A LEXICOGRAPHIC APPROACH TO HEALTH CARE OPTIMIZATION: CASE STUDIES

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ABSTRACT

The objective of this project is to solve two optimization multi-criteria models (a bi-criteria one and a triple objective one) using a Lexicographic approach. These models are about the assignment of workers to the different job or services of a real hospital, taking into account the available budget and requirements of each job. The software used for solving it is AMPL programming language with solver CPLEX v9.1 that uses the Branch and Bound method. Both, Lexicographic approach and Branch and Bound method are also explained in this paper.

Before the resolution of those models we show several optimization cases applied to health care institutions. We talk about nurse rostering problems, allocation and management problems.

1. TABLE OF CONTENTS

2. INTRODUCTION

The initial idea of this project was to build a model to optimize the nurse rostering of a hospital. However there was not enough time to work on it and solve it, so we finally decided to describe some optimization cases in health care and build and solve a model about optimization of job assignments in health care institutions.

At the beginning Raquel Villegas and I (Uxue Tornos) were going to do the whole project together but we finally decided to do it separately. Only one part of it was made by both authors.

Health is a very important question in our society and hospitals have to be ready to face all the problems that each of the millions of patients has. Diary, these health care institutions have to deal with different situations, surgeries, emergencies, patients, staff… It is really important to organize it in a way that it does not become a mess, and be able to attend all the situations in a fast and efficient way. By optimizing different aspects of the hospitals it is possible to increase their efficiency and to save a lot of lives. It is also important the schedules of the personnel. They should take into account the preferences of the workers and the needs of the hospital.

Because of the importance of the health care there is a lot of literature about different models of optimization of this topic. In the first part of this project we are doing a review of the most important authors in nurse rostering optimization.

After that we explain some cases. The first case explains how a centralized nurse scheduling system can reduce the costs and improve the nurse satisfaction. In the second we present a few models of nurse rostering and after that one we talk about the possibility of integrate nurse and surgery scheduling to obtain better results. The forth case of this project treats the multi-objective models for assignments of services in a health care institution. These cases and the literature review were prepared by both Raquel and me.

The fifth, sixth and seventh cases are more directed to the management of a hospital than to the scheduling systems. The first of them talks about how to improve the location of health care facilities using some optimization models. The second determines the efficiency of the perioperative services of a hospital using Data Envelopment Analysis

(DEA) and the third one is a model of casualty processing in major incident response. These three cases were prepared by each one of us separately.

During the realization of this project we both went to a hospital in Krzeszowice to see how it works and the real management problems that hospitals have. Some of those specific problems and possible solutions are narrated in this paper. We took some pictures to show how the hospital looks.

The final and most important part of this project is the resolution of two models to optimize the assignments of the jobs in a hospital with the help of the tutor Bartosz Sawik PhD using the CPLEX v9.1 software and the analysis of those results. They are multi-objective model (bi-objective and triple-objective) and we are going to solve them by using the "Lexicographic Approach". The CPLEX software uses the "Branch and Bound" method to solve the model. Inside this project we can find a little explanation of the method and the approach used.

The data used to solve the model was gathered from one hospital of Krakow by project supervisor – professor Bartosz Sawik PhD from Department of Applied Computer Science at Faculty of Management, AGH University of Science and Technology in Kraków. Some information about this hospital can be also found inside of this paper, before the resolution of the method.

The scripts of both models introduced in the software are attached on APENDIX A, and the solving characteristics on APENDIX B.

3. THE STATE OF THE ART OF NURSE ROSTERING [20]

3.1 INTRODUCTION

The term "rostering" has a lot of different meanings. Here, when we talk about nurse rostering, we are going to refer to the huge scheduling problem of the hospital personnel, and it means basically the allocation of nurses to periods of work over several weeks.

This domain usually covers different fields but although they are different, they are really connected. These fields are staffing, budgeting and short-term scheduling problems.

Until a few years ago the scheduling was made manually, that is called "self-scheduling". It was a very time consuming task. In most of the hospitals they still use this manually system, and in the ones that use computerized scheduling systems, they do not get the best results because they do not exploit all the capacities of the system.

In healthcare is really important for the system to create a good timetable because it is not possible to do not support patient care needs. The automatized programs are really useful for that, they give us many different solutions for the scheduling problems and also they can optimize the nurses' work by saving time.

3.2 DESCRIPTION OF MODELS

There are some important decisions that must be taken before using a scheduling method. It is significant to choose between manual or automated scheduling methods and also to decide to use cyclical or non-cyclical scheduling that will lead to different solutions. We will now focus on these decisions discussing about some surveys articles.

3.2.1 LITERATURE OVERVIEWS

We are going to talk about the view of different people during 30 years.

First of all, **Warner** (1976) talks about three areas of manpower decision research: staffing, reallocation and scheduling of nurses. He defines 5 criteria for the scheduling part of the problem: coverage, quality, stability, flexibility and cost. He compares three scheduling approaches to these 5 criteria and he takes the next conclusions: the only advantage that the Traditional Approach has is that is flexible because it is done by hand, on the other hand the Cyclical Scheduling provides good schedules but it cannot easily address personal requests and the Computer Aided Traditional Scheduling is the one that follows better all the criteria.

Fries (1976) shows a list of early methods based on manual procedures that follow some arbitrary rules. It would be difficult to use it in a modern complex hospital but it would be possible to talk about some kind of hybrid approach, mixing an early process with a modern one.

Tien and Kamiyama (1982) present some personnel scheduling algorithms concentrating in the USA hospitals but not restricted to healthcare. They refer to five different stages inside the manpower scheduling problem, two of them are management decisions that belong to the long-term staffing problem and the other includes short-term timetabling problem. However, these 5 stages are not enough to explain the modern nurse rostering.

Sitompul and Randhawa (1990) focus on reduce the personnel cost, the financial cost. They defend the approach of solve staffing and rostering problems at the same time. In a theory way this is true, however in practice it is not so good.

Warner, Keller and Martel (1990) discuss patient-oriented and employee-oriented issues in nurse management. They talk about the history of computerized nurse scheduling in the US and also they introduce a nurse scheduling system called ANSOS (Automated Nurse Scheduling Office Systems).

Bradley and Martin (1990) discuss about three different decisions about personnel scheduling: staffing, personal scheduling and allocation (that was started by Warner). The first problem is about determining the long-term number of personnel that must be employed. The second is the conversion of the expected daily work force into assignments, that is, personnel rostering. The allocation consists of assigning the scheduled personnel to actual work sites. They present a classification that can be summarized as: exact cyclical, heuristic cyclical, exact non-cyclical and heuristic non-cyclical.

Siferd and Benton (1992) did a review of the factors that influence hospital scheduling and staffing in the USA. They start talking about the importance that cost reduction is having and they discuss about the nurses' schedule too, with the constraints that they have. They discuss things like: it is more popular the full time work than a part time work, it is also strange to have the same nurses for day and night shifts, half of the hospitals have 3 shifts in weekdays and the 30% they have 5 shifts.

Hung (1995) did an overview from nurse scheduling, from the 60's up to 1994. It can be a bibliography or a useful collection of literature.

Cheang and others (2003) present a survey where they cover mathematical programming methods to show models and approaches for nurse rostering.

Ernst and others (2004) discuss about staff scheduling and rostering in general, they distributed the overview in three parts: the definitions and classification of problems, a classification of literature and the solution methods. They point out that mathematical programming and metaheuristic approaches are the most investigated techniques and that metaheuristics are promising for difficult problems and real world problems which solutions cannot be obtained with other approaches.

It is clear that nurse scheduling problems has been addressed across a lot of articles; some of them only talk about problems and propose solutions that cannot be applied in hospitals. This problem was treated by Warner in 1976 and it continues during the years. Siferd and Benton started to show solutions for the real world problem, a lot of people in the past did not address this complexity.

3.2.2 STAFFING

Hospital staffing involves determining the number of personnel of the required skills to meet predicted requirements. In real world staffing, budgeting and personnel rostering usually take place at different levels and horizons. As we have seen before, lots of researchers have decomposed the nurse rostering problem into phases but the interaction between levels, in practice, will be unworkable. A summary of some key articles is presented because of the impact that some input data can have on the short-term nurse rostering problem.

In 1963, **Wolfe and Young** presented a model to minimize the cost for assigning nurses with different skill categories to different tasks.

Warner (1976) did not only deal with rostering problem, he dealt with the staffing problem too. The staffing problem consists in determining an appropriate number of full time nurses for each skill category. He proposed a methodology and some hospitals accepted it. After that phase it comes the last one: reallocation of nurses. He is sure about that the combination of these three stages leads to a better scheduling policy.

De Vries (1987) developed a 'management control framework' to balance the supply and the demand for nursing care. It seems to be not a strict equilibrium but an acceptable range of balance. He divides the workload per hour by the available staff per hour and calculates by this way the actual capacity utilization.

Smith-Daniels and Schweikhart (1988) present a literature overview on capacity planning in healthcare. They talked about two different decisions: acquisition and allocation decisions. They predict that the strict staffing and timetabling of people and other resources will all be combined in an objective for new scale health organizations.

Easton, Rossin and Borders (1992) compare different staff policies during one month in a large hospital in USA. They see that in busy periods unscheduled nurses will be expected to work, and in slack periods, people will work too few hours to earn their full wages. They discussed the possibility of working with 'float' nurses, but they must not do some works that required a lot of experience. They finally have 12 different methods, and they conclude that the expected nursing expenses decrease as the scheduling alternatives increase. The research excludes overtime, part-time work and understaffing because it is very difficult to formalize them, but these extra parameters need to be into account.

The rostering algorithms have to handle the results of management decisions at a high level. In some hospitals staffing and scheduling information is used to support structural change.

3.2.3 ADMINISTRATIVE MODES OF OPERATION

There are really different administrative procedures in different hospitals that lead to different types of nurse rostering problems. Now, we are going to see the most important ones.

Centralized scheduling is a term used to describe the situation where one administrative department in a hospital carries out all the personnel scheduling (Easton, Rossin and Borders, 1992; Siferd and Benton, 1992; Smith-Daniels, Schweikhart, and Smith-Daniels, 1988; Warner, 1976). Head nurses do not have to construct the schedules. The advantage of this procedure is the opportunity for cost containment through better use of resources. On the other hand it has some limitations as being some kind of favoritism or that the rosters are unfair.

We talk about unit scheduling when the head nurses are given the responsibility to generate the schedules locally (Aickelin and Dowsland, 2000; Bradley and Martin, 1990; Dowsland, 1998; Meisels, Gudes, and Solotorevski, 1996; Meyer auf'm Hofe, 1997; Sitompul and Randhawa, 1990).

Self-scheduling is used to talk about the situation when the personnel roster is generated manually by the staff themselves. It is more time consuming than automatic scheduling but it has the advantage that the nurses co-operate and are asked for advice. It is usually performed by the personnel members themselves and coordinated by the head of the nurse of a ward. It is a very intensive procedure.

Miller (1984) and **Hung** (1992) they defend that type of scheduling, they say that the autonomy of the nurses increases, it reduces the head nurse's scheduling time and it improves cooperation and team work. They say this is more effective because the personnel know that their problems are taken into account.

Silvestro and **Silvestro** (2000) define three different types of scheduling that are departmental rostering, conducted by the charge nurse, team rostering, the staff is divided in teams with nominated leaders in each and self-rostering where the roster is prepared by the ward staff. They identify 4 key determinants of rostering problem complexity that are ward size, predictability of demand, demand variability and complexity of skill mix. They conclude that departmental rostering is more appropriate for large wards and team rostering is better for medium sized ward with easy problems. Self rostering is dangerous because unbalanced rosters can be generated.

3.2.4 CYCLICAL SCHEDULING

Cyclical scheduling concerns organizations in which each person works a cycle of n weeks, it is called "fixed" scheduling too, and the non-cyclical is known as "flexible" scheduling. This type is used when the day is divided in different shifts and the personnel requirements per shift and per day obey a cyclical pattern.

Warner (1976) said that it has some advantages as knowing the schedule a long time in advance or using “forward” rotation, which is when a schedule includes no shift starting at an earlier time than a shift on the day before.

Megeath (1978) proposed cyclical 7-days patterns of shifts and days off to allow for balanced shift coverage. It has benefits but it is not very flexible and it requires a higher level decision.

Burns and Koop (1987) developed a cyclical model for manpower scheduling with strict specifications on consecutive working days and days off. It uses only 3 shifts and it is not flexible at all.

Hung (1991) presents a cyclical pattern for short-term nurse scheduling with 4-day workweeks and 10-hour shifts. He said that it has benefits, as the people that work together are a real team and the people have more time for social activities. The algorithm is simple enough to be generated by hand. He found the limitations of this approach too. It is not directly applicable to many modern real nurse rostering situations. It would be a good idea to translate some of its benefits into constraint based methods. Cyclical personnel rostering are generated using constraint satisfaction by **Muslija, Gaertner and Slany** (2000).

Tour scheduling is a special case of cyclical scheduling and it is one of simultaneously determining days worked, start times and shift lengths worked over some plan horizon.

Bechtold and Showalter (1987) combine the problem of staffing and scheduling personnel in a tour scheduling problem. It is not so flexible so it cannot be applied in modern hospitals.

Cyclical schedules are not flexible when it comes to addressing slight changes in personnel demands or in expressing personal preferences.

3.3 NURSE ROSTERING APPROACHES

In the scientific literature there are several approaches to the nurse rostering problem. Most nurse rostering problems are extremely complex and difficult.

Here we are going to present them grouped to the type of method that is described.

3.3.1 OPTIMISING APPROACHES: MATHEMATICAL PROGRAMMING

Mathematical programming methods are very good for finding optimal solution. Most of the mathematical approaches are based on optimizing the value of a single objective function, so then, researchers simplify the problem of nurse rostering by considering only a few constrains in their model. These models are too simple to be useful into a real hospital situation.

However there have been done several experiments with goal programming or multi objective decision making.

3.3.1.1 LINEAR AND INTEGER PROGRAMMING

Abernathy et al. (1973) solved nurse scheduling problem by using mathematical programming techniques. They divide the staffing of hospitals into three decision levels: policy decisions, staff planning and short-term scheduling of available personnel taking into account the constrains imposed by the first two stages. This involves more management decisions than the nurse rostering problem. This approach has been only tested in a hypothetical example application.

Warner and Prawda (1972) presented a formulation to calculate the number of nurses from certain skill category to undertake a number of shifts per day. They use three shifts of 8 hours each one, and try to minimize the difference between the lower limit for the number of nurses and the nurses. By employing more people the cost for personnel shortage can be reduced. The problem is that in this model there is no way to include personal preferences, and it is not trustworthy for a period longer than four days due to the accurate forecast of personnel demand.

Warner (1976) worked with the previous formulation and introduced weights or fairness levels. He uses shift patterns of 2 weeks length and some flexible constrains. Certain parts of the scheduling are carried out once the optimization starts such as the weekends' assignation or the people who is going to rotate. Those things are done manually, and that simplifies the model. One of the most important things of this model is that allows a fair evaluation of the schedules.

Trivedi and Warner (1976) describe an algorithm to arrange the assignment of nurses from different units in moments that there is shortage of personnel, what is called float nurses. It is not relevant for modern nurse rostering environment because it only works with small-scale problems, but it is the first model that introduces the idea of float personnel. There isn't any methodology to deal with the floating staff, but it is still very used in modern hospitals nowadays, although it seems that float nurses "settle down" very quickly in a ward with a temporary shortage of personnel.

Warner, Keller and Martel (1990) presented a nurse scheduling system called ANSOS that consists in four modules: The Position Control Module (scheduling information for each employee), The Scheduling Module (specific rules with personal preferences), The Staffing Module (computes the staffing level required for each unit) and The Management Reporting Module (provides reports). This model takes into account a lot of personal and individual constrains, and it is used in real hospitals.

In the **Miller, Pierskalla and Rath** (1976) method there are no shifts specified. They formulate the personnel requirements and the number of personnel per day. In this model, they use an "aversion" index which measures how good or bad the previous nurse schedules were. It is used a cyclic coordinate descend algorithm in order to find the optimal solution.

Bailey and Field (1985) create a general mathematical model for the nurse scheduling problem. In this model the cost function is the sum of the cost for utilizing a shift type multiplied by the number of times that appears in the schedule. The length of the scheduling period is of 12 hours instead of 8, and it reduces the idle time.

Rosenbloom and Goertzen (1987) developed an integer programming algorithm to create cyclical scheduling. Optimal solutions are generated, but it only considers work stretches and days off. In the real world there are more constrains that only those ones.

Jaumard, Semet and Vovor (1998) proposed an exact solution approach for a flexible model. The linear programming model allows full exploration of the set of feasible solutions, although the conflicting nature of the nurse scheduling constraints makes it very difficult to find them. This approach allows for formulating coverage constraints in terms of time intervals.

Millar and Kiragu (1998) use a model that combines the possibilities of 2-shift patterns and 4 days length for cyclic and noncyclical nurse scheduling. However the results of the patterns do not deal with the requirements needed in large hospitals.

Isken (2004) tackles real hospital scheduling problems. It introduces a tour scheduling model that simplifies the problem for the mathematical programming techniques. The objective is to reduce labour costs while meeting the fluctuating coverage requirements over a one week planning period. It determines shift start times and full time and part time tours.

Moz and Pato (2004) talk about the problem of re-rostering nurse schedules, without using float nurses. They solve the problem by using replacement within the ward. They keep the hard constraints satisfied and try to minimize the difference between the original schedule and the new one, in order not to disturb the private life of the staff.

3.3.2 GOAL PROGRAMMING/MULTI-CRITERIA APPROACHES

Mathematical programming techniques sometimes are not flexible to deal with more than one goal, so they often optimize only one goal or one criterion. Most of the papers in this section apply mathematical programming but the latest research (Berrada, Ferland, and Michelon, 1996; Burke et al., 2002; Jaszkiewicz, 1997) tackles metaheuristics within a multi-objective framework.

Arthur and Ravindran (1981) propose a two phase goal programming heuristic to simultaneously optimize different goals in priority sequence: staff size, the number of staff with preferences, staff dissatisfaction, and the deviation between scheduling and desired staffing levels. The shifts are heuristically assigned at the end of the scheduling process.

Musa and Saxena (1984) use an interactive heuristic procedure. It is possible to change the relative weights given to the goals during the scheduling process, so then it is possible to take into account temporal conditions. It is very useful because the scheduling circumstances of the hospitals change regularly and they are very difficult to model mathematically. It is not possible to use it directly but it is worth to investigate it and incorporate some ideas into modern methods.

Ozkar ahan and Bailey (1988) define three basic objective functions. The first of them tries to minimize de deviation between the number of nurses scheduled and the demand of each day (time-of-day). The second works with the difference between the sum of days on work patterns and the size of the work force (day-of-week). The last one combines the day-of-week and time-of-day scheduling problems. Employing a heuristic assignment of schedules, the algorithm solves the most important shift times and days for individual nurses. **Ozkar ahan** (1991) creates a goal programming approach for a decision support system. It tries to maximize the utilization of the full time personnel, minimizing over- and understaffing and personnel costs. It helps in staffing decisions and provides support for nurses' preferences. This model works only with small size problems.

Franz et al. (1989) create a multi-objective integer linear program for health care staff working at different locations (multi-clinic health regions). It includes personnel with different skills, and tries to minimize the traveling costs and maximize the quality of the service by considering the personal preferences and requirements.

Chen and Yeung (1993) combine goal programming with expert systems. The first one assists in satisfying the time related constraints on the schedules and attempts to cover personnel demands at the same time. The second one does the assignment of shift types to personnel members. This paper allows flexibility and the relaxation of constraints.

Berrada, Ferland and Michelon (1996) combine multi-criteria approach and tabu search in a flexible tool. To obtain a feasible solution, it must be satisfied a set of hard constraints. To get that, a list of soft constraints is treated as a list of goals. In this model, every nurse works the same shift all the time, so the problem is reduced to 3 smaller problems. It produces satisfactory results. This is one of the first papers in which mataheruistics are applied to address a range of different goals.

Jaszkiewicz (1997) introduces a decision support system for the nurse scheduling problem in Polish hospitals. Working and free days are preferably grouped, and the shifts have to be divided evenly among the nurses. There are two stages to reach the solution. In the first one there is a combination of a multi-objective algorithm with a simulated annealing approach (Pareto-Simulated Approach). In this stage it is generated a set of good quality solutions. In the second stage a hospital planner evaluates these results in an interactive way. This model is actually applied in a hospital, but in order to be easily transferred to other hospitals it would require further work because it takes many constraints for granted.

Burke et al. (2002) present a new multi-criteria approach. The criteria space is mapped to a preference space with dimensionless units. This approach gives the possibility for users to express their preference for certain constrains. The weights can control the compensation of constraints. One advantage of multi-criteria method is that it facilitates a better handling of dissimilar constraints by considering possible ranges for the criteria.

Most mathematical approaches apply exact methods but the real world problem is so complex that most of the publications mention heuristic methods to tackle the problems.

