# A WEIGHTED-SUM APPROACH
# TO HEALTH CARE OPTIMIZATION:
# CASE STUDIES



FINAL PROJECT FOR THE BACHELOR

IN ENGINEERING DEGREE


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ABSTRACT

The aim of this project is to develop a program using the Branch and Bound method with the Weighted Sum Approach to solve the problem of allocation of personnel in a health care institution. To deal with this problem we use multi-criteria optimization and we compare and analyze the results obtained as we change the importance of the different objectives.

Before focusing in the most important part of the project, we also give some explanations about different cases related with health care institutions. Some of them are related with nurse rostering problems, others talk about management and capacity planning in hospitals and some of them are related with multi criteria optimization so they are really important to understand the core of the project. All these cases have been done to approach into the health institutions problems, as an introduction to the main part of the project.

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## 2   INTRODUCTION

In the beginning, this project was going to be focused on nurse rostering and scheduling problems and on the optimization models that can be used to solve them. However, we noticed that we did not have enough time to deal with that big issue and we changed it to solve staff problems in health care institutions. At first, we were going to do the project together, Uxue Tornos and me, Raquel Villegas, but finally we decided to do some parts together but the rest each one by ourselves.

Health care is a really important issue in the society. If health care systems are well developed and well organized, people will be glad with the attention received in hospitals and other medical centers. However, if there are any problems on them that might probably affect patient's health.

Hence, it is necessary to optimize the health care system. There is some stuff really important in health care institutions that can be studied and changed to have better performance and to increase the efficiency. Using optimization models and simulation programs it is possible to improve the work on a hospital. They can be used to reduce delays in Emergency Rooms (ER), to reduce the number of free beds or to improve the schedule to have more employees in agreement with it, being one of the most important issues the nurses scheduling. Here, we present a model that deals with the best assignment of hospital's personnel in different departments and different permanent jobs.

The project starts with four cases talking about nurse rostering. The first one offers a summary of how nurse rostering problem has been treated over the last years with different proposed models and different nurse rostering approaches. Then, the second one shows how centralized nurse scheduling can improve costs and increase nurses' satisfaction. After that, another case refers in a general way to nurse scheduling, dealing with nurse scheduling problems in a General Clinic and in an Operating Suite and making differences between single objective and multi-objective problems. The last one of this group gives a general idea of how can be the integration of nurse and surgery scheduling. This cases related with nurse rostering are the ones that I have done with the collaboration of Uxue Tornos.

Right after, there are two cases related with management and simulation in hospitals. One of them talks about management and capacity planning in hospitals, referring to emergency room delays, target occupancy levels and staffing levels across the day. The other basically treats some problems about waiting time in hospitals and refers to a simulation model for an acute care emergency department.

Then, we present a basic model of healthcare resource allocation using Multi-Criteria Optimization (MCO) and an example based on prioritizing treatments for depression.

The next is a case, which offers single and triple objective mathematical programming models for assignment of services in a healthcare institution. It shows the data gathered for the study, the optimization models used and the computational results. This case has also been made with the collaboration of Uxue Tornos.

After that, we discuss about some real problems of hospital's management, which is situated near Krakow, called Ośrodek Rehabilitacji Narządu Ruchu "Krzeszowice", and then we refer to the hospital Szpital Specjalistyczny Im. Ludwika Rydygiera, from where we took all the data used in this project.

The aim of the project is to design an optimization model of the assignment of the personnel to the previous hospital. The method used is the Branch and Bound method with the Weighted-Sum Approach. Two models are developed, one bi-objective and another triple-objective. The data has been gathered and then studied to make analysis and observations of them. The software used for making the analysis is called solver CPLEX 9.1 with AMPL programming language. Results and data collection have been done with the aid of the project supervisor – professor Bartosz Sawik PhD from Department of Applied Computer Science, Faculty of Management, AGH University of Science and Technology.

Finally we offer two appendixes where we show both models' program in AMPL language and the solving characteristics of them.

# 3   THE STATE OF THE ART OF NURSE ROSTERING [20]

## 3.1   INTRODUCTION

The term "rostering" has a lot of different meanings. Here, when we talk about nurse rostering, we are going to refer to the huge scheduling problem of the hospital personnel, and it means basically the allocation of nurses to periods of work over several weeks.

This domain usually covers different fields but although they are different, they are really connected. These fields are staffing, budgeting and short-term scheduling problems.

Until a few years ago the scheduling was made manually, that is called "self-scheduling". It was a very time consuming task. In most of the hospitals they still use this manually system, and in the ones that use computerized scheduling systems, they do not get the best results because they do not exploit all the capacities of the system.

In healthcare is really important for the system to create a good timetable because it is not possible to do not support patient care needs. The automatized programs are really useful for that, they give us many different solutions for the scheduling problems and also they can optimize the nurses' work by saving time.

## 3.2   DESCRIPTION OF MODELS

There are some important decisions that must be taken before using a scheduling method. It is significant to choose between manual or automated scheduling methods and also to decide to use cyclical or non-cyclical scheduling that will lead to different solutions. We will now focus on these decisions discussing about some surveys articles.

### 3.2.1   LITERATURE OVERVIEWS

We are going to talk about the view of different people during 30 years.

First of all, **Warner** (1976) talks about three areas of manpower decision research: staffing, reallocation and scheduling of nurses. He defines 5 criteria for the scheduling part of the problem: coverage, quality, stability, flexibility and cost. He

compares three scheduling approaches to these 5 criteria and he takes the next conclusions: the only advantage that the Traditional Approach has is that is flexible because it is done by hand, on the other hand the Cyclical Scheduling provides good schedules but it cannot easily address personal requests and the Computer Aided Traditional Scheduling is the one that follows better all the criteria.

**Fries** (1976) shows a list of early methods based on manual procedures, which follow some arbitrary rules. It would be difficult to use it in a modern complex hospital but it would be possible to talk about some kind of hybrid approach, mixing an early process with a modern one.

**Tien and Kamiyama** (1982) present some personnel scheduling algorithms concentrating in the USA hospitals but not restricted to healthcare. They refer to five different stages inside the manpower scheduling problem, two of them are management decisions, which belong to the long-term staffing problem, and the other includes short-term timetabling problem. However, these 5 stages are not enough to explain the modern nurse rostering.

**Sitompul and Randhawa** (1990) focus on reduce the personnel cost, the financial cost. They defend the approach of solve staffing and rostering problems at the same time. In a theory way this is true, however in practice it is not so good.

**Warner, Keller and Martel** (1990) discuss patient-oriented and employee-oriented issues in nurse management. They talk about the history of computerized nurse scheduling in the US and also they introduce a nurse scheduling system called ANSOS (Automated Nurse Scheduling Office Systems).

**Bradley and Martin** (1990) discuss about three different decisions about personnel scheduling: staffing, personal scheduling and allocation (that was started by Warner). The first problem is about determining the long-term number of personnel that must be employed. The second is the conversion of the expected daily work force into assignments, that is, personnel rostering. The allocation consists of assigning the scheduled personnel to actual work sites. They present a classification that can be summarized as: exact cyclical, heuristic cyclical, exact non-cyclical and heuristic non-cyclical.

**Siferd and Benton** (1992) did a review of the factors that influence hospital scheduling and staffing in the USA. They start talking about the importance that cost reduction is having and they discuss about the nurses' schedule too, with the constraints that they have. They discuss things like: it is more popular the full time work than a part time work, it is also strange to have the same nurses for day and night shifts, half of the hospitals have 3 shifts in weekdays and the 30% they have 5 shifts.

**Hung** (1995) did an overview from nurse scheduling, from the 60's up to 1994. It can be a bibliography or a useful collection of literature.

**Cheang and others** (2003) present a survey where they cover mathematical programming methods to show models and approaches for nurse rostering.

**Ernst and others** (2004) discuss about staff scheduling and rostering in general, they distributed the overview in three parts: the definitions and classification of problems, a classification of literature and the solution methods. They point out that mathematical programming and metaheuristic approaches are the most investigated techniques and that metaheuristics are promising for difficult problems and real world problems which solutions cannot be obtained with other approaches.

It is clear that nurse scheduling problems has been addressed across a lot of articles, some of them only talk about problems and propose solutions that cannot be applied in hospitals. This problem was treated by Warner in 1976 and it continues during the years. Siferd and Benton started to show solutions for the real world problem, a lot of people in the past did not address this complexity.

3.2.2   STAFFING

Hospital staffing involves determining the number of personnel of the required skills to meet predicted requirements. In real world staffing, budgeting and personnel rostering usually take place at different levels and horizons. As we have seen before, lots of researchers have decomposed the nurse rostering problem into phases but the interaction between levels, in practice, will be unworkable. A summary of some key articles is presented because of the impact that some input data can have on the short-term nurse rostering problem.

In 1963, **Wolfe and Young** presented a model to minimize the cost for assigning nurses with different skill categories to different tasks.

**Warner** (1976) did not only deal with rostering problem, he dealt with the staffing problem too. The staffing problem consists in determining an appropriate number of full time nurses for each skill category. He proposed a methodology and some hospitals accepted it. After that phase it comes the last one: reallocation of nurses. He is sure about that the combination of these three stages leads to a better scheduling policy.

**De Vries** (1987) developed a 'management control framework' to balance the supply and the demand for nursing care. It seems to be not a strict equilibrium but an acceptable range of balance. He divided the workload per hour by the available staff per hour and calculates by this way the actual capacity utilization.

**Smith-Daniels and Schweikhart** (1988) presented a literature overview on capacity planning in healthcare. They talked about two different decisions: acquisition and allocation decisions. They predict that the strict staffing and timetabling of people and other resources will all be combined in an objective for new scale health organizations.

**Easton, Rossin and Borders** (1992) compared different staff policies during one month in a large hospital in USA. They saw that in busy periods unscheduled nurses will be expected to work, and in slack periods, people will work too few hours to earn their full wages. They discussed the possibility of working with 'float' nurses, but they must not do some works that required a lot of experience. They finally have 12 different methods, and they conclude that the expected nursing expenses decrease as the scheduling alternatives increase. The research excludes overtime, part-time work and understaffing because it is very difficult to formalize them, but these extra parameters need to be into account.

The rostering algorithms have to handle the results of management decisions at a high level. In some hospitals staffing and scheduling information is used to support structural change.

### 3.2.3   ADMINISTRATIVE MODES OF OPERATION

There are really different administrative procedures in different hospitals that lead to different types of nurse rostering problems. Now, we are going to see the most important ones.

**Centralized scheduling** is a term used to describe the situation where one administrative department in a hospital carries out all the personnel scheduling (Easton, Rossin, and Borders, 1992; Siferd and Benton, 1992; Smith-Daniels, Schweikhart, and Smith-Daniels, 1988; Warner, 1976). Head nurses do not have to construct the schedules. The advantage of this procedure is the opportunity for cost containment through better use of resources. On the other hand it has some limitations as being some kind of favoritism or that the rosters are unfair.

We talk about **unit scheduling** when the head nurses are given the responsibility to generate the schedules locally (Aickelin and Dowsland, 2000; Bradley and Martin, 1990; Dowsland, 1998; Meisels, Gudes, and Solotorevski, 1996; Meyer auf'm Hofe, 1997; Sitompul and Randhawa, 1990).

**Self-scheduling** is used to talk about the situation when the personnel roster is generated manually by the staff themselves. It is more time consuming than automatic scheduling but it has the advantage that the nurses co-operate and are asked for advice. It is usually performed by the personnel members themselves and coordinated by the head of the nurse of a ward. It is a very intensive procedure.

**Miller** (1984) and **Hung** (1992) they defend that type of scheduling, they say that the autonomy of the nurses increases, it reduces the head nurse's scheduling time and it improves cooperation and team work. They say this is more effective because the personnel know that their problems are taken into account.

**Silvestro** and **Silvestro** (2000) define three different types of scheduling that are departmental rostering, conducted by the charge nurse, team rostering, the staff is divided in teams with nominated leaders in each and self-rostering where the roster is prepared by the ward staff. They identify 4 key determinants of rostering problem complexity that are ward size, predictability of demand, demand variability and complexity of skill mix. They conclude that departmental rostering is more appropriate

for large wards and team rostering is better for medium sized ward with easy problems. Self rostering is dangerous because unbalanced rosters can be generated.

### 3.2.4   CYCLICAL SCHEDULING

Cyclical scheduling concerns organizations in which each person works a cycle of n weeks, it is called "fixed" scheduling too, and the non-cyclical is known as "flexible" scheduling. This type is used when the day is divided in different shifts and the personnel requirements per shift and per day obey a cyclical pattern.

**Warner** (1976) said that it has some advantages as knowing the schedule a long time in advance or using "forward" rotation that is when a schedule includes no shift starting at an earlier time than a shift on the day before.

**Megeath** (1978) proposed cyclical 7-days patterns of shifts and days off to allow for balanced shift coverage. It has benefits but it is not very flexible and it requires a higher level decision.

**Burns and Koop** (1987) developed a cyclical model for manpower scheduling with strict specifications on consecutive working days and days off. It uses only 3 shifts and it is not flexible at all.

**Hung** (1991) presents a cyclical pattern for short-term nurse scheduling with 4-day workweeks with 10-hour shifts. He said that it has benefits, as the people that work together are a real team and the people have more time for social activities. The algorithm is simple enough to be generated by hand. He found the limitations of this approach too. It is not directly applicable to many modern real nurse rostering situations. It would be a good idea to translate some of its benefits into constraint based methods. Cyclical personnel rostering are generated using constraint satisfaction by **Muslija, Gaertner and Slany** (2000).

Tour scheduling is a special case of cyclical scheduling and it is one of simultaneously determining days worked, start times and shift lengths worked over some plan horizon.

**Bechtold and Showalter** (1987) combine the problem of staffing and scheduling personnel in a tour scheduling problem. It is not so flexible so it cannot be applied in modern hospitals.

Cyclical schedules are not flexible when it comes to addressing slight changes in personnel demands or in expressing personal preferences.

## 3.3 NURSE ROSTERING APPROACHES

In the scientific literature there are several approaches to the nurse rostering problem. Most nurse rostering problems are extremely complex and difficult.

Here we are going to present them grouped to the type of method that is described.

### 3.3.1 OPTIMIZING APPROACHES: MATHEMATICAL PROGRAMMING

Mathematical programming methods are very good for finding optimal solution. Most of the mathematical approaches are based on optimizing the value of a single objective function, so then, researchers simplify the problem of nurse rostering by considering only a few constrains in their model. These models are too simple to be useful into a real hospital situation.

However there have been done several experiments with goal programming or multi objective decision making.

#### 3.3.1.1 Linear and integer programming

**Abernathy et al.** (1973) solved nurse scheduling problem by using mathematical programming techniques. They divide the staffing of hospitals into three decision levels: policy decisions, staff planning and short-term scheduling of available personnel taking into account the constraints imposed by the first two stages. This involves more management decisions than the nurse rostering problem. This approach has been only tested in a hypothetical example application.

**Warner and Prawda** (1972) presented a formulation to calculate the number of nurses from certain skill category to undertake a number of shifts per day. They use

three shifts of 8 hours each one, and try to minimize the difference between the lower limit for the number of nurses and the nurses. By employing more people the cost for personnel shortage can be reduced. The problem is that in this model there is no way to include personal preferences, and it is not trustworthy for a period longer than four days due to the accurate forecast of personnel demand.

