot account for the stylized facts that characterize Canada/US capital flows after 2012 (because of either theoretical or empirical flaws), we have reviewed the circumstances of that time and built a combination of shocks that may represent the state of the economies at that time:

i) A positive global technology shock: both economies experienced an increase in total factor productivity in the years after the financial crisis (which might have been the consequence of a creative destruction that results from the process of business churning). Cao and Kozicki (2015) from the Bank of Canada and Fernald (2014) from the Federal Reserve of San Francisco obtain estimates of increasing total factor productivity after 2009 for, respectively, Canada and the US. A positive technology shock on both economies of size equivalent to one calibrated standard deviation (0.89%) is set to capture the global increase in total factor productivity.

ii) Looking at the time series of the nominal interest rate (Figure 1), the subperiod of the sample that runs from 2012 to 2018 coincides with massive balance sheet purchases of the Fed, commonly referred as the Quantitative Easing (QE) policies.[21] Actually, the third round of Quantitative Easing (QE3) was implemented during the sample period of upwards trend in the NFA position of Canada with the US.[22] Meanwhile, the Bank of Canada also increased significantly the size of its balance sheet, with the official policy rate at an annualized 1%. We replicate these monetary expansions with a negative shock entering the Taylor-type policy rule of value equivalent to one calibrated standard deviation for the US and half of it for Canada.

iii) Adverse (negative) wage-push shock in the US: the rates of growth of nominal wages in the US are, on average, lower after 2012 than in the years before and they also come smaller than the ones observed in Canada. Figure 2 shows two observations after 2012 with clearly negative values for US wage inflation (one of them at -2.30% quarterly). Table 5 reports a swap in the average wage inflation difference from -0.10% in the full sample (favorable to the US) to +0.10% in the period 2012-2018 (favorable to Canada). Average wage inflation falls in almost 20 basis points in the period 2012-2018 in comparison to the complete period. These are indications for the existence of an adverse wage push shock in the US. Hence, we introduce a single negative wage-push shock for the US equivalent to 3 times its calibrated standard deviation.

iv) The US government ran a fiscal consolidation program in response to the dramatic increase in public debt after the financial crisis (fiscal cliff). Thus, public deficit over GDP got reduced from -6.7% in 2012 to -2.4% in 2016. We introduce an adverse fiscal shock for the US of size equivalent to two calibrated standard deviations.

Figure 9 plots the impulse-response functions following a simultaneous combination of the four shocks defined above. Let us describe what we see and discuss why we see it. Both economies report

[21] The Wu and Xia (2013)'s shadow policy rate capture the impact of asset purchases on the nominal interest rate below the zero lower bound. It allows for the negative US nominal interest rates displayed in Figure 2.

[22] The period of purchases under Q3 is typically considered from September of 2012 to October of 2014. In December of 2015, the Fed decided to raise the official interest rate since 2006.

a fall in output produced at the time of the combined shocks, but it quickly goes up to the positive size. In the US, the economic expansion peaks four quarters after the shocks when output is almost 1% higher than its steady-state level. In Canada, the effects are much smaller. Such difference is due to the origin of the shocks: the fiscal (contractionary) shock only takes place in the US economy, the (QE) monetary shock in the US is twice the size of it in Canada, and the wage-push shock only enters the US wage indexation norm. The nominal interest rate moves down from the QE shocks, with a higher cut implemented by the Fed compared to the Bank of Canada. Wage inflation and the real wage fall by around 2% in the US (the wage-push shock cuts nominal wages down) whereas Canadian wages barely change. The lower cost of labor in the US has a significant implication for the comparison of firm dividends. Although both Canadian and US firms take advantage of lower interest rates and increasing sales, the US dividend displays a much higher increase than the Canadian one (14% *versus* 4%) because the US real wage drops significantly while the Canadian one reports a moderate increase. The combined shocks result in a real exchange rate appreciation and a trade deficit for Canada, where imports rise up to 1.5% while exports falls around 0.5%. As Figure 9 also shows, the NFA position of Canada rises persistently due to the valuation effects (and despite the negative net exports). The positive valuation effects come from two main sources: the Canadian ownership of US equity that yields a higher return and the reduction in the interest rate payments of Canadian bonds to US households. These valuation effects determine the upwards trend in the Canadian NFA position with the US.

Table 6a provides the asset returns. The combined shocks reduces the US bond interest rate to a negative value, -0.27%, while the Canadian real interest rate rises to almost 3%. Hence, the bond return differential is 3.26%. Most of this spread is due to the impact of unanticipated inflation, which lowers significantly the *ex post* real interest rate of the US bond (and it is not captured in the *ex ante* real interest rates plotted in Figure 9). This spread can motivate sales of US bonds to purchase Canadian bonds as actually occurred in the period of the quantitative expansion in the US (Canada bond outflows to the US). Regarding equity returns, the combined shocks bring a positive impact on firm profitability both in the US and Canada. Both equity returns rise to rates around 8%. The spread in equity returns is small (0.24%) and favorable to the US. Meanwhile, firm dividends rise significantly more in the US than Canada (see Figure 9). The equity gain is higher for US firms, because they take advantage of the lower cost of labor that comes with the negative wage-push shock. Such dividend gap can also explain the purchases of US equity by Canadian investors (US equity inflows to Canada).