3.3.3 ARTIFICIAL INTELLIGENCE METHODS

3.3.3.1 DECLARATIVE AND CONSTRAIN PROGRAMMING

Okada and Okada (1988) use the program Prolog with a formal core that assists in the assignment to shifts to nurses. The importance of the requirements can change during the planning period, so then not all the constraints must be strictly satisfied. The approach is much stricter than most others, it distinguish between the scheduling task and the general requirements that must be fulfilled. Assignments are carried out in a manual-like manner following a strict procedure. In **Okada's** method (1992) there are a set of "role sequences" as a language in which the constraints are presented as a grammar, and individual preferences are constraints in strings. There are multiple criteria to evaluate, and the system tries to discover the best schedules. This system allows for flexible definition of the soft constraints by the users of different types of hospitals.

Weil et al. (1995) reduces the complexity of a constraint satisfaction problem by joining some constrains and reducing the domains by eliminating interchangeable values. He solved quite simple problems with this method.

Darmoni et al. (1995) describe a software system called Horoplan for large hospitals. It is useful for rostering and for some short term staffing decisions. It creates nursing schedules with a step by step procedure, reflecting the way that head nurses create their schedules manually.

Meisels, Gudes, and Solotorevski (1995) describe an approach that is implemented in a commercial software package called TORANIT. It is very flexible with respect to defining constraints and shifts. For the constraint programming approach they separate them into 3 groups: mutual exclusion constraints (one job at time for nurses), finite capacity of employees (limited hours of work in a determinate period of time) and objectives (distribution of the employee assignments per shift). This combines assignment rules and constraint rules, and personal preferences for certain shifts are tackled by the assignment rules. On the other hand, the constraint rules handle the demand for certain types of nurses or for individual nurses, in addition to personal constraints. **Meisels and Lusternik** (1998) also investigate constraint networks for employee timetabling problems. The approach consists in standard constraint processing techniques, which solve randomly generated test problems.

Cheng, Lee, and Wu (1997) invented a nurse rostering system for solving one hospital problem in particular. They use ILOG solver to create a schedule that satisfies a set of rules. Those rules are divided into hard and soft constraints, and the solutions are generated in 4 steps. The problem of this method is that is designed for one particular situation of one determinate hospital.

Meyer auf'm Hofe (1997) talked about the nurse rostering problem as a hierarchical constraint satisfaction problem. It enables the use of overlapping shifts to the traditional ones. The created schedules have also to meet requirements like legal regulations, personnel costs, flexibility with respect to the actual expenditure of work, and the consideration of special qualities. Some of these requirements are from higher decision level, and it is not clear how staffing decisions are implemented in the model. In the practice, it is impossible to satisfy all the constraints, and that is the reason for what requirements are treated in order of importance. It is very complex to generate a satisfactory schedule, but this method allows user to alter the result of the algorithm by hand. **Meyer auf'm Hofe** (2001) maintains the hierarchical level of constraints and constraints weights. As we have said it is not possible to satisfy all the constraints, so now, instead of considering the constraint satisfaction he considers the nurse rostering problem as a constraint optimization. He introduces fuzzy constraints than can be partially violated. A mix of iterative improvement and branch and bound are used in a constraint propagation algorithm that deals with the fuzzy constraints.

Abdennadher and Schlenker (1999a, b) present an interactive program (INTERDIP) that has been tested in a real hospital environment and it is based on constraint programming.

Muslija, Gaertner and Slany (2000) attempt to generate cyclical solutions for a simplified version of general workforce scheduling problems. This kind of schedule is beneficial for the employees' health and satisfaction. Some important characteristics of the schedules are the length of work blocks and "optimal" weekend characteristics. This method is good for generating schedules very quickly, but it is too simple to be used in large scale healthcare environments.

Li, Lim and Rodrigues (2003) tackle a problem from a real world hospital situation applying a hybrid of constraint satisfaction and local search technique. The soft constraints in this model are called preference rules and they consist on personal preferences or general preferences for shift sequences. They first solve a problem in which some of the constraints are relaxed. After that they apply local search techniques to improve the solution trying to satisfy as many preference constraints as possible.

3.3.3.2 EXPERT SYSTEMS AND KNOWLEDGE BASED SYSTEMS

Expert systems approaches give the possibility of developing user-interactive integrated decision support methodologies for nurse scheduling problems.

Smith, Bird, and Wiggins (1979) created a "what-if" decision support system where the user can change the weights of the objectives and take into account personal preferences.

Bell, Hay, and Liang (1986) made a visual and very interactive decision support system. **Ozkar ahan** (1991) uses a goal programming model and kept the dimensions of the problem very small. This model is too simple to tackle with most of the problems. **Ozkar ahan and Bailey** (1988) describe three objectives in the goal programming approach.

Chen and Yeung (1993) use a hybrid expert system to create full time schedules for nurses. This system handles constrains related to the work time conditions of the employees and other fairness measures. At the same time the program tries to use the minimum staff level by applying a goal module, but it is not a hard constraint for them. They define aspiration levels for each goal.

Scott and Simpson (1998) reduce the search space by using constraint elements in a case-base. They imitate the approaches of manual roster planners and generate good solutions with it in a limited time.

Petrovic, Beddoe and Vanden Berghe (2003) also use a case-based reasoning approach to nurse rostering problems. It is being used in a UK hospital. Hospital planners combine partial rosters that have been done with the personal preferences and requests. This methodology solves problem by using the experience, what means that similar problems have similar solutions.

3.3.4 HEURISTICS

The size of the rostering problems and the lack of knowledge about the structure of most of them hinder the applicability of exact optimization methods. The applicability of heuristics scheduling algorithms requires a clear formulation of the hospital requirements, so then it is possible to obtain high quality schedules in an acceptable computation time.

Smith (1976) creates an interactive algorithm to help to construct a cyclical schedule. The algorithm determines the number of personnel members, but not all of them can have rotating schedules. **Smith and Wiggins** (1977) use list-processing techniques that generate non-cyclical schedules for each month, and each skill category. Schedules are developed per person with a considerable number of constraints taken into account. Its interactivity allows users to make manual changes into the generated schedules.

Blau and Sear (1983) work with a two week period and generates all the possible schedules. In a second step, a cyclic descent algorithm is used to find an optimal schedule for each nurse. This approach is developed for wards with three skills categories hierarchical replaceable. **Blau** (1985) tries to distribute the unpopular work in addition to the frequency with which employees are granted requests for shift or days.

Anzai and Miura (1987) present a cyclic descent algorithm for a ward in which the personnel members have the same skills. It is a model too simple for practical applications.

Kostreva and Jennings (1991) use two phases. First, they calculate groups of feasible schedules (respecting the minimum staffing requirements and meeting as much constraints as possible). In the second phase the "aversion score" is calculated based in the nurses' preferences (Kostreva and Genevier, 1989).

Schaerf and Meisels (1999) define the problem of assigning employees to tasks in shifts. The shifts are predefined time periods that can reside anywhere on the time axis. This model is strict with the coverage constraints, but flexible with the time related constraints. It is introduced a general local search that allows partial assignments, making use of a larger search space.

3.3.5 METAHEURISTICS SCHEDULING

3.3.5.1 SIMULATED ANNEALING

Isken and Hancok (1991) allow variable starting times instead of three fixed shifts per day. The problem is formulated as an integer program in which under- and overstaffing are allowed but penalized. This model was not intended to address the nurse rostering problem.

Brusco and Jacobs (1995) combine simulated annealing and simple local search heuristic to generate cyclical schedules for continuously operating organizations. That kind of organizations allows their workers' schedules to begin at any hour of the day. This is the reason that makes the problem complex. This problem is called by Brusco and Jacobs "tour scheduling problem". One alternative for pure tour scheduling is the use of mixture of both full-time and part-time workers.

3.3.5.2 TABU SEARCH

Berrada, Ferland and Michelon (1996) combine tabu search with multi-objective approach. It is interesting that metaheuristic is applied instead of a mathematical programming approach.

Dowsland (1998) use different neighborhood search strategies in a tabu search algorithm. The heuristic oscillates between feasible solutions meeting the personnel requirements and schedules concentrating on the nurses' preferences. This algorithm must provide enough personnel with the request qualities and at the same time satisfy personal requests in a fair manner.

Dowsland and Thompson (2000) mix integer programming model with a modern heuristic. The algorithms are implemented in CARE (Computer Aided Rostering Environment). Papers describing algorithms for real hospital use tend to cope with similar issues and come up with similar solutions. They use a pre-processing phase to see if the number of nurses is enough to cover the demand. That part of the problem is tackled by a knapsack model. Due to the reduced search space in pre- and post-processing integer programming approaches solutions are obtained considerably faster than with an IP package.

Burke, De Causmaecker and Vanden Berghe (1999) hybridize a tabu search approach with algorithms that are based upon human-inspired improvement techniques. Some of that hybridization work as diversifications for the tabu search algorithm. Users of the software can define their own shift types, work regulations, and more. The algorithms attempt to modify the roaster in order to reduce the number of violations of time-related constrains on the personal schedules. This model is being used in some Belgian hospitals.

Valouxis and Housos (2000) also use the three-shift schedules, but they propose some hybrid optimization techniques. Only looking at forward rotation it creates a list of feasible schedules. They integrate tabu search in an integer linear programming model.

Ikegami and Niwa (2003) introduce a mathematical programming formulation and solve the problem of nurse rostering with metaheuristics. This model covers all the characteristics that seem unavoidable for real world applications. It also distinguishes between nurse constraints and shift constraints. This algorithm provides very promising results but extra heuristics are needed to speed the algorithm up.

Bellanti et al. (2004) deal with a real problem of an Italian hospital, so the constraints are so detailed and specific. Some unavoidable relaxations have been incorporated in the model. The computation time of this algorithm is acceptably low to be used in real hospitals. They explain that different initial solutions are generated by applying different multi-start procedure. From that set of solutions, the best is taken for the local search procedure. The multi-start local search approach and the iterated local search seem to be better than tabu search.

3.3.5.3 GENETIC ALGORITHMS

Easton and Mansour (1993) developed a distributed genetic algorithm for the ‘tour scheduling’. It tries to minimize the number of personnel members to fulfill the demands, but is too simple to be applied to a real problem.

Tanomaru (1995) presents a genetic algorithm to solve staff scheduling problem. It minimizes the total wage cost in a situation where the number of personnel is not fixed. His heuristic mutation operators might be too time consuming and not general enough to deal with real problems.

Aickelin and Dowsland (2000) talked about a cooperative genetic algorithm. They said that the cyclical schedule cannot be performed because nurses' preferences can change. They divided the problems into sub-problems to deal with them better. In 2003, they applied an indirect genetic algorithm to the same nurse rostering to better deal with the necessary constraints.

Aickelin and White (2004) dealt with the same problem as in 2000. They introduced two different algorithms: a genetic algorithm with an encoding (based on an integer programming formulation) and an 'indirect' genetic algorithm (with a separate heuristic decoder function).

Jan, Yamamoto and Ohuchi (2000) developed a less cooperative genetic algorithm which solves a 3-shift problem. Feasible schedules satisfy the hard constraints, which are coverage constraints and personal requests for days off.

Kawanaka and others (2001) propose a genetic algorithm for scheduling nurses under various constrains that can be 'absolute' or 'desirable' constraints. One absolute is the minimum coverage per skill category and one absolute is, for example, the number of new nurses. It is better than other methods because it do not only implement the absolute constraints.

Burke et al. (2001a) developed a set of genetic and memetic algorithms. The coverage constraints are satisfied throughout the search space. By the recombination of two solutions it is impossible to create a feasible solution. This is the reason for what repair procedures have been developed. Thanks to evaluating a population of solutions instead of one single solution the new approach overcomes inappropriate choices for the planning order of skill categories.

3.4 CONCLUSIONS

In these pages we have discussed lots of nurse scheduling articles talking about these nurse rostering problems and we have been able to see how this problem has attracted the attention of scientists for about 40 years.

The automatic generation of high quality nurse schedules can lead to improvements in hospital resource efficiency, staff and patient safety and satisfaction and administrative workload. Researchers have constructed so many different models and they have developed many different techniques. However, very few of the developed approaches are suitable for directly solving difficult real problems because some of them are really simple. We can see in the next table the approaches that have been implemented in practice (in one hospital or in multiple) and the ones that have been tested on real data. The nurse rostering approaches that do not address real problems and those which are only concerned with modeling issues are not included in this table.

Not applied in practice but tested on real data	Applied in practice
Abernathy et al. (1973)	**Approaches applied in just one hospital**
Berrada, Ferland, and Michelon (1996)	Easton, Rossin, and Borders (1992): staffing
Petrovic, Beddoe and Vanden Berghe (2003)	Jaszkiewicz (1997)
Cheng, Lee, and Wu (1996, 1997)	Smith and Wiggins (1977)
Okada and Okada (1988) and Okada (1992)	Bellanti et al. (2004)
Abdennadher and Schlenker (1999a, 1999b): INTERDIP	
de Vries (1987)	**Approaches applied in multiple hospitals**
Warner and Prawda (1972) and Trivedi and Warner (1976)	Warner, Keller, and Martel (1990): ANSOS, Warner (1976)
Miller, Pierskalla, and Rath (1976)	Jelinek and Kavois (1992): Medicus
Muslija, Gaertner, and Slany (2000)	Darmoni et al. (1995) Horoplan
Isken and Hancock (1991, 1998) and Isken (2004)	Meisels, Gudes, and Solotorevski (1997): EasyStaff
Jaumard, Semet, and Vovor (1998)	Meisels, Gudes, and Solotorevski (1996): TORANIT
Aickelin and Dowsland (2000) and Dowsland (1998)	Dowsland and Thompson (2000): CARE
Moz and Pato (2004)	Meyer auf'm Hofe (1997): ORBIS Dienstplan
Burke et al. (2001a, 2002, 2003)	Burke et al. (2001b, 2001, 1999), De Causmaecker and Vanden Berghe (2003): PLANE

Table 1: Applicability of the approach. (Edmund K. Burke, Patrick De Causmaecker Greet Vanden Berghe, And Hendrik Van Landeghem "The State of the Art of Nurse Rostering")

Table 1 clearly demonstrates that modern hybridized artificial intelligence and operations research techniques form the basis of the most successful real world implementations. There is a research in hybridizing exact methods with heuristic approaches. The current state of art is represented by interactive approaches that incorporate problem specific methods and heuristics with powerful modern metaheuristics, constraint based approaches and other search problems.

Here we list some of the future search directions that represent promising paths for nurse scheduling research:

- *Multi-criteria Reasoning:* Nurse scheduling presents a lot of objectives and requirements (Burke, De Causmaecker, and Vanden Berghe, 2004). Many of these are conflicting and it is clear that there is so much to investigate in this area.

- *Flexibility and Dynamic Reasoning:* The personnel schedule usually has to be changed to deal with some circumstances like staff sickness or emergencies (Burke et al., 2001c; De Causmaecker and Vanden Berghe, 2003). There is an amount of promise in investigating confused methodologies as an attempt to address the dynamic nature of the problem.

- *Robustness:* It is clear that robustness has not played an important role in the scientific research literature. However, it is a really important area that must be taken into account more than before.

- *Ease of use:* It is an important feature in the uptake of a decision support system. Many of the algorithmic methods that we have discussed in this review require significant research expertise to employ.

- *Human/computer interaction:* In this article we have seen that most of the models are too simply to be applied in hospitals and that the measures of evaluating algorithmic approaches do not consider some issues that are really important in the real world. To apply the algorithms they have to interface with the administrators or nurses that will use them. This item needs to be more considerate by scientists.

- *Problem decomposition:* To make the work easier is important to decompose the large problems into small ones to treat them better and to solve problems quicker. It is an important thing to take into account.

- *Exploitation of Problem Specific Information:* The full range of constraints that are generated by real hospital problems has not been address enough in the scientific literature on nurse scheduling and it should be more investigated.

- *Hybridization:* It is clear that no one method is going to increase the uptake of nurse scheduling by its own. Progress will be made by drawing on the capabilities of a range of methods, approaches and research advances.

This list is only an indication of some of the directions that may lead to significant research advances in this area. It is clear that all the above cannot be tackled in isolation of each other.

In summary, we need to pay more attention to the issues that are important to modern hospital administrators to increase the uptake of nurse rostering research in the real world. We need to be less rigid when we are talking about algorithms.

4. CENTRALIZED NURSE SCHEDULING TO SIMULTANEOUSLY IMPROVE SCHEDULE COST AND NURSE SATISFACTION [45]

4.1 INTRODUCTION

A recent survey of registered nurses reveals that one-third of the nurses try to leave their position within a year due to many reasons such as high workload, stress, non-patient duties and the most important reason, the scheduling policies. In this case a nurse scheduling model that assists managers in achieving more desirable schedules and reducing wage costs simultaneously is proposed.

Hospitals with decentralized nurse scheduling usually have to call to their nurses to work mandatory overtime. This contributes to less desirable schedules and additional costs for the hospital. By centralizing scheduling decisions across departments it is possible to reduce overtime. The idea of centralizing is interesting when duties are similar from one unit to the next so then it is possible to use cross-trained employees across multiple units (cross-utilization).

This case is based on real nurse data from two hospitals in the United States. The study shows how desirability of nurse scheduling improves by approximately 34%, reducing overtimes by 80% and costs by 11%.

4.2 DATA COLLECTION

Data from two medium to large medical and surgical hospitals of the United States (Hospital 1 and Hospital 2) is used. They have 247 and 526 total beds, respectively. Each hospitals consists in several units, but this model is focused and designed for the general medical/surgical type units (Medical, Surgical, and Orthological-Neurological units) because they have similar nurse duties.

These hospitals have made an effort to standardize procedures across many of their units, so then minimal training is required for nurses to be utilized across the units.

The nurse types used at both hospitals are “Registered Nurses” (RN) and “Nurse Aides” (NA), who are paid hourly at a base rate determined by experience. There is also a premium added to all shifts except the easier to staff day shift.

	Beds	Nurses		Beds	Nurses
Hospital 1			**Hospital 2**		
Medical	32	54	Medical	23	38
Surgical	30	43	Surgical	24	38
Ortho-Neuro	34	46	Ortho-Neuro	28	41

Table 2: Unit Characteristics. (P. Daniel Wright, Stephen Mahar "Centralized nurse scheduling to simultaneously improve schedule cost and nurse satisfaction")

4.3 CENTRALIZED NURSE SCHEDULING MODEL

This is a bicriteria integer scheduling model with objectives for schedule cost and schedule desirability. The centralized scheduling model includes constraints that control the service levels on each unit based on nurse-to-patient ratios. It takes into account the RNs and NAs and it could be expanded if necessary to accommodate additional nurse types. This model is nonlinear, but is presented a linearization of it. The notation used in the model is outlined below followed by the model.