**Warner** (1976) worked with the previous formulation and introduced weights or fairness levels. He uses shift patterns of 2 weeks length and some flexible constrains. Certain parts of the scheduling are carried out once the optimization starts such as the weekends' assignation or the people who is going to rotate. Those things are done manually, and that simplifies the model. One of the most important things of this model is that allows a fair evaluation of the schedules.

**Trivedi and Warner** (1976) described an algorithm to arrange the assignment of nurses from different units in moments that there is shortage of personnel, what is called float nurses. It is not relevant for modern nurse rostering environment because it only works with small-scale problems, but it is the first model that introduces the idea of float personnel. There is not any methodology to deal with the floating staff, but it is still very used in modern hospitals nowadays, although it seems that float nurses "settle down" very quickly in a ward with a temporary shortage of personnel.

**Warner, Keller and Martel** (1990) presented a nurse scheduling system called ANSOS that consists in four modules: The Position Control Module (scheduling information for each employee), The Scheduling Module (specific rules with personal preferences), The Staffing Module (computes the staffing level required for each unit) and The Management Reporting Module (provides reports). This model takes into account a lot of personal and individual constrains, and it is used in real hospitals.

In the **Miller, Pierskalla and Rath** (1976) method there are no shifts specified. They formulate the personnel requirements and the number of personnel per day. In this model, they use an "aversion" index which measures how good or bad the previous nurse schedules were. It is used a cyclic coordinate descend algorithm in order to find the optimal solution.

**Bailey and Field** (1985) create a general mathematical model for the nurse scheduling problem. In this model the cost function is the sum of the cost for utilizing a

shift type multiplied by the number of times that appears in the schedule. The length of the scheduling period is of 12 hours instead of 8, and it reduces the idle time.

**Rosenbloom and Goertzen** (1987) developed an integer programming algorithm to create cyclical scheduling. Optimal solutions are generated, but it only considers work stretches and days off. In the real world there are more constrains that only those ones.

**Jaumard, Semet and Vovor** (1998) proposed an exact solution approach for a flexible model. The linear programming model allows full exploration of the set of feasible solutions, although the conflicting nature of the nurse scheduling constraints makes it very difficult to find them. This approach allows for formulating coverage constraints in terms of time intervals.

**Millar and Kiragu** (1998) used a model that combines the possibilities of 2-shift patterns and 4 days length for cyclic and noncyclic nurse scheduling. However the results of the patterns do not deal with the requirements needed in large hospitals.

**Isken** (2004) tackles real hospital scheduling problems. It introduces a tour scheduling model that simplifies the problem for the mathematical programming techniques. The objective is to reduce labor costs while meeting the fluctuating coverage requirements over a one week planning period. It determines shift start times and full time and part time tours.

**Moz and Pato** (2004) talk about the problem of re-rostering nurse schedules, without using float nurses. They solve the problem by using replacement within the ward. They keep the hard constraints satisfied and try to minimize the difference between the original schedule and the new one, in order not to disturb the private life of the staff.

### 3.3.2   GOAL PROGRAMMING/MULTI-CRITERIA APPROACHES

Mathematical programming techniques sometimes are not flexible to deal with more than one goal, so they often optimize only one goal or one criterion. Most of the papers in this section apply mathematical programming but the latest research (Berrada,

Ferland, and Michelon, 1996; Burke et al., 2002; Jaszkiewicz, 1997) tackles metaheuristics within a multi-objective framework.

**Arthur and Ravindran** (1981) propose a two phase goal programming heuristic to simultaneously optimize different goals in priority sequence: staff size, the number of staff with preferences, staff dissatisfaction, and the deviation between scheduling and desired staffing levels. The shifts are heuristically assigned at the end of the scheduling process.

**Musa and Saxena** (1984) use an interactive heuristic procedure. It is possible to change the relative weights given to the goals during the scheduling process; so then it is possible to take into account temporal conditions. It is very useful because the scheduling circumstances of the hospitals change regularly and they are very difficult to model mathematically. It is not possible to use it directly but it is worth to investigate it and incorporate some ideas into modern methods.

**Ozkar ahan and Bailey** (1988) define three basic objective functions. The first of them tries to minimize de deviation between the number of nurses scheduled and the demand of each day (time-of-day). The second works with the difference between the sum of days on work patterns and the size of the work force (day-of-week). The last one combines the day-of-week and time-of-day scheduling problems. Employing a heuristic assignment of schedules, the algorithm solves the most important shift times and days for individual nurses. **Ozkar ahan** (1991) creates a goal programming approach for a decision support system. It tries to maximize the utilization of the full time personnel, minimizing over- and understaffing and personnel costs. It helps in staffing decisions and provides support for nurses' preferences. This model works only with small size problems.

**Franz et al.** (1989) create a multi-objective integer linear program for health care staff working at different locations (multi-clinic health regions). It includes personnel with different skills, and tries to minimize the traveling costs and maximize the quality of the service by considering the personal preferences and requirements.

**Chen and Yeung** (1993) combine goal programming with expert systems. The first one assists in satisfying the time related constraints on the schedules and attempts to cover personnel demands at the same time. The second one does the assignment of

shift types to personnel members. This paper allows flexibility and the relaxation of constraints.

**Berrada, Ferland and Michelon** (1996) combine multi-criteria approach and tabu search in a flexible tool. To obtain a feasible solution, it must be satisfied a set of hard constraints. To get that, a list of soft constraints is treated as a list of goals. In this model, every nurse works the same shift all the time, so the problem is reduced to 3 smaller problems. It produces satisfactory results. This is one of the first papers in which metaheuristics are applied to address a range of different goals.

**Jaszkiewicz** (1997) introduces a decision support system for the nurse scheduling problem in Polish hospitals. Working and free days are preferably grouped, and the shifts have to be divided evenly among the nurses. There are two stages to reach the solution. In the first one there is a combination of a multi-objective algorithm with a simulated annealing approach (Pareto-Simulated Approach). In this stage it is generated a set of good quality solutions. In the second stage a hospital planner evaluates these results in an interactive way. This model is actually applied in a hospital, but in order to be easily transferred to other hospitals it would require further work because it takes many constraints for granted.

**Burke et al.** (2002) present a new multi-criteria approach. The criteria space is mapped to a preference space with dimensionless units. This approach gives the possibility for users to express their preference for certain constrains. The weights can control the compensation of constraints. One advantage of multi-criteria method is that it facilitates a better handling of dissimilar constraints by considering possible ranges for the criteria.

Most mathematical approaches apply exact methods but the real world problem is so complex that most of the publications mention heuristic methods to tackle the problems.

### 3.3.3  ARTIFICIAL INTELLIGENCE METHODS

3.3.3.1  Declarative and constrain programming

**Okada and Okada** (1988) use the program Prolog with a formal core, which assists in the assignment to shifts to nurses. The importance of the requirements can

change during the planning period, so then not all the constraints must be strictly satisfied. The approach is much stricter than most others, it distinguish between the scheduling task and the general requirements that must be fulfilled. Assignments are carried out in a manual-like manner following a strict procedure. In **Okada's** method (1992) there are a set of "role sequences" as a language in which the constraints are presented as a grammar, and individual preferences are constraints in strings. There are multiple criteria to evaluate, and the system tries to discover the best schedules. This system allows for flexible definition of the soft constraints by the users of different types of hospitals.

**Weil et al.** (1995) reduces the complexity of a constraint satisfaction problem by joining some constrains and reducing the domains by eliminating interchangeable values. He solved quite simple problems with this method.

**Darmoni et al.** (1995) describe a software system called Horoplan for large hospitals. It is useful for rostering and for some short term staffing decisions. It creates nursing schedules with a step by step procedure, reflecting the way that head nurses create their schedules manually.

**Meisels, Gudes, and Solotorevski** (1995) describe an approach that is implemented in a commercial software package called TORANIT. It is very flexible with respect to defining constraints and shifts. For the constraint programming approach they separate them into 3 groups: mutual exclusion constraints (one job at time for nurses), finite capacity of employees (limited hours of work in a determinate period of time) and objectives (distribution of the employee assignments per shift). This combines assignment rules and constraint rules, and personal preferences for certain shifts are tackled by the assignment rules. On the other hand, the constraint rules handle the demand for certain types of nurses or for individual nurses, in addition to personal constraints. **Meisels and Lusternik** (1998) also investigate constraint networks for employee timetabling problems. The approach consists in standard constraint processing techniques, which solve randomly generated test problems.

**Cheng, Lee, and Wu** (1997) invented a nurse rostering system for solving one hospital problem in particular. They use ILOG solver to create a schedule that satisfies a set of rules. Those rules are divided into hard and soft constraints, and the solutions are

generated in 4 steps. The problem of this method is that is designed for one particular situation of one determinate hospital.

**Meyer auf'm Hofe** (1997) talked about the nurse rostering problem as a hierarchical constraint satisfaction problem. It enables the use of overlapping shifts to the traditional ones. The created schedules have also to meet requirements like legal regulations, personnel costs, flexibility with respect to the actual expenditure of work, and the consideration of special qualities. Some of these requirements are from higher decision level, and it is not clear how staffing decisions are implemented in the model. In the practice, it is impossible to satisfy all the constraints, and that is the reason for what requirements are treated in order of importance. It is very complex to generate a satisfactory schedule, but this method allows user to alter the result of the algorithm by hand. **Meyer auf'm Hofe** (2001) maintains the hierarchical level of constraints and constraints weights. As we have said it is not possible to satisfy all the constraints, so now, instead of considering the constraint satisfaction he considers the nurse rostering problem as a constraint optimization. He introduces fuzzy constraints than can be partially violated. A mix of iterative improvement and branch and bound are used in a constraint propagation algorithm that deals with the fuzzy constraints.

**Abdennadher and Schlenker** (1999a, b) present an interactive program (INTERDIP) that has been tested in a real hospital environment and it is based on constraint programming.

**Muslija, Gaertner and Slany** (2000) attempt to generate cyclical solutions for a simplified version of general workforce scheduling problems. This kind of schedule is beneficial for the employees' health and satisfaction. Some important characteristics of the schedules are the length of work blocks and "optimal" weekend characteristics. This method is good for generating schedules very quickly, but it is too simple to be used in large scale healthcare environments.

**Li, Lim and Rodrigues** (2003) tackle a problem from a real world hospital situation applying a hybrid of constraint satisfaction and local search technique. The soft constraints in this model are called preference rules and they consist on personal preferences or general preferences for shift sequences. They first solve a problem in

which some of the constraints are relaxed. After that they apply local search techniques to improve the solution trying to satisfy as many preference constraints as possible.

### 3.3.3.2   Expert systems and knowledge based systems

Expert systems approaches give the possibility of developing user-interactive integrated decision support methodologies for nurse scheduling problems.

**Smith, Bird, and Wiggins** (1979) created a "what-if" decision support system where the user can change the weights of the objectives and take into account personal preferences.

**Bell, Hay, and Liang** (1986) made a visual and very interactive decision support system. **Ozkar ahan** (1991) uses a goal programming model and kept the dimensions of the problem very small. This model is too simple to tackle with most of the problems. **Ozkar ahan and Bailey** (1988) describe three objectives in the goal programming approach.

**Chen and Yeung** (1993) use a hybrid expert system to create full time schedules for nurses. This system handles constrains related to the work time conditions of the employees and other fairness measures. At the same time the program tries to use the minimum staff level by applying a goal module, but it is not a hard constraint for them. They define aspiration levels for each goal.

**Scott and Simpson** (1998) reduce the search space by using constraint elements in a case-base. They imitate the approaches of manual roster planners and generate good solutions with it in a limited time.

**Petrovic, Beddoe and Vanden Berghe** (2003) also use a case-based reasoning approach to nurse rostering problems. It is being used in a UK hospital. Hospital planners combine partial rosters that have been done with the personal preferences and requests. This methodology solves problem by using the experience, what means that similar problems have similar solutions.

### 3.3.4　HEURISTICS

The size of the rostering problems and the lack of knowledge about the structure of most of them hinder the applicability of exact optimization methods. The applicability of heuristics scheduling algorithms requires a clear formulation of the hospital requirements, so then it is possible to obtain high quality schedules in an acceptable computation time.

**Smith** (1976) creates an interactive algorithm to help to construct a cyclical schedule. The algorithm determines the number of personnel members, but not all of them can have rotating schedules. **Smith and Wiggins** (1977) use list-processing techniques that generate non-cyclical schedules for each month, and each skill category. Schedules are developed per person with a considerable number of constraints taken into account. Its interactivity allows users to make manual changes into the generated schedules.

**Blau and Sear** (1983) work with a two week period and generates all the possible schedules. In a second step, a cyclic descent algorithm is used to find an optimal schedule for each nurse. This approach is developed for wards with three skills categories hierarchical replaceable. **Blau** (1985) tries to distribute the unpopular work in addition to the frequency with which employees are granted requests for shift or days.

**Anzai and Miura** (1987) present a cyclic descent algorithm for a ward in which the personnel members have the same skills. It is a model too simple for practical applications.

**Kostreva and Jennings** (1991) use two phases. First, they calculate groups of feasible schedules (respecting the minimum staffing requirements and meeting as much constraints as possible). In the second phase the "aversion score" is calculated based in the nurses' preferences (Kostreva and Genevier, 1989).

**Schaerf and Meisels** (1999) define the problem of assigning employees to tasks in shifts. The shifts are predefined time periods that can reside anywhere on the time axis. This model is strict with the coverage constraints, but flexible with the time related constraints. It is introduced a general local search that allows partial assignments, making use of a larger search space.

3.3.5   METAHEURISTICS SCHEDULING

3.3.5.1   Simulated annealing

**Isken and Hancok** (1991) allow variable starting times instead of three fixed shifts per day. The problem is formulated as an integer program in which under- and overstaffing are allowed but penalized.  This model was not intended to address the nurse rostering problem.

**Brusco and Jacobs** (1995) combine simulated annealing and simple local search heuristic to generate cyclical schedules for continuously operating organizations. That kind of organizations allows their workers' schedules to begin at any hour of the day. This is the reason that makes the problem complex. This problem is called by Brusco and Jacobs "tour scheduling problem". One alternative for pure tour scheduling is the use of mixture of both full-time and part-time workers.

3.3.5.2   Tabu search

**Berrada, Ferland and Michelon** (1996) combine tabu search with multi-objective approach. It is interesting that metaheuristic is applied instead of a mathematical programming approach.

**Dowsland** (1998) use different neighborhood search strategies in a tabu search algorithm. The heuristic oscillates between feasible solutions meeting the personnel requirements and schedules concentrating on the nurses' preferences. This algorithm must provide enough personnel with the request qualities and at the same time satisfy personal requests in a fair manner.

**Dowsland and Thompson** (2000) mix integer programming model with a modern heuristic. The algorithms are implemented in CARE (Computer Aided Rostering Environment). Papers describing algorithms for real hospital use tend to cope with similar issues and come up with similar solutions. They use a pre-processing phase to see if the number of nurses is enough to cover the demand. That part of the problem is tackled by a knapsack model. Due to the reduced search space in pre- and post-processing integer programming approaches solutions are obtained considerably faster than with an IP package.

**Burke, De Causmaecker and Vanden Berghe** (1999) hybridize a tabu search approach with algorithms that are based upon human-inspired improvement techniques. Some of that hybridization work as diversifications for the tabu search algorithm. Users of the software can define their own shift types, work regulations, and more. The algorithms attempt to modify the roaster in order to reduce the number of violations of time-related constrains on the personal schedules. This model is being used in some Belgian hospitals.