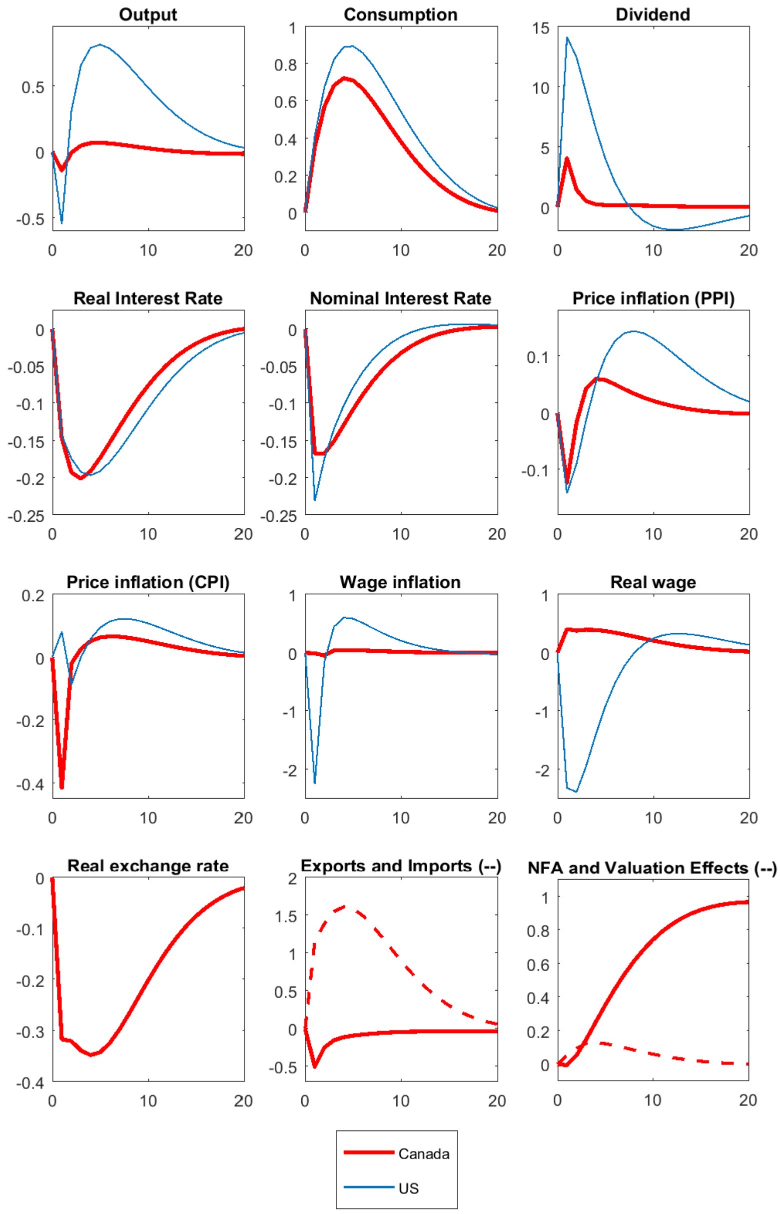

Figure 9: Impulse-response functions following a combination of shocks describing the 2012 scenario for Canada and the US

Table 6a. Asset returns following shocks in the 2012-18 scenario, % annualized

	Canada		US		Differentials	
$r_{ss} = 1.32\%$	Bonds, r_B^{CAN}	Equity, r_E^{CAN}	Bonds, r_B^{US}	Equity, r_E^{US}	$r_B^{CAN} - r_B^{US}$	$r_E^{US} - r_E^{CAN}$
Four shocks	2.993	7.928	-0.267	8.168	3.260	0.240
eliminating technology shock	2.529	7.321	-0.720	7.493	3.249	0.172
eliminating QE shocks	2.161	2.871	2.218	3.527	-0.057	0.657
eliminating US wage-push shock	2.761	7.158	-0.807	6.868	3.568	-0.290
eliminating US fiscal shock	2.859	7.766	-0.160	7.946	3.019	0.180

Table 6b. Responses of Canadian NFA following shocks in the 2012-18 scenario, % of output

	Quarter of the shock	4 quarters after the shock	12 quarters after the shock
Four shocks	-0.005	0.352	0.870
eliminating technology shock	-0.043	0.274	0.743
eliminating QE shocks	0.224	0.137	0.278
eliminating US wage-push shock	-0.182	0.328	0.749
eliminating US fiscal shock	-0.015	0.317	0.841

As for the Canadian NFA position with the US, Table 6b reports a initial tiny drop followed by a persistent and substantial increase that becomes 0.87% of output twelve quarters after the shock. Since net exports are negative, the positive variation of NFA must be accounted for gains in the value of existing assets. The relative position of bonds and equity determines this positive valuation effects: Canadian households own US equity and the Canadian government is being financed by US households. Since equity returns are higher than bond returns, the valuation effects have been positive for Canada. Therefore, the combined shocks described above can explain: i) Canadian capital outflows to purchase US equity, ii) US capital inflows to buy Canadian bonds and iii) a persistent increase in the NFA position of Canada with the US. These effects correspond to the stylized facts that describe the Canada/US capital flows from 2012 to 2018 and jointly satisfy (20) and (21).

Finally, let us carry out some robustness checks to distinguish which individual shocks play a major role in the reversed two-way capital flows. Tables 6a and 6b collect, respectively, the return differentials and the NFA variations when one of the contributing shocks is eliminated. The absence of a technology shock would reduce the returns of all assets and also the spreads would be smaller, but the conditions stated in (20) are met. The response of NFA without the technology shock is qualitatively similar but also small. Hence, the combined shocks without the technology shock still can explain the reversed two-way capital flows at a smaller magnitude. If the monetary shocks are eliminated the picture changes dramatically. The bond return differential would switch from $r_B^{CAN} - r_B^{US} > 0$ to $r_B^{CAN} - r_B^{US} < 0$, and equity returns would be much lower. These changes would reduce significantly the valuation effects and the NFA position of Canada with the US would only rise by 0.28% after twelve periods. Hence, monetary shocks are necessary to explain the reversed two-way capital flows as they play a major role to explain US purchases of Canadian bonds and a significant increase in the Canada NFA position. The adverse US wage-push shocks turns also a necessary contributor. If the negative US wage shock is dropped, the US equity premium vanishes as $r_E^{US} - r_E^{CAN} = -0.29\%$. The valuation effects and the variation of the NFA would also be significantly diminished. Without the contractionary US fiscal shock, the signs of the return differentials (20) are satisfied and the evolution of the NFA position is similar to the all-shock case. The absence of the US fiscal contraction just reduces, in a minor way, the quantitative implications of the shocks.

In summary, monetary (QE) shocks and adverse wage-push shocks in the US are the major contributors to the model-based explanation of the capital flows and the NFA position of Canada with the US.

7 Conclusions

This paper comes motivated by some interesting empirical facts observed in the asset trading between Canada and the US. There is a reversed two-way capital flows that characterize the evolution of net foreign assets positions between these two countries from 2012 to the present time. The US is a net creditor for Canadian risk-free bonds while Canada has increased significantly the purchases of US equity.

We have built a two-economy structural model with international portfolio choice and nominal rigidities. The model introduces both sticky prices and sticky wages for the real effects of monetary and demand shocks. In addition the role of international trading is crucial for aggregate fluctuations through the impact of net exports on the determination of the aggregate demand. The portfolio choice setup describes the household decision of international purchases and sales of bonds and equity. The general equilibrium model brings a dynamic equation that determines the evolution of the Net Foreign Asset position across the two economies where valuation effects matter.