Sets

T	the set of weeks in the scheduling horizon
N	the set of all nurses
K	the set of all units
L	the set of all nurse types
NW	the set of nurses who don't want to work weekends
N^{lj}	the set of nurses of type l available for shift j
S	the set of all shifts
SA^{i}	the set of all shifts across all units that nurse i is available to work
S^{it}	the set of all shifts across all units that nurse i is available to work in week t
WS^{it}	the set of all weekend shifts across all units that nurse i is available to work in week t

Subscripts

i	nurse i
j	shift j
t	week t
k	unit k
l	nurse type l (RN or NA)

Decision variables

x_{ijk}	1 if nurse i works shift j at regular time wages on unit k, 0 otherwise
y_{ijk}	1 if nurse i works shift j at overtime wages on unit k, 0 otherwise
z_{it}	1 if nurse i works any weekend shifts during week t, 0 otherwise
b_{jkl}	the number of nurses of type l required for eight-hours shift j on unit k.
g_{jhk}	1 if h beds exceed the number of beds that nurses can handle according to the nurse-to-patient ratio for unit k on shift j, 0 otherwise
γ_{ik}	1 if nurse i is assigned to unit k, 0 otherwise

Parameters

w	the number of weeks in the scheduling horizon
c_{ijk}	regular time wages if nurse i works shift j on unit k
d_{ijk}	overtime wages if nurse i works on shift j on unit k

$\bar{n}_i$ maximum number of shifts for each week for nurse i
$\underline{n}_i$ minimum number of shifts for each week for nurse i
$\bar{p}_i$ upper limit on the number of units to which nurse i can be assigned

Parameters (con't)

a_{ijk} the undesirability that nurse i has for shift j on unit k ($0 \le a_{ijk} \le 1$, where 1 is the most undesirable)
q_i upper limit on number of weekends worked by nurse i over the scheduling horizon
r_i upper limit on the number of undesirable shifts assigned to nurse i
f_i upper limit on the number of overtime shifts assigned to nurse i
R_i the number of patients per nurse type l as determined by the nurse-to-patient ratio
λ_{jk} mean patient arrival rate during shift j on unit k
μ_{jk} mean unit service rate during shift j on unit k
$P_h(\lambda_{jk}, \mu_{jk})$ probability of h occupied beds during shift j on unit k
u_l maximum allowable single-period average proportion of time in violation of the nurse-to-patient ratio for nurse type l
v_l maximum allowable aggregate period average proportion of time in violation of the nurse-to- patient ratio for nurse type l
s_k number of beds (servers) on unit k

Model

$$(P) \quad \text{Min} \sum_{i \in N} \sum_{j \in SA^i} \sum_{k \in K} \left(c_{ijk} x_{ijk} + d_{ijk} y_{ijk} \right) \qquad (1)$$

$$\text{Min} \sum_{i \in N} \sum_{j \in SA^i} \sum_{k \in K} \left(a_{ijk} x_{ijk} + a_{ijk} y_{ijk} \right) + \sum_{i \in NW} \sum_{t \in T} z_{it} \qquad (2)$$

$$st \sum_{i \in N^{lj}} \left(x_{ijk} + y_{ijk} \right) b_{jkl}, \; j \in S, \; k \in K, \; l \in L \qquad (3)$$

$$\underline{n}_i \le \sum_{j \in S^{it}} \sum_{k \in K} x_{ijk} \le \bar{n}_i, \; i \in N, \; t \in T \qquad (4)$$

$$\sum_{j \in SA^i} \sum_{k \in K} a_{ijk} \left(x_{ijk} + y_{ijk} \right) \le r_i, \; i \in N \qquad (5)$$

$$\sum_{j \in SA^i} \sum_{k \in K} y_{ijk} \le f_i, \; i \in N \qquad (6)$$

$$\sum_{t \in T} z_{it} \le q_i, \; i \in NW \qquad (7)$$

$$\sum_{k \in K} \gamma_{ik} \le p_i, \; i \in N \qquad (8)$$

$$\sum_{k \in K} \left(x_{ijk} + y_{ijk} \right) + \sum_{k \in K} \left(x_{i(j+1)k} + y_{i(j+1)k} \right) + \sum_{k \in K} \left(x_{i(j+2)k} + y_{i(j+2)k} \right) \le 1,$$
$$i \in N, \; j \in SA^i < |S| - 1 \qquad (9)$$

$$\sum_{k \in K} \left(x_{ijk} + y_{ijk} \right) z_{it}, \; i \in NW, \; t \in T, \; j \in WS^{it} \qquad (10)$$

[illegible] (11)

$$x_{ijk} \in \{0,1\}, y_{ijk} \in \{0,1\}, 0 \ge a_{ijk} \ge 1, \; i \in N, \; j \in SA^i, \; k \in K \qquad (12)$$

$$z_{it} \in \{0,1\}, \; i \in NW, \; t \in T \qquad (13)$$

$$\gamma_{ik} \in \{0,1\}, \; i \in N, \; k \in K \qquad (14)$$

$$b_{jkl} \ge 0 \text{ and integer}, \; j \in S, \; k \in K, \; l \in L \qquad (15)$$

In the above formulation, shifts are numbered j=1, …, (3x7)w, where w is the number of weeks in the scheduling horizon. The regular time and overtime nurse wages are minimized in objective (1), and objective (2) minimizes the total number of undesirable regular time shifts, undesirable overtime shifts and undesirable weekend shifts. Nurses that don't want to work on weekends (set NW) are prohibited from specifying those shifts as undesirable, to avoid to count them as double undesirable. Constraint set (3) ensures that there would be a sufficient number of nurses of each type in each shift and unit. Constraint set (4) limits the total number of regular time shifts for a nurse in one week. Constraints sets (5) - (7) enforce upper limits on undesirable regular or overtime shifts, the number of overtime shifts, and the number of weekends worked for each nurse, respectively. Constraint (8) limits the number of units in which a nurse can be cross-utilized. Constraint set (9) prohibits backward rotation for any nurse. Backwards rotation is defined as a nurse being scheduled for a shift that starts less than 24 h after the starting time of the previous shift worked. This can be controlled through the composition of sets SA_i by not including those day or evening shifts in the SA_i set for that nurse. Constraint set (10) forces z_{it} to be 1 if nurse i is assigned to work any weekend shifts during week t. Constraints (11) force γ_{ik} to be 1 if nurse i is assigned to any shifts on unit k so that the level of cross-utilization (from constraint set (8)) can be enforced. Constraints (12)–(15) enforce integrality and non negativity conditions.

The centralized scheduling model (P) is difficult to solve because of its large problem size, and it is most dependent on the number of units scheduled.

4.4 SOLUTION METHODOLOGY

The main objectives of this model are the cost and the desirability of the schedule. Both are solved. First an optimal labor cost is obtained, and then the second objective about the desirability is solved with an additional constraint that is the solution of the first objective.

4.4.1 NURSE-TO-PATIENT RATIOS AND SERVICE LEVELS

To accommodate nurse-to-patient ratios and satisfy the minimum service levels, a manager could multiply the total number of beds on a unit by the minimum nurse-to-patient ratio and then staff to that level. The problem is that there is a daily and weekly variation in patient census patterns, so with this approach, some shifts are going to be overstaffed. A more efficient approach is to staff so that the probability of violating the minimum ratios is kept below a certain critical level, e.g. 5%. Nursing managers have several options for adding staff and avoid violating that minimum ratio, such as the use of excess nurses from other units, on-call workers (Vaughan 2000), travel nurses, and outside agency nurses. The centralized scheduling model is perfect for using excess nurses from other units. Without centralization this cross-utilization is not possible.

Constraint set (16) limits the expected probability that the number of occupied beds exceeds the nurse-to-patient ratio on a unit during each shift for each nurse type. Constraint set (17) controls the upper limit on the aggregate proportion of time over the scheduling horizon that the nurse-to-patient ratio is violated in a unit for each nurse type. In this case, the single-period service level corresponds to each particular shift, while the aggregate service level corresponds to the 5-week scheduling horizon.

$$\sum_{h=R_l b_{jkl}+1}^{s_k} P_h\left(\lambda_{jk},\mu_{jk}\right) \leq u_l,\ j \in S,\ k \in K,\ l \in L \tag{16}$$

$$\left(\sum_{j \in S} \sum_{h=R_l b_{jkl}+1}^{s_k} P_h(\lambda_{jk},\mu_{jk}))/|S| \leq v_l,\ k \in K,\ l \in L\right) \tag{17}$$

Data on patient arrivals to the various hospital units was collected over a 90-day period. Using historical length of stay data and discussions with unit managers, service rates were estimated for each unit (μjk). The average service rate at the subject hospitals is 0.4 patients per shift. Let m denote an index ranging from 0 to sk. Based on Erlang's loss model, the probability of h occupied beds in the system during shift j on unit k is calculated as

$$P_h\left(\lambda_{jk},\mu_{jk}\right) = \frac{\left(\lambda_{jk}/\mu_{jk}\right)^h/h!}{\sum_{m=0}^{s_k}\left(\lambda_{jk}/\mu_{jk}\right)^m/m!} \tag{18}$$

where s_k is the number of beds on unit k. This equation for $P_h(\lambda_{jk},\mu_{jk})$ is used to calculate the service levels in constraint sets (16) and (17).

4.4.2 LINEARIZATION OF THE WORKLOAD MODEL CONSTRAINTS (19) AND (17)

Here a linearization of the workload model is presented so that the centralized nurse scheduling model can be solved as an integer program. To perform the linearization, we define a new decision variable, g_{jhk}, that is set to 1 when there are more beds occupied than the number of beds (patients) that the nurses can handle based on the nurse-to-patient ratio. Constraints (16) and (17) are then modified.

$$1-\frac{(b_{jkl}R_l)}{h} \leq g_{jhk},\ j \in S,\ h = 1..s_k,\ k \in K,\ l \in L \tag{19}$$

$$\sum_{h \in s_k} P_h\left(\lambda_{jk},\mu_{jk}\right) g_{jhk} \leq u_l,\ j \in S,\ k \in K \tag{16a}$$

$$\left(\sum_{j \in S^8} \sum_{h \in s_k} P_h\left(\lambda_{jk},\mu_{jk}\right) g_{jhk}\right)/|S| \leq v_l,\ j \in S,\ k \in K \tag{17a}$$

Once it is linearized it can be solved using CPLEX optimization software.

4.5 RESULTS

Computational results indicate that this centralized scheduling model outperforms decentralized scheduling by 10.7% for labor costs and 34% for undesirable shifts. The amount of overtime is reduced by 80%. The results are not depending on high levels of cross-utilization, it also works well with low levels of it.

By centralizing scheduling decisions hospitals can reduce costs and at the same time give a quality schedules to nurses with less amount of overtime.

4.5.1 SCHEDULE DESIRABILITY

Many hospital administrators are looking for ways to increase the job satisfaction of their nursing staff. By reducing the number of undesirable shifts and overtime, they improve the overall desirability of the schedule. Undesirable shifts are reduced as the number of units in which a nurse can work increases, but the efficiency sometimes is reduced too. Under decentralized scheduling a unit may be forced to schedule many overtime shifts and by centralizing decisions it is reduced. The centralized model is flexible to accommodate additional scheduling policies and nurse types that may exist at other hospitals.

5. NURSE SCHEDULING [27]

The costs related with the health care are rising every year in USA due to the new technologies or the increased demand of the health care. One of the aims of the healthcare systems is to provide high quality services at lower costs to patients. Operations Research techniques are useful to provide optimal or efficient schedules to improve aspects of healthcare systems such as medical supply chain or staff scheduling. Here we are going to discuss nurse scheduling.

There are some common rules related with the staff that can be applied to most departments of the hospital except for operating suits. They are the most important places in the hospital and they have different schedules. That is why we talk about Nurse Scheduling Problem in a General Clinic and in an Operating Suite.

5.1 NURSE SCHEDULING PROBLEMS (NSP)

5.1.1 NURSE SCHEDULING PROBLEM IN A GENERAL CLINIC

A general clinic is an area in a hospital that provides healthcare services to patients with needs. The patients can be divided in inpatients, the ones who are hospitalized overnight, and outpatients, the ones that do not need that. The nurses can be divided in different groups too. They can work for full time or part time, for example, but basically nurses are distinguished from each other by their area of specialty. If it is necessary higher skilled nurses are assigned to shifts that lower skilled nurses are capable to perform, however the reverse is not possible.

In a general clinic there must be available nurses at all times responding to patient workload although they cannot work more hours that their scheduled hours.

5.1.2 NURSE SCHEDULING PROBLEM IN AN OPERATING SUITE

An operating suite is an area of the hospital that provides surgical procedures and consists of many operating rooms (OR). The nurses that work in an operating suite can have different roles some of the most important are *circulation* and *scrub*. Operating suites have some predefined shifts during a working day and all nurses are assigned to these shifts based on their contracts. Operating rooms have a lot of variabilities that must be taken into account in a complicated model such as human behavior or nurse availability.

5.1.3 PROBLEM STATEMENT

Researchers might develop different nurse scheduling tools by finding proper answers for some questions related with the input parameters, the associated goals, the limitations and constraints or the proposed methods.

The problem in a general clinic is to develop a decision-making tool that assigns nurses to shifts based on nurse preferences and patient workload requirements. In an operating suite is the same but assign nurses to surgery cases not to shifts.

5.2 A REVIEW OF OPTIMIZATION APPLICATIONS AND METHODS

Mathematical and heuristic approaches can provide efficient solutions to the scheduling problem. Several nurse scheduling models are introduced, decision variables and constraints are expressed and solution algorithms are discussed for each nurse scheduling problem.

5.2.1 NURSE SCHEDULING PROBLEM IN A GENERAL CLINIC

5.2.1.1 PRIMARY NURSE SCHEDULING PROBLEMS

This model was introduced by Warner and Prawda (1972), and it uses a mixed-integer quadratic programming formulation to calculate the nurses of a certain category to undertake a number of shifts per day (8 hour shifts).

Optimization model

$$\text{Min}\, C(U|R) = \sum_{i\in I}\sum_{n\in N}\sum_{t\in T} W_{\text{int}}(R_{\text{int}} - \sum_{m} Q_{\text{imnt}}U_{\text{imnt}})$$

$$\sum_{p} U_{\text{inpt}} - X_{\text{int}} \leq 0, \qquad \forall i\in I, n\in N, t\in T$$

$$\sum_{i\in I}\sum_{t\in T} X_{\text{int}} \leq B_n, \qquad \forall n\in N$$

$$X_{\text{int}} \geq A_{\text{int}}R_{\text{int}}, \qquad \forall i\in I, n\in N, t\in T$$

$$X_{\text{int}} \leq R_{\text{int}} + e_{\text{int}}, \qquad \forall i\in I, n\in N, t\in T, e_{\text{int}}\geq 0$$

$$X_{\text{int}}\ \text{integer}, \qquad \forall i\in I, n\in N, t\in T$$

$$U_{\text{imnt}} \geq 0, \qquad \forall i\in I, m, n\in N, t\in T$$

The constraints make sure that the total number of nurses with skill class *n* cannot exceed the total number of skill class *n* nurses assigned; they also ensure a minimum amount of nursing care services of each skill class and limit the amount of substitution of nursing tasks among skill classes.

This early approach could not consider the needs of the hospital and there were no possibility to include personnel preferences in this model. An excess of nursing supply for a particular skill category could absorb the shortage of other skills.

5.2.1.2 SINGLE-OBJECTIVE NURSE SCHEDULING PROBLEMS

Minimizing costs and maximizing nurse preferences are two of the goals of nurse scheduling.

-Cost as an objective function

Bai et al. (2010) developed an optimization model to minimize the cost of assignment nurses to shifts patterns. This paper presents a hybrid algorithm for nurse rostering problem, which is flexible and easily adaptable to other constraints.

This model was motivated by a real problem in UK. The problem is to make weekly schedules for 30 nurses with day and night shifts and with three different grades. Some nurses have only a few hours per week and on certain shifts and the schedule has to satisfy the "day-off" requests by nurses.

Optimization model

$$\text{Min } f = \sum_{i=1}^{n} \sum_{j \in F_i} p_{ij} x_{ij}$$

$$\sum_{j \in F_i} x_{ij} = 1, \qquad \forall i \in \{1, \cdots, n\}$$

$$\sum_{i \in G_r} \sum_{j \in F_i} a_{jk} x_{ij} \geq R_{kr}, \qquad \forall k, r$$

The hybrid algorithm combines a genetic algorithm and a simulated annealing hyper heuristic (SAHH). A stochastic ranking method was used to improve the constraint handling capability of the genetic algorithm while an SAHH procedure was incorporated in order to locate local optima more efficiently. The stochastic method has demonstrated better performance.

-Nurse preferences as an objective function

Purnomo and Bard (2006) proposed a new integer programming model for cyclic preference scheduling and hourly workers. The aim is to generate a set of rosters that minimizes the number of uncovered shifts. The problem combines elements of both cyclic and preference approaches and includes five different shift types. The first three are 8 hours shifts non-overlapped, day (D), evening (E) and night (N). The other two are both of 12 hours called a.m. and p.m. The objective function aims to minimize the weighted sum of preference violations and the cost of covering gaps with outside nurses.

Optimization model

$$\theta_{ip} = \text{Min} \sum_{i \in N} \sum_{a=1}^{V_{\max}} r_a v_{ia} + \sum_{d \in D} \sum_{t \in T} M_t y_{dt},$$

$$\sum_{i \in N} x_{idt} - s_{dt} + y_{dt} = LD_{dt}, \qquad \forall d \in D, t \in T$$

$$\sum_{d \in D} x_{idt} \geq P_{it}, \qquad \forall i \in N_R, t \in T_i$$

$$\sum_{d \in D} \sum_{t \in T_i} h_t x_{idt} = H_i, \qquad \forall i \in N$$

$$\sum_{t \in T_i} x_{idt} \leq 1, \qquad \forall i \in N, d \in D$$

$$x_{idt_2} + x_{i,d+1,t_1} \leq 1, \qquad \forall i \in N_{BB}, d \in D$$

$$\sum_{l=d}^{d+D_i^{\text{maxon}}} \sum_{t \in T_i} x_{ilt} \leq D_i^{\text{maxon}}, \qquad \forall i \in N, d \in D$$

$$\sum_{d \in D_w} \sum_{t \in T_i} x_{idt} = W_i^{\max} w_{im}, \qquad \forall i \in N, m \in W$$

$$\sum_{m \in W} w_{im} = 1, \qquad \forall i \in N$$

$$\sum_{t \in T_i} x_{idt} + \left(1 - \sum_{t \in T_i} x_{i,d+1,t}\right) + \sum_{t \in T_i} x_{i,d+2,t} + p_{id} \geq 1, \qquad \forall i \in N, d \in D$$

$$\left(1 - \sum_{t \in T_i} x_{idt}\right) + \sum_{t \in T_i} x_{i,d+1,t} + \left(1 - \sum_{t \in T_i} x_{i,d+2,t}\right) + q_{id} \geq 1, \qquad \forall i \in N, d \in D$$

$$1 - x_{idt_\alpha} + 1 - x_{i,d+1,t_\beta} + b_{id} \geq 1, \qquad \forall d \in D, \alpha \neq \beta$$

$$\sum_{d \in D} b_{id} \leq TR_{\max}, \qquad \forall i \in N_R$$

$$\sum_{d \in D} (p_{id} + q_{id} + b_{id}) = \sum_{a=1}^{V_{\max}} a v_{ia}, \qquad \forall i \in N$$

$$\sum_{a=1}^{V_{max}} v_{ia} \leq 1, \qquad \forall i \in N$$

$$0 \leq s_{dt} \leq UD_{dt} - LD_{dt}, 0 \leq y_{dt} \leq O_{dt}^{\max}, \qquad \forall t, d$$

$$b_{id}, p_{id}, q_{id} \geq 0, \forall i, t, d; \quad v_{ia} \in \{0,1\}, \forall i, a; \quad w_{im} \in \{0,1\}, \forall i, m$$

$$x_{idt} \in \{0,1\}, \forall i, t, d, \quad \text{where } x_{i,14+l,t} \equiv x_{ilt}, l = 1, \ldots, D_i^{\text{maxon}}$$

A branch-and-price algorithm was developed for solving the problem. It was decomposed and two approaches were investigated to modify branching rules, one based on the master problem variables, which is better for a large number of nurses, and the other based on sub-problem variables, more efficient for a less number of them. It is important to say also that the variables are integer and not binary.

MULTI-OBJECTIVE NURSE SCHEDULING PROBLEMS

NSP is actually a multi-objective optimization problem (Burke et al. 2010; Maenhout and Vanhoucke 2010; Parr and Thompson 2007). Some of the goals may have conflicts of interests. Hadwan and Ayob (2010) proposed a constructive heuristic algorithm based on the idea of generating the most required shift patterns to solve the nurse rostering problem in UKMMC, one University of Malaysia. The complexity of the solution search space was reduced by generating all the allowed two-day and three-day shift patterns to build up the roster. The objective function aimed to minimize the total penalty cost that occurs due to the violations of soft constraints.

Optimization model

$$\text{Min } w_1 \sum_{i\in I}(d1_i^+ + d1_i^-) + w_2 \sum_{i\in I} d2_i^+ + w_3 \sum_{i\in I}\sum_{d\in D} d3_{id}^- + w_4 \sum_{i\in I}\sum_{d\in D} d4_{id}^- + w_5 \sum_{p\in P} d5_p^+$$

Subject to:

$$\sum_{i\in I} X_{id(1)} \geq R_{d(1)}, \quad \forall d \in D$$

$$\sum_{i\in I} X_{id(2)} \geq R_{d(2)}, \quad \forall d \in D$$

$$\sum_{i\in I} X_{id(3)} \geq R_{d(3)}, \quad \forall d \in D$$

$$\sum_{s\in S} X_{ids} = 1, \quad \forall i \in I, d \in D$$

$$\sum_{i\in I_s} X_{ids} \geq 1, \quad \forall d \in D, s = 1\ldots3$$

$$X_{id(4)} + X_{i(d+1)(1)} + X_{i(d+1)(2)} + X_{i(d+1)(3)} + X_{i(d+2)(4)} \leq 2, \quad \forall i \in I, \; d = 1\ldots|D|-2$$

$$\sum_{d\in D}\sum_{s\in S-\{4\}} X_{ids} \leq 12, \quad \forall i \in I$$

$$\sum_{d\in D}\sum_{s\in S-\{4\}} X_{ids} \geq 10, \quad \forall i \in I$$

$$X_{id4} + X_{i(d+1)4} + X_{i(d+2)4} + X_{i(d+3)4} + X_{i(d+4)4} \geq 1, \quad \forall i \in I, \quad d = 1\ldots|D|-4$$

$$\sum_{i\in I} Y_{pi} = 1, \quad p \in P$$

$$\sum_{p\in P} Y_{pi} \leq 1, \quad i \in I$$

$$\sum_{i\in I} Y_{pi}\left(\sum_{l=0}^{l=3} X_{i(d_p+l)(3)} + \sum_{l=4}^{l=5} X_{i(d_p+l)(4)}\right) = 6, \quad p = 1\ldots9$$

$$\sum_{i\in I} Y_{pi}(X_{id_p3} + X_{i(d_p+1)3}) = 2, \quad p = 10, 11$$

$$\sum_{d\in D} X_{id4} \geq 2, \quad \forall i \in I$$

$$\sum_{d\in D}\sum_{s\in S-\{4\}} X_{ids} + (d1_i^+ - d1_i^-) = 11, \quad \forall i \in I$$

$$X_{i(6)(4)} + X_{(i)(7)(4)} + X_{i(13)(4)} + X_{i(14)(4)} + (d2_i^+ - d2_i^-) = 1, \quad \forall i \in I$$

$$X_{id(1)} + X_{i(d+1)(2)} + (d3_{id}^+ - d3_{id}^-) = 1, \quad \forall i \in I, \quad d = 1\ldots|D|-1$$

$$X_{id(2)} + X_{i(d+1)(1)} + (d4_{id}^+ - d4_{id}^-) = 1, \quad \forall i \in I, \quad d = 1\ldots|D|-1$$

$$\left(\sum_{i\in I}[Y_{pi} \times (X_{i(7)(2)} + X_{i(7)(4)})]\right) + (d5_p^+ - d5_p^-) = 1, \quad p \in \{1,2,3\}$$

$$\left(\sum_{i\in I}[Y_{pi} \times (X_{i(11)(2)} + X_{i(11)(4)})]\right) + (d5_p^+ - d5_p^-) = 1, \quad p \in \{4,5,6\}$$

A modified shift pattern approach was proposed, that was divided into three groups: (1) Initialize the problem by reducing search space and generating valid shifts sequence patterns; (2) Construct feasible initial solution; and (3) Optimize the initial solution that was constructed in the previous two stages to get the optimal solution.