**Valouxis and Housos** (2000) also use the three-shift schedules, but they propose some hybrid optimization techniques. Only looking at forward rotation it creates a list of feasible schedules. They integrate tabu search in an integer linear programming model.

**Ikegami and Niwa** (2003) introduce a mathematical programming formulation and solve the problem of nurse rostering with metaheuristics. This model covers all the characteristics that seem unavoidable for real world applications. It also distinguishes between nurse constraints and shift constraints. This algorithm provides very promising results but extra heuristics are needed to speed the algorithm up.

**Bellanti et al.** (2004) deal with a real problem of an Italian hospital, so the constraints are so detailed and specific. Some unavoidable relaxations have been incorporated in the model. The computation time of this algorithm is acceptably low to be used in real hospitals. They explain that different initial solutions are generated by applying different multi-start procedure. From that set of solutions, the best is taken for the local search procedure. The multi-start local search approach and the iterated local search seem to be better than tabu search.

3.3.5.3    Genetic algorithms

**Easton and Mansour** (1993) developed a distributed genetic algorithm for the 'tour scheduling'. It tries to minimize the number of personnel members to fulfill the demands, but is too simple to be applied to a real problem.

**Tanomaru** (1995) presents a genetic algorithm to solve staff scheduling problem. It minimizes the total wage cost in a situation where the number of personnel

is not fixed. His heuristic mutation operators might be too time consuming and not general enough to deal with real problems.

**Aickelin and Dowsland** (2000) talked about a cooperative genetic algorithm. They said that the cyclical schedule couldn't be performed because nurses' preferences can change. They divided the problems into sub-problems to deal with them better. In 2003, they applied an indirect genetic algorithm to the same nurse rostering to better deal with the necessary constraints.

**Aickelin and White** (2004) dealt with the same problem as in 2000. They introduced two different algorithms: a genetic algorithm with an encoding (based on an integer programming formulation) and an 'indirect' genetic algorithm (with a separate heuristic decoder function).

**Jan, Yamamoto and Ohuchi** (2000) developed a less cooperative genetic algorithm which solves a 3-shift problem. Feasible schedules satisfy the hard constraints, which are coverage constraints and personal requests for days off.

**Kawanaka and others** (2001) propose a genetic algorithm for scheduling nurses under various constrains that can be 'absolute' or 'desirable' constraints. One absolute is the minimum coverage per skill category and one absolute is, for example, the number of new nurses. It is better than other methods because it do not only implement the absolute constraints.

**Burke et al.** (2001a) developed a set of genetic and memetic algorithms. The coverage constraints are satisfied throughout the search space. By the recombination of two solutions it is impossible to create a feasible solution. This is the reason for what repair procedures have been developed. Thanks to evaluating a population of solutions instead of one single solution the new approach overcomes inappropriate choices for the planning order of skill categories.

## 3.4    CONCLUSIONS

In these pages we have discussed lots of nurse scheduling articles talking about these nurse rostering problems and we have been able to see how this problem has attracted the attention of scientists for about 40 years.

The automatic generation of high quality nurse schedules can lead to improvements in hospital resource efficiency, staff and patient safety and satisfaction and administrative workload. Researchers have constructed so many different models and they have developed many different techniques. However, very few of the developed approaches are suitable for directly solving difficult real problems because some of them are really simple. We can see in the next table the approaches that have been implemented in practice (in one hospital or in multiple) and the ones that have been tested in real world. The nurse rostering approaches, which do not address real problems and those, which are only concerned with modeling issues are not included in this table.

| Not applied in practice but tested on real data | Applied in practice |
|---|---|
| Abernathy et al. (1973) | **Approaches applied in just one hospital** |
| Berrada, Ferland, and Michelon (1996) | Easton, Rossin, and Borders (1992): staffing |
| Petrovic, Beddoe and Vanden Berghe (2003) | Jaszkiewicz (1997) |
| Cheng, Lee, and Wu (1996, 1997) | Smith and Wiggins (1977) |
| Okada and Okada (1988) and Okada (1992) | Bellanti et al. (2004) |
| Abdennadher and Schlenker (1999a, 1999b): INTERDIP | |
| de Vries (1987) | **Approaches applied in multiple hospitals** |
| Warner and Prawda (1972) and Trivedi and Warner (1976) | Warner, Keller, and Martel (1990): ANSOS, Warner (1976) |
| Miller, Pierskalla, and Rath (1976) | Jelinek and Kavois (1992): Medicus |
| Muslija, Gaertner, and Slany (2000) | Darmoni et al. (1995) Horoplan |
| Isken and Hancock (1991, 1998) and Isken (2004) | Meisels, Gudes, and Solotorevski (1997): EasyStaff |
| Jaumard, Semet, and Vovor (1998) | Meisels, Gudes, and Solotorevski (1996): TORANIT |
| Aickelin and Dowsland (2000) and Dowsland (1998) | Dowsland and Thompson (2000): CARE |
| Moz and Pato (2004) | Meyer auf'm Hofe (1997): ORBIS Dienstplan |
| Burke et al. (2001a, 2002, 2003) | Burke et al. (2001b, 2001, 1999), De Causmaecker and Vanden Berghe (2003): PLANE |

*Table1: Applicability of the approach. (Edmund K. Burke, Patrick De Causmaecker Greet Vanden Berghe, And Hendrik Van Landeghem "The State of the Art of Nurse Rostering")*

Table 1 clearly demonstrates that modern hybridized artificial intelligence and operations research techniques form the basis of the most successful real world implementations. There is a research in hybridizing exact methods with heuristic approaches. The current state of art is represented by interactive approaches, which incorporate problem specific methods and heuristics with powerful modern metaheuristics, constraint based approaches and other search problems.

Here we list some of the future search directions that represent promising paths for nurse scheduling research:

*-Multi-criteria Reasoning:* Nurse scheduling presents a lot of objectives and requirements (Burke, De Causmaecker, and Vanden Berghe, 2004). Many of these are conflicting and it is clear that there is so much to investigate in this area.

*-Flexibility and Dynamic Reasoning:* The personnel schedule usually has to be changed to deal with some circumstances like staff sickness or emergencies (Burke et al., 2001c; De Causmaecker and Vanden Berghe, 2003). There is an amount of promise in investigating confused methodologies as an attempt to address the dynamic nature of the problem.

*-Robustness:* It is clear that robustness has not played an important role in the scientific research literature. However, it is a really important area that must be taken into account more than before.

*-Ease of use:* It is an important feature in the uptake of a decision support system. Many of the algorithmic methods that we have discussed in this review require significant research expertise to employ.

*-Human/computer interaction:* In this article we have seen that most of the models are too simply to be applied in hospitals and that the measures of evaluating algorithmic approaches do not consider some issues that are really important in the real world. To apply the algorithms they have to interface with the administrators or nurses that will use them. This item needs to be more considerate by scientists.

*-Problem decomposition:* To make the work easier is important to decompose the large problems into small ones to treat them better and to solve problems quicker. It is an important thing to take into account.

*-Exploitation of Problem Specific Information:* The full range of constraints that are generated by real hospital problems has not been address enough in the scientific literature on nurse scheduling and it should be more investigated.

*-Hybridization:* It is clear that no one method is going to increase the uptake of nurse scheduling by its own. Progress will be made by drawing on the capabilities of a range of methods, approaches and research advances.

This list is only an indication of some of the directions that may lead to significant research advances in this area. It is clear that all the above cannot be tackled in isolation of each other.

In summary, we need to pay more attention to the issues that are important to modern hospital administrators to increase the uptake of nurse rostering research in the real world. We need to be less rigid when we are talking about algorithms.

# 4 CENTRALIZED NURSE SCHEDULING TO SIMULTANEOUSLY IMPROVE SCHEDULE COST AND NURSE SATISFACTION [39]

## 4.1 INTRODUCTION

A recent survey of registered nurses reveals that one-third of the nurses try to leave their position within a year due to many reasons such as high workload, stress, non-patient duties and the most important reason, the scheduling policies. In this case, a nurse scheduling model that assists managers in achieving more desirable schedules and reducing wage costs simultaneously is proposed.

Hospitals with decentralized nurse scheduling usually have to call to their nurses to work mandatory overtime. This contributes to less desirable schedules and additional costs for the hospital. By centralizing scheduling decisions across departments it is possible to reduce overtime. The idea of centralizing is interesting when duties are similar from one unit to the next so then it is possible to use cross-trained employees across multiple units (cross-utilization).

This case is based on real nurse data from two hospitals in the United States. The study shows how desirability of nurse scheduling improves by approximately 34%, reducing overtimes by 80% and costs by 11%.

## 4.2 DATA COLLECTION

Real data from two medium to large medical and surgical hospitals of the United States (Hospital 1 and Hospital 2) is used. They have 247 and 526 total beds, respectively. Each hospitals consists in several units, but this model is focused and designed for the general medical/surgical type units (Medical, Surgical, and Orthological-Neurological units) because they have similar nurse duties.

These hospitals have made an effort to standardize procedures across many of their units, so then minimal training is required for nurses to be utilized across the units.

The nurse types used at both hospitals are "Registered Nurses" (RN) and "Nurse Aides" (NA), who are paid hourly at a base rate determined by experience. There is also a premium added to all shifts except the easier to staff day shift.

|  | Beds | Nurses |
|---|---|---|
| Hospital 1 |  |  |
| Medical | 32 | 54 |
| Surgical | 30 | 43 |
| Ortho-Neuro | 34 | 46 |
|  |  |  |
| Hospital 2 |  |  |
| Medical | 23 | 38 |
| Surgical | 24 | 38 |
| Ortho-Neuro | 28 | 41 |

*Table 2: Unit Characteristics. (P. Daniel Wright, Stephen Mahar "Centralized nurse scheduling to simultaneously improve schedule cost and nurse satisfaction")*

## 4.3    CENTRALIZED NURSE SCHEDULING MODEL

This is a bi-criteria integer scheduling model with objectives for schedule cost and schedule desirability. The centralized scheduling model includes constraints that control the service levels on each unit based on nurse-to-patient ratios. It takes into account the RNs and NAs and it could be expanded if necessary to accommodate additional nurse types. This model is nonlinear, but is presented a linearization of it. The notation used in the model is outlined below followed by the model.

*Sets*
T       the set of weeks in the scheduling horizon
N      the set of all nurses
K      the set of all units
L       the set of all nurse types
NW    the set of nurses who don't want to work weekends
$N^{lj}$      the set of nurses of type l available for shift j
S       the set of all shifts
$SA^{i}$     the set of all shifts across all units that nurse i is available to work
$S^{it}$      the set of all shifts across all units that nurse i is available to work in week t
$WS^{it}$   the set of all weekend shifts across all units that nurse i is available to work in week t

*Subscripts*
i       nurse i
j       shift j
t       week t
k      unit k
l       nurse type l (RN or NA)

*Decision variables*

$x_{ijk}$      1 if nurse i works shift j at regular time wages on unit k, 0 otherwise

$y_{ijk}$      1 if nurse i works shift j at overtime wages on unit k, 0 otherwise

$z_{it}$      1 if nurse i works any weekend shifts during week t, 0 otherwise

$b_{jkl}$      the number of nurses of type l required for eight-hours shift j on unit k.

$g_{jhk}$      1 if h beds exceed the number of beds that nurses can handle according to the nurse-to-patient ratio for unit k on shift j, 0 otherwise

$\gamma_{ik}$      1 if nurse i is assigned to unit k, 0 otherwise

### *Parameters*

w      the number of weeks in the scheduling horizon

$c_{ijk}$      regular time wages if nurse i works shift j on unit k

$d_{ijk}$      overtime wages if nurse i works on shift j on unit k

$\bar{n}_i$      maximum number of shifts for each week for nurse i

$n_i$      minimum number of shifts for each week for nurse i

$\bar{p}_i$      upper limit on the number of units to which nurse i can be assigned

### *Parameters (con't)*

$a_{ijk}$      the undesirability that nurse i has for shift j on unit k ($0 \leq a_{ijk} \leq 1$, where 1      is the most undesirable)

$q_i$      upper limit on number of weekends worked by nurse i over the scheduling horizon

$r_i$      upper limit on the number of undesirable shifts assigned to nurse i

$f_i$      upper limit on the number of overtime shifts assigned to nurse i

$R_i$      the number of patients per nurse type l as determined by the nurse-to-patient ratio

$\lambda_{jk}$      mean patient arrival rate during shift j on unit k

$\mu_{jk}$      mean unit service rate during shift j on unit k

$P_h(\lambda_{jk}, \mu_{jk})$      probability of h occupied beds during shift j on unit k

$u_l$      maximum allowable single-period average proportion of time in violation of the nurse-to-patient ratio for nurse type l

$v_l$      maximum allowable aggregate period average proportion of time in violation of the nurse-to-patient ratio for nurse type l

$s_k$      number of beds (servers) on unit k

### *Model*

$$(P) \quad \text{Min} \sum_{i \in N} \sum_{j \in SA^i} \sum_{k \in K} \left( c_{ijk} x_{ijk} + d_{ijk} y_{ijk} \right) \tag{1}$$

$$\text{Min} \sum_{i \in N} \sum_{j \in SA^i} \sum_{k \in K} \left( a_{ijk} x_{ijk} + a_{ijk} y_{ijk} \right) + \sum_{i \in NW} \sum_{t \in T} z_{it} \tag{2}$$

$$st \sum_{i \in N^\delta} \left( x_{ijk} + y_{ijk} \right) b_{jkl}, \; j \in S, \; k \in K, \; l \in L \tag{3}$$

$$\underline{n_i} \leq \sum_{j \in S^{it}} \sum_{k \in K} x_{ijk} \leq \bar{n}_i, \; i \in N, \; t \in T \tag{4}$$

$$\sum_{j \in SA^i} \sum_{k \in K} a_{ijk} \left( x_{ijk} + y_{ijk} \right) \leq r_i, \; i \in N \tag{5}$$

$$\sum_{j \in SA^i} \sum_{k \in K} y_{ijk} \leq f_i, \; i \in N \tag{6}$$

$$\sum_{t \in T} z_{it} \leq q_i, \; i \in NW \tag{7}$$

$$\sum_{k \in K} \gamma_{ik} \leq p_i, \; i \in N \tag{8}$$

$$\sum_{k \in K} \left( x_{ijk} + y_{ijk} \right) + \sum_{k \in K} \left( x_{i(j+1)k} + y_{i(j+1)k} \right) + \sum_{k \in K} \left( x_{i(j+2)k} + y_{i(j+2)k} \right) \leq 1,$$
$$i \in N, \; j \in SA^i < |S| - 1 \tag{9}$$

$$\sum_{k \in K} \left( x_{ijk} + y_{ijk} \right) z_{it}, \; i \in NW, \; t \in T, \; j \in WS^{it} \tag{10}$$

$$\left( x_{ijk} + y_{ijk} \right) \leq \gamma_{ik}, \; i \in N, \; j \in SA^i, \; k \in K \tag{11}$$

$$x_{ijk} \in \{0,1\}, y_{ijk} \in \{0,1\}, 0 \geq a_{ijk} \geq 1, \; i \in N, \; j \in SA^i, \; k \in K \tag{12}$$

$$z_{it} \in \{0,1\}, \; i \in NW, \; t \in T \tag{13}$$

$$\gamma_{ik} \in \{0,1\}, \; i \in N, \; k \in K \tag{14}$$

$$b_{jkl} \geq 0 \text{ and integer}, \; j \in S, \; k \in K, \; l \in L \tag{15}$$

In the above formulation, shifts are numbered j=1, …, (3x7)w, where w is the number of weeks in the scheduling horizon. The regular time and overtime nurse wages are minimized in objective (1), and objective (2) minimizes the total number of undesirable regular time shifts, undesirable overtime shifts and undesirable weekend

shifts. Nurses that don't want to work on weekends (set NW) are prohibited from specifying those shifts as undesirable, to avoid to count them as double undesirable. Constraint set (3) ensures that there would be a sufficient number of nurses of each type in each shift and unit. Constraint set (4) limits the total number of regular time shifts for a nurse in one week. Constraints sets (5) - (7) enforce upper limits on undesirable regular or overtime shifts, the number of overtime shifts, and the number of weekends worked for each nurse, respectively. Constraint (8) limits the number of units in which a nurse can be cross-utilized. Constraint set (9) prohibits backward rotation for any nurse. Backwards rotation is defined as a nurse being scheduled for a shift that starts less than 24 h after the starting time of the previous shift worked. This can be controlled through the composition of sets $SA_i$ by not including those day or evening shifts in the $SA_i$ set for that nurse. Constraint set (10) forces $z_{it}$ to be 1 if nurse i is assigned to work any weekend shifts during week t. Constraints (11) force $\gamma_{ik}$ to be 1 if nurse i is assigned to any shifts on unit k so that the level of cross-utilization (from constraint set (8)) can be enforced. Constraints (12)–(15) enforce integrality and non-negativity conditions.