The calibration of the model has initially assumed a symmetric institutional framework and similar preferences and technology for households and firms. Then, some asymmetric patterns were introduces to replicate long-run properties and business cycle patterns observed empirically in the relationships between Canada and the US. The paper finds a reasonably good fit between the model and actual data, both in terms of long-run ratios and second-moment statistics.

The analysis of impulse-response functions has included both global and country-specific shocks. A global technology shock turns more expansionary to Canada than to the US due to its lower inflation and a positive trade balance for Canadian goods. Likewise, a global monetary (interest-rate) shock brings a larger and more persistent recession in Canada than in the US due to its higher wage rigidities and the reduction of Canadian net exports. We have also discussed the implication of country-specific shocks. A positive shock on government spending results in a exchange rate appreciation that increases the external demand and aggregate output of the foreign economy. Either price-push or wage-push shocks originate changes in relative prices and a substitution between domestic and foreign goods. Aggregate output falls in the economy hit by the inflationary shock whereas the other economy observes higher external demand and output produced.

The variability of asset returns has been examined following either global or individual shocks. The only exogenous perturbation that can explain both the increase in the NFA position of Canada with the US and the two-way reversed capital flows is a producer price shock (either a negative shock on Canadian inflation or a positive shock on US inflation). However, there is no apparent evidence of actual price inflation shocks in the corresponding period from 2012 to 2018.

So, we have design a combination of shocks that may describe some of the stylized facts of the 2012-2018 period: a positive global technology shock that raises total factor productivity after the financial crisis, monetary shocks that capture the QE stimulus, a contractionary fiscal shock that reflects the US fiscal cliff and a negative shock on US wage inflation consistent with the lower relative wages observed in the data. It turns out that the effective real interest rate of US bonds turns negative and creates a substantial spread between the returns of the Canadian bond and the US bond. Moreover, equity value rises in the US more than in Canada. These responses are a correct characterization of the two-way reversed capital flows between the US and Canada documented after 2012. Furthermore, the model simulation indicates that the NFA position of Canada with the US rises steadily in the quarters after the US monetary expansion. Both the monetary shock and the adverse US wage inflation shock are crucial to key ingredients the two-way reversed capital flows.

References

Adolfson, M, S. Laséen, J. Lindé and M. Villani (2005). The Role of Sticky Prices in an Open Economy DSGE Model: A Bayesian Investigation, *Journal of the European Economic Association* 3, 444-457.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992). International Real Business Cycles, *Journal* *of Political Economy* 100, 745-775.

Benigno, G., and C. Thoenissen (2003). Equilibrium Exchange Rates and Supply-Side Performance, *Economic Journal* 113, 103-124.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* 12, 383-398.

Cao, S., and S. Kozicki (2015). A New Data Set of Quarterly Total Factor Productivity in the Canadian Business Sector, Staff Working Papers 15-6, Bank of Canada.

Coeurdacier, N., and H. Rey (2013). Home Bias in Open Economy Financial Macroeconomics, *Journal of Economic Literature* 51, 63-115.

Corsetti, G., and P. Pesenti (2001). International Dimensions of Optimal Monetary Policy, NBER Working Papers, 8230.

De Walque, G., Smets, F., and R. Wouters (2005). An estimated two-country DSGE model for the Euro area and the US economy. European Central Bank, mimeo.

Del Villar, A. (2018). Gross capital flows between emerging and advanced economies: A two-country portfolio choice model, manuscript.

Devereux, M. B., and A. Sutherland (2008). Financial Globalization and Monetary Policy, *Journal of Monetary Economics* 55(8), 1363-1375.

Devereux, M. B., and A. Sutherland (2009). A portfolio model of capital flows to emerging markets, *Journal of Development Economics* 89 (2), 181-193.

Devereux, M. B., and A. Sutherland (2010). Country portfolio dynamics, *Journal of Economic Dynamics and Control* 34(7), 1325-1342.

Devereux, M. B., and A. Sutherland (2011). Country portfolios in open economy macro-models, *Journal of the European Economic Association* 9, 337-369.

Dixit, A.K., and J.E. Stiglitz (1977). Monopolistic competition and optimum product diversity, *American Economic Review* 67, 297-308.

Engel, C. and A. Matsumoto (2009). The International Diversification Puzzle When Goods Prices Are Sticky: It's Really about Exchange-Rate Hedging, Not Equity Portfolios, *American Economic Journal: Macroeconomics* 1(2), 155-188.

Erceg, C.J., L. L. Guerrieri and C. Gust (2006). SIGMA: A New Open Economy Model for Policy Analysis, *International Journal of Central Banking,* 2(1).

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000). Optimal Monetary Policy with Staggered Wage and Price Contracts, *Journal of Monetary Economics* 46, 281-313.

Fernald, J. (2014). A Quarterly, Utilization-Adjusted Series on Total Factor Productivity, FRBSF

Working Paper 2012-19.

Galí, J., and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy, *Review of Economic Studies* 72(3), 707-734.

Ghironi F. (1999). Towards New Open Economy Macroeconometrics, mimeo, Boston College.

Gourinchas, P. O., and H. Rey (2007). From world banker to world venture capitalist: US external adjustment and the exorbitant privilege. In G7 current account imbalances: sustainability and adjustment, pages 11-66. University of Chicago Press.

Gourinchas, P. O., and H. Rey (2014). External adjustment, global imbalances, valuation effects. In Handbook of International Economics, vol. 4, pages 585-645. Elsevier.

Justiniano A. and B. Preston (2004), Small open economy DSGE models specification, estimation and model fit, manuscript, IMF.

Justiniano, A., and B. Preston (2010). Can structural small open-economy models account for the influence of foreign disturbances?, *Journal of International Economics* 81(1), 61-74.

Kollmann, R. (2002). Monetary policy rules in the open economy: effects on welfare and business cycles, *Journal of Monetary Economics* 49, 989-1015.

Kortelainen, M. (2002). Edge: a model of the euro area with applications to monetary policy, Bank of Finland Studies, E:23.

Laxton, D., and P. Pesenti (2003). Monetary rules for small, open, emerging economies, *Journal of Monetary Economics* 50, 1109-1146.