CONSTRAINT-BASED AND HEURISTIC-ORIENTED NURSE SCHEDULING PROBLEMS

We describe a hybrid model of integer programming and variable neighborhood search (VNS) for highly constrained nurse scheduling problems developed by Burke et al. (2010). They divided the constraint sets in a way that those with lower complexity and higher importance have more priority to be included in the sub-problem solved using IP.

Optimization model

$$\mathrm{Min}\,G(x) = [g_1(x), g_2(x), g_3(x), g_4(x), g_5(x), g_6(x), g_7(x), g_8(x)]$$

Where:

$$g_1(x) = \sum_{i\in I}\sum_{j\in J}(s_{ij}^1 + s_{ij}^2), \qquad g_5(x) = \sum_{i\in I}\sum_{j=2}^{7|J|-1}\sum_{k'\in\{1,3\}} s_{ijk'}^6,$$

$$g_2(x) = \sum_{i\in I}\sum_{j=2}^{7|J|-1} s_{ij}^3, \qquad g_6(x) = \sum_{t=1}^{3}\sum_{i\in I_t}\sum_{w=1}^{|J|}(s_{tiw}^7 + s_{tiw}^8),$$

$$g_3(x) = \sum_{i\in I}\sum_{j=2}^{7|J|-1} s_{ij}^4, \qquad g_7(x) = \sum_{i\in I_1}\sum_{r=1}^{7|J|-3} s_{ir}^9,$$

$$g_4(x) = \sum_{i\in I}\sum_{r=1}^{7|J|-3}\sum_{k\in\{1,3\}} s_{ijk}^5, \qquad g_8(x) = \sum_{i\in I}\sum_{j=1}^{7|J|-1}\sum_{k'\in K'} s_{ijk'}^{10}.$$

Subject to:

$$\sum_{i\in I} x_{ijk} = d_{jk}, \qquad \forall j\in\{1,\ldots,7|J|\}, k\in\mathscr{K}$$

$$\sum_{k\in K} x_{ijk} \le 1, \qquad \forall i\in I, j\in\{1,\ldots,7|J|\}$$

$$\sum_{j=1}^{7|J|}\sum_{k\in K} x_{ijk} \le m_i, \qquad \forall i\in I$$

$$\sum_{j\in J}\sum_{k\in K} x_{ijk} \le 3, \qquad \forall i\in I$$

$$\sum_{j=1}^{7|J|} x_{ij4} \le 3, \qquad \forall i\in I$$

$$x_{i(j-1)4} - x_{ij4} + x_{i(j+1)4} \ge 0 \qquad \forall i\in I,\ j\in\{2,\ldots,7|J|-1\}$$

$$x_{i(j-1)4} - \sum_{k=1}^{3} x_{ijk} + \sum_{k=1}^{3} x_{i(j+1)k} \le 1, \qquad \forall i\in I,\ j\in\{2,\ldots,7|J|-1\}$$

$$x_{i(j-1)4} + \sum_{k=1}^{3} x_{ijk} - \sum_{k=1}^{3} x_{i(j+1)k} \le 1, \qquad \forall i\in I,\ j\in\{2,\ldots,7|J|-1\}$$

$$x_{i(j-1)4} + \sum_{k=1}^{3} x_{ijk} + \sum_{k=1}^{3} x_{i(j+1)k} \le 2, \qquad \forall i\in I,\ j\in\{2,\ldots,7|J|-1\}$$

$$\sum_{j=r}^{r+n_1} x_{ij4} \le n_1, \qquad \forall i\in I,\ r\in\{1,\ldots,7|J|-n_1\}$$

$$\sum_{j=r}^{r+n_2}\sum_{k\in K} x_{ijk} \le n_2, \qquad \forall i\in I,\ r\in\{1,\ldots,7|J|-n_2\}$$

$$x_{16(j)3} = 0, \qquad \forall j\in\{1,\ldots,7|J|\}$$

$$\sum_{k\in K}[x_{i(j-1)k} - x_{ijk}] + s_{ij}^1 - s_{ij}^2 = 0, \qquad \forall i\in I,\ j\in J$$

$$\sum_{k\in K}[x_{i(j-1)k} - x_{ijk}] + s_{ij}^1 - s_{ij}^2 = 0, \qquad \forall i\in I,\ j\in J$$

$$\sum_{k\in K}[x_{i(j-1)k} - x_{ijk} + x_{i(j+1)k}] + s_{ij}^3 \ge 0, \qquad \forall i\in I,\ j\in\{2,\ldots,7|J|-1\}$$

$$\sum_{k\in K}[x_{i(j-1)k} - x_{ijk} + x_{i(j+1)k}] - s_{ij}^4 \le 1, \qquad \forall i\in I,\ j\in\{2,\ldots,7|J|-1\}$$

$$\sum_{j=r}^{r+3} x_{ijk} - s_{irk}^5 \le c_k, \qquad \forall i\in I,\ r\in\{1,\ldots,7|J|-3\},\ k\in\{1,3\}$$

$$x_{i(j-1)k} - x_{ijk} + x_{i(j+1)k} + s_{ijk}^6 \ge 0, \qquad \forall i\in I,\ j\in\{2,\ldots,7|J|-1\},\ k\in\{1,3\}$$

$$\sum_{j=7w-6}^{7w}\sum_{k\in K} x_{ijk} - s_{tiw}^7 \le g_t, \qquad \forall t\in\{1,2,3\},\ i\in I_t,\ w\in\{1,\ldots,|J|\}$$

$$\sum_{j=7w-6}^{7w}\sum_{k\in K} x_{ijk} + s_{tiw}^8 \ge h_t, \qquad \forall t\in\{1,2,3\},\ i\in I_t,\ w\in\{1,\ldots,|J|\}$$

$$\sum_{j=r}^{r+3}\sum_{k\in K} x_{ijk} - s_{ir}^9 \le 3, \qquad \forall i\in I_1,\ r\in\{1,\ldots,7|J|-3\}$$

$$x_{ijk_1} + x_{i(j+1)k_2} - s_{ijk'}^{10} \le 2, \qquad \forall i\in I,\ j\in\{1,\ldots,7|J|-1\},\ k' = (k_1,k_2)\in K'$$

The advantages of IP and VNS are combined. First IP solves the sub-problem with a full set of high constraints and a subset of soft constraints, and then VNS improves the IP solution. This hybrid model was able to handle all the requirements of nurse rostering in a complex hospital environment.

5.2.2 NURSE SCHEDULING PROBLEM IN AN OPERATING SUITE

Mobasher and Lim (2011) based on the nurse scheduling problem in an operating suite by developing nurse scheduling optimization models for an actual operating suite in Texas, USA. They introduced a multi-objective integer programming nurse scheduling model in an operating suite which considers different aspects of the scheduling problem such as demand satisfaction, idle time and over time. The model called “Nurse Assignment Model”, assigns nurses to different surgery cases based on their specialties and competency levels. Real data are gathered from a cancer center in Texas and used to show the efficiency of the optimization models and solution methods. The objective is to determine which nurses should be assigned to each surgery case. It is essential to provide schedules with minimum overtime, non-consecutive and minimum changes during surgery procedures as well as maximum demand satisfaction.

Because the model has multiple objectives a Solution Pool Method (SPM) appears. The solution pool feature generates and stores multiple solutions to the mixed integer programming (MIP) model for each deviation and then chooses the solution among these optimal solutions that have the smallest deviations. The solution pool-based method has three steps. In the first step, a single objective optimization model is developed for each deviation containing all hard constraints and soft constraints related to the current deviation. In the second step, the solution pool approach is applied to generate alternate good feasible solutions (with the absolute gap of less than 0.01) for each single objective model developed in Step 1. Finally, in Step 3, the comparison indexes can be compared and the best solution introduced for NAM. Numerical results indicated that this model can provide more efficient and reliable nurse schedules for operating suites.

6. INTEGRATING NURSE AND SURGERY SCHEDULING [30]

A simple and effective way to achieve considerable saving in staff costs is to integrate the operation room scheduling process with the nurse scheduling process.

6.1 MODEL DESCRIPTION

6.1.1 GENERAL IDEA

During the nurse scheduling process we have to take into account the collective agreement requirements that are rules that define acceptable schedules for individual nurses, and the workload that depends of the master surgery schedule.

6.1.2 THE NURSE SCHEDULING PROBLEM (NSP)

The nurse scheduling problem consists of generating a configuration of individual schedules over a given time horizon. It is generated so as to fulfill collective agreement requirements and the hospital staffing demand coverage while minimizing the salary cost.

We present an integrated model that can be used to find optimal schedules for a homogeneous set of nurses.

Let J be the set o feasible roster lines j and I be the set of demand period i. Let $d_i \in \Re^+, \forall i \in I$, denote the required number of nurses scheduled during period i. Furthermore, let a_{ij} be 1 if roster line j contains an active shift during period i and 0 otherwise. The general integer decision variable x_j, $\forall j \in J$, indicates the number of individual nurses which are scheduled by roster line j. Then, the nurse scheduling problem (NSP) can be stated as follows:

$$\text{Minimize} \sum_{j \in J} x_j \qquad (1)$$

With these constraints:

$$\sum_{j \in J} a_{ij} x_j \geq d_i \qquad \forall i \in I \qquad (2)$$

$$x_j \in \{0, 1, 2, \ldots\} \qquad \forall j \in J \qquad (3)$$

6.1.3 SOLUTION PROCEDURE FOR NURSE SCHEDULING PROBLEM

This problem is solved by first solving the linear programming relaxation and then using a branching scheme to bring the solution into integrality. Then column generation is often applied to solve the LP relaxation. Let $\pi_i, \forall i \in I$, denote the dual price of constraint (2). Then, the reduced cost of a new column (roster line) j is given by:

$$1 - \sum_{i \in I} a_{ij} \pi_i \tag{4}$$

6.1.4 THE GENERALIZED NURSE SCHEDULING PROBLEM (GNSP)

The right hand side values of the coverage constraints are consider to be fixed, but the coverage constraints are based on workload estimations, and that is determined by the patient type. The number and the type of the patients are determined by the operation room schedule. This is taken into account in the generalized nurse scheduling problem, instead of assuming the demand values to be fixed.

Let K denote the set of possible workload patterns that could be generated by modifying the surgery schedule. Each workload pattern k is described by a number of periodic demands $d_{ik} \in \{0,1,2,\dots\}, \forall i \in I$. Let z_k be 1 if the surgery schedule that corresponds to workload k is chosen and 0 otherwise.

$$\text{Minimize} \sum_{j \in J} x_j \tag{5}$$

Subject to:

$$\sum_{j \in J} a_{ij} x_j \geq \sum_{k \in K} d_{ik} z_k \qquad \forall i \in I \tag{6}$$

$$\sum_{k \in K} z_k = 1 \tag{7}$$

$$x_j \in \{0, 1, 2, \dots\} \qquad \forall j \in J \tag{8}$$

$$z_k \in \{0, 1\} \qquad \forall k \in K \tag{9}$$

The constraint (7) is called the workload convexity constraint and implies that only one workload pattern has to be chosen.

6.1.5 SOLUTION PROCEDURE FOR THE GENERALIZED NURSE SCHEDULING PROBLEM

The column generation approach used to solve the LP relaxation of NSP can easily be extended to cope with the GNSP. Here the possible workload patterns is too large, so the process start with a limited subset and new patterns are added as needed. So then a new master surgery schedule has to be constructed. Let γ denote the dual price of the workload pattern convexity constrain (7). The reduced cost of the new workload pattern *k* is given by:

$$0 - \gamma + \sum_{i \in I} \pi_i d_{ik} \tag{10}$$

6.2 PRICING PROBLEMS

6.2.1 GENERATING A NEW ROSTER LINE

There are several requirements that have to be satisfied when building a new roster line. One nurse cannot work more than one shift per day, and the overall number of active days cannot exceed certain limit. The maximum number of consecutive working days and consecutive resting days is constrained. The unpopular shifts are limited per roster line, and in a block (a sequence of working days), certain shift transitions are not allowed. To generate a new roster line is common to use a dynamic programming recursion.

6.2.2 GENERATING A NEW WORKLOAD PATTERN

As each workload pattern corresponds a particular surgeon schedule, if you build a new surgery schedule you are generating a new workload pattern. Here a new surgery schedule is built by solving an integer program. The objective function minimizes the dual price vector of the demand constraints (6) multiplied by the new demands. There are surgery constraints that determine how many blocks must be preserved for each surgeon, and capacity constraints that ensure that the number of blocks assigned for each period do not exceed the available capacity.

Let y_{rt} $(\forall r \in R \; and \; t \in T)$ be the number of blocks assigned to surgeon r in period t. T represents the set of active periods and R the set of surgeons. Let q_r be the number of blocks required by each surgeon r. Let b_t be the maximal number of blocks available in period t. Let $w_{rti} \in \Re^+$ denote the contribution to the workload of demand period *i* of assigning one block to surgeon *r* in period *t*. Then, the integer program to construct a new surgery schedule (and at the same time price out a new workload pattern *k*) is as follows:

$$\text{Minimize} \sum_{i \in I} \pi_i d_{ik} \tag{11}$$

Subject to:

$$\sum_{t \in T} y_{rt} = q_r \qquad \forall r \in R \tag{12}$$

$$\sum_{r \in R} y_{rt} \leq b_t \qquad \forall t \in T \tag{13}$$

$$\sum_{r \in R} \sum_{t \in T} w_{rti} y_{rt} \leq d_{ik} \qquad \forall i \in I \tag{14}$$

$$y_{rt} \in \{0, 1, 2, \ldots, \min(q_r, b_t)\} \qquad \forall r \in R \text{ and } \forall t \in T \tag{15}$$

$$d_{ik} \in \{0, 1, 2, \ldots\} \qquad \forall i \in I \tag{16}$$

The objective function (11) minimizes the reduced cost of a new workload pattern. Now, the periodic demands d_{ik} are part of the decision process, instead of being only coefficients. With the constraint set (12) each surgeon obtains the number of required blocks, and with constraint set (13) we are sure that the number of blocks assigned to all surgeons, does not exceed the maximal number of blocks available for that period. Constraint set (4) triggers the d_{ik}s to the appropriate integer values and constraints (5) and (6) define y_{rt} and d_{ik} as integer values.

6.3 COMPUTATIONAL RESULTS

6.3.1 TEST SET

The surgery schedule is set in cycles of 7 days, and the last two days are not available to allocate OR time. There are different problems depending on 5 different factors: the number of time blocks per day; the number of surgeons; the division of requested blocks per surgeon; the number of operated patients per surgeon; and the length of stay (LOS) distribution. If we consider two different values for each factor and repeat the combination 3 times, we obtain $2^5 * 3 = 96$ test instances.

Factor setting	Nr. blocks per day	Nr. surgeons	Division req. blocks	Nr. patients per surgeon	LOS
1	3-6	3-7	evenly distributed	3-5	2-5
2	7-12	8-15	not evenly distributed	3-12	2-12

Table 3: Factor settings in surgery scheduling test set. (Jeroen BeliÄen, Erik Demeulemeester, "Integrating nurse and surgery scheduling")

A block is defined as the smallest time unit for which a specific operating room can be allocated to a specific surgeon (or surgical group). In real-life applications the number of blocks per day in one operating room is usually 1 or 2 but here, considering more blocks can be seen as a way of considering more operating rooms, because there is no difference from a computational point of view. The third factor indicates whether or not the requested blocks are evenly distributed among all surgeons.

For the LOS in factor 5 we simulated exponential distributions (made discrete by use of binomial distributions) with mean dependent on the factor setting.

Next, we generated some weights w_{rti} defining the contributions to the workload of period i of allocating a block to surgeon r in period t. These weights vary linearly with the number of patients of surgeon r operated in period t that are still in the hospital in period i. The patient's workload contribution generally decreases the longer the patient has already recovered in the hospital. In our test set the workload demand periods coincide with the shifts. Furthermore, we set the contribution to a "day" shift two times as large as the one to an "evening" shift and four times as large as the one to a "night" shift. Obviously, although attempting to represent realistic scenarios, these contributions are chosen somewhat arbitrarily.

Thirdly, we composed a set of collective agreement rules that apply on individual roster lines. The scheduling horizon amounted to 4 weeks or 28 days (= n). For each person, the maximum days an active shift could be scheduled ("day", "evening" or "night") was set to 20 (= $fmax$). Shifts during the weekends were marked as unpopular shifts. The maximum number of consecutive working days was set to 6 ($=h_1^{max} = h_{max}$) and the maximum number of consecutive rest days was set to 3 (= h_2^{max}). Furthermore, we distinguished between two scenarios: a hard constrained scenario and a flexible one.

In the hard constrained scenario, there is only one shift type allowed within each block. In the flexible scenario, all shift transitions are allowed, except the backwards transitions. In the hard constrained scenario, the maximal penalty with respect to unpopular shifts is set to 4, whereas in the flexible scenario it is set to 8 (=g_{max}).

6.3.2 SAVINGS

Having a look at the upper bounds we can see that those for the GNSP are generally better than those for the NSP. The same results are obtained for the lower bounds. The GNSP lower bounds are guaranteed to be at least as good as the NSP lower bounds. It is not the same for the upper bounds. Looking at the table we can see that the average lower bound for the NSP is 5lower than the average upper bound for the GNSP in the flexible scenario, and that the average upper bound for the GNSP is lower than the average lower bound for the NSP in the hard constraint scenario.

The stricter the constraints are, the harder it is to fit the nurse rosters into the required workload pattern in the NSP. As the workload pattern can be adapted in the GNSP, it includes more possible savings in the case of severe collective agreement requirements.

		Flexible scenario				Hard constrained scenario			
		NSP		GNSP		NSP		GNSP	
Nr.	Problem	lb	ub	lb	ub	lb	ub	lb	ub
1	d00000_0	15	17	13	15	21	22	14	14
2	d00000_1	26	28	25	27	32	33	25	26
3	d00000_2	25	27	23	25	32	33	24	26
4	d00001_0	40	42	39	41	59	59	51	51
5	d00001_1	45	47	44	46	65	66	56	57
6	d00001_2	94	96	92	94	145	146	125	125
7	d00010_0	34	36	32	35	36	37	33	33
8	d00010_1	40	42	38	40	44	45	38	39
9	d00010_2	28	30	26	27	33	34	26	27
...	...	...	...	...	...	...	...	...	...
88	d11101_0	96	98	94	96	149	150	129	129
89	d11101_1	99	102	97	99	140	141	115	116
90	d11101_2	122	125	119	121	179	180	149	149
91	d11110_0	83	85	80	82	104	105	80	81
92	d11110_1	111	113	109	111	149	149	109	110
93	d11110_2	58	60	56	58	76	77	56	56
94	d11111_0	252	254	249	252	368	368	315	315
95	d11111_1	119	122	116	119	175	175	124	125
96	d11111_2	135	137	131	133	184	184	131	132
	Average	70.18	72.43	68.33	70.44	99.99	100.66	80.55	81.25

Table 4: Lower and upper bounds for the NSP and the GNSP. (Jeroen BeliÄen, Erik Demeulemeester, "Integrating nurse and surgery scheduling")

6.3.3 INTERPRETATION OF THE SAVINGS

In the obtained results we can see that integrating the surgery scheduling process with the nurse scheduling process may yield important savings in terms of required nurses to hire.

The origin of the waste is twofold. The first reason is an unfavorable workload pattern that contains many workload demands that slightly exceed the workforce of x nurses but that are dramatically inferior to the workforce of $x+1$ nurses. It is called “Waste due to the workforce surplus per shift”. The second reason is the inflexibility of the roster lines due to strict general agreement requirements. Because of this, no set of roster lines can be found that perfectly fit with the workload demand. This source of waste is further referred to as “Waste due to the inflexibility of roster lines”.

There are three columns for each scenario. The first one is the total waste in terms of overstaffing in the NSP compared with the GNSP. These numbers are obtained by subtracting the upper bounds for the NSP from those for the GNSP. The second and third column indicate the parts of this total waste that are due to the “workforce surplus per shift” and to the “inflexibility of roster lines”. These numbers can easily be calculated as follows. Firstly, the total required workforce is calculated by making the sum of the (integral) demands of the chosen workload pattern for both the NSP and the GNSP. Next, divide this number by the workforce per nurse and round up the result to the next integer. This gives the minimal number of nurses that would be needed and can be obtained in the case of fully flexible roster lines. The difference between these numbers for the NSP and GNSP is the waste due to the workforce surplus per shift. The difference between the total waste and the waste due to the workforce surplus per shift is the waste due to the inflexibility of roster lines.

The stricter the general agreement requirements are, the larger is the share of the waste due to the inflexibility of the roster lines.