The centralized scheduling model (P) is difficult to solve because of its large problem size, and it is most dependent on the number of units scheduled.

## 4.4   SOLUTION METHODOLOGY

The main objectives of this model are the cost and the desirability of the schedule. Both are solved. First an optimal labor cost is obtained, and then the second objective about the desirability is solved with an additional constraint that is the solution of the first objective.

### 4.4.1   NURSE-TO-PATIENT RATIOS AND SERVICE LEVELS

To accommodate nurse-to-patient ratios and satisfy the minimum service levels, a manager could multiply the total number of beds on a unit by the minimum nurse-to-patient ratio and then staff to that level. The problem is that there is a daily and weekly variation in patient census patterns, so with this approach, some shifts are going to be overstaffed. A more efficient approach is to staff so that the probability of violating the minimum ratios is kept below a certain critical level, e.g. 5%. Nursing managers have several options for adding staff and avoid violating that minimum ratio, such as the use

of excess nurses from other units, on-call workers (Vaughan 2000), travel nurses, and outside agency nurses. The centralized scheduling model is perfect for using excess nurses from other units. Without centralization this cross-utilization is not possible.

Constraint set (16) limits the expected probability that the number of occupied beds exceeds the nurse-to-patient ratio on a unit during each shift for each nurse type. Constraint set (17) controls the upper limit on the aggregate proportion of time over the scheduling horizon that the nurse-to-patient ratio is violated in a unit for each nurse type. In this case, the single-period service level corresponds to each particular shift, while the aggregate service level corresponds to the 5-week scheduling horizon.

$$\sum_{h = R_l b_{jkl} + 1}^{s_k} P_h\left(\lambda_{jk}, \mu_{jk}\right) \leq u_l, \; j \in S, \; k \in K, \; l \in L \tag{16}$$

$$\left(\sum_{j \in S} \sum_{h = R_l b_{jkl} + 1}^{s_k} P_h(\lambda_{jk}, \mu_{jk})) / |S| \leq v_l, \; k \in K, \; l \in L\right) \tag{17}$$

Data on patient arrivals to the various hospital units was collected over a 90-day period. Using historical length of stay data and discussions with unit managers, service rates were estimated for each unit ($\mu$jk). The average service rate at the subject hospitals is 0.4 patients per shift. Let m denote an index ranging from 0 to sk. Based on Erlang's loss model, the probability of h occupied beds in the system during shift j on unit k is

$$P_h\left(\lambda_{jk}, \mu_{jk}\right) = \frac{\left(\lambda_{jk} / \mu_{jk}\right)^h / h!}{\sum_{m=0}^{s_k} \left(\lambda_{jk} / \mu_{jk}\right)^m / m!} \tag{18}$$

calculated as

where $s_k$ is the number of beds on unit k. This equation for $P_h(\lambda_{jk}, \mu_{jk})$ is used to calculate the service levels in constraint sets (16) and (17).

## 4.4.2   LINEARIZATION OF THE WORKLOAD MODEL CONSTRAINTS (19) AND (17)

Here a linearization of the workload model is presented so that the centralized nurse scheduling model can be solved as an integer program. To perform the linearization, we define a new decision variable, $g_{jhk}$, that is set to 1 when there are more beds occupied

than the number of beds (patients) that the nurses can handle based on the nurse-to-patient ratio. Constraints (16) and (17) are then modified.

$$1 - \frac{(b_{jkl}R_l)}{h} \leq g_{jhk}, \; j \in S, \; h = 1..s_k, \; k \in K, \; l \in L \qquad (19)$$

$$\sum_{h \in s_k} P_h\left(\lambda_{jk}, \mu_{jk}\right) g_{jhk} \leq u_l, \; j \in S, \; k \in K \qquad (16a)$$

$$\left(\sum_{j \in S^8} \sum_{h \in s_k} P_h\left(\lambda_{jk}, \mu_{jk}\right) g_{jhk}\right) / |S| \leq v_l, \; j \in S, \; k \in K \qquad (17a)$$

Once it is linearized it can be solved using CPLEX optimization software.

## 4.5  RESULTS

Computational results indicate that this centralized scheduling model outperforms decentralized scheduling by 10.7% for labor costs and 34% for undesirable shifts. The amount of overtime is reduced by 80%. The results are not depending on high levels of cross-utilization, it also works well with low levels of it.

By centralizing scheduling decisions hospitals can reduce costs and at the same time give a quality schedules to nurses with less amount of overtime.

### 4.5.1  SCHEDULE DESIRABILITY

Many hospital administrators are looking for ways to increase the job satisfaction of their nursing staff. By reducing the number of undesirable shifts and overtime, they improve the overall desirability of the schedule. Undesirable shifts are reduced as the number of units in which a nurse can work increases, but the efficiency sometimes is reduced too. Under decentralized scheduling a unit may be forced to schedule many overtime shifts and by centralizing decisions it is reduced. The centralized model is flexible to accommodate additional scheduling policies and nurse types that may exist at other hospitals.

# 5    NURSE SCHEDULING [25]

The costs related with the health care are rising every year in USA due to the new technologies or the increased demand of the health care. One of the aims of the healthcare systems is to provide high quality services at lower costs to patients. Operations Research techniques are useful to provide optimal or efficient schedules to improve aspects of healthcare systems such as medical supply chain or staff scheduling. Here we are going to discuss nurse scheduling.

There are some common rules related with the staff that can be applied to most departments of the hospital except for operating suits. They are the most important places in the hospital and they have different schedules. That is why we talk about Nurse Scheduling Problem in a General Clinic and in an Operating Suite.

## 5.1    NURSE SCHEDULING PROBLEMS (NSP)

### 5.1.1    NURSE SCHEDULING PROBLEM IN A GENERAL CLINIC

A general clinic is an area in a hospital that provides healthcare services to patients with needs. The patients can be divided in inpatients, the ones who are hospitalized overnight, and outpatients, the ones that do not need that. The nurses can be divided in different groups too. They can work for full time or part time, for example, but basically nurses are distinguished from each other by their area of specialty. If it is necessary, higher skilled nurses are assigned to shifts that lower skilled nurses are capable to perform, however the reverse is not possible.

In a general clinic there must be available nurses at all times responding to patient workload although they cannot work more hours that their scheduled hours.

### 5.1.2    NURSE SCHEDULING PROBLEM IN AN OPERATING SUITE

An operating suite is an area of the hospital which provides surgical procedures and consists of many operating rooms (OR). The nurses that work in an operating suite can have different roles some of the most important are *circulation* and *scrub*. Operating suites have some predefined shifts during a working day and all nurses are assigned to these shifts based on their contracts. Operating rooms have a lot of

variabilities that must be taken into account in a complicated model such as human behavior or nurse availability.

### 5.1.3    PROBLEM STATEMENT

Researchers might develop different nurse scheduling tools by finding proper answers for some questions related with the input parameters, the associated goals, the limitations and constraints or the proposed methods.

The problem in a general clinic is to develop a decision-making tool that assigns nurses to shifts based on nurse preference and patient workload requirements. In an operating suite is the same but assign nurses to surgery cases not to shifts.

## 5.2    A REVIEW OF OPTIMIZATION APPLICATIONS AND METHODS

Mathematical and heuristic approaches can provide efficient solutions to the scheduling problem. Several nurse scheduling models are introduced, decision variables and constraints are expressed and solution algorithms are discussed for each nurse scheduling problem.

### 5.2.1    NURSE SCHEDULING PROBLEM IN A GENERAL CLINIC
### 5.2.1.1    PRIMARY NURSE SCHEDULING PROBLEMS

This model was introduced by Warner and Prawda (1972), and it uses a mixed-integer quadratic programming formulation to calculate the nurses of a certain category to undertake a number of shifts per day (8 hour shifts).

*Optimization model*

$$\text{Min } C(U|R) = \sum_{i \in I} \sum_{n \in N} \sum_{t \in T} W_{int}\left(R_{int} - \sum_m Q_{imnt} U_{imnt}\right)$$

$$\sum_p U_{inpt} - X_{int} \leq 0, \qquad \forall i \in I, n \in N, t \in T$$

$$\sum_{i \in I} \sum_{t \in T} X_{int} \leq B_n, \qquad \forall n \in N$$

$$X_{int} \geq A_{int} R_{int}, \qquad \forall i \in I, n \in N, t \in T$$

$$X_{int} \leq R_{int} + e_{int}, \qquad \forall i \in I, n \in N, t \in T, e_{int} \geq 0$$

$$X_{int} \text{ integer}, \qquad \forall i \in I, n \in N, t \in T$$

$$U_{imnt} \geq 0, \qquad \forall i \in I, m, n \in N, t \in T$$

The constraints make sure that the total number of nurses with skill class *n* cannot exceed the total number of skill class *n* nurses assigned; they also ensure a

minimum amount of nursing care services of each skill class and limit the amount of substitution of nursing tasks among skill classes.

This early approach could not consider the needs of the hospital and there were no possibility to include personnel preferences in this model. An excess of nursing supply for a particular skill category could absorb the shortage of other skills.

### 5.2.1.2    SINGLE-OBJECTIVE NURSE SCHEDULING PROBLEMS

Minimizing costs and maximizing nurse preferences are two of the goals of nurse scheduling.

**-Cost as an objective function**

Bai et al. (2010) developed an optimization model to minimize the cost of assignment nurses to shifts patterns. This paper presents a hybrid algorithm for nurse rostering problem, which is flexible and easily adaptable to other constraints.

This model was motivated by a real problem in UK. The problem was to make weekly schedules for 30 nurses with day and night shifts and with three different grades. Some nurses had only a few hours per week and on certain shifts and the schedule had to satisfy the "day-off" requests by nurses.

*Optimization model*

$$\text{Min } f = \sum_{i=1}^{n} \sum_{j \in F_i} p_{ij} x_{ij}$$

$$\sum_{j \in F_i} x_{ij} = 1, \qquad \forall i \in \{1, \cdots, n\}$$

$$\sum_{i \in G_r} \sum_{j \in F_i} a_{jk} x_{ij} \geq R_{kr}, \qquad \forall k, r$$

The hybrid algorithm combines a genetic algorithm and a simulated annealing hyper heuristic (SAHH). A stochastic ranking method was used to improve the constraint handling capability of the genetic algorithm while a SAHH procedure was incorporated in order to locate local optima more efficiently. The stochastic method has demonstrated better performance.

**-Nurse preferences as an objective function**

Purnomo and Bard (2006) proposed a new integer programming model for cyclic preference scheduling and hourly workers. The aim is to generate a set of rosters that minimizes the number of uncovered shifts. The problem combines elements of both cyclic and preference approaches and includes five different shift types. The first three are 8 hours shifts non-overlapped, day (D), evening (E) and night (N). The other two are both of 12 hours called a.m. and p.m. The objective function aims to minimize the weighted sum of preference violations and the cost of covering gaps with outside nurses.

*Optimization model*

$$\theta_{ip} = \text{Min} \sum_{i \in N} \sum_{a=1}^{V_{\max}} r_a v_{ia} + \sum_{d \in D} \sum_{t \in T} M_t y_{dt},$$

$$\sum_{i \in N} x_{idt} - s_{dt} + y_{dt} = LD_{dt}, \qquad \forall d \in D, t \in T$$

$$\sum_{d \in D} x_{idt} \geq P_{it}, \qquad \forall i \in N_R, t \in T_i$$

$$\sum_{d \in D} \sum_{t \in T_i} h_t x_{idt} = H_i, \qquad \forall i \in N$$

$$\sum_{t \in T_i} x_{idt} \leq 1, \qquad \forall i \in N, d \in D$$

$$x_{idt_2} + x_{i,d+1,t_1} \leq 1, \qquad \forall i \in N_{BB}, d \in D$$

$$\sum_{l=d}^{d+D_i^{\max on}} \sum_{t \in T_i} x_{ilt} \leq D_i^{\max on}, \qquad \forall i \in N, d \in D$$

$$\sum_{d \in D_w} \sum_{t \in T_i} x_{idt} = W_i^{\max} w_{im}, \qquad \forall i \in N, m \in W$$

$$\sum_{m \in W} w_{im} = 1, \qquad \forall i \in N$$

$$\sum_{t \in T_i} x_{idt} + \left(1 - \sum_{t \in T_i} x_{i,d+1,t}\right) + \sum_{t \in T_i} x_{i,d+2,t} + p_{id} \geq 1, \qquad \forall i \in N, d \in D$$

$$\left(1 - \sum_{t \in T_i} x_{idt}\right) + \sum_{t \in T_i} x_{i,d+1,t} + \left(1 - \sum_{t \in T_i} x_{i,d+2,t}\right) + q_{id} \geq 1, \qquad \forall i \in N, d \in D$$

$$1 - x_{idt_\alpha} + 1 - x_{i,d+1,t_\beta} + b_{id} \geq 1, \qquad \forall d \in D, \alpha \neq \beta$$

$$\sum_{d \in D} b_{id} \leq TR_{\max}, \qquad \forall i \in N_R$$

$$\sum_{d \in D} (p_{id} + q_{id} + b_{id}) = \sum_{a=1}^{V_{\max}} a v_{ia}, \qquad \forall i \in N$$

$$\sum_{a=1}^{V_{\max}} v_{ia} \leq 1, \qquad \forall i \in N$$

$$0 \leq s_{dt} \leq UD_{dt} - LD_{dt}, 0 \leq y_{dt} \leq O_{dt}^{\max}, \qquad \forall t, d$$

$$b_{id}, p_{id}, q_{id} \geq 0, \forall i, t, d; \quad v_{ia} \in \{0,1\}, \forall i, a; \quad w_{im} \in \{0,1\}, \forall i, m$$

$$x_{idt} \in \{0,1\}, \forall i, t, d, \quad \text{where } x_{i,14+l,t} \equiv x_{ilt}, l = 1, \dots, D_i^{\max on}$$

A branch-and-price algorithm was developed for solving the problem. It was decomposed and two approaches were investigated to modify branching rules, one based on the master problem variables, which is better for a large number of nurses, and the other based on sub-problem variables, more efficient for a less number of them. It is important to say also that the variables are integer and not binary.