Murchison, S., A. Rennison, and Z. Zhu (2004). A Structural Small Open-Economy Model for Canada, Staff Working Papers 04-4, Bank of Canada.

Obstfeld, M., and K. Rogoff (1995). Exchange Rate Dynamics Redux, *Journal of Political Economy* 103, 624-660.

Schmitt-Grohé, S. (1998). The international transmission of economic fluctuations: Effects of US business cycles on the Canadian economy, *Journal of international Economics* 44, 257-287.

Schmitt-Grohé, S., and M. Uribe (2003). Closing small open economy models, *Journal of International Economics* 61, 163-185.

Schmitt-Grohé, S. and M. Uribe (2006). Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In NBER Macroeconomics Annual 2005, vol. 20, pages 383-462. MIT Press.

Smets, F. and R. Wouters (2002). Openness, imperfect exchange rate pass-through and monetary policy, *Journal of Monetary Economics* 49, 947-981.

Smets, F. and R. Wouters (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach, *American Economic Review* 97, 586-606.

Taylor, J.B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214.

Tille, C. and E. Van Wincoop (2008). International capital flows under dispersed information: Theory and evidence. National Bureau of Economic Research, working paper, No. 14390.

Tille, C. and E. Van Wincoop (2010). International capital flows, *Journal of International Economics* 80, 157-175.

Woodford, M. (2003). Optimal interest-rate smoothing, *Review of Economic Studies* 70, 861-886.

Wu, J. C., and F. D. Xia (2016). Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound, *Journal of Money, Credit, and Banking* 48, 253-291.

APPENDIX

I. A detailed description of the equations of the model

The model comprises a set of behavioral equations providing solution paths for the 58 endogenous variables. Each economy is described with 26 endogenous variables which for the domestic economy would be c_t, $c_{H,t}$, $c_{F,t}$, β_{t+1}/β_t, n_t, $\widetilde{W_t}(j)$, w_t, π_t^w, $A_{W,t}$, $B_{W,t}$, $\widetilde{P_{H,t}}(j)$, $A_{P,t}$, $B_{P,t}$, π_t^{PPI}, π_t^{CPI}, RP_t, Θ_{t+1}/Θ_t, d_t, v_t, mc_t, PD_t, PDD_t, PD_t^*, y_t, R_t, and r_t. Foreign endogenous variables are completely analogous. There are 6 variables that simultaneously relate or affect to both economies, e_t, q_t, NX_t, NFA_t, WD_t and VAL_t. Finally, there are 8 exogenous variables: 3 country-specific AR(1) processes to autonomous (government) spending, price and wage indexation rules, along with 2 global (common) shocks to technology and monetary policy.

The following 26 equations describe only the home economy (foreign-economy equations are totally analogous):

Intertemporal consumption (Euler) equation

$$\frac{(c_t - hc_{t-1})^{-\sigma}}{(1+r_t)} = \frac{\beta_{t+1}}{\beta_t}\mathbb{E}_t[(c_{t+1} - hc_t)^{-\sigma}]$$

Equilibrium condition for home equity assets

$$v_t = \mathbb{E}_t\left[\frac{d_{t+1} + v_{t+1}}{1+r_t}\right]$$

Fisher equation

$$1 + r_t = \frac{1+R_t}{\mathbb{E}_t\left(1+\pi_{t+1}^{CPI}\right)}$$

Endogenous discount factor

$$\frac{\beta_{t+1}}{\beta_t} = (1+c_t)^{-\varsigma}$$

Domestic consumption of domestically produced goods

$$c_{H,t} = (1-\alpha)\, RP_t^{-\theta} c_t$$

Domestic consumption of foreign produced goods (imports)

$$c_{F,t} = \alpha \left(RP_t^* q_t\right)^{-\theta} c_t$$

Plugging the optimal allocation of both domestic and foreign consumption, $c_{H,t}$ and $c_{F,t}$, in the aggregate consumption definition, $c_t \equiv \left[(1-\alpha)^{\frac{1}{\theta}}(c_{H,t})^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}}(c_{F,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$, it is obtained

$$1 = (1-\alpha)\left(RP_t\right)^{1-\theta} + \alpha\left(RP_t^* q_t\right)^{1-\theta}$$

Optimal wage setting. We use $A_{W,t}$ and $B_{W,t}$ variables to solve the infinite sum

$$\widetilde{W}_t(j)B_{W,t} = \left(\frac{\theta_w}{\theta_w - 1}\right) A_{W,t}$$

with

$$A_{W,t} = \psi\left(\left(\widetilde{W}_t(j)\right)^{-\theta_w} n_t\right)^{\gamma} n_t + \eta_w \frac{\beta_{t+1}}{\beta_t}\mathbb{E}_t\left[A_{W,t+1}\left(\frac{(1+\pi^w_{t+1})}{(1+\pi^{CPI}_t)^{\kappa_w}(1+\pi+\varepsilon^W_{t+1})^{1-\kappa_w}}\right)^{\theta_w}\right]$$

and

$$B_{W,t} = (c_t - hc_{t-1})^{-\sigma} w_t n_t + \eta_w \frac{\beta_{t+1}}{\beta_t}\mathbb{E}_t\left[B_{W,t+1}\left(\frac{(1+\pi^w_{t+1})}{(1+\pi^{CPI}_t)^{\kappa_w}(1+\pi+\varepsilon^W_{t+1})^{1-\kappa_w}}\right)^{\theta_w-1}\right]$$

Dixit-Stiglitz aggregator for relative wages

$$\widetilde{W}_t(j)^{(\theta_w-1)} = (1-\eta_w) + \eta_w\left[(1+\pi^{CPI}_{t+k-1})^{\kappa_w}(1+\pi^{CPI}_{ss}+\varepsilon^W_t)^{1-\kappa_w}\right]^{(1-\theta_w)}(1+\pi^w_t)^{\theta_w-1}\widetilde{W}_t(j)^{(\theta_w-1)}$$

Nominal wage inflation from the definition of the real wage ($w_t = W_t/P_t$)

$$(1+\pi^w_t) = \frac{w_t}{w_{t-1}}(1+\pi^{CPI}_t)$$

Firm's discount factor (α^{equity} is the home agent ownership of foreign firm)

$$\frac{\Theta_{t+1}}{\Theta_t} = (1-\alpha^{equity})\frac{\beta_{t+1}}{\beta_t} + \alpha^{equity}\frac{\beta^*_{t+1}}{\beta^*_t}$$