		Flexible scenario			Hard constrained scenario		
Nr.	Problem	Total waste	Waste due to workforce surplus per shift	Waste due to inflexibility of roster lines	Total waste	Waste due to workforce surplus per shift	Waste due to inflexibility of roster lines
1	d00000_0	2	1	1	2	1	1
2	d00000_1	1	1	0	4	2	2
3	d00000_2	1	2	-1	3	1	2
4	d00001_0	1	2	-1	2	0	2
5	d00001_1	2	1	1	1	0	1
6	d00001_2	2	1	1	3	0	3
7	d00010_0	1	2	-1	3	1	2
8	d00010_1	1	2	-1	3	2	1
9	d00010_2	1	2	-1	2	0	2
...	...	...	...	...	...	...	...
88	d11101_0	2	1	1	2	1	1
89	d11101_1	2	2	0	4	0	4
90	d11101_2	1	2	-1	3	0	3
91	d11110_0	2	2	0	6	1	5
92	d11110_1	2	1	1	7	1	6
93	d11110_2	1	2	-1	6	1	5
94	d11111_0	2	1	1	7	0	7
95	d11111_1	2	2	0	4	-1	5
96	d11111_2	1	2	-1	6	1	5
	Average	1.58	1.48	0.10	4.11	0.29	3.82

Table 5: Interpretation of the savings. (Jeroen BeliÄen, Erik Demeulemeester, "Integrating nurse and surgery scheduling")

6.4 CONCLUSIONS

We have conclude that considerable savings could be achieved by using this approach to build nurse and surgery schedules. We simulated problems for a large range of surgery scheduling instances and distinguished between a flexible and a hard constrained scenario with respect to the collective agreement requirements.

Obviously, in real-life hospital environments it is not so easy to modify the master surgery schedule. As the surgery schedule can be considered to be the main engine of the hospital, it not only has an impact on the workload distribution for nurses, but also on several other resources throughout the hospital.

7. A SINGLE AND TRIPLE-OBJECTIVE MATHEMATICAL PROGRAMMING MODELS FOR ASSIGNMENT OF SERVICES IN A HEALTHCARE INSTITUTION [60]

The assignment of service positions plays a really important role in healthcare institutions. Poorly assigned positions or over-employment may result in increased costs. In hospital departments, the supporting services include financial management, logistics, inventory management, analytic laboratories, etc. An application of operations research model for optimal supporting service jobs allocation in a public hospital is presented. Most of the real problems are formulated as multi-objective mathematical programming models that want to minimize operations costs of a supporting service subject to some constraints. The problem is formulated as a mixed integer program known as the assignment problem. The results of some computational experiments modeled after a real data from a selected Polish hospital are informed.

7.1 DATA USED FOR COMPUTATIONS

Data was gathered from a selected Polish public healthcare institution from one month period and they were used for computations. The data included 17 supporting service hospital departments in which there are 74 types of supporting service jobs. Permanent employment is defined as a percentage of permanent post between 25% and 100% according to the size of a job position (par time or full time) for a selected job in a selected department. For instance, it is possible to have two full time permanent employees or four part time permanent employees for two full permanent employments. Supporting service departments consists of 78.5 permanent employments with 192 employees before the optimization. Specific data consists of the average salaries for selected jobs in departments defined as costs of assignment of workers to jobs. Moreover, the average of money paid monthly for services in each department was used. Additional parameters include the number and size of permanent employments in each department for each job defined as partial or full time. Furthermore, the minimum number of permanent employments for each job in each department and the maximal number of positions that can be assigned to a single worker were given (Sawik, 2008b, 2010a, 2010b, 2012a, 2012b; Sawik and Mikulik, 2008a, 2008b).

Supporting service departments	*Number of workers*	*Number of jobs*
Central Heating Department	16	5
Power Department	15	3
Medical Bottled Gases Department	6	2
Ventilation and Air-condition Department	8	4
Heating and Air-condition Department	11	4
Distribution Department	6	3
Medical Equipment Department	8	4
Technical Department	11	5
Economy Department	21	5
Hospital Pharmacy	20	11
Sterilisation Department	27	5
Material Monitoring Department	13	5
Information Department	7	4
Business Executive Department	8	5
Technical Executive Department	4	4
Law Regulation Department	7	3
Attorneys-at-Law Department	4	2
Number of workers in all department	192	74

Table 6: Number of workers and service jobs in the hospital departments before optimization. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)

Supporting service departments	*Number of types of permanent employments*	*Amount of money paid for services [PLN]*
Central Heating Department	5	29,250
Power Department	3	31,050
Medical Bottled Gases Department	2	11,400
Ventilation and Air-condition Department	4	16,650
Heating and Air-condition Department	4	21,200
Distribution Department	3	13,600
Medical Equipment Department	4	17,500
Technical Department	5	20,950
Economy Department	5	31,360
Hospital Pharmacy	11	43,400
Sterilisation Department	5	41,500
Material Monitoring Department	5	27,150
Information Department	4	16,100
Business Executive Department	5	15,450
Technical Executive Department	4	7,150
Law Regulation Department	7	16,100
Attorneys-at-Law Department	2.5	7,950
Money paid for services in all departments	78.5	367,760

Table 7: Number of permanent employments and the maximum amount of money paid for services in the hospital departments before optimization. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)

7.2 PROBLEM FORMULATION

Mathematical programming approach deals with optimization problems of maximizing or minimizing a function of many variables subject to inequality and equality constraints and integrality restrictions on some or all of the variables. Variables can be, for instance, binary but, in this case, the model presented is defined as a mixed integer programming problem.

Suppose there are m people and p jobs, where $m \neq p$. Each job must be done by at least one person and each person can do at least one job. The cost of person i doing job k is c_{ik}. The objective is to assign people to jobs in the right way to minimize the total cost of completing all of the jobs.

The optimal criterion of the problem is to minimize operations costs of a supporting service subject to some specific constraints that represent specific conditions for resource allocation in a hospital. The overall problem is formulated as a modified assignment problem.

	Indices
i	Worker, $i \in I = \{1, \ldots, m\}$
j	Supporting service hospital department, $j \in J = \{1, \ldots, n\}$
k	Type of supporting service job, $k \in K = \{1, \ldots, q\}$
	Input parameters
c_{ik}	Cost of assignment of a worker i to job k (i.e., monthly salary)
C_j	Maximal monthly budget for salaries in a department j
e_k	Size of permanent (partial or full time) employment for job k (i.e., e_k = 0.25 or 0.50 or 0.75 or 1.00)
E_j	Maximal number of permanent employments in a department j
h_{jk}	Minimal number of permanent employments for job k in a department j
β_i	Weight of the objective functions f_i, i = 1, 2, 3
γ	Small positive value
f_1^{opt}	Ideal solution value of number of workers selected for an assignment to any job in any department
f_2^{opt}	Ideal solution value of operational costs of the supporting services
f_3^{opt}	Ideal solution value of number of permanent employments for all jobs in all departments

Table 8: Notations for mathematical models M1, M2,M3. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)

7.3 OPTIMIZATION MODELS

The problem of optimal assignment is formulated as a single objective (M1,M2) or triple objective mixed integer program (M3), which allows commercially available software (e.g., AMPL/CPLEX; Fourer et al., 1990) to be applied for solving practical instances (Sawik, 2008b, 2010a, 2010b, 2012a, 2012b; Sawik and Mikulik, 2008a, 2008b).

Variables	
x_{ijk}	1 if worker i is assigned to job k in department j, 0 otherwise
y_i	1 if worker i is assigned to any job in any department, 0 otherwise
g_{jk}	Number of permanent employments for job k in department j
z	Total number of workers assigned to any job in any department
δ	Deviation from the reference solution

Table 9: Variables for mathematical models M1, M2, M3. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution",)

MODEL M1

The optimal criterion of the single objective mixed integer program M1 is to minimize operational costs of the supporting services.

$$minimize \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} c_{ik} \cdot x_{ijk} \qquad (1)$$

Subject to:

$$\sum_{i=1}^{m}\sum_{k=1}^{q} c_{ik} \cdot x_{ijk} \leq C_j \ , j \ \in J \qquad (2)$$

$$\frac{\sum_{j=1}^{n}\sum_{k=1}^{q} x_{ijk}}{\sum_{j=1}^{n} E_j} \leq y_i \leq \sum_{j=1}^{n}\sum_{k=1}^{q} x_{ijk} \ , i \ \in I \qquad (7)$$

$$\sum_{i=1}^{m} e_k \cdot x_{ijk} \leq E_j \ , j \ \in J \qquad (3)$$

$$x_{ijk} \ \in \ \{0,1\} \, , i \ \in I \, , j \ \in J \, , k \ \in K \qquad (8)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{q} e_k \cdot x_{ijk} \leq 2\, , i\ \in I \tag{4}$$

$$y_i\ \in\ \{0,1\}\, , i\ \in I \tag{9}$$

$$\sum_{i=1}^{m} x_{ijk} \geq h_{jk}\ , j\ \in J, k\ \in K \tag{5}$$

$$g_{jk}\ \geq 0, j\ \in J\, , k\ \in K \tag{10}$$

$$g_{jk} \geq h_{jk}\ , j\ \in J, k\ \in K \tag{6}$$

Constraint (2) ensures that the cost of workers assignment to service jobs in each department must be less than or equal to the maximum amount of money paid regularly for services in the department (monthly salaries).

Constraint (3) ensures that the total size of permanent employment (partial or full time) for each job (i.e., 0.25 or 0.50 or 0.75 or 1.00) in each department must be less than or equal to the maximal number of permanent employments in this department.

Constraint (4) ensures that each worker can be assigned to a maximum two full time positions in parallel.

Constraint (5) is responsible for an assignment of workers on at least minimal level requirements, e.g., the number of permanent employments on a selected service jobs.

Constraint (6) is responsible for obtaining only the results that will not lead to solutions without any assignment to some jobs. It compares real and minimal accepted number of permanent employments.

Constraint (7) ensures that worker *i* is taken (*yi* = 1) if he gets assignment to any job in any department (*xijk* = 1 for any *j* and *k*). It defines the relation between binary decision variables *xijk* and *yi*.

Constraints (8) and (9) define binary decision variables *xijk* and *yi*.

Constraint (10) defines continuous decision variable *gjk*.

MODEL M2

It is the same as Model 1 but replacing constraint 7 for constraint 12, they can be use alternatively.

$$\sum_{j=1}^{n}\sum_{k=1}^{q} x_{ijk} = y_i \,, i \in I \tag{12}$$

Constraint (12) ensures that each worker can be assigned to at most one job.

MODEL M3

The triple-objective optimization model can be formulated by reference point method. The optimal criteria (13) are to minimize total number of workers selected for an assignment to any job in any department and also to minimize operational costs of the supporting services and finally to minimize the number of permanent employments for all jobs in all departments.

$$\delta + \gamma\left(z + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} c_{ik}\cdot x_{ijk} + \sum_{j=1}^{n}\sum_{k=1}^{q} g_{jk}\right) \tag{13}$$

Subject to (2) to (10) and

$$\beta_1\left(z - f_1^{opt}\right) \le \delta \tag{14}$$

$$\beta_2\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} c_{ik}\cdot x_{ijk} - f_2^{opt}\right) \le \delta \tag{15}$$

$$\beta_3\left(\sum_{j=1}^{n}\sum_{k=1}^{q} g_{jk} - f_3^{opt}\right) \le \delta \tag{16}$$

$$\sum_{i=1}^{m} y_i \le z \tag{17}$$

$$z \ge 0, integer \tag{18}$$

$$\delta \ge 0 \tag{19}$$

Constraints (14), (15) and (16) define the deviation from the reference solution.

Constraint (17) defines the relation between binary decision variables *yi* and *z*.

Constraints (18) and (19) define integer variable z and continuous variable δ.

7.4 COMPUTATIONAL RESULTS

In this section, numerical examples and some computational results are presented to illustrate possible applications of the proposed formulations of integer programming of optimal assignment of service positions. Selected problem instances with the examples are modeled on a real data from a Polish hospital.

Scenario	Operational costs [PLN]	Number of assigned workers	MIP simplex iteration	CPU*	Constraint
A	153,251	77	2	10.49	(7)
A	153,251	77	0	12.25	(12)
B	209,751	108	3	12.46	(7)
B	209,751	108	0	12.08	(12)
C	248,951	131	3	9.17	(7)
C	248,951	131	0	12.80	(12)
D	311,651	166	4	7.47	(7)
D	311,651	166	0	7.47	(12)

Note: *CPU seconds for proving optimality on Pentium III, RAM 512MB/CPLEX v.9.1.

Table 10: Comparison of computational result(models M1 and M2) with alternative constraints. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution",.)

Scenario	f_1^{opt}	f_2^{opt}	f_3^{opt}	All variables	Binary variables	Constraints
A	70	150,000	75	4,448	3,188	566
B	110	200,000	105	4,416	3,156	526
C	130	250,000	120	4,416	3,156	526
D	160	300,000	155	4,404	3,144	512
$\gamma = 0.01$		$\beta_1 = 0.33 \cdot 1000$		$\beta_2 = 0.34$	$\beta_3 = 0.33 \cdot 1000$	

Table 11: The values of parameters for computational experiments and the size of adjusted problem. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution",.)

Scenario	δ	Number of workers	Operational costs [PLN]	Number of permanent employments	MIP simplex iterations	B-&-B nodes	CPU* seconds
A	1,320.00	74	147,201	71.50	297	6	0.265
B	3,842.51	109	211,302	105.75	161	0	0.202
C	330.00	131	248,952	121.75	433	0	0.296
D	1,802.51	162	305,302	159.00	310	0	0.171

Note: *CPU seconds for proving optimality on IntelCore 2 Duo T9300 processor running at 2.5G Hz, 4 GB RAM/CPLEX v.11.

Table 12: Comparison of computational results (model M·) with alternative scenarios. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)

	Assignment of workers in departments according to scenario							
Supporting service departments	*Workers*	*Permanent employments*	*Workers*	*Permanent employments*	*Workers*	*Permanent employments*	*Workers*	*Permanent employments*
Number of workers and permanent employments in all departments	74 (A*)	71.5 (A)	109 (B*)	105.75 (B)	131 (C*)	121.75 (C)	162 (D*)	159 (D)
Attorneys-at-Law Department	2 (A)	1.5 (A)	3 (B)	2.5 (B)	3 (C)	2 (C)	3 (D)	2.5 (D)
Law Regulation Department	3 (A)	3 (A)	5 (B)	5 (B)	5 (C)	4 (C)	5 (D)	5 (D)
Technical Executive Department	4 (A)	3.5 (A)	4 (B)	3.5 (B)	4 (C)	3.5 (C)	4 (D)	3.5 (D)
Business Executive Department	5 (A)	5 (A)	6 (B)	6 (B)	6 (C)	6 (C)	7 (D)	7 (D)
Information Department	4 (A)	3.5 (A)	5 (B)	4.5 (B)	5 (C)	4.5 (C)	6 (D)	5.5 (D)
Material Monitoring Department	5 (A)	5 (A)	7 (B)	7 (B)	8 (C)	8 (C)	11 (D)	11 (D)
Sterilisation Department	5 (A)	5 (A)	8 (B)	8 (B)	14 (C)	13.5 (C)	21 (D)	21 (D)
Hospital Pharmacy	11 (A)	10.5 (A)	15 (B)	14.5 (B)	17 (C)	14.5 (C)	19 (D)	18.5 (D)
Economy Department	5 (A)	5 (A)	9 (B)	9 (B)	14 (C)	13.5 (C)	18 (D)	18 (D)
Technical Department	5 (A)	5 (A)	8 (B)	8 (B)	8 (C)	7.5 (C)	9 (D)	9 (D)
Medical Equipment Department	4 (A)	4 (A)	6 (B)	5.75 (B)	6 (C)	5.75 (C)	7 (D)	6.5 (D)
Distribution Department	3 (A)	3 (A)	5 (B)	5 (B)	5 (C)	4.5 (C)	5 (D)	5 (D)
Heating and Air-condition Department	4 (A)	4 (A)	5 (B)	5 (B)	7 (C)	7 (C)	9 (D)	9 (D)
Ventilation and Air-condition Department	4 (A)	3 (A)	6 (B)	6 (B)	6 (C)	5.5 (C)	7 (D)	7 (D)
Medical Bottled Gases Department	2 (A)	2 (A)	3 (B)	3 (B)	4 (C)	3.5 (C)	5 (D)	5 (D)
Power Department	3 (A)	3 (A)	5 (B)	5 (B)	8 (C)	8 (C)	12 (D)	12 (D)
Central Heating Department	5 (A)	4.5 (A)	9 (B)	8 (B)	11 (C)	10.5 (C)	14 (D)	13.5 (D)

Note: *Scenarios A, B, C and D considered subject to hospital authority requirements.

Table 13: Assignment of workers in departments according to scenarios. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)

As it has been recommended by the hospital manager's four different scenarios of the assignment have been implemented. In scenario A, a minimal number of people are employed in each supporting service department so that each type of a job has at least one worker assigned. This rule is implemented in input parameter *hjk*. In scenario B at least two workers were assigned to each job. Scenario C secured the level of supporting service workers. In each department, there are at least two workers assigned to each job, but for some special cases, more than two workers are assigned to each job. Finally, scenario D presents the optimal assignment of workers to jobs with a high service level with all currently employed workers. The results obtained have indicated the problem of over-employment in the hospital.

7.5 CONCLUSIONS

This document shows how important and useful are the mathematical programming approaches to optimize the work on a hospital. With the results, it is shown that in almost all the departments the number of hired workers can be reduced.

The proposed solution for the single and triple-objective assignments problems can be used in other institutions, not only in healthcare. The results consist of monthly expenses for salaries, the number of workers and the number of permanent employments in all considered departments. Computational time takes only a fraction of a second to find the optimal solution because of the small size of the input data.

8. LOCATION OF HEALTH CARE FACILITIES [30]

8.1 INTRODUCTION

It is very important the location of facilities in health care because even if the correct number of facilities is used, if they are not well located it will result in increases in mortality (death) and morbidity (disease).

We are going to talk about three basic facility location models: the set covering model, the maximal covering model and the P-*median* model.

8.2 BASIC LOCATION MODELS

The models that we are going to talk about are in the class of discrete facility location models. They assume that demands can be aggregated to a finite number of discrete points (nodes) and that there is a finite set of candidate locations at which facilities can be sited.

In these models we define an indicator variable as follows:

$$a_{ij} = \begin{cases} 1 \text{ if demand node } i \text{ can be covered by a facility at candidate site } j \\ 0 \text{ if not} \end{cases}$$

Demands at node are said to be covered by a facility located at some other node if the distance between the two nodes is less than or equal to some specified coverage distance.

The set covering model tries to minimize the cost of the facilities that are selected so that all demand nodes are covered. These are the sets, inputs and the decision variable:

$$I = \text{set of demand nodes}$$
$$J = \text{set of candidate facility sites}$$
$$f_j = \text{fixed cost of locating a facility at candidate site } j$$
$$X_j = \begin{cases} 1 \text{ if we locate at candidate site } j \\ 0 \text{ if not} \end{cases}$$

The set covering model is:

$$Minimize \quad \sum_{j \in J} f_j \, X_j \tag{1}$$

Subject to:

$$\sum_{j \in J} a_{ij} \, X_j \geq 1 \qquad \forall i \in I \tag{2}$$

$$X_j \in \{0,1\} \qquad \forall j \in J \tag{3}$$

The objective function minimizes the total cost of all selected facilities. Constraint (2) means that each demand node must be covered by at least one of the selected facilities. The left hand side of that constraint represents the total number of selected facilities that can cover demand node i.

When the fixed facility costs are similar and the dominant costs are the operating costs that depend on the number of located facilities, it is interesting to minimize the number of facilities that are located, instead of minimizing the cost of locating them. This variant is the *location* set covering problem and the objective function becomes:

$$Minimize \quad \sum_{j \in J} X_j \tag{4}$$

There are some problems, with this model. If objective function number 1 is used the cost of covering all demands is huge, and if number 4 is used the number of facilities required is too large. Also, this model does not see the difference between demand nodes that generate a lot of demand per unit time and those that generate relatively little demand. Then, a second model, the maximal covering problem, was formulated by Church and Re Velle (Church, R. and C. ReVelle (1974)):

$$h_i = demand\ at\ node\ i$$

$$P = number\ of\ facilities\ to\ locate$$

$$Z_i = \begin{cases} 1\ if\ demand\ node\ i\ is\ covered \\ 0\ if\ not \end{cases}$$

The objective function is:

$$Maximize \quad \sum_{i \in I} h_i \, Z_i \tag{5}$$

Subject to:

$$Z_i - \sum_{j \in J} a_{ij} X_j \leq 0 \qquad \forall i \in I \qquad (6)$$

$$\sum_{j \in J} X_j = P \qquad (7)$$

$$X_j \in \{0,1\} \qquad \forall j \in J \qquad (8)$$

$$Z_i \in \{0,1\} \qquad \forall i \in I \qquad (9)$$

The objective function maximizes the number of covered demands. Constraint (6) guarantees that demand node i can not be counted as covered unless we locate at least one facility that is able to cover the demand node. Constraint number (8) restricts the number of facilities to P. Several heuristic and exact algorithms have been proposed for this model, but Lagrangian Relaxation (Fisher, M.L. (1981), Fisher, M.L. (1985)) provides the most effective way of solving the problem.