### 5.2.1.3　MULTI-OBJECTIVE NURSE SCHEDULING PROBLEMS

NSP is actually a multi-objective optimization problem (Burke et al. 2010; Maenhout and Vanhoucke 2010; Parr and Thompson 2007). Some of the goals may have conflicts of interests. Hadwan and Ayob (2010) proposed a constructive heuristic

algorithm based on the idea of generating the most required shift patterns to solve the nurse rostering problem in UKMMC, one University of Malaysia. The complexity of the solution search space was reduced by generating all the allowed two-day and three-day shift patterns to build up the roster. The objective function aimed to minimize the total penalty cost that occurs due to the violations of soft constraints.

*Optimization model*

$$\text{Min } w_1 \sum_{i \in I} (d1_i^+ + d1_i^-) + w_2 \sum_{i \in I} d2_i^+ \\ + w_3 \sum_{i \in I} \sum_{d \in D} d3_{id}^- + w_4 \sum_{i \in I} \sum_{d \in D} d4_{id}^- + w_5 \sum_{p \in P} d5_p^+$$

Subject to:

$$\sum_{i \in I} X_{id(1)} \geq R_{d(1)}, \quad \forall d \in D$$

$$\sum_{i \in I} X_{id(2)} \geq R_{d(2)}, \quad \forall d \in D$$

$$\sum_{i \in I} X_{id(3)} \geq R_{d(3)}, \quad \forall d \in D$$

$$\sum_{s \in S} X_{ids} = 1, \quad \forall i \in I, d \in D$$

$$\sum_{i \in I_s} X_{ids} \geq 1, \quad \forall d \in D, s = 1 \dots 3$$

$$X_{id(4)} + X_{i(d+1)(1)} + X_{i(d+1)(2)} + X_{i(d+1)(3)} + X_{i(d+2)(4)} \leq 2, \quad \forall i \in I, \\ d = 1 \dots |D| - 2$$

$$\sum_{d \in D} \sum_{s \in S-\{4\}} X_{ids} \leq 12, \quad \forall i \in I$$

$$\sum_{d \in D} \sum_{s \in S-\{4\}} X_{ids} \geq 10, \quad \forall i \in I$$

$$X_{id4} + X_{i(d+1)4} + X_{i(d+2)4} + X_{i(d+3)4} + X_{i(d+4)4} \geq 1, \quad \forall i \in I, \quad d = 1 \dots |D| - 4$$

$$\sum_{i \in I} Y_{pi} = 1, \quad p \in P$$

$$\sum_{p \in P} Y_{pi} \leq 1, \quad i \in I$$

$$\sum_{i \in I} Y_{pi} \left( \sum_{l=0}^{l=3} X_{i(d_p+l)(3)} + \sum_{l=4}^{l=5} X_{i(d_p+l)(4)} \right) = 6, \quad p = 1 \dots 9$$

$$\sum_{i \in I} Y_{pi}(X_{id_p3} + X_{i(d_p+1)3}) = 2, \quad p = 10, 11$$

$$\sum_{d \in D} X_{id4} \geq 2, \quad \forall i \in I$$

$$\sum_{d \in D} \sum_{s \in S-\{4\}} X_{ids} + (d1_i^+ - d1_i^-) = 11, \quad \forall i \in I$$

$$X_{i(6)(4)} + X_{(i)(7)(4)} + X_{i(13)(4)} + X_{i(14)(4)} + (d2_i^+ - d2_i^-) = 1, \quad \forall i \in I$$

$$X_{id(1)} + X_{i(d+1)(2)} + (d3_{id}^+ - d3_{id}^-) = 1, \quad \forall i \in I, \quad d = 1 \dots |D| - 1$$

$$X_{id(2)} + X_{i(d+1)(1)} + (d4_{id}^+ - d4_{id}^-) = 1, \quad \forall i \in I, \quad d = 1 \dots |D| - 1$$

$$\left( \sum_{i \in I} [Y_{pi} \times (X_{i(7)(2)} + X_{i(7)(4)})] \right) + (d5_p^+ - d5_p^-) = 1, \quad p \in \{1,2,3\}$$

$$\left( \sum_{i \in I} [Y_{pi} \times (X_{i(11)(2)} + X_{i(11)(4)})] \right) + (d5_p^+ - d5_p^-) = 1, \quad p \in \{4,5,6\}$$

A modified shift pattern approach was proposed, that was divided into three groups: (1) Initialize the problem by reducing search space and generating valid shifts sequence patterns; (2) Construct feasible initial solution; and (3) Optimize the initial solution that was constructed in the previous two stages to get the optimal solution.

## 5.2.1.4 CONSTRAINT-BASED AND HEURISTIC-ORIENTED NURSE SCHEDULING PROBLEMS

We describe a hybrid model of integer programming and variable neighborhood search (VNS) for highly constrained nurse scheduling problems developed by Burke et

al. (2010). They divided the constraint sets in a way that those with lower complexity and higher importance have more priority to be included in the sub-problem solved using IP.

*Optimization model*

$$\text{Min}\, G(x) = [g_1(x), g_2(x), g_3(x), g_4(x), g_5(x), g_6(x), g_7(x), g_8(x)]$$

Where:

$$g_1(x) = \sum_{i \in I} \sum_{j \in J} (s_{ij}^1 + s_{ij}^2), \qquad g_5(x) = \sum_{i \in I} \sum_{j=2}^{7|J|-1} \sum_{k' \in \{1,3\}} s_{ijk'}^6,$$

$$g_2(x) = \sum_{i \in I} \sum_{j=2}^{7|J|-1} s_{ij}^3, \qquad g_6(x) = \sum_{t=1}^{3} \sum_{i \in I_t} \sum_{w=1}^{|J|} (s_{tiw}^7 + s_{tiw}^8),$$

$$g_3(x) = \sum_{i \in I} \sum_{j=2}^{7|J|-1} s_{ij}^4, \qquad g_7(x) = \sum_{i \in I_1} \sum_{r=1}^{7|J|-3} s_{ir}^9,$$

$$g_4(x) = \sum_{i \in I} \sum_{r=1}^{7|J|-3} \sum_{k \in \{1,3\}} s_{ijk}^5, \qquad g_8(x) = \sum_{i \in I} \sum_{j=1}^{7|J|-1} \sum_{k' \in K'} s_{ijk'}^{10}.$$

Subject to:

$$\sum_{i \in I} x_{ijk} = d_{jk}, \qquad \forall j \in \{1, \ldots, 7|J|\}, k \in \mathcal{K}$$

$$\sum_{k \in K} x_{ijk} \leq 1, \qquad \forall i \in I, j \in \{1, \ldots, 7|J|\}$$

$$\sum_{j=1}^{7|J|} \sum_{k \in K} x_{ijk} \leq m_i, \qquad \forall i \in I$$

$$\sum_{j \in J} \sum_{k \in K} x_{ijk} \leq 3, \qquad \forall i \in I$$

$$\sum_{j=1}^{7|J|} x_{ij4} \leq 3, \qquad \forall i \in I$$

$$x_{i(j-1)4} - x_{ij4} + x_{i(j+1)4} \geq 0 \qquad \forall i \in I, j \in \{2, \ldots, 7|J| - 1\}$$

$$x_{i(j-1)4} - \sum_{k=1}^{3} x_{ijk} + \sum_{k=1}^{3} x_{i(j+1)k} \leq 1, \qquad \forall i \in I, j \in \{2, \ldots, 7|J| - 1\}$$

$$x_{i(j-1)4} + \sum_{k=1}^{3} x_{ijk} - \sum_{k=1}^{3} x_{i(j+1)k} \leq 1, \qquad \forall i \in I, j \in \{2, \ldots, 7|J| - 1\}$$

$$x_{i(j-1)4} + \sum_{k=1}^{3} x_{ijk} + \sum_{k=1}^{3} x_{i(j+1)k} \leq 2, \qquad \forall i \in I, j \in \{2, \ldots, 7|J| - 1\}$$

$$\sum_{j=r}^{r+n_1} x_{ij4} \leq n_1, \qquad \forall i \in I, r \in \{1, \ldots, 7|J| - n_1\}$$

$$\sum_{j=r}^{r+n_2} \sum_{k \in K} x_{ijk} \leq n_2, \qquad \forall i \in I, r \in \{1, \ldots, 7|J| - n_2\}$$

$$x_{16(j)3} = 0, \qquad \forall j \in \{1, \ldots, 7|J|\}$$

$$\sum_{k \in K} [x_{i(j-1)k} - x_{ijk}] + s_{ij}^1 - s_{ij}^2 = 0, \qquad \forall i \in I, j \in J$$

$$\sum_{k \in K} [x_{i(j-1)k} - x_{ijk}] + s_{ij}^1 - s_{ij}^2 = 0, \qquad \forall i \in I, j \in J$$

$$\sum_{k \in K} [x_{i(j-1)k} - x_{ijk} + x_{i(j+1)k}] + s_{ij}^3 \geq 0, \qquad \forall i \in I, j \in \{2, \ldots, 7|J| - 1\}$$

$$\sum_{k \in K} [x_{i(j-1)k} - x_{ijk} + x_{i(j+1)k}] - s_{ij}^4 \leq 1, \qquad \forall i \in I, j \in \{2, \ldots, 7|J| - 1\}$$

$$\sum_{j=r}^{r+3} x_{ijk} - s_{irk}^5 \leq c_k, \qquad \forall i \in I, r \in \{1, \ldots, 7|J| - 3\}, k \in \{1,3\}$$

$$x_{i(j-1)k} - x_{ijk} + x_{i(j+1)k} + s_{ijk}^6 \geq 0, \qquad \forall i \in I, j \in \{2, \ldots, 7|J| - 1\}, k \in \{1,3\}$$

$$\sum_{j=7w-6}^{7w} \sum_{k \in K} x_{iiw} - s_{tiw}^7 \leq g_t, \qquad \forall t \in \{1,2,3\}, i \in I_t, w \in \{1, \ldots, |J|\}$$

$$\sum_{j=7w-6}^{7w} \sum_{k \in K} x_{ijk} + s_{tiw}^8 \geq h_t, \qquad \forall t \in \{1,2,3\}, i \in I_t, w \in \{1, \ldots, |J|\}$$

$$\sum_{j=r}^{r+3} \sum_{k \in K} x_{ijk} - s_{ir}^9 \leq 3, \qquad \forall i \in I_1, r \in \{1, \ldots, 7|J| - 3\}$$

$$x_{ijk_1} + x_{i(j+1)k_2} - s_{ijk'}^{10} \leq 2, \qquad \forall i \in I, j \in \{1, \ldots, 7|J| - 1\}, k' = (k_1, k_2) \in K'$$

The advantages of IP and VNS are combined. First IP solves the sub-problem with a full set of high constraints and a subset of soft constraints, and then VNS improves the IP solution. This hybrid model was able to handle all the requirements of nurse rostering in a complex hospital environment.

## 5.2.2    NURSE SCHEDULING PROBLEM IN AN OPERATING SUITE

Mobasher and Lim (2011) based on the nurse scheduling problem in an operating suite by developing nurse scheduling optimization models for an actual operating suite in Texas, USA. They introduced a multi-objective integer programming nurse scheduling model in an operating suite which considers different aspects of the scheduling problem such as demand satisfaction, idle time and over time. The model called "Nurse Assignment Model", assigns nurses to different surgery cases based on their specialties and competency levels. Real data are gathered from a cancer center in Texas and used to show the efficiency of the optimization models and solution methods. The objective is to determine which nurses should be assigned to each surgery case. It is essential to provide schedules with minimum overtime, non-consecutive and minimum changes during surgery procedures as well as maximum demand satisfaction.

Because the model has multiple objectives a Solution Pool Method (SPM) appears. The solution pool feature generates and stores multiple solutions to the mixed integer programming (MIP) model for each deviation and then chooses the solution among these optimal solutions that have the smallest deviations. The solution pool-based method has three steps. In the first step, a single objective optimization model is developed for each deviation containing all hard constraints and soft constraints related to the current deviation. In the second step, the solution pool approach is applied to generate alternate good feasible solutions (with the absolute gap of less than 0.01) for each single objective model developed in Step 1. Finally, in Step 3, the comparison indexes can be compared and the best solution introduced for NAM. Numerical results indicated that this model can provide more efficient and reliable nurse schedules for operating suites.

# 6 INTEGRATING NURSE AND SURGERY SCHEDULING [30]

A simple and effective way to achieve considerable saving in staff costs is to integrate the operation rooms' scheduling process with the nurse scheduling process.

## 6.1 MODEL DESCRIPTION

### 6.1.1 GENERAL IDEA

During the nurse scheduling process we have to take into account the collective agreement requirements that are rules that define acceptable schedules for individual nurses, and the workload that depends of the master surgery schedule.

### 6.1.2 THE NURSE SCHEDULING PROBLEM (NSP)

The nurse scheduling problem (NSP) consists of generating a configuration of individual schedules over a given time horizon which have to fulfill collective agreement requirements and the hospital staffing demand coverage while minimizing the salary cost.

Here we present an integrated model that can be used to find optimal schedules for a homogeneous set of nurses.

Let *J* be the set of feasible roster lines *j* and *I* be the set of demand period *i*. Let $d_i \in R^+, \forall i \in I$ , denote the required number of nurses scheduled during period *i*. Furthermore, let $a_{ij}$ be 1 if roster line *j* contains an active shift during period *i* and 0 otherwise. The general integer decision variable $x_j$, $\forall j \in J$ , indicates the number of individual nurses which are scheduled by roster line *j*. Then, the nurse scheduling problem (NSP) can be stated as follows:

$$\text{Minimize} \sum_{j \in J} x_j \tag{1}$$

With these constraints:

$$\sum_{j \in J} a_{ij} x_j \geq d_i \qquad\qquad \forall i \in I \tag{2}$$

$$x_j \in \{0, 1, 2, \dots\} \qquad\qquad \forall j \in J \tag{3}$$

## 6.1.3   SOLUTION PROCEDURE FOR NURSE SCHEDULING PROBLEM

This problem is solved by first solving the linear programming relaxation and then using a branching scheme to bring the solution into integrality. Then column generation is often applied to solve the LP relaxation. Let $\pi_i, \forall i \in I$, denote the dual price of constraint (2). Then, the reduced cost of a new column (roster line) $j$ is given by:

$$1 - \sum_{i \in I} a_{ij} \pi_i \tag{4}$$

## 6.1.4   THE GENERALIZED NURSE SCHEDULING PROBLEM (GNSP)

The right hand sides values of the coverage constraints are consider to be fixed, but the coverage constraints are based on workload estimations, and that is determined by the patient type. The number and the type of patients are determined by the operation room schedule. This is taken into account in the generalized nurse scheduling problem, instead of assuming the demand values to be fixed.