Optimal price function for which we use $A_{P,t}$ and $B_{P,t}$ variables to solve the infinite sum.

$$\widetilde{P_{H,t}}(j)B_{P,t} = \left(\frac{\epsilon}{\epsilon-1}\right)A_{P,t}$$

with

$$A_{P,t} = y_t mc_t + \frac{\Theta_{t+1}}{\Theta_t}\eta_P\mathbb{E}_t\left[A_{P,t+1}\left(\frac{(1+\pi^{PPI}_{t+1})}{(1+\pi^{PPI}_t)^{\kappa_p}(1+\pi+\varepsilon^P_{t+1})^{1-\kappa_p}}\right)^{\epsilon}\right]$$

and

$$B_{P,t} = y_t + \frac{\Theta_{t+1}}{\Theta_t}\eta_P\mathbb{E}_t\left[B_{P,t+1}\left(\frac{(1+\pi^{PPI}_{t+1})}{(1+\pi^{PPI}_t)^{\kappa_p}(1+\pi+\varepsilon^P_{t+1})^{1-\kappa_p}}\right)^{\epsilon-1}\right]$$

Dixit-Stiglitz price aggregator

$$\widetilde{P_{H,t}}(j)^{(\epsilon-1)} = (1-\eta_p) + \eta_p\left[(1+\pi^{PPI}_{t-1})^{\kappa_p}(1+\pi^{PPI}_{ss}+\varepsilon^P_t)^{1-\kappa_p}\right]^{(1-\epsilon)}(1+\pi^{PPI}_t)^{\epsilon-1}\widetilde{P_{H,t}}(j)^{(\epsilon-1)}$$

Relationship between CPI-inflation, π^{CPI}_t, and producer price-inflation, π^{PPI}_t, through relative prices, RP_t,

$$\frac{(1+\pi^{PPI}_t)}{(1+\pi^{CPI}_t)} = \frac{RP_t}{RP_{t-1}}$$

Real marginal cost (labour demand function)

$$mc_t = \left(\frac{P_t}{P_{H,t}}\right) \frac{w_t}{e^{\varepsilon_t^A} A}$$

Average dividend

$$d_t = y_t \left[RP_t PD_t - \left(\frac{w_t}{e^{\varepsilon_t^A} A}\right) PDD_t \right]$$

with these price dispersion measures

$$PD_t = \left(1 - \eta_p\right) \left(\widetilde{P_{H,t}(j)}\right)^{1-\epsilon} + \eta_p \left(\widetilde{P_{H,t-1}(j)(1 + idx_t^p)}\right)^{1-\epsilon} PD_{t-1}$$

and

$$PDD_t = \left(1 - \eta_p\right) \left(\widetilde{P_{H,t}(j)}\right)^{-\epsilon} + \eta_p \left(\widetilde{P_{H,t-1}(j)(1 + idx_t^p)}\right)^{-\epsilon} PDD_{t-1}$$

Aggregate production function

$$PDD_t y_t = e^{\varepsilon_t^A} n_t$$

Resources constraint equilibrium condition

$$PD_t y_t = PD_t c_{H,t} + PD_t^* c_{H,t}^* + g_t$$

with another price dispersion indicator

$$PD_t^* = \left(1 - \eta_p\right) \left(\widetilde{P_{H,t}(j)}\right)^{1-\epsilon^*} + \eta_p \left(\widetilde{P_{H,t-1}(j)(1 + idx_t^p)}\right)^{1-\epsilon^*} PD_{t-1}$$

Monetary policy rule *a la* Taylor (1993)

$$1 + R_t = \left((1+r)\,(1+\pi)^{(1-\mu_\pi)}\right)^{(1-\mu_R)} (1 + R_{t-1})^{\mu_R} \left(1 + \pi_t^{CPI}\right)^{(1-\mu_R)\mu_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{(1-\mu_R)\mu_y} e^{\varepsilon_t^R}$$

The following 6 equations are related to both economies

Real exchange rate

$$\frac{q_t}{q_{t-1}} = \frac{e_t}{e_{t-1}} \frac{1 + \pi_t^{CPI,*}}{1 + \pi_t^{CPI}}$$

Uncover interest rate parity condition

$$(1 + r_t) = (1 + r_t^*) \mathbb{E}_t \left[\frac{q_{t+1}}{q_t}\right]$$

Net Exports

$$NX_t = \alpha^* \left(\frac{RP_t}{q_t}\right)^{-\theta^*} c_t^* - \alpha \left(RP_t^* q_t\right)^{-\theta} c_t$$

Net Foreign Assets

$$NFA_t - \left(\frac{d_t + v_t}{v_{t-1}}\right) NFA_{t-1} = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t + r'_{x,t} \alpha_{t-1} - g_t - c_t$$

with wage dispersion computed as follows

$$\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj \equiv WD_t = (1-\eta_w)\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} + \eta_w \left(\frac{W_{t-1}(j)}{W_{t-1}}(1+idx_t^w)\right)^{1-\theta_w} WD_{t-1}$$

Valuation effects

$$VAL_t = r'_{x,t}\alpha_{t-1} + \left(\frac{v_t}{v_{t-1}} - 1\right) NFA_{t-1}$$

where $r'_{x,t}\alpha_{t-1}$ is defined as a function of asset holdings and return differentials.

II. Derivation of Net Foreign Assets (NFA) dynamic equation

Let us introduce the variable α for asset holdings where the time subscript is identified with two letters: either E or B refers to equity and bond assets respectively, while either H or F refers to origin (issuing) from either the home or foreign agents. Recalling the household budget constraint

$$\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + (d_t + v_t) S_{H,t} + q_t (d_t^* + v_t^*) S_{F,t} + b_{H,t} + q_t b_{F,t} - tax_t =$$
$$c_t + v_t S_{H,t+1} + q_t v_t^* S_{F,t+1} + (1+r_t)^{-1} b_{H,t+1} + (1+r_t^*)^{-1} q_t b_{F,t+1}, \quad \text{(A0)}$$

the following definitions bring the domestic household asset holdings in period t