The P-center model finds the location of P facilities to minimize the coverage distance subject to a requirement that all demands are covered. This model, as the two explained before treat service as binary: a demand node is either covered or not covered. Sometimes we are interested in the average distance that a client has to travel to receive service or the average distance a provider has to travel to reach his patients. To deal with this the P-median problem is formulated to minimize the demand weighted total distance.

$$d_{ij} = distance\ from\ demand\ node\ i\ to\ candidate\ location\ j$$

$$Y_{ij} = \begin{cases} 1\ if\ demands\ at\ node\ i\ are\ assigned\ to\ a\ facility\ at\ candidate\ site\ j \\ 0\ if\ not \end{cases}$$

$$Minimize \quad \sum_{j \in J} \sum_{i \in I} h_i\, d_{ij}\, Y_{ij} \qquad (10)$$

Subject to:

$$\sum_{j \in J} Y_{ij} = 1 \qquad \forall i \in I \qquad (11)$$

$$Y_{ij} - X_j \leq 0 \qquad \forall i \in I; \forall j \in J \qquad (12)$$

$$\sum_{j \in J} X_j = P \qquad (13)$$

$$X_j \in \{0,1\} \qquad \forall j \in J \qquad (14)$$

$$Y_{ij} \in \{0,1\} \qquad \forall i \in I; \forall j \in J \qquad (15)$$

The objective function of the P-median problem minimizes the demand weighted total distance. Constraint (11) states that each demand node must be assigned to exactly one facility site. Constraint (12) stipulates that demand nodes can only be assigned to open facility sites. Constraint (13) states that we are to locate exactly P facilities. Constraint (15) can be relaxed to a simple non-negativity constraint since each demand node will naturally be assigned to the closest open facility.

The two best algorithms to solve this problem are the neighborhood search algorithm (Maranzana, F.E. (1964)) and the exchange algorithm (Tietz, M.B. and P. Bart (1968)).

Some authors have transformed the maximal covering problem into a P-median formulation by replacing the distance between demand node i and candidate site j by the following modified distance:

$$\hat{d}_{ij} = \begin{cases} 1 \; if \; d_{ij} < D_c \\ 0 \; if \; not \end{cases}$$

where D_c denotes the coverage distance. The effect of this formula is to minimize the total uncovered demand that is the same as maximize the covered demand.

8.3 LOCATION MODELS IN HEALTH CARE

It is relatively easy to site facilities based on past conditions, but there is no guarantee that in the future will replicate the past. Recent applications and modeling efforts have focused on identifying solutions that can be implemented in the short term but can adapt to changing future conditions relatively easily.

The health care location literature has tended to address three major topics: accessibility, adaptability and availability.

8.3.1 ACCESSIBILITY MODELS AND APPLICATIONS

Accessibility models attempt to find facility locations that perform well with respect to static inputs. Demand, cost and travel distance or travel time data are generally assumed to be fixed in this class of models.

Federal legislation has encouraged the use of such models: the EMS (Emergency Medical Services) Act of 1973 stipulated that 95% of service requests had to be served within 30 minutes in a rural area and within 10 minutes in an urban area. This encouraged the use of models like the maximal covering model. For instance, Adenso-Diaz and Rodriguez (Adenso-Díaz, B. and F. Rodríguez (1997)) used the model to locate ambulances in Leon, Spain.

Sinuany-Stern et al. (Sinuany-Stern, Z., A. Mehrez, A.-G. Tal, and B. Shemuel (1995)) and Mehrez et al. (Mehrez, A., Z. Sinuany-Stern, A.-G. Tal, and B. Shemuel (1996)) used the P-median model and other discrete one for the assignment of demand to a hospital in excess of the hospital's capacity.

In hierarchical location modeling a number of different services are simultaneously located where the low level facilities are assigned to lower numbers and high level facilities to the higher ones. There are at least three factors that need to be considered in hierarchical problems. The first one is that if level m factor can provide only level m service or it also can provide lower level services. The second issue is, in a successively inclusive service, if a level m facility can provide m all m levels of service to all demand nodes, or only to the node where is located and only level m of service to the rest of the nodes. This is called local inclusive service hierarchy. The third one is that generally there are fewer high level facilities than low level facilities. If high-level facilities can only be located at sites housing a lower facility, the system is called nested.

8.3.2 ADAPTABILITY MODELS

Location decisions must be robust with respect to uncertain future conditions particularly for facilities as hospitals that are difficult to relocate as conditions change. A lot of approaches have been developed to deal with future uncertainty. A number of future conditions are defined and plans are developed that work well in all scenarios.

In location planning, the facility sites must generally be chosen before we know which scenario will evolve. But designing a robust system often carries compromises because the best plan may not be optimal under a particular scenario, but work well across all scenarios. Three performance measures are often used in scenario planning: optimizing the expected performance, minimizing the worst case performance, and minimizing the worst case regret. The regret measures the difference between the performance measure using the compromise solution for that scenario and the performance measure when the optimal solution is used for that scenario. Minimizing the expected regret is identical to optimizing the expected performance.

8.3.3 MODELS OF FACILITY AVAILABILITY

Adaptability reflects long-term uncertainty about the conditions under which a system will operate. Availability, in contrast, addresses very short-term changes in the condition of the system that result from facilities being busy. Such models are most applicable to emergency service systems (ambulances) in which a vehicle may be busy serving one demand at the time it is needed to respond to another emergency.

There are two kinds of models: Deterministic and probabilistic models.

Deterministic models: One simple way to deal with vehicle busy periods is to find solutions that cover demand nodes multiple times. The Hierarchical Objective Set Covering (HOSC) model (Daskin, M. and E. Stern (1981)) first minimizes the number of facilities needed to cover all demand nodes and then the model selects the solution that maximizes the system-wide multiple coverage. The multiple coverage of a node is given by the total number of times the node is covered in addition to the one time needed to satisfy the set covering requirement. The system-wide multiple coverage is the sum of the nodal multiple coverage over all nodes.

Probabilistic models: Two different probabilistic approaches have been developed, the first one based on queuing theory and the second on Bernoulli trials. The queuing model ensures that a proper level of service is provided in terms of response time requirements. With this model Fitzsimmons (Fitzsimmons, J.A. (1973)) approximated the number of busy ambulances. A Bernoulli trial is a random experiment in which there are only two possible outcomes, usually called failure or success with respect probabilities p and q. It is less exact than the other approach. Here, the outcome of a Bernoulli trial represents the probability that a vehicle at any site will be available.

8.4 OTHER APPLICATIONS OF LOCATION MODELS IN HEALTH CARE

These models are very useful and can be used in many different applications. Here we show one of them.

The location set covering model could be also used to determine the minimum number of fields of view to read a cytological sample. A field of view is the area that a microscope can see without moving the slide that is being analyzed. All areas must be examined, and by minimizing the number of required fields of view we can reduce the time needed to analyze each sample (Laporte, G., F. Semet, V.V. Dadeshidze, and L.J. Olsson (1998)).

Another application could be to localize radioactive sources or seeds in the treatment of prostate cancer. Applications of facility location-like models in the diagnosis and treatment of medical conditions is likely to be an important area of future work (Lee, E.K., R.J. Gallagher, D. Silvern, C.-S. Wuu, and M. Zaider (1999)).

9. EVALUATING THE EFFICIENCY OF HOSPITALS' PERIOPERATIVE SERVICES USING DEA [30]

9.1 INTRODUCTION

Elective surgery is the surgery that has been scheduled in advance. It is the opposite of urgent surgery, such as trauma and transplant surgery. Perioperative care begins once the decision of undergoing a surgery has been taken. Perioperative service (POS) is defined as the set of sub-systems that produces elective surgery, pre-operative and post-operative care. We are going to talk about a method to identify best practices among hospitals' perioperative services using Data Envelopment Analysis (DEA).

POS is practically and physically isolated from the rest of the hospital. The Director of POS is typically a nursing or medical director, and if a medical director is usually an anesthesiologist (Mazzei, W.J. (1998)).

9.1.1 EVALUATING THE PERFORMANCE OF PERIOPERATIVE SERVICES

A number of practical difficulties arise in evaluating the performance of POS across institutions. Hospitals differ significantly in factors such as the number of staffed beds, technological services offered, and size of the market. Therefore, a good evaluation method should compare a hospital's POS with peer entities that operate in a similar environment and use similar combination of resources.

9.2 DATA ENVELOPMENT ANALYSIS

Data envelopment analysis (DEA) is a linear-programming-based technique to measure the technical efficiency of Decision-Making-Units (DMUs). DEA works by estimating a piece-wise linear envelopment surface, known as the *best-practice frontier*. DEA is a deterministic, non-parametric technique, and thus makes no assumptions about the underlying form of the production function or the distribution of error terms. This technique accommodates multiple inputs and multiple outputs without prior knowledge of their relative prices. DEA has been applied extensively in health care; here we are going to extend the use of DEA in health care to perioperative services.

To estimate the efficiency of surgical hospitals, the CCR (Charnes, Cooper and Rhodes) input-oriented model was used (Charnes, A., W.W. Cooper, and E. Rhodes (1978); Charnes, A., W. Cooper, A. Lewin, and L. Seiford (1994)).

To obtain additional information about the hospitals we studied, we incorporated extensions to basic DEA including super-efficiency, known as the AP (Anderson and Peterson) model and multifactor efficiency (MFE). The AP model has been used to identify potential data errors and to rank efficient DMUs.

9.3 DATA AND METHODS

Patient data on inpatient admissions during 1998 from all non-Federal Pennsylvania hospitals were obtained from the Pennsylvania Health Care Cost Containment Council. The study sample consisted of 53 Pennsylvania hospitals that have at least 200 staffed beds and are located in non metropolitan areas (less than one million people).

9.3.1 DEFINING INPUTS AND OUTPUTS

Eight surgical procedures were selected to measure the surgical output. The procedures studied were those that are performed once per hospitalization. The number of hospitalizations is then proportional to resource use. For instance, hip replacement was included but nor knee replacement, since some patients could undergo both knee replacement procedure instead of only one knee replacement. Also, each procedure is performed by only one specialty. The eight procedures studied represent the 7.5% of all inpatient discharges in the State of Pennsylvania. There are more outputs that are not included in this study, because this study is focused on the outputs of elective, scheduled perioperative care.

Hospital size and capacity were measured by the number of staffed beds ("BEDS") and the use of technological services ("TECH"). The hospital technology was measured by the number of high technology services offered, such as cardiac catheterization, cardiac surgery, shock-wave urological lithotripsy, megavoltage radiation therapy, magnetic resonance imaging, organ transplantation, neonatal intensive care, cardiac intensive care, and certified trauma care. A constant (c=1) was added to the technology variable to prevent unbounded solutions in the AP model.

The input "SURGEONS" is the number of surgeons who generated at least one hospital discharge for any of the above procedures. Surgeons determine the volume and the type of procedures that the hospital can perform.

The demand of surgery depends on the number of surgeons, population size, and population demographics, such as age and gender. County demand ("COUNTY") was measured as the number of the aforementioned procedures performed on residents of each hospital's county, weighted by DRG case-mix index. The number of procedures performed on residents of all those counties sharing a common border with the hospital's county is the demand form contiguous counties ("CONTIGUOUS COUNTY").

9.3.2 EXPLANATORY VARIABLES

Surgeons are required on several hospitals. As the number of hospital affiliations increases, the surgeon is available less often at each hospital. This causes unused OR time and empty beds. Then we would expect the efficiency of POS to decrease at facilities where the surgeons operate at many other hospitals. To test this, we take into account the number of hospital affiliations per surgeon ("AFFIL") and hospitals' market share among its surgeons ("HOSP-SURG"). This last concept is defined as the sum of the eight procedures performed at the hospital divided by the sum of the eight procedures performed by all surgeons with privileges at that hospital.

Another explanatory variable is if the hospital is located in a rural county ("RURAL").

Variable	Description	Specialty	DRG Weight index
Outputs			
AAA	Abdominal Aortic Aneurysm resection	Vascular	4.08
CABG	Coronary Artery Bypass Graft	Cardiac	5.65
COLO	Colorectal Resection	General	3.13
CRAN	Craniotomy not for trauma	Neurological	3.23
HIP	Hip Replacement	Orthopedic	2.37
HYS	Hysterectomy	Gynecology	0.77
LUNG	Lung Resection	Thoracic	3.04
NEPH	Nephrectomy	Urology	2.65
Inputs			
BEDS	Num. staffed beds at the hospital		
TECH	Num. high-tech services		
SURGEONS	Num. of surgeons who did at least one of the procedures above		
COUNTY	Num. of procedures done on residents of hospital's region, weighted by case-mix		
CONTIG. CONUNTY	Num. of procedures done on residents of the contiguous region of the hospital		
Explanatory			
AFFIL	Average number of hospital affiliations per surgeon		
HOSP-SURG	Hospital's market share among its surgeons		
RURAL	Hospital located in a rural county		

Table 14: Description of input and output variables (Margaaret L. Brandeau, François Saintfort, William P.Pierskalla, "Operations Research and Health Care")

9.4 RESULTS

Variable	Mean	Standard deviation	Minimum	Maximum
Outputs				
AAA	25	20	0	85
CABG	229	290	0	1.113
COLO	138	81	11	381
CRAN	34	48	0	261
HIP	131	77	26	367
HYS	226	173	27	893
LUNG	28	24	0	110
NEPH	20	20	0	119
Inputs				
BEDS	312	112	203	659
TECH	6	2	1	10
SURGEONS	66	39	9	214
COUNTY	6.295	3.924	406	14.2
CONTIG. CONUNTY	39.821	29.986	3.464	83.841
Explanatory				
AFFIL	1.64	0.46	1.03	3.20
HOSP-SURG	0.72	0.21	0.19	1.00
RURAL	0.19	0.39	0	1

Table 15: Descriptive statistics for input and output variables (Margaaret L. Brandeau, François Saintfort, William P.Pierskalla, "Operations Research and Health Care")

The results that they obtained in this research show that three of the eight procedures were performed by every hospital. The average number of surgeons per hospital was 66, and the average number of hospital affiliations per surgeon 1.64.

DEA identified 24 hospitals as efficient and 29 as inefficient, with an average efficiency of 0.91, based on the CCR model. For inefficient hospitals, DEA provided information on the sources of inefficiency.

We now focus on three of the 53 hospitals in more detail and compare the most important results: Hospital 38, Hospital 48 and Hospital 10. The results of all the hospitals can be consulted in the book: *"Operation Research and Health Care. A Hand Book of Methods and Applications." (Margaret L. Brandeau, François Sainfort and William P. Pierskalla).*

Hospital 38 is a tertiary facility with more than two million enrollees. Hospital volumes were high for complex, resource-intensive procedures, such as craniotomy. Many of its surgery patients were from outside of the Hospital's county. The craniotomy volumes were in the 96th percentile and the hospital's market share among its surgeons was 97 percent (the 5th highest in the sample).

Hospital 48 was an efficient, "maverick" hospital. It had only 9 surgeons, 204 beds and 0 technological services but despite of the difficult operating environment it was so efficient. It is focused on three procedures: colorectal resection, hip replacement, and hysterectomy. The market share of this hospital among surgeons was 94 percent, what means that the 94 percent of the cases performed by these nine surgeons were performed at that hospital.

Hospital 10 is located in a competitive market with four other hospitals within its county. Surgeons of this hospital had 2.8 hospital affiliations on average. It is the second higher in the sample. The market share among its surgeons was 51 percent, what means that if all the surgeons with privileges could be persuaded to admit all their patients to this hospital, then the surgical volume would almost double the actual one.

After looking all the results, it can be concluded that as expected, efficient hospitals had fewer affiliations per surgeon than inefficient hospitals. Only two of the 24 efficient hospitals had an average of more than two affiliations per surgeon.

However, eight of the 29 inefficient hospitals averaged more than two affiliations per surgeon.

If we have a look to the rest of the variables, we are able to see that there are no significant differences between efficient and inefficient hospitals. Hospital POS efficiency was not associated with the size of the hospital or market.

9.5 DISCUSSIONS AND CONCLUSSIONS

This study has demonstrated the usefulness of DEA in capturing the complexity and managerial tradeoffs that characterize perioperative services. This study also includes market factors, what is very helpful to predict the surgical demand.

We conclude that the strength of the relations of a hospital with its surgeons is an important predictor of POS efficiency. The stronger the relations between the hospital and the surgeons are, the higher the POS efficiency is likely to be. These are not unexpected results. In rural areas, surgeons' choices about the hospital where they want to work are limited by the geography and other factors. In more competitive markets, hospitals have to work to satisfy their surgeons and keep a steady stream of patients.

The results of this study can be used by hospital managers to improve the efficiency of the hospitals. For inefficient hospitals, DEA suggests ways to increase hospital volume. For instance, a positive slack of number of surgeons means that the hospital has more surgeons than would be expected given its current surgical volume.

Further research should adapt this model to metropolitan areas where competition among hospitals for surgeons and patients is more intense.

10. A MULTI-OBJECTIVE MODEL OF CASUALTY PROCESSING IN MAJOR INCIDENT RESPONSE [18]

10.1 INTRODUCTION

The response in disaster management is a very important phase but it has received a little attention from the operation room (OR) research community.

10.1.1 CASUALTY PROCESSING IN MCI RESPONSE

One significant component of the mass casualty incident (MCI) response is the delivery of the casualties to a hospital. To delivery each casualty, several tasks have to be done, for instance, if it is trapped or in an unstable condition. Then the casualties should be taken to a safe area called Casualty Clearing Station (CCS) where they receive the necessary treatments to be transported without any risk to the hospital. This sequence of tasks is called casualty processing and is shown in figure 1.

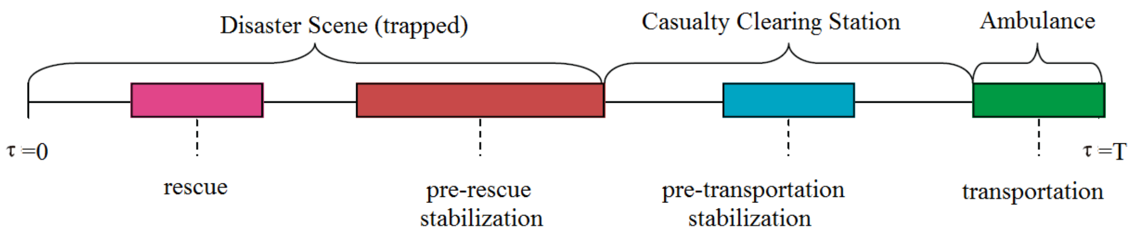

Figure 1: An example of the processing of a single casualty in an MCI (Duncan T. Wilson, Glenn I. Hawe, Graham Coates, Roger S. Crouch).

There are several models that do not speak about the processing of casualties and they only focus their attention on either the distribution of emergency responder units to areas that require their attention, or the distribution of some vital commodities such as food and medicines. Here the Flexible Job-Shop Problem (FJSP) is adapted to accommodate the characteristics of the casualty processing problem.

10.2 MODEL

10.2.1 THE FLEXIBLE JOB-SHOP SCHEDULING PROBLEM

The flexible job shop scheduling problem is a variant of the Job Shop Scheduling Problem (JSP) and is defined as follows:

We are given a set of machines $M = \{M_k\}, 1 \leq k \leq m$ and a set of Jobs $J = \{J_i\},\ 1 \leq i \leq n$. Each job J_i consists of a sequence of n_i ordered operations $O_{ij}, 1 \leq j \leq n_i$. Each operation may be processed by a subset of machines $M_{ij} \subseteq M$, with machine $M_k \in M_{ij}$ having a fixed and predetermined processing time $P_{ijk}$for the operation O_{ij}. It is assumed that all machines are available from time zero and that a machine can process at most one operation at a time.

The objective of the standard FJSP is to minimize the makespan, that is, the latest completion time of all jobs, by finding the optimal allocation of operations to machines and the correct ordering of operations on these machines.

Now we have to adapt this model to the casualty processing problem.

10.2.2 CASUALTY PROCESSING AS A FJSP VARIANT

We can identify some variables of the original problem with some of the casualty processing problem:

- Jobs → Casualties, $c_i \in C, 1 \leq i \leq n_c$, where n_c is the total number of casualties;
- Operations → Tasks, $t_{ij} \in T, 1 \leq j \leq n_{ti}$, where n_{ti}, is the total number of tasks associated with casualty c_i;
- Machines → Responder units, $r_k \in R, 1 \leq k \leq n_r$, where n_r is the total number of responder units.

A set of hospitals is also added to this model: $H = \{h_l\}, 1 \leq l \leq n_h$.

A solution to the casualty processing problem must include an ordered list of tasks to be allocated to each responder, as is the case with the basic FJSP, together with an assignment of each casualty to a hospital.

10.2.3 EVALUATING SCHEDULES

The standard FJSP objective function is not an appropriate measure of quality for the casualty processing problem, where the time taken to finish a response operation is incidental when compared with the resulting number of facilities and the level of suffering endured by survivors. Here is proposed a multi-objective approach with five objectives: (f_1) the expected number of facilities, (f_2) the measure of how quickly casualties are delivered to hospital, (f_3) the measure of how appropriate the hospital allocation choice is, (f_4) the total time spent idle by responders and (f_5) the latest time at which a casualty arrives at a hospital.

10.2.3.1 (F1) FACILITIES

A discrete state Markov chain model is proposed (N.B.-B. Saoud, T.B. Mena, J. Dugdale, B. Pavard, M.B. Ahmed) with a state space L={T1, T2, T3, D} denoting the four triage levels of the casualties: "Immediate" that require immediate life-saving procedure, "Urgent" that require surgical or medical intervention within 2-4 hours, "Delayed" that are the less serious cases whose treatment can safely be delayed beyond 4 hours and "Dead". This approach allows the calculation of the probability that casualty c_i will be in state $T \in L$ at time τ under the proposed solution, which we shall denote by $P_i^T(\tau)$.