Let $K$ denote the set of possible workload patterns that could be generated by modifying the surgery schedule. Each workload pattern $k$ is described by a number of periodic demands $d_{ik} \in \{0,1,2,...\}, \forall i \in I$. Let $z_k$ be 1 if the surgery schedule that corresponds to workload $k$ is chosen and 0 otherwise.

$$\text{Minimize} \sum_{j \in J} x_j \tag{5}$$

Subject to:

$$\sum_{j \in J} a_{ij} x_j \geq \sum_{k \in K} d_{ik} z_k \qquad \forall i \in I \tag{6}$$

$$\sum_{k \in K} z_k = 1 \tag{7}$$

$$x_j \in \{0,1,2,...\} \qquad \forall j \in J \tag{8}$$

$$z_k \in \{0,1\} \qquad \forall k \in K \tag{9}$$

The constraint (7) is called the workload convexity constraint and implies that only one workload pattern has to be chosen.

### 6.1.5  SOLUTION PROCEDURE FOR THE GENERALIZED NURSE SCHEDULING PROBLEM

The column generation approach used to solve the LP relaxation of NSP can easily be extended to cope with the GNSP. Here the possible workload patterns is too large, so the process start with a limited subset and new patterns are added as needed. So then a new master surgery schedule has to be constructed. Let $\gamma$ denote the dual price of the workload pattern convexity constrain (7). The reduced cost of the new workload pattern $k$ is given by:

$$0 - \gamma + \sum_{i \in I} \pi_i d_{ik} \tag{10}$$

## 6.2   PRICING PROBLEMS

### 6.2.1   GENERATING A NEW ROSTER LINE

There are several requirements that have to be satisfied when building a new roster line. One nurse cannot work more than one shift per day, and the overall number of active days cannot exceed certain limit. The maximum number of consecutive working days and consecutive resting days is constrained. The unpopular shifts are limited per roster line, and in a block (a sequence of working days), certain shift transitions are not allowed. To generate a new roster line is common to use a dynamic programming recursion.

### 6.2.2   GENERATING A NEW WORKLOAD PATTERN

As each workload pattern corresponds a particular surgeon schedule, if you build a new surgery schedule you are generating a new workload pattern. Here a new surgery schedule is built by solving an integer program. The objective function minimizes the dual price vector of the demand constraints (6) multiplied by the new demands. There are surgery constraints that determine how many blocks must be preserved for each surgeon, and capacity constraints that ensure that the number of blocks assigned for each period do not exceed the available capacity.

Let $y_{rt}$ ($\forall r \in R \; and \; t \in T$) be the number of blocks assigned to surgeon $r$ in period $t$. $T$ represents the set of active periods and $R$ the set of surgeons. Let $q_r$ be the number of blocks required by each surgeon $r$. Let $b_t$ be the maximal number of blocks

available in period $t$. Let $w_{rti} \in \Re^+$ denote the contribution to the workload of demand period $i$ of assigning one block to surgeon $r$ in period $t$. Then, the integer program to construct a new surgery schedule (and at the same time price out a new workload pattern $k$) is as follows:

$$\text{Minimize} \sum_{i \in I} \pi_i d_{ik} \tag{11}$$

Subject to:

$$\sum_{t \in T} y_{rt} = q_r \qquad\qquad \forall r \in R \tag{12}$$

$$\sum_{r \in R} y_{rt} \leq b_t \qquad\qquad \forall t \in T \tag{13}$$

$$\sum_{r \in R} \sum_{t \in T} w_{rti} y_{rt} \leq d_{ik} \qquad\qquad \forall i \in I \tag{14}$$

$$y_{rt} \in \{0, 1, 2, \ldots, \min(q_r, b_t)\} \qquad \forall r \in R \text{ and } \forall t \in T \tag{15}$$

$$d_{ik} \in \{0, 1, 2, \ldots\} \qquad\qquad \forall i \in I \tag{16}$$

The objective function (11) minimizes the reduced cost of a new workload pattern. Now, the periodic demands $d_{ik}$ are part of the decision process, instead of being only coefficients. With the constraint set (12) each surgeon obtains the number of required blocks, and with constraint set (13) we are sure that the number of blocks assigned to all surgeons, does not exceed the maximal number of blocks available for that period. Constraint set (4) triggers the $d_{ik}$s to the appropriate integer values and constraints (5) and (6) define $y_{rt}$ and $d_{ik}$ as integer values.

## 6.3   COMPUTATIONAL RESULTS

### 6.3.1   TEST SET

The surgery schedule is set in cycles of 7 days, and the last two days are not available to allocate OR time. There are different problems depending on 5 different factors: the number of time blocks per day; the number of surgeons; the division of requested blocks per surgeon; the number of operated patients per surgeon; and the length of stay (LOS) distribution. If we consider two different values for each factor and repeat the combination 3 times, we obtain $2^5 * 3 = 96$ test instances.

| Factor setting | Nr. blocks per day | Nr. surgeons | Division req. blocks | Nr. patients per surgeon | LOS |
|----------------|--------------------|--------------|----------------------|--------------------------|-----|
| 1 | 3-6 | 3-7 | evenly distributed | 3-5 | 2-5 |
| 2 | 7-12 | 8-15 | not evenly distributed | 3-12 | 2-12 |

*Table 3: Factor settings in surgery scheduling test set. (Jeroen BeliÄen, Erik Demeulemeester, "Integrating nurse and surgery scheduling")*

A block is defined as the smallest time unit for which a specific operating room can be allocated to a specific surgeon (or surgical group). In real-life applications the number of blocks per day in one operating room is usually 1 or 2 but here, considering more blocks can be seen as a way of considering more operating rooms, because there is no difference from a computational point of view. The third factor indicates whether or not the requested blocks are evenly distributed among all surgeons.

For the LOS in factor 5 we simulated exponential distributions (made discrete by use of binomial distributions) with mean dependent on the factor setting.

Next, we generated some weights $w_{rti}$ defining the contributions to the workload of period $i$ of allocating a block to surgeon $r$ in period $t$. These weights vary linearly with the number of patients of surgeon $r$ operated in period $t$ that are still in the hospital in period $i$. The patient's workload contribution generally decreases the longer the patient has already recovered in the hospital. In our test set the workload demand periods coincide with the shifts. Furthermore, we set the contribution to a "day" shift two times as large as the one to an "evening" shift and four times as large as the one to a "night" shift. Obviously, although attempting to represent realistic scenarios, these contributions are chosen somewhat arbitrarily.

Thirdly, we composed a set of collective agreement rules which apply on individual roster lines. The scheduling horizon amounted to 4 weeks or 28 days ($= n$). For each person, the maximum days an active shift could be scheduled ("day", "evening" or "night") was set to 20 ($= fmax$). Shifts during the weekends were marked as unpopular shifts. The maximum number of consecutive working days was set to 6 ($= h_1^{max} = h_{max}$) and the maximum number of consecutive rest days was set to 3 ($= h_2^{max}$

). Furthermore, we distinguished between two scenarios: a hard constrained scenario and a flexible one.

In the hard constrained scenario, there is only one shift type allowed within each block. In the flexible scenario, all shift transitions are allowed, except the backwards transitions. In the hard constrained scenario, the maximal penalty with respect to unpopular shifts is set to 4, whereas in the flexible scenario it is set to 8 ($=g_{max}$).

### 6.3.2   SAVINGS

Having a look at the upper bounds we can see that those for the GNSP are generally better than those for the NSP. The same results are obtained for the lower bounds. The GNSP lower bounds are guaranteed to be at least as good as the NSP lower bounds. It is not the same for the upper bounds. Looking at the table we can see that the average lower bound for the NSP is lower than the average upper bound for the GNSP in the flexible scenario, and that the average upper bound for the GNSP is lower than the average lower bound for the NSP in the hard constraint scenario.

The stricter the constraints are, the harder it is to fit the nurse rosters into the required workload pattern in the NSP. As the workload pattern can be adapted in the GNSP, it includes more possible savings in the case of severe collective agreement requirements.

| | | Flexible scenario | | | | Hard constrained scenario | | | |
| | | NSP | | GNSP | | NSP | | GNSP | |
| Nr. | Problem | lb | ub | lb | ub | lb | ub | lb | ub |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | d00000_0 | 15 | 17 | 13 | 15 | 21 | 22 | 14 | 14 |
| 2 | d00000_1 | 26 | 28 | 25 | 27 | 32 | 33 | 25 | 26 |
| 3 | d00000_2 | 25 | 27 | 23 | 25 | 32 | 33 | 24 | 26 |
| 4 | d00001_0 | 40 | 42 | 39 | 41 | 59 | 59 | 51 | 51 |
| 5 | d00001_1 | 45 | 47 | 44 | 46 | 65 | 66 | 56 | 57 |
| 6 | d00001_2 | 94 | 96 | 92 | 94 | 145 | 146 | 125 | 125 |
| 7 | d00010_0 | 34 | 36 | 32 | 35 | 36 | 37 | 33 | 33 |
| 8 | d00010_1 | 40 | 42 | 38 | 40 | 44 | 45 | 38 | 39 |
| 9 | d00010_2 | 28 | 30 | 26 | 27 | 33 | 34 | 26 | 27 |
| … | … | … | … | … | … | … | … | … | … |
| 88 | d11101_0 | 96 | 98 | 94 | 96 | 149 | 150 | 129 | 129 |
| 89 | d11101_1 | 99 | 102 | 97 | 99 | 140 | 141 | 115 | 116 |
| 90 | d11101_2 | 122 | 125 | 119 | 121 | 179 | 180 | 149 | 149 |
| 91 | d11110_0 | 83 | 85 | 80 | 82 | 104 | 105 | 80 | 81 |
| 92 | d11110_1 | 111 | 113 | 109 | 111 | 149 | 149 | 109 | 110 |
| 93 | d11110_2 | 58 | 60 | 56 | 58 | 76 | 77 | 56 | 56 |
| 94 | d11111_0 | 252 | 254 | 249 | 252 | 368 | 368 | 315 | 315 |
| 95 | d11111_1 | 119 | 122 | 116 | 119 | 175 | 175 | 124 | 125 |
| 96 | d11111_2 | 135 | 137 | 131 | 133 | 184 | 184 | 131 | 132 |
| | Average | 70.18 | 72.43 | 68.33 | 70.44 | 99.99 | 100.66 | 80.55 | 81.25 |

*Table 4: Lower and upper bounds for the NSP and the GNSP. (Jeroen BeliÄen, Erik Demeulemeester,"Integrating nurse and surgery scheduling")*

6.3.3   INTERPRETATION OF THE SAVINGS

In the obtained results we can see that integrating the surgery scheduling process with the nurse scheduling process may yield important savings in terms of required nurses to hire.

The origin of the waste is twofold. The first reason is an unfavorable workload pattern that contains many workload demands that slightly exceed the workforce of $x$ nurses but that are dramatically inferior to the workforce of $x+1$ nurses. It is called "Waste due to the workforce surplus per shift". The second reason is the inflexibility of the roster lines due to strict general agreement requirements. Because of this, no set of roster lines can be found that perfectly fit with the workload demand. This source of waste is further referred to as "Waste due to the inflexibility of roster lines".

There are three columns for each scenario. The first one is the total waste in terms of overstaffing in the NSP compared with the GNSP. These numbers are obtained by subtracting the upper bounds for the NSP from those for the GNSP. The second and third column indicate the parts of this total waste that are due to the "workforce surplus per shift" and to the "inflexibility of roster lines". These numbers can easily be calculated as follows. Firstly, the total required workforce is calculated by making the sum of the (integral) demands of the chosen workload pattern for both the NSP and the GNSP. Next, divide this number by the workforce per nurse and round up the result to the next integer. This gives the minimal number of nurses that would be needed and can be obtained in the case of fully flexible roster lines. The difference between these numbers for the NSP and GNSP is the waste due to the workforce surplus per shift. The difference between the total waste and the waste due to the workforce surplus per shift is the waste due to the inflexibility of roster lines.

The stricter the general agreement requirements are, the larger is the share of the waste due to the inflexibility of the roster lines.

| Nr. | Problem | Flexible scenario | | | Hard constrained scenario | | |
| | | Total waste | Waste due to workforce surplus per shift | Waste due to inflexibility of roster lines | Total waste | Waste due to workforce surplus per shift | Waste due to inflexibility of roster lines |
|---|---|---|---|---|---|---|---|
| 1 | d00000_0 | 2 | 1 | 1 | 2 | 1 | 1 |
| 2 | d00000_1 | 1 | 1 | 0 | 4 | 2 | 2 |
| 3 | d00000_2 | 1 | 2 | -1 | 3 | 1 | 2 |
| 4 | d00001_0 | 1 | 2 | -1 | 2 | 0 | 2 |
| 5 | d00001_1 | 2 | 1 | 1 | 1 | 0 | 1 |
| 6 | d00001_2 | 2 | 1 | 1 | 3 | 0 | 3 |
| 7 | d00010_0 | 1 | 2 | -1 | 3 | 1 | 2 |
| 8 | d00010_1 | 1 | 2 | -1 | 3 | 2 | 1 |
| 9 | d00010_2 | 1 | 2 | -1 | 2 | 0 | 2 |
| ... | ... | ... | ... | ... | ... | ... | ... |
| 88 | d11101_0 | 2 | 1 | 1 | 2 | 1 | 1 |
| 89 | d11101_1 | 2 | 2 | 0 | 4 | 0 | 4 |
| 90 | d11101_2 | 1 | 2 | -1 | 3 | 0 | 3 |
| 91 | d11110_0 | 2 | 2 | 0 | 6 | 1 | 5 |
| 92 | d11110_1 | 2 | 1 | 1 | 7 | 1 | 6 |
| 93 | d11110_2 | 1 | 2 | -1 | 6 | 1 | 5 |
| 94 | d11111_0 | 2 | 1 | 1 | 7 | 0 | 7 |
| 95 | d11111_1 | 2 | 2 | 0 | 4 | -1 | 5 |
| 96 | d11111_2 | 1 | 2 | -1 | 6 | 1 | 5 |
| | Average | 1.58 | 1.48 | 0.10 | 4.11 | 0.29 | 3.82 |

*Table 5: Interpretation of the savings. (Jeroen BeliÄen, Erik Demeulemeester,"Integrating nurse and surgery scheduling")*

## 6.4    CONCLUSIONS

We have conclude that considerable savings could be achieved by using this approach to build nurse and surgery schedules. We simulated problems for a large range of surgery scheduling instances and distinguished between a flexible and a hard constrained scenario with respect to the collective agreement requirements.

Obviously, in real-life hospital environments it is not so easy to modify the master surgery schedule. As the surgery schedule can be considered to be the main engine of the hospital, it not only has an impact on the workload distribution for nurses, but also on several other resources throughout the hospital.

# 7   CAPACITY PLANNING AND MANAGEMENT IN HOSPITALS [33]

## 7.1   INTRODUCTION

Since a few years ago hospitals have used medical and technical innovations successfully to develop more effective clinical treatments while reducing patients' time spent on them. However, they often have inefficiencies and delays and that produces problems like patients having to wait for beds, surgeries cancelled and rescheduled or inpatients placed in inappropriate beds and moved many times from one unit to another.

These problems are related to the regulatory and financing environment in which hospitals existed until recently. Until 1985 U.S. hospitals were paid by insurers on a "fee for service" basis and expansions were subsidized by state governments. With the increased prevalence of managed care and reduced government subsidies, hospital managers have been under pressure to cut costs and have undertaken large-scale changes to do it. Hospitals have been merged, downsized, and in many cases, closed.