$$\alpha_{BH,t} \equiv \frac{b_{H,t+1}}{(1+r_t)} \quad \text{(A1)}$$

$$\alpha_{BF,t} \equiv \frac{q_t b_{F,t+1}}{(1+r_t^*)} \quad \text{(A2)}$$

$$\alpha_{EH,t} \equiv v_t S_{H,t+1} \quad \text{(A3)}$$

$$\alpha_{EF,t} \equiv q_t v_t^* S_{F,t+1} \quad \text{(A4)}$$

Meanwhile, the definition of Net Foreign Assets (NFA) implies

$$NFA_t = [\alpha_{EF,t} + \alpha_{BF,t} - \alpha_{EH,t}^* - \alpha_{BH,t}^*] \quad \text{(A5)}$$

where it should be noticed that the foreign holdings of home assets, $\alpha_{EH,t}^*$ and $\alpha_{BH,t}^*$, are expressed in terms of the domestic bundle of consumption goods

$$\alpha_{EH,t}^* \equiv v_t S_{H,t+1}^* \text{ and } \alpha_{BH,t}^* \equiv \frac{B_{H,t+1}^*/P_{t+1}}{(1+r_t)} = \frac{b_{H,t+1}^*}{(1+r_t)}.$$

The financial market clearing conditions are, for the equity markets,

$$S_{H,t+1} + S_{H,t+1}^* = S_{F,t+1} + S_{F,t+1}^* = 1 \quad \text{(A6)}$$

whereas for the government bond markets

$$b_{H,t+1} + b_{H,t+1}^* = b_{F,t+1} + b_{F,t+1}^* = 0 \quad \text{(A7)}$$

So, that the following conditions hold

$$\alpha_{EH,t}^{*} = v_t(S_{H,t+1}^{*}) = v_t(1 - S_{H,t+1}) = v_t - v_t S_{H,t+1} = v_t - \alpha_{EH,t} \tag{A8}$$

and

$$\alpha_{BH,t}^{*} = \frac{b_{H,t+1}^{*}}{(1+r_t)} = \frac{-b_{H,t+1}}{(1+r_t)} = -\alpha_{BH,t} \tag{A9}$$

and inserting both (A8) and (A9) in (A5) gives

$$NFA_t = [\alpha_{EF,t} + \alpha_{BF,t} - v_t + \alpha_{EH,t} + \alpha_{BH,t}]$$

or, alternatively

$$NFA_t + v_t = \alpha_{EF,t} + \alpha_{BF,t} + \alpha_{EH,t} + \alpha_{BH,t} \tag{A10}$$

Next, let us define the returns in domestic consumption bundles

$$1 + r_{BH,t} \equiv 1 + r_t \tag{A11}$$

$$1 + r_{BF,t} \equiv 1 + r_t^{*} \tag{A12}$$

$$1 + r_{EH,t} \equiv \left(\frac{d_{t+1} + v_{t+1}}{v_t}\right) \tag{A13}$$

$$1 + r_{EF,t} \equiv \left(\frac{d_{t+1}^{*} + v_{t+1}^{*}}{v_t^{*}}\right) \tag{A14}$$

Both the definitions of the holdings, (A1)-(A4), and the returns, (A11)-(A14), can be combined to find the following key relationships for terms that belong to the left hand side of the household budget constraint

$$\alpha_{BH,t-1}\left(1 + r_{BH,t-1}\right) = b_{H,t} \tag{A16}$$

$$\alpha_{BF,t-1}\left(1 + r_{BF,t-1}\right) = \frac{q_{t-1}}{q_t} q_t b_{F,t} \tag{A17}$$

$$\alpha_{EH,t-1}\left(1 + r_{EH,t-1}\right) = \left(d_t + v_t\right) S_{H,t} \tag{A18}$$

$$\alpha_{EF,t-1}\left(1 + r_{EF,t-1}\right) = \frac{q_{t-1}}{q_t} q_t \left(d_t^{*} + v_t^{*}\right) S_{F,t} \tag{A19}$$

Hence, inserting (A16) to (A19) on the left side of the household budget constraint (A0) gives

$$\begin{aligned}\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + \alpha_{EH,t-1}\left(1 + r_{EH,t-1}\right) + \frac{q_t}{q_{t-1}}\alpha_{EF,t-1}\left(1 + r_{EF,t-1}\right) + \alpha_{BH,t-1}\left(1 + r_{BH,t-1}\right) \\ + \frac{q_t}{q_{t-1}}\alpha_{BF,t-1}\left(1 + r_{BF,t-1}\right) - tax_t = \\ c_t + v_t S_{H,t+1} + q_t v_t^{*} S_{F,t+1} + \left(1 + r_t\right)^{-1} b_{H,t+1} + \left(1 + r_t^{*}\right)^{-1} q_t b_{F,t+1},\end{aligned}$$

and, furthermore, plugging (A1)-(A4) on the right side of the resulting expression brings

$$\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + \alpha_{EH,t-1}\left(1+r_{EH,t-1}\right) + \frac{q_t}{q_{t-1}}\alpha_{EF,t-1}\left(1+r_{EF,t-1}\right) + \alpha_{BH,t-1}\left(1+r_{BH,t-1}\right) + \frac{q_t}{q_{t-1}}\alpha_{BF,t-1}\left(1+r_{BF,t-1}\right) - tax_t = c_t + \alpha_{EH,t} + \alpha_{EF,t} + \alpha_{BH,t} + \alpha_{BF,t}, \quad \text{(A20)}$$

Using (A10) and a lagged version of it in (A20) introduces both current and lagged NFA

$$\left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + NFA_{t-1} + v_{t-1} + r_{EH,t-1}\alpha_{EH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{EF,t-1}\right) - 1\right)\alpha_{EF,t-1} + r_{BH,t-1}\alpha_{BH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{BF,t-1}\right) - 1\right)\alpha_{BF,t-1} - tax_t = c_t + NFA_t + v_t, \quad \text{(A21)}$$

or, alternatively

$$NFA_t - NFA_{t-1} + \left(v_t - v_{t-1}\right) = \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t - c_t - tax_t + r_{EH,t-1}\alpha_{EH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{EF,t-1}\right) - 1\right)\alpha_{EF,t-1} + r_{BH,t-1}\alpha_{BH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{BF,t-1}\right) - 1\right)\alpha_{BF,t-1}$$