Denoting by τ_i^C the time at which casualty c_i arrives at Casualty Clearing Station, the Markov chain model is used to calculate $P_i^D(\tau_i^C)$ for each casualty. The objective function looks like this:

$$f_1(s) = \sum_{i=1}^{n_c} P_i^D\left(\tau_i^C\right) \qquad (1)$$

10.2.3.2 (F2) HOSPITAL ARRIVAL TIME

Here we sum the completion times of each casualty's processing that is the time at which they are delivered to a hospital, denoted τ_i^H for casualty c_i. Each time is weighted by the parameter W_T, where $T \epsilon L = \{T1, T2, T3, D\}$.

$$f_2(s) = \sum_{T \in L} W_T \left(\sum_{i=1}^{n_c} P_i^T(\tau_i^H) \right) \qquad (2)$$

10.2.3.3 (F3) HOSPITAL ALLOCATION

To quantify how well casualties have been allocated to hospitals we consider two factors: the dynamic capacity of each hospital and the effect of oversubscription; and the pairing of specific injuries of individual casualties to the corresponding specialist treatment facilities.

Given a specific injury type denoted I, the model tries to ensure that each injury type matches with the treatment facility where it has been allocated. To measure that a set of penalty terms are defined ($penalty_{IT}$) which can be defined as "the maximum delay in the treatment of a casualty with injury I and health state T which could be tolerated in order to ensure they are treated at an appropriate specialist facility". We calculate for each casualty c_i the value β_i, where $\beta_i = -penalty_{IT}$ if casualty c_i has an injury of type I and a triage level T but is not taken to a hospital with the corresponding treatment facilities and 0 otherwise.

Denoting Q^{T1}, Q^{T2} and Q^{T3} the total untreated waiting time of casualties grouped by triage level and Q_l^T the specific value for hospital h_l, the objective function is:

$$f_3(s) = \left(\sum_{T \in L} \sum_{l=1}^{n_h} W_T Q_l^T \right) + \left(\sum_{i=1}^{n_c} \beta_i \right) \tag{3}$$

10.2.3.4 (F4) IDLENESS

The total idleness can be calculated easily from a given schedule by summing all intervals between the end of one task and the time where the responder leaves to travel to the site of their next task.

10.2.3.5 (F5) MAKESPAN

The makespan is not an appropriate measure when is the only objective of the model. However, when there are more objectives like the previously described, it is desirable to give some consideration to makespan, because low levels correspond to an early finish of the response operation.

10.2.4 THE CASUALTY PROCESSING MODEL

We can separate the objectives in three categories:

- Fatalities - f_1
- Suffering - f_2, f_3
- Efficiency - f_4, f_5

This model is formulated as follows:

$$\min_{s \in S} g_K(s) = \left(\sum_{i \in K} W_i |f_i(s) - z_i^*|^2 \right)^{\frac{1}{2}} \tag{4}$$

In order to be able to use this model we have to set the utopia point z^*. For objectives f_1, f_2 and f_4 we simply set $z_i^* = 0.$ For hospital arriving time (f_2), we suppose that each casualty arrives to the hospital at the earliest possible time and at the same triage state as at $\tau = 0$. For (f_5) the makespan we use the latest arriving time of that idealized situation.

There are some weights to give more importance to those objectives that we consider more critical, for instance in the suffering category is more important the hospital arriving time, because it is preferable to spend the same time waiting at a hospital than at a disaster scene. So the full multi-objective can be defined as:

$$\min_{s \in S} g_{\{1\}}(s), g_{\{2,3\}}(s), g_{\{4,5\}}(s). \tag{5}$$

Regarding model fitting, we note that the lexicographic approach employed helps to minimize the need for setting weights.

10.3 CONCLUSION

This is a novel approach to delivering decision support to the emergency services during mass casualty incidents. Thanks to this multi-objective model, it is taken into account not only the time used in the response of an emergency, but also the needs of the casualties based in their triage state.

One problem of this model is that it is assumed that the full triage operation has been completed so then all the information to build the model is known. In reality, triage values are not known from the very beginning and they are also changing as the response operations proceeds. A possible solution should be to modify the model and create a dynamic nature model that can receive information from the problem in a real time and update the values of the existing variables. However, we have to assume that processing information in a disaster response environment is a challenging task, and that the supply of regular and accurate information is almost impossible to reach.

11. OŚRODEK REHABILITACJI NARZĄDU RUCHU "KRZESZOWICE"

To appreciate the real problems of a hospital we have been in Ośrodek Rehabilitacji Narządu Ruchu "Krzeszowice". It is a hospital in Krzeszowice, a small village near Krakow. There we met a manager from the hospital and he explained to us some problems of it.

This institution has two main different parts: rehabilitation and neurology. They have a lot of treatments for rehabilitation in different places of the hospital. The employees that work at the hospital are doctors, nurses, psychologists and physical therapists. There are also people that work at the administration section and one of them is the person who does the schedules of the employees and the treatments of the patients by hand.

When one patient needs some treatments, the doctor has to prescribe them. And, after that, the person who deals with the scheduling has to find one time period for each patient.

11.1 AREAS OF THE HOSPITAL

In the pictures below, we can see some different areas of the hospital.

First of all, we can observe the outside of the hospital which now has one part temporary, it also has its own garden where the patients can rest and be in contact with the environment.

Figure 2: Hospital of Krzeszowice temporary part (Tornos U. and Villegas R.)

Figure 3: Hospital's garden (Tornos U. and Villegas R.)

Inside the hospital there are some rooms where the physical therapists can treat the patients and others where they can do exercises by themselves.

Figure 4 Massage room (Tornos U. and Villegas R.)

Figure 5: Massage room (Tornos U. and Villegas R.)

Figure 6: Patient's Gym (Tornos U. and Villegas R.)

Figure 7: Patient's Gym (Tornos U. and Villegas R.)

They offer treatments like magnetic fields applied to different parts of the body, massages, and hydrothermal therapy. It is possible to do rehabilitation inside of a swimming pool, but we could not take pictures of it.

Figure 8: Magnetic field cabin (Tornos U. and Villegas R.)

Figure 9: Hydrothermal therapy (Tornos U. and Villegas R.)

In the section of neurology there are some rooms dedicated to speech treatments and a multi-activities room where patients can cook, paint, read…

The hospital also has a place dedicated to investigation.

Figure 10: Multi-activities room (Tornos U. and Villegas R.)

Figure 11: Investigation room (Tornos U. and Villegas R.)

HOSPITAL'S PATIENTS

In this hospital, there are around 400 patients who are divided in two groups, the ones that are entered called inpatients and the ones that go to the hospital to receive the treatments called outpatients. They can receive the same treatments but there are some important differences between them. For instance, the waiting list is four years for an inpatient and one year and a half for an outpatient. Another important thing is that they do not pay in the same way, inpatients pay an amount of money for being in the hospital but outpatients have to pay for each treatment. That is why, from the management department, they tend to assign the most expensive treatments to outpatients.

11.2 ORGANIZATIONAL AND SCHEDULING PROBLEMS

One of the main problems is the way in which the hospital is distributed. As this is a rehabilitation hospital, there are a lot of patients that are disabled and they need the help of a nurse for going to one place to another to receive the treatment. Although it can surprise, this is not taken into account in the scheduling process. The person who works there assigns treatments to periods of time without any knowledge of the patient and this usually creates problems as patients arriving late to the treatments. When this happens some patients lose their citation and then the insurance does not pay to the hospital because the average number of treatments per day is not enough. This also could happen if one patient becomes ill and it is impossible for him to go to the different treatment rooms.

The same person that organizes the schedules often reduces the number of treatments of the patient when there is not enough time for all patients without having any medical knowledge.

Furthermore, it will be better if the scheduling system is done automatically instead of manually.

11.3 POSSIBLE SOLUTION

In order to reduce displacements in the hospital one possible solution could be to reorganize the different treatment rooms. By doing a research we will be able to know which treatments are usually done together and to make groups of them. In that case we will have the treatments of each group in the same room or in rooms that will be very close, and this will end with the problem of displacement of disabled people in the hospital. However, this solution is not so easy to carry out; there are some problems that make this difficult.

Basically there are four main problems. First of all, doctors refuse to choose between different groups of treatments for each patient, they want to make a list of them for each one personalized. The second problem is that the hospital is a place that must be operative all the time, it cannot be closed while a reorganization is been doing. Moreover the hospital has only one room that can be used for making changes provisionally. Thirdly, the distribution of the rooms has to be made under some rules. For instance, the rooms where they provide hydrothermal therapy must be in the ground floor and there are some restrictions about magnetic field machines such as the need of separate cabins. Finally and not less important than the other problems is that some patients could have the feeling that they are treated like objects and could be humiliate for them.

11.4 CONCLUSIONS

In this hospital we can see how it is a real problem and how difficult is to deal with it. It will be great for the hospital using an optimization model to find the best distribution of the rooms of the hospital and to improve the scheduling of employees and treatments.

12. SZPITAL SPECJALISTYCZNY IM. LUDWIKA RYDYGIERA

The data that we have used for doing the approaches of the model has been gathered from the hospital S*zpital Specjalistyczny im. Ludwika Rydygiera*, which is in Krakow. This hospital is a modern unit that offers medical care to a huge population. The 97.02% of all the admissions of the Hospital are patients from the Malopolska Region and in the last few years the admissions have reach the number of 28000-30000 patients per year.

It offers services in clinics and several surgical processes that are carried out by high qualified personnel. Some of the clinics that this hospital has are Neurological Clinic, Multiple Sclerosis Clinic, Dermatological Clinic, Clinic of Plastic Surgery, Otolaryngology Clinic, Radiotherapy Clinic, Urological Clinic, Clinic of Neonatology, etc…

Figure 12: Hospital Ludwika Rydygiera (www.szpitalrydygier.pl)

The hospital provides a lot of services. First of all, it has a lot of different departments, such as department of anesthesiology and intensive care, oncology, obstetrics or radiotherapy, among others.

Then, for having a diagnostic they have a laboratory, with a pathology sub-department. They offer several fields of investigation:

- General analysis
- Clinical chemistry
- Hematology and coagulation

- Immunochemistry
- Microbiology
- Toxicology
- Gynecological cytology
- Blood group serology

They also have some imaging machines that are basically radiology, mammography, ultrasound and computer tomography machines.

Figure 13: Medical technology (www.szpitalrydygier.pl)

Figure 14: Medical technology (www.szpitalrydygier.pl)

The hospital also has a pharmacy were they provided you some basics medicaments.

Moreover, there is a conference room, where employees of the hospital can do seminaries and also people who work in other places can book a period of time for do that.

In this hospital, as in most of hospitals, patients can enter there because of an urgency surgery or because of an elective surgery.

13. BRANCH-AND-BOUND METHOD AND WEIGHTED AND LEXICOGRAPHIC APPROACH

13.1 BRANCH AND BOUND METHOD

The Branch and Bound method is used to find the optimal solution of an integer problem. It is an enumerative relaxation algorithm. It is used when is too difficult to find an optimal solution over the whole set. In that case, divisions of the bigger set are made. When all the intersections of each division with the rest of divisions are empty they are called partitions. What the method does is to optimize the result over smaller sets or partitions and then put the results together.

The division is frequently done recursively, where the sons of a given node represent a division of their father. The extreme divisions can be viewed as total enumeration of elements of the initial set. It is usually schematically represented as a tree, where we have to avoid dividing the set into too many subsets. When it is established that no further division is necessary the enumeration tree can be pruned at that node.

Given a partial tree the question is to decide which node should be examined in detail next using one of these two options: a priori rules, to determine in advance the order on which the tree will be developed and adaptive rules, that helps to choose a node using the information about the status of the rest of the active nodes.

One a priori rule that is commonly used in the Branch and Bound method is the depth-first search plus backtracking. It is also known as last in, first out (LIFO).

13.2 WEIGHTING AND LEXICOGRAPHIC APPROACH FOR LINEAR AND MIXED INTEGER MULTI-CRITERIA OPTIMIZATION MODELS FORMULATIONS

Mathematical programming approach deals with optimization problems of maximizing or minimizing a function of many variables subject to inequality and equality constraints and integrality (being, containing, or relating to one or more mathematical integers or relating to or concerned with mathematical integrals or integration) restrictions on some or all of the variables (Crescenzi and Kann, 2005; Merris, 2003; Nemhauser and Wolsey, 1999). In particular model equations consist of linear, integer and (representing binary choice) 0-1 variables. Therefore, the optimization models presented in this project are defined as mixed integer or linear programming problems.

The lexicographic optimization generates efficient solutions that can be found by sequential optimization with elimination of the dominating functions. The weighted objective functions also generate various efficient solutions. It provides a complete parametrization of the efficient set for multi-objective mixed integer programs.

An efficient solution to the multi-criteria optimization problem can be found by applying the weighting and lexicographic approach (Ehrgott, 2000; Sawik, 2007b, 2008d, 2009e, 2009g, 2010b, 2013a, 2013b; Steuer, 1986; Wiecek, 2007).

The non-dominated solution set of multi-objective mixed integer, linear or quadratic program models *M* can be partially determined by the parametrization on λ of the following weighted-sum program.

Model M_λ

Maximization or minimization

$$\sum_{l=1}^{m} \lambda_l f_l$$

subject to some specific model constraints, where

$$\lambda_1 > \lambda_2 > \cdots > \lambda_m, \qquad \lambda_1 + \lambda_2 + \cdots + \lambda_m = 1$$

It is well known, however, that the non-dominated solution set of a multi-objective mixed integer or linear or quadratic program such as M_λ cannot by fully determined even if the complete parametrization on λ is attempted (e.g., Steuer, 1986). To compute unsupported non-dominated solutions, some upper bounds on the objective functions should be added to M_λ (e.g., Alves and Climaco, 2007).

Considering the relative importance of the two or the three objective function the multi-criteria mixed integer or linear or quadratic program *M* can be replaced with

M_ι, where $\iota \in 1,2$ in case of two objective functions or $\iota \in 1,2,3$ in case of three objectives, that could be solved subsequently.

Model $M_\iota, \iota = 1,2,3$

Maximization or minimization f_ι

subject to some specific model equations with additional constraints, in which upper or lower bounds are the optimal solution values of all objectives except the one with highest priority (f_ι) - objective actually optimized:

$f_l = f_\iota^*; l < \iota: \iota > 1$, where f_ι^* is the optimal solution value to the mixed integer or linear or quadratic program $M_\iota, \iota = 1,2,3$ (considering three objective lexicographic problems).

14. A LEXICOGRAPHIC APPROACH TO HEALTH CARE OPTIMIZATION MODELS

14.1 INTRODUCTION

In this section we present two models to deal with the problem of the assignment of the services of a public hospital of Krakow. Based on the models developed by Bartosz Sawik (Sawik B., 2013c, 2013d). The hospital has been described in the chapter number 12. The data was gathered from that hospital by Bartosz Sawik PhD, professor in the Management Faculty of the AGH University of Krakow.

The first model is a bi-criteria model, what means that it has two objectives. The second model is a triple objective model. The multi-criteria models are solved using the lexicographic approach. The procedure of this approach is to solve the problem using only one objective and leaving the other as constraints. After obtaining a solution we should change the structure of the model and solve it again. This time we choose other objective, and the previously used objective is now another constraint, limited with the previously obtained value. We should keep solving the problem until all the objectives have been evaluated. We have to take into account that in each case there are some constraints that are not needed depending on which objective has been used to solve that case.

Both models have been solved using CPLEX v9.1 and the results are presented in this paper.

14.2 INPUT DATA TO THE MODEL

The Table 16 shows the data used in the resolution of the problem. As we have said it was gathered from a public hospital of Krakow, Poland. There are 20 different departments in which there are 88 types of supporting service jobs. Permanent employment is defined as a percentage of permanent post between 25% and 100% according to the size of a job position (par time or full time) for a selected job in a selected department. Supporting service departments consist of 214 permanent employments with 221 employees before the optimization. There is specific data about the maximum amount of money paid monthly for services in each department.

Supporting Services Hospital Departments	Number of types of supporting services position in department	Number of permanent jobs in department	Number of employees in department (before optimization)	Maximal amount of money monthly paid for services in department
Central Heating Department	5	15.5	16	29250
Power Department	3	15	15	31050
Medical Bottled Gases Department	2	6	6	11400
Ventilation & Air-condition Department	4	8	8	16650
Heating & Hydraulic Department	4	11	11	21200
Distribution Department	3	6	6	13600
Medical Equipment Department	4	6.75	8	17500
Technical Department	5	1	11	20950
Economy Department	5	21	21	31360
Hospital Pharmacy	11	19.5	20	43400
Sterilization Department	5	27	27	41500
Stuff Monitoring Department	5	13	13	27150
Information Department	4	6.5	7	16100
Business Executive Department	5	8	8	15450
Technical Executive Department	4	3.5	4	7150
Law Regulation Department	3	7	7	16100
Attorneys-at-law Department	2	3,5	4	7950
Hospital Management Cost Section	5	9	9	15550
Salary Section	5	6,75	9	15800
Accounting Section	4	11	11	26950
TOTAL	**88**	**214**	**221**	**426060**

Table 16: Input data to the model (Szpital Specjalistyczny Im. Ludwika Rydygiera)

14.3 FORMULATION OF THE MODEL

These are the index, parameters and variables used in the model:

Indices	
i	Worker $i \epsilon M = \{1 \dots m\}$
j	Supporting service hospital department $j \epsilon N = \{1 \dots n\}$
k	Type of supporting service job $k \epsilon P = \{1 \dots p\}$
Input parameters	
d_j	Minimal labor cost in department j.
c_{ik}	Cost of assignment of a worker *i* to job *k* (monthly salary)
C_j	Maximal monthly budget for salaries in a department *j*
CB	Maximal total budget
f_k	Minimal size of permanent employment for job *k*
e_{ik}	Size of permanent (partial or full time) employment for job *k* (e_k= 0.25 or 0.50 or 0.75 or 1.00)
E_j	Maximal number of permanent employments in a department *j*
EP	Maximal number of permanent employments.
W	Maximal number of employees
h_{jk}	Maximal number of permanent employments in department *j* and job *k*
Variables	
x_{ijk}	1 if worker i is assigned to permanent job k in department j; 0 otherwise
y_i	1 if worker i is assigned to any job in any department; 0 otherwise
g_k	Number of permanent employments for a job k

Table 17: Indices, parameters and variables of the model

14.3.1 BI-CRITERIA MODEL

There are two objectives in this model that are the total budget of the hospital and the total number of workers. We want to optimize them in order to save money and hire all the workers that are necessary to cover all the needs of the health care institution.

Objective 1: This is the objective that minimizes the total budget:

$$maximize\ -\left(\sum_{i\in M}\sum_{j\in N}\sum_{k\in P} c_{ik}\, x_{ijk}\right) \tag{1}$$

Objective 2: This is the objective that maximizes the total number of hired workers:

$$maximize \sum_{i\in M} y_i \tag{2}$$

Subject to:

$$\sum_{i\in M}\sum_{k\in P} c_{ik}\, x_{ijk} \le C_j \qquad \forall j \in N \tag{3}$$

$$\sum_{i\in M}\sum_{j\in N}\sum_{k\in P} d_j\, x_{ijk} \le CB \qquad \forall i \in M, \forall j \in N, \forall k \in P \tag{4}$$

$$\sum_{i\in M}\sum_{k\in P} e_{ik}\, x_{ijk} \le E_j \qquad \forall j \in N \tag{5}$$

$$\sum_{i\in M}\sum_{j\in N}\sum_{k\in P} f_k\, x_{ijk} \le EP \tag{6}$$

$$\sum_{i\in M}\sum_{j\in N} x_{ijk} \le \sum_{j\in N} h_{jk} \qquad \forall k \in P \tag{7}$$

$$\sum_{j\in N}\sum_{k\in P} x_{ijk} \ge y_i \qquad \forall i \in M \tag{8}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in M, \forall j \in N, \forall k \in P \tag{9}$$

$$y_i \in \{0,1\} \qquad \forall i \in M \tag{10}$$

Constraint (3) ensures that the cost of assigning one worker to a job on each department is less than the total budget assigned to that department.

Constraint (4) ensures that the sum over all the departments of assigning one worker to one department is less or equal to the total budget.

Constraint (5) ensures that the total number of permanent employments in one department is less or equal to E_j, that is the maximal number of those permanent employments in that determinate department.

Constraint (6) says that the minimal number of permanent employments is less or equal to a given number EP, which is the maximal number of permanent employments.

Constraint (7) ensures that the assignment of workers to selected permanent job is less or equal to the sum over all departments of the maximal permanent employments h_{jk}.

Constraint (8) makes a relation between both variables, and ensures that each worker is assigned to at least one job in one department if he has been hired.

Constraints (9) and (10) define x_{ijk} and y_i as binary variables.

As it has been explained, when objective 1 is used, the other objective should be added as a constraint:

$$\sum_{i \in M} y_i \geq W \quad (11)$$

Constraint (11) is the second objective written as a constraint, and limited by the computed number of workers W. It ensures that the number of workers is at least the given number W.