It is really important for hospital managers to identify ways to "right-size" their facilities and use their resources more effectively.

### 7.1.1   CAPACITY PLANNING IN HOSPITALS: OVERVIEW

The most important measure of hospital capacity is the number of inpatient beds. Hospital bed capacity decisions have traditionally been made based on target occupancy levels - the average percentage of occupied beds- which usually has been 85%.

Until recently, the number of hospital beds was regulated in most states under the Certificate of Need (CON) process, under which hospitals could not be built or expanded without state review and approval, but nowadays this process have almost disappear. Although there has been a lot of literature talking about optimization models and simulation, target occupancy levels continue being the most fundamental way to measure the capacity. Due to the pressure of being more efficient, most hospitals have changed it from 85% to 90%.

Another relevant component of capacity is personnel, particularly nurses. They have a lot of impact on clinical outcomes and on hospital budget. Usually, a ratio of

patients to nurses is used to assign nurses to a unit, and it can changes from 8:1 to 1:1 in intensive care units, and it can be also exceed. Although there are many articles on the use of optimization models to determine nurse staffing, hospitals usually do not use basic data, like patient census time of day, and this must be taken into account in these models (Green, L.V. and J. Meissner (2002)).

Operating rooms are another component of capacity in hospitals as well as diagnostic equipment like magnetic resonance imaging devices (MRIs), because they are really important but very expensive. To avoid an excess of capacity and unnecessary usage, these purchases are regulated by the CON.

## 7.2   AN ILLUSTRATION OF THE ISSUES: EMERGENCY ROOM DELAYS

Many patients that arrive to an Emergency Department (ED) are "non-urgent" and are not harmed for waiting for a doctor. However there are other patients "emergent patients", needing an immediate care or "urgent patients", requiring care in a short period of time". It is not easy to classify people who go to an ED and this ends up with delays. The delays are also due to the telephone calls and the waiting of a patient for having a bed.

From a capacity planning perspective all the process from patient arrival in the ED to placement in a bed must be examined. The process starts with the triage nurse, who determines the acuity of the patient's condition. After that the patient is seen by an ED doctor and he or she asks for the necessary tests that the patient needs. When all of them are done and the doctor has the results the patient might go home or stay in hospital. If he or she has to be entered, a bed is requested in the appropriate nursing unit. The availability of a bed is affected not only by the capacity of the unit but also by the admission and scheduling policies of patients.

This description can show how complex is the hospital capacity planning and management. We can see the variety of both fixed capacity, like inpatients beds, and variable capacity, as nurses or physicians. In order to create a true emergency response system, capacity needs must be considered on a regional basis, and ambulance dispatch and diversion policies must be developed to assure timely access to care for the most urgent patients.

**7.3   HOW MANY HOSPITAL BEDS?**

As mentioned before, hospitals usually rely on target occupancy levels to plan and evaluate bed capacity. However, this process has some handicaps. First of all, occupancy levels are based on average "midnight census" which usually measures the lowest occupancy level of the day, because for example in the middle of the day the occupancy can be easily 20% higher than at midnight (LaPierre, S.D., D. Goldsman, R. Cochran, and J. Dubow (1999)). In addition, occupancy levels vary seasonally. For instance, data from Beth Israel Deaconess Hospital in Boston revealed that the average occupancy levels varied from about 68% in January to about 88% in July. Moreover, bed occupancy levels do not measure patients' delays for beds. Hospitals neither measure bed delays nor use simulation models to estimate delays that would happen from changes in demand.

7.3.1   TARGET OCCUPANCY LEVELS, BED DELAYS AND SIZE

Obstetrics is usually independent from other units so its capacity needs can be determined without taking into account other parts of the hospital. It is also one for which the use of a standard M/M/s queueing model is quite good. This model describes a system where arrivals form a single queue and are governed by a Poisson process, there are c servers and job service times are exponentially distributed. Most obstetrics patients are unscheduled and the assumption of Poisson arrivals has been shown to be a good one in some studies (Young, J.P. (1965)). In addition, the coefficient of variation (CV) of length of stay (LOS), which is defined as the ratio of the standard deviation to the mean, is typically very close to 1.0 satisfying the service time assumption of the M/M/s model (Green, L.V. and V. Nguyen (2001)). Obstetrics patients are considered emergent that is why the American College of Obstetrics and Gynecology (ACOG) recommends that occupancy levels of obstetrics units not exceed 75% (Freeman, R.K. and R.L. Poland (1992)). Many hospitals have them operating below this level.

Although there is no standard delay target, Schneider (Schneider, D. (1981)) suggested that the probability of delay for an obstetrics bed should not exceed 1%. Applying this criterion and using the Institutional Cost Report (ICR) data in an M/M/s model, we see that in 40% of the hospitals they have insufficient capacity. The main reason for this is size. From queueing theory, we know that larger service systems can

operate at higher utilization levels than smaller ones while attaining the same level of delays (Whitt, W. (1992)). Of the New York State hospitals represented in this data, more than 50% had maternity units with 25 or fewer beds. They need at least 67 beds to operate at a 75% occupancy level and probability of delay not exceeding 1%. Only 3 of the 148, or 2% of the New York hospitals represented in the 1997 ICR reports had units at least this large.

### 7.3.2   THE IMPACT OF CLINICAL ORGANIZATIONS

In most hospitals, beds are organized into nursing units. A nursing unit generally corresponds to a specific physical location with a dedicated nursing staff headed by a general nurse manager and it is used for one or more clinical services, with some exceptions. Hospitals usually have one or more intensive care units (ICUs). One important distinction of ICUs is that all beds have telemetry so that vital functions can be continually monitored.

Hospital managers are usually aware that higher occupancy levels can be achieved if beds are used more flexibly. Small clinical services are often combined with other services because of physical constraints and overhead considerations. For example, cardiac and thoracic surgery patients are often treated in a single unit because thoracic patients are few and require similar nursing skills as cardiac ones. From an operational point of view, is it always beneficial to combine clinical services?

It might be beneficial to operate the two services in one unit but employing a policy, such as a dynamic priority scheme, that would better balance the delays experienced by the two patient types. The degree of disparity of the patients has to be considered too.

### 7.3.3   THE SEVEN DAY HOSPITAL?

In most hospitals, elective procedures and diagnostic testing are different on weekends and therefore average bed occupancy levels are lower. Hospitals are starting to think about the benefits of a "seven-day hospital". They should need additional staff for that.

To show the possible impact of this we consider the case of a surgical intensive care unit whose patients are elective. With some data and solving some differential equations we see that 17 beds are needed to assure that the daily probability of delay is below 10%. Now if we assume that the same number of admissions is smoothed over the entire seven-day week, the M/M/s model indicates that only 15 beds are now needed to meet this target performance. If 15 beds are used but the demand is not smoothed over the week the average probability of delay over the week would be about 11% and on Fridays would be 18%. That might end in a backup of patients in recovery rooms and cancelled surgeries.

Therefore, optimal capacity could be determined by additional staffing costs and by weighing the expected income loss against the alternatives of expanded bed capacity.

## 7.4    STAFFING THE ED: HOW SHOULD LEVELS VARY ACROSS THE DAY?

### 7.4.1   USING QUEUEING MODELS TO DETERMINE PHYSICIAN STAFFING

In the busiest weekday of an ED in a mid-sized urban medical center they have about 0.9 arrivals per hour in the middle of the night and over 5 in the middle of the day. They did not have any data neither of the duration of the examinations nor patients' delay before seeing a physician, which was very long. This resulted in a high rate of "walkouts" - patients who leave after registering but before being seen by a physician - a matter of concern to the ED directors.

At the time of this study, a request for additional physician hours was under consideration by senior hospital officials. To determine the appropriateness of using queueing models to guide the allocation of any additional staffing, current performance was estimated by using the empirical demand data, the mean physician exam time (estimated to be 45 minutes) and the staffing levels. The realized demand for physicians was derived from the arrival data by adjusting for walkouts. The walkout rate was about 14.1% over the day. They adopted as their primary performance measure the probability of delay exceeding one hour, or $Pr(D > 1)$. The results, showing $Pr(D > 1)$ changing from 0.25 at 5 a.m. to 0.87 at 1 a.m., were considered by the ED managers as consistent with empirical observations.

Traditionally, in a service system with time-varying arrivals, the desired staffing levels would be determined by the Stationary Independent Period by Period or SIPP approach which starts by dividing the workday into planning periods, like shifts, hours… Then a series of stationary queueing models, most often M/M/s type models, are constructed, one for each planning period. Each of these period-specific models is independently solved for the minimum number of servers needed to meet the service target in that period. When the mean service times are high and planning periods are long the SIPP approach underestimate the number of servers needed to meet a given delay performance target. Is then when it is used the lag SIPP which correct the time lag between the peak in the arrival rate and the peak in the system congestion.

### 7.4.2   TRANSPORT STAFFING: ANOTHER POTENCIAL SOURCE OF DELAYS

One of the two most frequent times for ED overcrowding is from midnight to 2 a.m. However this is the time period when hospital occupancy levels are lowest. In one large New York hospital, a data collection showed that the time between bed assignment and patient leaving the ED peaked from an average of 2.1 hours to 3 or 4 hours between 12 a.m. and 4 a.m. This occurred because of three reasons.

First, the demand for transports peaked to about 8 patients per hour starting at midnight from a daytime average of about 7. However hospital managers had decided that transport staff should be reduced starting at midnight from two to one, because ED arrival rates are lower at that time. In addition, it was found that while the average transport during daytime hours was about 20 minutes, this increased to 27 minutes starting at midnight. Furthermore, most hospitals reduce other support staff at midnight.

This demonstrates the need to identify and analyze the impact of time-varying effects of both demands and processing times in the hospital in order to reduce ED overcrowding.

### 7.5   FUTURE RESEARCH OPPORTUNITIES AND CHALLENGES
### 7.5.1   CREATING FLEXIBILITY

As indicated above, patients often experience serious delays due to highly variable patient demands and capacity constraints. It is really important to use existing capacity more efficiently because hospitals are usually unable or reluctant to add

capacity because of stuffs like costs. One way of doing that could be increasing bed flexibility, but no comprehensive analysis has evaluated alternatives regarding bed flexibility. Two approaches that have been used in some hospitals are worthy of comprehensive analysis.

From a medical perspective, there may be benefits derived from having specialized nurses such as shorter LOS or fewer readmits. Yet, many hospitals believe that nurses can be successfully cross-trained and that the increase of bed flexibility can be reached by increasing speedy access and minimizing the number of bed transfers.

Another approach for increasing bed flexibility is the use of "overflow" units or "swing" beds, these are beds that are not normally staffed but used in case of a demand increase. These often exist in hospitals that have downsized by closing units. A related strategy is to use units that generally have more predictable demand and lower occupancy levels to serve as overflow units for those that frequently fill up.

The above strategies increase "horizontal" bed flexibility. Some hospitals have increased "vertical" bed flexibility by reducing the number of different areas in which certain categories of patients reside during their stay. For example, some maternity units have combined labor, delivery and recovery rooms.

OR-based analyses could help shed light on which of these alternatives is more attractive and under what conditions.

## 7.5.2   ALLOCATING CAPACITY AMONG COMPETING PATIENT GROUPS

In hospitals, there are usually three types of patients: inpatients, outpatients and emergency patients. They have differences but they all require the same set of resources including laboratories, imaging facilities and ORs. MRIs are expensive pieces of equipment and they are critical in diagnosing illnesses and significant delays are common in its use. Delays are compounded by late arrivals, cancellations and "no-shows". Operating rooms have very similar characteristics.

Research on operational policies for these types of shared resources could be very useful in increasing their efficiency and service performance.

### 7.5.3 REGIONAL CAPACITY PLANNING

In 1990's some hospitals, which were near geographically, were merged and they have some commitments to coordinate their planning and activities. First, these associations were done to enhance hospitals' bargaining power but in some cases their goal is to improve the delivery of health care.

It could be better to offer a particular clinical service in a single location, for example, obstetrics. Consolidating obstetrics units across two or more hospitals in a region would result in bed savings and in greater administrative and staffing efficiencies. However, patient travel distances and times must be considered too. OR-based analyses could be very helpful in identifying candidate services for regionalization.

Another dimension of regional planning is emergency responsiveness. Hospitals are coordinating efforts to respond to unanticipated spikes in demand for emergency department services and inpatient capacity. This has become more important since the events of September 11th, 2001. First, hospitals have focused on developing better communications and information systems between them. Little attention has been given to identifying which hospitals, clinical units and resources might be vulnerable given sudden unanticipated surges in demand within and across a given region. Emergency planning is a complex, multi-dimensional issue involving a high degree of unpredictability.

### 7.5.4 CONCLUSIONS

Effective capacity management is really important for optimize hospital resources and to assure that hospitals will survive and prosper. It must deal with complexities such as tradeoffs between bed flexibility and quality of care, demands from competing sources and types of patients, time-varying demands, and the differing perspectives of administrators, physicians, nurses and patients. These challenges affect the ability of hospital managers to control the cost and improve the quality of healthcare delivery. From an analytical perspective, these capacity management issues involve complex dynamics that will require the development of new optimization, queueing and simulation models in order to gain insights to guide strategies and decisions.

# 8    USING SIMULATION IN AN ACUTE-CARE HOSPITAL [33]

## 8.1    INTRODUCTION

Simulation has a huge range of application in health care. In this few pages we try to give an idea of the issues that arise when an operations research technique is applied to a health care setting.

These pages include: a study to evaluate the link between inpatient census and the surgical schedule; a study to evaluate the causes of, and solutions to, emergency room wait time in a pediatrics hospital and a generalized simulation model for an acute care emergency department.

## 8.2    EVALUATION THE IMPACT OF THE ELECTIVE SURGERY
##       SCHEDULE ON RESOURCE ALLOCATION

### 8.2.1    DESCRIPTION OF THE APPLICATION

Nursing, as a profession, has a number of unique characteristics that make human resource planning more difficult, for instance, most of the nurses are woman and they have other activities like child rearing. As in other professions, general economic conditions, quality of work-life issues, and random fluctuations in the labor market also appear.

In 1989, Toronto experienced a shortage of qualified nurses. Due to a good economy lots of nurses went from downtown to suburban institutions. For solving this problem, nursing leaders from five urban Toronto hospitals collaborated to discuss possible ways to attract and retain nurses in their institutions. Nurses complained about their salary and their excess of work on weekends. One interest idea was to change the surgical schedule to reduce the weekend ward census. A project was funded by a grant from the Ontario Ministry of Health and five Toronto teaching hospitals.

The study lasted for two years in 1991-93 and involved developing a simulation model to use as a decision support tool (Blake, J.T., M.W. Carter, L.L. O'Brien-Pallas, and L. McGillis-Hall (1995); Carter, M.W., L.L. O'Brien-Pallas, J.T. Blake, L. McGillis, and S. Zhu (1992)). The model included the operating rooms, the recovery room, intensive care units and regular inpatient wards.

In all of the hospitals in the study, operating room time was assigned on a "block booking" basis. Surgeons received blocks of operating room (OR) time (e.g., every Monday morning for three hours) and were free to schedule patients in any order within their assigned blocks. Usually, elective surgery took place Monday to Friday on the day shift with one or two rooms available nights and weekends for emergencies, so by changing the weekly OR schedule, we could influence in the rest of the hospital. It should be possible to determine a schedule that would be optimal from a staffing perspective. The schedule would not have any impact on patients, only a minor impact on surgeons who might have their block time inside the master surgical schedule.