Without a loss of generalization, we take the return of domestic equity, $r_{EH,t-1}$, as the reference one and both add it and subtract it, multiplied by $(\alpha_{EH,t-1} + \alpha_{EF,t-1} + \alpha_{BH,t-1} + \alpha_{BF,t-1})$, in the previous expression to obtain

$$NFA_t - NFA_{t-1} + \left(v_t - v_{t-1}\right) = \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t - c_t - tax_t + \left(\alpha_{EH,t-1} + \alpha_{EF,t-1} + \alpha_{BH,t-1} + \alpha_{BF,t-1}\right) r_{EH,t-1} - \left(\alpha_{EH,t-1} + \alpha_{EF,t-1} + \alpha_{BH,t-1} + \alpha_{BF,t-1}\right) r_{EH,t-1} + r_{EH,t-1}\alpha_{EH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{EF,t-1}\right) - 1\right)\alpha_{EF,t-1} + r_{BH,t-1}\alpha_{BH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{BF,t-1}\right) - 1\right)\alpha_{BF,t-1}$$

Using the lagged expression from (A10), and cancelling terms on $\alpha_{EH,t-1}r_{EH,t-1}$, and grouping terms on $\alpha_{EF,t-1}$, $\alpha_{BH,t-1}$, and $\alpha_{BF,t-1}$, we reach

$$NFA_t - NFA_{t-1} + \left(v_t - v_{t-1}\right) = \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t - c_t - tax_t + \left(NFA_{t-1} + v_{t-1}\right) r_{EH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{EF,t-1}\right) - 1 - r_{EH,t-1}\right)\alpha_{EF,t-1} + \left(r_{BH,t-1} - r_{EH,t-1}\right)\alpha_{BH,t-1} + \left(\frac{q_t}{q_{t-1}}\left(1+r_{BF,t-1}\right) - 1 - r_{EH,t-1}\right)\alpha_{BF,t-1} \quad \text{(A22)}$$

To save some space, let us putting together the return differentials as

$$r'_{x,t-1}\alpha_{t-1} = \alpha_{EF,t-1}\left(\frac{q_t}{q_{t-1}}\left(1+r_{EF,t-1}\right) - 1 - r_{EH,t-1}\right) + \alpha_{BH,t-1}\left(r_{BH,t-1} - r_{EH,t-1}\right) + \alpha_{BF,t-1}\left(\frac{q_t}{q_{t-1}}\left(1+r_{BF,t-1}\right) - 1 - r_{EH,t-1}\right)$$

which requires that $r_{x,t-1}$ is the column vector

$$r_{x,t-1} = \begin{bmatrix} \frac{q_t}{q_{t-1}} \left(1 + r_{EF,t-1}\right) - 1 - r_{EH,t-1} \\ r_{BH,t-1} - r_{EH,t-1} \\ \frac{q_t}{q_{t-1}} \left(1 + r_{BF,t-1}\right) - 1 - r_{EH,t-1} \end{bmatrix} = \begin{bmatrix} \frac{q_t}{q_{t-1}} \left(\frac{d_t^* + v_t^*}{v_{t-1}^*}\right) - \left(\frac{d_t + v_t}{v_{t-1}}\right) \\ r_{t-1} - \left(\left(\frac{d_t + v_t}{v_{t-1}}\right) - 1\right) \\ \frac{q_t}{q_{t-1}} \left(1 + r_{t-1}^*\right) - \left(\frac{d_t + v_t}{v_{t-1}}\right) \end{bmatrix}$$

while α_{t-1} is also a column vector

$$\alpha_{t-1} = \begin{bmatrix} \alpha_{EF,t-1} \\ \alpha_{BH,t-1} \\ \alpha_{BF,t-1} \end{bmatrix} = \begin{bmatrix} q_{t-1} v_{t-1}^* S_{F,t} \\ \left(1 + r_{t-1}\right)^{-1} b_{H,t} \\ \left(1 + r_{t-1}^*\right)^{-1} q_{t-1} b_{F,t} \end{bmatrix}$$

Replacing $r'_{x,t-1}\alpha_{t-1}$ in the (A22) and connecting terms on both NFA_{t-1} and v_{t-1} yield

$$NFA_t - \left(1 + r_{EH,t-1}\right) NFA_{t-1} + v_t - \left(1 + r_{EH,t-1}\right) v_{t-1} = \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t - c_t - tax_t + r'_{x,t-1}\alpha_{t-1} \tag{A22}$$

Finally, reinserting the definition of the domestic equity return $\left(1 + r_{EH,t-1}\right) = \left(\frac{d_t + v_t}{v_{t-1}}\right)$ implies

$$NFA_t - \left(\frac{d_t + v_t}{v_{t-1}}\right) NFA_{t-1} + v_t - \left(\frac{d_t + v_t}{v_{t-1}}\right) v_{t-1} = \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t - c_t - tax_t + r'_{x,t-1}\alpha_{t-1}$$

which simplifies to

$$NFA_t - \left(\frac{d_t + v_t}{v_{t-1}}\right) NFA_{t-1} = \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + d_t - c_t - tax_t + r'_{x,t-1}\alpha_{t-1} \tag{A24}$$

The government budget constraint is

$$g_t = tax_t + \frac{b_{H,t+1}}{1 + r_t} - b_{H,t} + \frac{b_{H,t+1}^*}{1 + r_t} - b_{H,t}^*$$

where inserting the bonds market-clearing conditions, $b_{H,t+1} = -b_{H,t+1}^*$ and $b_{H,t} = -b_{H,t}^*$, we have

$$g_t = tax_t \tag{A25}$$

Using (A25) in (A24) yields

$$NFA_t - \left(\frac{d_t + v_t}{v_{t-1}}\right) NFA_{t-1} = \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} w_t n_t + d_t - c_t - g_t + r'_{x,t-1}\alpha_{t-1} \tag{A26}$$

Finally, the aggregation of (A26) across households implies

$$NFA_t - \left(\frac{d_t + v_t}{v_{t-1}}\right) NFA_{t-1} = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t - c_t - g_t + r'_{x,t-1}\alpha_{t-1}$$

Let us find the change in net foreign assets, $\triangle NFA_t = NFA_t - NFA_{t-1}$, implied by the last expression

$$\triangle NFA_t = \left(\frac{d_t + v_t}{v_{t-1}}\right) NFA_{t-1} - NFA_{t-1} + w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t - g_t - c_t + r'_{x,t}\alpha_{t-1}$$

which slightly simplifies to

$$\triangle NFA_t = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t + \left(\frac{d_t + v_t}{v_{t-1}} - 1\right) NFA_{t-1} - g_t - c_t + r'_{x,t}\alpha_{t-1}$$