When the second objective is used, the first one is added as a constraint. So in this case, the constraint needed is:

$$\sum_{i \in M} \sum_{j \in N} \sum_{k \in P} c_{ik}\, x_{ijk} \leq CB \quad (12)$$

Constraint (12) is the first objective written as a constraint, and limited by the computed budget CB. It ensures that the cost of assigning each worker to a permanent position is less than the given number CB that is the total maximum budget.

14.3.2 TRIPLE OBJECTIVE MODEL

In this model the third variable g_k is used, and is the number of permanent employments of a job. This model has some equations in common with the previous one. To avoid repeating the equations here we indicate them with their associated number.

The three objectives of this model are: the first one the minimization of the budget; the second one the maximization of the number of workers and the third one the maximization of the number of permanent employments.

Objective 1: This is the objective that minimizes the total budget: (1).

Objective 2: This is the objective that maximizes the total number of hired workers: (2).

Objective 3: This is the objective that maximizes the number of permanent employments for each job:

$$maximize \sum_{k \in P} g_k \tag{13}$$

Subject to (3), (4), (5), (6), (7), (8), (9), (10) and:

$$g_k \leq h_{jk} \qquad \forall j \in N, \forall k \in P \tag{14}$$

$$\sum_{i \in M} \sum_{j \in N} \frac{x_{ijk}}{f_k} \geq g_k \qquad \forall k \in P \tag{15}$$

$$g_k \geq 0 \qquad \forall k \in P \tag{16}$$

Constraint (14) makes the number of permanent employments of a specific job less or equal to the maximum number of permanent employments of that job in a determinate department.

Constraint (15) gives the relation between the variable x_{ijk} and the variable g_k .

The rest of the constraints of this model depend on the objective function that is been used in each case.

So, when objective 1 is used (minimize the total budget), the other objectives must be transformed to constraints, which are equation (11) and:

$$\sum_{k \in P} g_k \geq EP \tag{17}$$

Constraint (17) makes the total number of permanent employments of all the jobs is at least equal to the maximum number of permanent employment defined by EP.

When the problem is solved for the second objective (maximize the number of workers) the constraints that must be used are (12) and:

$$\sum_{k \in P} g_k \leq EP \tag{18}$$

Constraint (18) ensures that the maximum number of permanent employments is less or equal to EP.

We can observe that when we have to minimize the objective the constraint is lower bounded and when we have to maximize the objective, the constraint is upper bounded.

When objective 3 is used (maximize the number of permanent employments), the constraints that must be used are (12) and:

$$\sum_{i \in M} y_i \leq W \tag{19}$$

The values of the total budget CB, the total number of workers W, and the total number of permanent employments $\sum_{k \in P} g_k$ obtained in objective 3, can be updated each time we solve the problem to reach to an optimal solution.

14.4 RESULTS

The model and the data were introduced in the software CPLEX v9.1 with the help of project supervisor professor Bartosz Sawik PhD and these are the results that were obtained.

14.4.1 RESULTS OF THE BI-CRITERIA MODEL

The problem has been solved twice, once with each objective.

14.4.1.1 FIRST SOLUTION (OBJECTIVE 1)

The model was first solved to minimize the budget of the hospital. Then the total number of workers was used as a constraint and the value given to W was 221 workers, which is the actual value of the total number of workers.

The solution of the minimization of the total budget was 426060 PLN. The comparison of the budget before and after the optimization is shown in the Table 18:

	Before optimization	**After optimization**
Total budget of the hospital	426060	426060

Table 18:Comparison of the input data of the total budget and the optimized value (bi-criteria model)

The assignment of the workers to the different departments is shown in the Table 19:

Supporting Services Hospital Departments	Number of employees in department (before optimization)	Number of employees in department (after optimization)
Central Heating Department	16	16
Power Department	15	15
Medical Bottled Gases Department	6	6
Ventilation & Air-condition Department	8	8
Heating & Hydraulic Department	11	11
Distribution Department	6	6
Medical Equipment Department	8	8
Technical Department	11	11
Economy Department	21	21
Hospital Pharmacy	20	20
Sterilization Department	27	27
Stuff Monitoring Department	13	13
Information Department	7	7
Business Executive Department	8	8
Technical Executive Department	4	4
Law Regulation Department	7	7
Attorneys-at-law Department	4	4
Hospital Management Cost Section	9	9
Salary Section	9	9
Accounting Section	11	11
TOTAL	**221**	**221**

Table 19:Comparison of the input data and the optimal number of workers assigned to each department after minimizing the total budget(bi-criteria model)

As we can see the budget does not change. This is because of the constraint that limits the minimum number of hired workers. We have set that number to the real one (221), so is not possible to reduce the cost of assignments, because we cannot hire fewer workers than the actual number. The Table 19 shows that the assignment of workers to departments does not change at all.

14.4.1.2 SECOND SOLUTION (OBJECTIVE 2)

The second solution was obtained using the second objective as objective function. Therefore, the aim of this problem is to maximize the total number of workers, but having a budget restriction. The restriction value given to the parameter CB is the actual total budget 426060 PLN. Hence, we are trying to maximize the number of workers using the same budget that we had.

The solution of the maximization of the number of workers was 221. The assignation of the workers to the different departments is shown in the Table 20:

Supporting Services Hospital Departments	**Number of employees in department (before optimization)**	**Number of employees in department (after optimization)**
Central Heating Department	16	16
Power Department	15	15
Medical Bottled Gases Department	6	6
Ventilation & Air-condition Department	8	8
Heating & Hydraulic Department	11	11
Distribution Department	6	6
Medical Equipment Department	8	8
Technical Department	11	11
Economy Department	21	21
Hospital Pharmacy	20	20
Sterilization Department	27	27
Stuff Monitoring Department	13	13
Information Department	7	7
Business Executive Department	8	8
Technical Executive Department	4	4
Law Regulation Department	7	7
Attorneys-at-law Department	4	4
Hospital Management Cost Section	9	9
Salary Section	9	9
Accounting Section	11	11
TOTAL	**221**	**221**

Table 20:Comparison of the input data and the optimal number of workers assigned to each department after maximizing the total number of workers (bi-criteria model)

As we can see the number of hired workers does not change. This is because of the budget constraint. It limits the maximum amount of money that we have to hire workers. We have set that number to the real one, so is not possible to hire more workers with the same budget.

14.4.2 RESULTS OF THE TRIPLE OBJECTIVE MODEL

14.4.2.1 FIRST SOLUTION (OBJECTIVE 1)

The tri-objective model was first solved with the objective of minimize the budget of the hospital.

The total number of workers that was used as a constraint was W=221 workers (lower bound), which is the actual value of the total number of workers. The total number of permanent employments EP is set to 214 (lower bound), which is the real and actual value of that parameter.

The solution of the minimization of the total budget was 426060 PLN. We also obtained that the total number of workers was 221, and the number of permanent employments 214. These results will be used when the problem is solved with the second objective.

In the next table (Table 21) we can observe the variation of the budget after the optimization:

	Before optimization	**After optimization**
Total budget of the hospital	426060	426060

Table 21:Comparison of the input data of the total budget and the optimized value (triple objective model)

The assignation of the workers to the different departments is shown in the Table 22:

Supporting Services Hospital Departments	Number of employees in department (before optimization)	Number of employees in department (after optimization)
Central Heating Department	16	16
Power Department	15	15
Medical Bottled Gases Department	6	6
Ventilation & Air-condition Department	8	8
Heating & Hydraulic Department	11	11
Distribution Department	6	6
Medical Equipment Department	8	8
Technical Department	11	11
Economy Department	21	21
Hospital Pharmacy	20	20
Sterilization Department	27	27
Stuff Monitoring Department	13	13
Information Department	7	7
Business Executive Department	8	8
Technical Executive Department	4	4
Law Regulation Department	7	7
Attorneys-at-law Department	4	4
Hospital Management Cost Section	9	9
Salary Section	9	9
Accounting Section	11	11
TOTAL	**221**	**221**

Table 22:Comparison of the input data and the optimal number of workers assigned to each department after minimizing the total budget (triple objective model)

As we can see neither the budget nor the total number of workers nor the number of permanent employments change. This is because we have a lower bounded constraint that limits the minimum number of workers that must be hired. We have set that limit number to the real one, so is the same problem as in the bi-criteria model: it is not possible to reduce the cost of assignments, because we cannot hire fewer workers than the actual number.

14.4.2.2 SECOND SOLUTION (OBJECTIVE 2)

The second objective function maximizes the number of workers.

The total budget that was used as a constraint was the value obtained in the previous calculations CB=426060 PLN (upper bound). The total number of permanent employments EP is set to 214 (upper bound), which is also the value of permanent employments obtained in the previous calculations.

The solution of the maximization of the objective function was 221 workers. We also obtained that the total budget was 426060 PLN, and the number of permanent employments is 214. These results will be used when the problem is solved with the third objective.

The assignation of the workers to the different departments is shown in the Table 23:

Supporting Services Hospital Departments	**Number of employees in department (before optimization)**	**Number of employees in department (after optimization)**
Central Heating Department	16	16
Power Department	15	15
Medical Bottled Gases Department	6	6
Ventilation & Air-condition Department	8	8
Heating & Hydraulic Department	11	11
Distribution Department	6	6
Medical Equipment Department	8	8
Technical Department	11	11
Economy Department	21	21
Hospital Pharmacy	20	20
Sterilization Department	27	27
Stuff Monitoring Department	13	13
Information Department	7	7
Business Executive Department	8	8
Technical Executive Department	4	4
Law Regulation Department	7	7
Attorneys-at-law Department	4	4
Hospital Management Cost Section	9	9
Salary Section	9	9
Accounting Section	11	11
TOTAL	**221**	**221**

Table 23:Comparison of the input data and the optimal number of workers assigned to each department after maximizing the number of workers (triple objective model)

In this case the number of workers does not change at all. This is again due to the strict constraints that have been used to solve the problem. If we cannot use more money than the amount that we actually have it is impossible to hire more workers. The upper bounded constraint of the permanent employments does not let us to increase the number of them.

Now, these values are going to be used in the next and last resolution of the problem.

14.4.2.3 THIRD SOLUTION (OBJECTIVE 3)

In this last resolution we want to maximize the number of permanent employments.

The values used in the constraints are the following ones: the total number of workers W=221 (upper bound); and the total budget CB=426060 PLN (upper bound).

The solution of the maximization of the number of permanent jobs was 214. We also obtained that the total number of workers was 221, and the total budget 426060 PLN.

These values are the definitive and optimized values of this problem.

In the next table (Table 24) we can observe the variation of the number of permanent jobs after the optimization:

	Before optimization	**After optimization**
Total permanent jobs	214	214

Table 24:Comparison of the input data of the total permanent jobs and the optimized value (triple objective model)

The assignation of the workers to the different departments is shown in the Table 25:

Supporting Services Hospital Departments	Number of employees in department (before optimization)	Number of employees in department (after optimization)
Central Heating Department	16	16
Power Department	15	15
Medical Bottled Gases Department	6	6
Ventilation & Air-condition Department	8	8
Heating & Hydraulic Department	11	11
Distribution Department	6	6
Medical Equipment Department	8	8
Technical Department	11	11
Economy Department	21	21
Hospital Pharmacy	20	20
Sterilization Department	27	27
Stuff Monitoring Department	13	13
Information Department	7	7
Business Executive Department	8	8
Technical Executive Department	4	4
Law Regulation Department	7	7
Attorneys-at-law Department	4	4
Hospital Management Cost Section	9	9
Salary Section	9	9
Accounting Section	11	11
TOTAL	**221**	**221**

Table 25:Comparison of the input data and the optimal number of workers assigned to each department after maximizing the total number of permanent employments (triple objective model)

We can see in the Table 24 that the total number of permanent employment remains the same before and after the optimization. The number of workers and the total budget is the same.

14.5 CONCLUSIONS

Due to the very restrictive constraints and to the real data used there are no changes after the optimization.

All these results are equal to the input data; it could mean that the number of workers, of permanent employments or the money that this hospital uses is the correct one because is the optimal according to this model.

15. CONCLUSIONS

Optimizing different aspects of health care institutions is possible to save several resources, such as money or the quantity of facilities. In this project we have seen how optimization helps us to build better schedules, to allocate the facilities in an efficient way, to understand which the best way to proceed is or to assign workers to hospital services and departments.

The importance of the good operation of hospitals is huge. There are a lot of people who depends on the services that they offer. Some people may need surgeries, other people treatments and others just a consult. People go to hospitals for many different reasons and workers have to attend all of them. That is why hospitals are places where there are a lot of people and therefore, must be clean and organized. Optimization is a big help with all the management of the hospital.

As we have seen we can choose by defining one objective function what issue or issues do we want to improve. Constraints are very important tools to ensure that all the requirements, needs and service levels of the hospitals are covered.

APENDIX A: SCRIPTS OF OPTIMIZATION MODELS (LEXICOGRAOHIC APPROACH)

- BI-OBJECTIVE MODEL

#MEDICAL SERVICES OPTIMIZATION MODEL ©Bartosz Sawik PhD

#--------------------------Sets--------------------------

param m:=221; *# Number of employees i*
set M:=1..m; *# Set of employees*

param n:=20; *# Number of departments j*
set N:=1..n; *# Set of hospital's departments*

param p:=88; *# Number of jobs k*
set P:=1..p; *# Set of jobs*

#--------------------------Parameters--------------------------

param d{j in N}; *# Minimal labor cost in department j.*

param c{i in M, k in P}; *# Cost of assignment of a worker i to job k (monthly salary)*

param C{j in N}; *Maximal monthly budget for salaries in a department j*

param CB; *# Maximal total budget*

param f{k in P}; *# Minimal size of permanent employment for job k*

param e{i in M, k in P}; *# Size of permanent employment for job k and employee i.*

param E{j in N}; *# Maximal number of permanent employments in a department j*

param EP; *# Maximal number of permanent employments.*

param W; *# Maximal number of employees*

param h{j in N, k in P}; *# Maximal number of permanent employments in department j and job k*

#--------------------------Variables--------------------------

var x{i in M, j in N, k in P} binary; *# 1 if worker i is assigned to job k in department j, 0 otherwise.*

var y{i in M} binary; *# 1 if worker i is assigned to any job in any department, 0 otherwise.*

#--------------------------Objective--------------------------

#OBJECTIVE FUNCTION#

maximize FC:

-sum{i in M, j in N, k in P}c[i,k]*x[i,j,k]; *#OBJECTIVE1 minimize the budget*

sum{i in M}y[i]; *#OBJECTIVE2 maximize the number of workers*

#---------------Lexicographic approach constraints----------------

#LEX1

subject to OBJ2: sum{i in M}y[i]>=W; *#computed number of workers*

#LEX2

subject to OBJ1: sum{i in M, j in N, k in P}c[i,k]*x[i,j,k]<=CB*;* *#computed budget*

#--------------------------Constraints--------------------------

subject to Budget{j in N}: sum{i in M, k in P}c[i,k]*x[i,j,k]<=C[j]; *# Maximal budget for each department j*

subject to Budget1: sum{i in M, j in N, k in P}d[j]*x[i,j,k]<=CB; *# Maximal - total budget for all departments*

subject to Posts{j in N}: sum{i in M, k in P}e[j,k]*x[i,j,k]<=E[j]; *# Maximal number of permanent positions in each department j*

subject to Posts1: sum{i in M, j in N, k in P}f[k]*x[i,j,k]<=EP; *# Maximal - total number of positions in all departments*

subject to WPosts{k in P}: sum{i in M, j in N}x[i,j,k]<=sum{j in N}h[j,k]; *# Maximal requirements for positions*

#--------------------------Binary variable constraint--------------------------

subject to OGR_y{i in M}:sum{j in N, k in P}x[i,j,k]>=y[i]; *# Relation between variables*

end;

- TRIPLE OBJECTIVE MODEL

#MEDICAL SERVICES OPTIMIZATION MODEL ©Bartosz Sawik PhD

```
#--------------------------Sets--------------------------

param m:=221;                          # Number of employees i
set M:=1..m;                           # Set of employees

param n:=20;                           # Number of departments j
set N:=1..n;                           # Set of hospital's departments

param p:=88;                           # Number of jobs k
set P:=1..p;                           # Set of jobs

#--------------------------Parameters--------------------------

param d{j in N};                       # Minimal labor cost in department j.

param c{i in M, k in P};               # Cost of assignment of a worker i to job k
                                       (monthly salary)

param C{j in N};                       # Maximal monthly budget for salaries in a
                                       department j

param CB;                              # Maximal total budget

param f{k in P};                       # Minimal size of permanent employment
                                       for job k

param e{i in M, k in P};               # Size of permanent employment for job k
                                       and employee i.

param E{j in N};                       # Maximal number of permanent
                                       employments in a department j

param EP;                              # Maximal number of permanent
                                       employments.

param W;                               # Maximal number of employees

param h{j in N, k in P};               # Maximal number of permanent
                                       employments in department j and job k

#--------------------------Variables--------------------------

var x{i in M, j in N, k in P} binary;  # 1 if worker i is assigned to job k in
                                       department j, 0 otherwise.

var y{i in M} binary;                  # 1 if worker i is assigned to any job
                                       in any department, 0 otherwise.

var g{k in P}>=0;                      # Number of permanent
                                       employments for a job k, integer
                                       variable.
```

```
#--------------------------Objective--------------------------

#OBJECTIVE FUNCTION#

maximize  FC:

- sum{i in M, j in N, k in P}c[i,k]*x[i,j,k];   #OBJ1: minimize budget

sum{i in M} y[i];       #OBJ2: maximize number of workers

sum{k in P} g[k];       #OBJ3: maximize number of permanent employment

#-------------Lexicographic approach constraints--------------------
#LEX1
subject to OBJ2: sum{i in M}y[i]>=W;        #computed number of workers
subject to OBJ3: sum{k in P} g[k]>=EP;      # computed number of permanent
                                            possitions.

#LEX2
subject to OBJ1: sum{i in M, j in N, k in P}c[i,k]*x[i,j,k]<=CB;#computed
                                                                 budget
subject to OBJ3: sum{k in P} g[k]<=EP;      # computed number of permanent
                                            possitions.

#LEX3
subject to OBJ1: sum{i in M, j in N, k in P}c[i,k]*x[i,j,k]<=CB;   #computed
                                                                   budget
subject to OBJ2: sum{i in M}y[i]<=W;        #computed number of workers

#--------------------------Constraints--------------------------
subject to Budget{j in N}: sum{i in M, k in P}c[i,k]*x[i,j,k]<=C[j]; # Maximal
budget for each department j

subject to Budget1: sum{i in M, j in N, k in P}d[j]*x[i,j,k]<=CB; # Maximal -
total budget for all departments

subject to Posts{j in N}: sum{i in M, k in P}e[j,k]*x[i,j,k]<=E[j]; # Maximal
number of permanent positions in each department j

subject to Posts1: sum{i in M, j in N, k in P}f[k]*x[i,j,k]<=EP; # Maximal - total
number of positions in all departments

subject to WPosts{k in P}: sum{i in M, j in N}x[i,j,k]<=sum{j in N}h[j,k];
# Maximal requirements for positions

subject to WPosts_g{j in N, k in P}: g[k]<=h[j,k]; # Maximal requirements for
positions

subject to OGR_g{k in P}: sum{i in M, j in N}x[i,j,k]/f[k]>=g[k]; # Relation
between variables
#--------------------------Binary variable constraint--------------------------
subject to OGR_y{i in M}:sum{j in N, k in P}x[i,j,k]>=y[i]; # Relation between
variables
end;
```

APENDIX B: SOLVING CHARACTERISTICS OF THE MODELS

- BI-OBJECTIVE MODEL

Presolve eliminates 1 constraint and 221 variables.
Adjusted problem:
388960 variables, all binary
330 constraints, all linear; 1171300 nonzeros
1 linear objective; 388960 nonzeros.

CPLEX 9.1.0: mipdisplay=3 timing=1 Dual steepest-edge pricing selected. MIP emphasis: balance optimality and feasibility Root relaxation solution time = 0.06 sec.
Times (seconds): Input = 0.592 Solve = 6.194 Output = 0.265
CPLEX 9.1.0: optimal integer solution within mipgap or absmipgap; objective
2468 MIP simplex iterations
946 branch-and-bound nodes

Presolve eliminates 1 constraint and 221 variables.
Adjusted problem:
388960 variables, all binary
330 constraints, all linear; 1171300 nonzeros
1 linear objective; 388960 nonzeros.

- TRIPLE-OBJECTIVE MODEL

Presolve eliminates 1762 constraints and 221 variables.
Adjusted problem:
389048 variables:
 388960 binary variables
 88 linear variables
331 constraints, all linear; 1560348 nonzeros
1 linear objective; 388960 nonzeros.

CPLEX 9.1.0: mipdisplay=3 timing=1 Dual steepest-edge pricing selected. MIP emphasis: balance optimality and feasibility Root relaxation solution time = 0.06 sec. Times (seconds): Input = 0.904 Solve = 6.256 Output = 0.296
CPLEX 9.1.0: optimal integer solution within mipgap or absmipgap; objective
2468 MIP simplex iterations
946 branch-and-bound nodes

Presolve eliminates 1762 constraints and 221 variables.
Adjusted problem:
389048 variables:
 388960 binary variables
 88 linear variables
331 constraints, all linear; 1560348 nonzeros
1 linear objective; 388960 nonzeros.

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