Valuation effects, VAL_t, correspond to the sum of the return differentials and the gains in value of previous NFA holdings

$$VAL_t = r'_{x,t}\alpha_{t-1} + \left(\frac{v_t}{v_{t-1}} - 1\right) NFA_{t-1}$$

which comprises the excess returns that domestic households get of foreign equity holdings, domestic bond holdings and foreign bond holdings, (adjusted with real exchange rate variations). Having valuation effects recognized, the dynamic equation for Net Foreign Assets becomes

$$\triangle NFA_t = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t + \left(\frac{d_t}{v_{t-1}}\right) NFA_{t-1} - (g_t + c_t) + VAL_t$$

III. Data sources

Canada

Data series are retrieved from FRED, Federal Reserve Bank of St. Louis and main source is OCDE "Main Economic Indicators - complete database". Otherwise it is stated.

Y is real GDP obtained as the ratio between the *Current Price Gross Domestic Product* (Series ID: CANGDPNQDSMEI) and the GDP implicit price deflator with base year in 2010 (Code CANGDPDEFQISMEI). Original series presented at annual rates are transformed into quarterly rates. Presented in per capita terms using Pop is *working age population, Aged 15-64,* (Series ID: LFWA64TTCAM647S). Quarters from 1990-1995 are missing from original source, and they are computed using the linear trend.

W are *monthly earnings in manufacturing.* Data are presented at monthly rates and transform into quarterly rate. (Series ID: LCEAMN01CAM189S).

R is the *Interest Rates, Government Securities, Treasury Bills.* Original data are presented in percent per annum, monthly and not seasonally adjusted, and transform into percent per quarter.

π is the quarterly rate of producer price inflation measured as the rate of growth of the GDP implicit price deflator (P).

G are *governments final consumption expenditures.* (Series ID: CANGFCEQDSMEI).

NT, EXP and IMP are *net exports, exports and imports of goods and services.* Data are taken from Statistics Canada: Table CANSIM 380-0070). Ratio over Canadian GDP.

$IMP(US)$ are in theory equal to $EXP(CA)$ see United States data for source. This data series is converted into Canadian dollars by using the Canadian Nominal Exchange Rate with the United States. We compute $NX(US)$ using these two variables.

NFA refers to *International investment position in Canada* in market values, expressed in quarterly millions of Canadian dollars. We compute Canadian NFA for the US and for the rest of the world. NFA data are provided by assets-liabilities basis, and we make use of its main components: Foreign Direct Investment, Portfolio equity and Portfolio Debt. Data series comes from STACAN Table: 36-10-0485-01 (formerly CANSIM 376-0142).

US

Data series are retrieved from FRED, Federal Reserve Bank of St. Louis and main source is U.S. Bureau of Economic Analysis. Otherwise it is stated.

Y^* is the *real gross domestic product* (Series ID GDPC1). These data series are computed in per capita terms using working age population: Monthly Ratio of Civilian Labor Force (in thousands of persons 16 years of age and older (Series ID CLF16OV)) and civilian labor force participation rate in percentage (Series ID CIVPARTT, it comes from the 'Current Population Survey (Household Survey))

W^* are *Average Hourly Earnings of Production and Nonsupervisory Employees for Total Private* in Dollars per Hour. (Series ID AHETPI. The series comes from the Current Employment Statistics (Establishment Survey)). Also, we have computed Median usual weekly real earnings for those employed full time. Workers 16 years and over. 1982-84 CPI Adjusted Dollars. (Series ID LES1252881600Q)

π^* is the quarterly rate of producer price inflation measured as the rate of growth of the GDP implicit price deflator (P^*), Series ID: GDPDEF.

G^* are *Government total expenditures.* (Series ID, W068RCQ027SBEA).

R^* is the *Wu-Xia Shadow Federal Funds Rate* obtained as described in Wu and Xia (2016). Unlike the observed short-term interest rate, the shadow rate is not bounded below by 0 percent.

NX^*, EXT^*, and IMP^* are *US total net exports, exports and imports of goods and services.* (Series ID NETEXP, EXPGS and IMPGS)

$EXP(CA)$ are monthly *US Exports of Goods by free Along side Ship Basis to Canada* (Not Seasonally Adjusted in millions of US dollars, Series ID EXPCA). This data series is converted to quarterly basis to be consistent with the model.

$IMP(CA)$ are monthly *US Imports of Goods by Customs Basis from Canada* (Not Seasonally Adjusted in millions of US dollars, Series ID IMPCA).

IV. The dynamics for portfolio holdings for valuation effects

We could have incorporated endogenously the portfolio choice for asset holdings following Devereux and Sutherland (2009, 2011)'s solution method. A first step in their process would be to recall the balance of payments accounts obtained in the model

$$\triangle NFA_t = w_t n_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} dj + d_t + \left(\frac{d_t}{v_{t-1}}\right) NFA_{t-1} - (g_t + c_t) + VAL_t$$

where valuation effects include the return differencials (weighted by the asset holdings) and the revaluation of lagged NFA

$$VAL_t = r'_{x,t}\alpha_{t-1} + \left(\frac{v_t}{v_{t-1}} - 1\right) NFA_{t-1}$$

The column vector of asset holding is $\alpha_{t-1} = [\alpha_{EF,t-1}\ \alpha_{BH,t-1}\ \alpha_{BF,t-1}]'$. Following Devereux and Sutherland (2009, 2011) we need to solve a first order Taylor-approximation of our model, so that the term $r'_{x,t}\alpha_{t-1}$ rests as follows,

$$r_{x,\hat{t}-1}{}'\bar{\alpha} + \bar{r_x}{}'\hat{\alpha_{t-1}}$$

in which the bar refers to steady state value and the hat to first order deviation with respect to the value in the steady state solution. Since there is no risk in steady state, the differential in the rates of return are 0, $\bar{r_x} = 0$, and the second term disappears. Moreover, the solution for the steady state portfolio holding vector, $\bar{\alpha}$, can take any value because there is no unique solution for this term in the steady state. Since there is no risk in equilibrium, any asset allocation would be valid. Furthermore, since we are empirically testing our theoretical model to the case of Canada and the US, we have set the values for the α_{t-1} vector from the historical ratios observed in the data. Those are described in the Section 4.