With all these assumptions they built a simulation model, a database and user interface for the simulation. The model included all scheduled surgical patients and the database included a nursing workload model that estimated the total hours in each ward. If there were no beds available or not enough nursing hours when a patient was to be admitted, elective surgery would be canceled. The model generally used a first-come-first-served logic for allocating scarce resources.

They used a two-pronged approach to collecting data for the model. One was to spend several months in each hospital analyzing the process, creating charts and collecting data. The other was an existing database of discharge records, The Canadian Institute of Health Information (CIHI).

With that interface the user was able to set the surgical schedule, make adjustments to the surgeons' case mix, specify the number of beds and nurses on each ward and change a variety of control parameters.

## 8.2.2  CHALLENGES ENCOUNTERED

**Timing/project cycle time :**  The project was designed to be done in 2 years but actually it took more than 4 years. Moreover in 1993 the reality was much different from 1989. In 1989 was the high point of an economic cycle while in 1993 was a low point. When the program was ready the government was cutting health care budgets and lots of nurses were losing their jobs. However the project did not lose its value. The simulation allowed users to experiment with various allocations of OR time and forecast the impact of ward census, nursing workload, ICU beds and recovery rooms.

**Data collection :** In simulations the most important issues are data collection, verification and validation. Health care information systems are designed to follow clinical requirements not administrative needs.

The source data sent to CIHI was not always reliable for the institutions. This is not surprising because institutions themselves are responsible for abstracting and summarizing the data that is sent to the CIHI.

Finally, the lag between when data was collected, abstracted, and made available to CIHI meant that it was better to use patient abstracts that were at least a year old in the model.

**Every hospital was different :** This model was designed to be flexible but in practice it is too difficult to create a single, generic, general purpose patient simulation. Each institution had a unique combination of services, programs and quirks that made difficult to directly move a model from one location to another.

**Stakeholders :** In health care working is really important to have the acceptance of all stakeholders groups. The schedule rearrangement is not a minor issue. In practice, in most hospitals, the administration only allocates total OR time to each service and then doctors decide among themselves how to allocate specific blocks of time.

## 8.3   CHILDREN'S HOSPITAL OF EASTERN ONTARIO (CHEO)
### 8.3.1   DESCRIPTION OF THE APPLICATION

In 1993, the CHEO's Emergency Department expressed their concern that more than 20% of patients were forced to wait at least 2 hours before being seen by a physician. Some staff members sent to the Vice President of Ambulatory Care (VPAC) process improvements suggestions such as reforming patient flow or making changes to the physical layout of treatments rooms. It was necessary a mechanism to provide quantitative analysis of these options. Therefore, a simulation model was developed (Blake, J.T., M.W. Carter, and S. Richardson (1996)). It included all the processes in the ED, but the most important criteria were the average wait time and the distribution of these times for each of the four triage categories (emergency, urgent, deferrable and medical walk in).

In terms of modeling effort, the simulation itself was relatively simple. However, data collection, model validation, and output analysis required significant effort. This hospital had a highly fractured nature of work in the ED. The ED was staffed by one to three physicians, called Casualty Officers (COs), five to seven nurses, and some residents, as CHEO is a teaching hospital. Usually, each patient was seen by a nurse, a resident, and the CO who reviewed the resident's assessment. On any given shift there were ten patient treatment rooms available for use. Patients in these rooms were under the care of one CO who might also have had responsibility for providing medical education to one or two medical residents.

Moreover, it was not common for any worker to complete a work cycle on one patient from start to end without interruptions. A physician had five to ten patients at the same time so work cycles became fractured.

When the model was working and validated the experiment design started. The factors that were varied were the number of COs on shift, the number of residents on shift, and the queuing discipline used to select patients from the waiting list. It was seen a strong negative effect for the numbers of COs on shift and a strong positive effect for the number of residents on shift. The experiment showed that adding one additional CO or eliminating all residents would result approximately the same improvement in waiting time. Obviously that is not a solution but the results indicated that waiting time could be impacted by a number of different scenarios, including different numbers of physicians, different shift schedules, and/or the addition of a hospital "walk-in clinic" to treat patients with minor injuries.

In the experiment they prepared a plot of patient arrival times for each day of the week and they compared it to the CO's shift schedule and it showed that demand peak often occurred several hours before staffing peak. It was possible to make some improvements by staggering the doctors' start times. Other recommendations were adding an additional 4 hours of CO time daily to the main ED and implementing a fast-track clinic for low-acuity patients. That would reduce patient wait time by 20%. The process was carried out and that took over a year.

8.3.2   CHALLENGES ENCOUNTERED

**Data collection :** The fractured nature of work in the ED presented a data collection problem. There are no workload standards for physicians and manual data collection is a difficult task too. However, it was possible to satisfy the data requirements with a combination of statistical work sampling and job shadowing, this last done by some nurses.

In this process we saw that the length of time required for any particular task is extremely variable. When things get busy in the ED everyone tends to work faster and CO spend less time teaching. One way to avoid this was to use process times based on data collected during the busy times. As a result, simulated patients were treated faster than the real patients during relatively quiet times.

**Time frame :** At first the project was going to last two weeks but it finally took almost one year. Building the simulation model took only two weeks but the data collection was more time consuming.

Once the model was ended, hospital managers asked to run the model under different assumptions and scenarios. It made increasing the understanding of the process and resulted in a collaborative arrangement between management and modelers which extended the project completion date.

## 8.4   THE CROWDED STUDY: CAUSES AND RELATIONSHIPS OF OVERCROWDING AND WAITING IN DIFFERENT EMERGENCY DEPARTMENTS

8.4.1   DESCRIPTION OF THE APPLICATION

In US and in Europe, waiting times and overcrowding in ED have become a serious problem over the past several years. To understand how the ED functions it is necessary to develop a detailed process analysis specifically focusing on the impact of bed blockers.

In 2002, a team including operations researchers, ED physicians, a statistician and an epidemiologist received funding for a two-year study to analyze the detailed processes in ten Ontario hospital emergency departments. The Causes and Relationships

of Overcrowding and Waiting in Different Emergency Departments (CROWDED) study was designed to include detailed data to promote better allocation decisions for scarce resources such as doctors, nurses, and examination rooms. The hospitals were selected to represent a cross-section of geography and clientele. Two full time research assistants were hired for one year to collect data by directly observing patients, doctors and nurses. There was a pre-visit of 2-3 days to study the layout, understand the policies, meet people, and put up posters to educate and inform people about the study. Data collection was conducted in two separate one week periods at different times of year to get a sense of pattern changes over time. The project was designed to construct a generic model of an ED that can provide detailed decision support for a wide range of process flow issues.

### 8.4.2   CHALLENGES ENCOUNTERED

**Doctors are difficult to track :** Since doctors are probably the scarcest ED resource, it is important to determine accurate workload information for them. In the CHEO study, we chose to implement a work sampling method supplemented with a job shadow provided by a small group of nurses. In the CROWDED project, we had significantly more resources at our disposal, and we were determined to get very accurate workload information.

The observers needed to use indirect means of observation, like consulting the patient chart, the "white board" that keeps track of patient progress in the ED or having access to the hospital's electronic systems.   However, in both, it was found that recorded times did not usually reflect the actual time or duration of a process.

**Missing data :** It is too difficult to collect complete, accurate flow data on all ED patients. Some data was missing for approximately 10-15% of patients in study.

For instance, trauma cases have to start really fast and they are generally handled behind closed doors. So it was decided to forego direct observation of trauma cases.

Similarly, acute patients may also receive treatment according to medical directives behind closed doors. In these cases, the observers used the charts after the fact to determine which processes had occurred. This usually provided reasonable results in terms of what happened, but not always when it was done.

In addition to "closed door" treatments carried out by staff on trauma and acute patients, another challenge was that processes for many patients happen simultaneously. The research assistants were only able to observe the processes of one patient at any given time.

**Layout issues :** The layout sometimes created problems for data collection. Some EDs are really expanded and in that situation is difficult what is happening to a patient. Other EDs has some different areas and this segregation made tracking difficult for the observers.

**Fast-track clinic :** Some hospitals had an off-site "fast-track" clinic (FTC) separate from the ED. Again, this physical separation made it difficult to track patients.

**Wait time before triage:** None of the studied hospitals had data on time before triage. Some observers were looking to what happens on the waiting room and the results of the preliminary studies showed that patients frequently line up to be triaged but critically ones were not overly delayed.

**Unplanned critical events :** In any study, blind luck sometimes comes into play. In the CROWDED study some events happened. First, after three days of collecting data the observers discovered that a bug in the program blocked the transfer of all patient demographic information to the database. In addition, some public health issues appeared during the collection process, for instance one ED had to close for one week. But the worst happened in March 2003 when Toronto was hit with the SARS virus (Severe Acute Respiratory Syndrome). We had to pull the observers out of all study hospitals for almost two months. Things started to become normal after a period, but they have to extend the period of data collection.

**Administrative issues :** Many staff members at these hospitals though that the study would never be used to benefit health care. The research assistant tried to tell them that this study was funded by CIHR and that they wanted to find a way to improve patient flow without compromising patient care. Moreover in some hospitals, managers had no knowledge about the project and they needed to begin the sales pitch again.

## 8.5    CONCLUSIONS

Health care is a huge business which offers lots of applications for simulation and other operations research techniques (Carter, M.W. (2002)). Analysts and clinicians speak different languages and that is why operational research has made fewer inroads into this field. However, we have seen that OR techniques can be successfully applied in health care.

# 9  AVERSION TO HEALTH INEQUALITIES IN HEALTHCARE PRIORITIZATION: A MULTICRITERIA OPTIMIZATION PERSPECTIVE [2]

## 9.1  INTRODUCTION

How to take into account health inequalities is always an important issue in health economics. One difficulty in connecting theory to practice is that most health planners work with aggregate data and population averages, and do not have access to information about individual members of the population. However, welfare economic models construct their models of societal value by building upwards from an individual base.

In these few pages we talk about how to prioritize some information about values and aggregated information about population health thanks to Multi-Criteria Optimization (MCO). MCO deals with the formulation of and solution procedures for optimization problems where there are multiple conflicting objective functions which cannot be completely traded off against each other. Solving a MCO involves identifying all solutions that are efficient what means that they are optimal with respect to some aggregate objective function within a family of possible functions, rather than optimizing a single unique objective function.

MCO procedures side step the problem of explicitly parameterizing an objective function– the client for the analysis is presented with a number of "efficient", but possibly very different solutions, between which they can choose directly.

However, MCO has not yet been proposed in a health economics context.

In the next pages we are going to present a basic model of healthcare resource allocation, some key MCO concepts and an example based on prioritizing treatments for depression in England, ending up with a conclusion obviously.

## 9.2 THE MODEL

### 9.2.1 THE (HEALTH-RELATED) WELFARE ECONOMIC FRAME

A common approach in health economics is to assume that investment decisions are only value relevant if they have influence on the health population captured through a Health-Related Social Welfare Function or HR-SWF.

The main idea of HR-SWF is presented as follows. Let $\mathcal{N} = \{1, \ldots, i, \ldots, N\}$ be the index set for the members of the population. A general form for the HR-SWF is:

$$\sum_{j \in \mathcal{N}} w_j u(h_j) \qquad (1)$$

$h_j \in \mathbb{R}_+$: Variable that measures the lifetime health for person j.

$u : \mathbb{R}_+ \mapsto \mathbb{R}_+$ : Concave increasing function which captures the idea that the healthier someone is, the less valuable a marginal increase is.

$w_j$ : Scaling factor which reflects that the health of some people might be valued more than other people because of some characteristics.

If $w_j = w \forall j$, this HR-SWF is interpersonally anonymous which means that the same health benefit is valued the same when different people of the same level of health receive it.

This model has a particularly practical difficulty: it seems to necessitate measuring the health of every individual in a population and planning based on that individual level data. Clearly, this is unlikely to be possible. One possible solution is to work with a simpler function. For health improvements $\delta = (\delta_1, \ldots, \delta_y, \ldots, \delta_N)$ that are small in the sense that the first order Taylor series is a good approximation:

$$\sum_{j \in \mathcal{N}} w_j u(h_j) + \sum_{j \in \mathcal{N}} \frac{du(h_j^0)}{dh} w_j u(\delta_j) = \sum_{j \in \mathcal{N}} w_j u(h_j) + \sum_{j \in \mathcal{N}} \alpha_j u(\delta_j) \qquad (2)$$

Maximizing this equation is the same as maximizing the second term of it. $h^0 = (h_1^0, \ldots, h_j^0, \ldots, h_N^0)$ is required only for the derivatives.

Healthcare consumption occurs disproportionately late in life and healthcare investments decisions will usually be incremental, that is why we say that investments usually have a small impact on individual health.

Now, we consider a social planner who has to choose between possible healthcare investments alternatives $\mathcal{M} = \{1, \dots, i, \dots, M\}$ by choosing $x = \{x_1, \dots, x_i, \dots, x_M\}$ in some feasible set $X \subseteq [0,1]^M$. Where $x_i = 0$ means $i$ is unfunded and $x_i = 1$ $i$ is funded. $x_i$ is a set of health improvements $\delta_{ij}$ for each $j$. Social Planner's problem can be written as:

$$\max_{x} \sum_{j \in \mathcal{N}} \alpha_j \sum_{i \in \mathcal{M}} \delta_{ij} x_i \qquad where\ x \in X \qquad (3)$$

This last equation can be considered in two different ways: in the continuous version, where $x_i$ is between [0, 1] and in the discrete version where $x_i$ is {0,1}. Both situations can appear and also it can appear a combination of them.

We can reframe (3) in terms of aggregates instead of individuals. Let the index set of populations be $\mathcal{P} = \{1, \dots, k, \dots, P\}$ and write the index of sets of members of the subpopulations as $\mathcal{N}_1 = \{1, \dots, N_1\}$ ; $\mathcal{N}_2 = \{1, \dots, N_2\}$ , $\dots$ , $\mathcal{N}_P = \{N_{p-1} + 1, \dots, N_p\}$ for suitable numbers $0 < N_1 < N_2 < \dots N_p$. After that (3) is equivalent to:

$$\max_{x} \sum_{k \in \mathcal{P}} \sum_{j \in \mathcal{N}_k} \alpha_j \sum_{i \in \mathcal{M}} \delta_{ij} x_i \qquad where\ x \in X \qquad (4)$$

If all members j of subpopulation k have the same weight $A_k$ then (4) can be rewritten as:

$$\max_{x} \sum_{k \in \mathcal{P}} \sum_{j \in \mathcal{N}_k} A_k \sum_{i \in \mathcal{M}} \delta_{ij} x_i \qquad where\ x \in X \qquad (5)$$

This model is unlikely to be usable to support resource allocation in practical settings.

## 9.3   MULTICRITERIA OPTIMIZATION

MCO is a generalization of mathematical programming in which, rather than a unique objective function, there are multiple, possibly conflicting "criteria functions".

In the multi-criteria setting, one way to define the concept of efficiency is the next one:

Definition 1. A solution $x'$ is efficient if there is a monotonically increasing functional v($\cdot$). Where $v(W(x)) \leq v(W(x'))\forall x \in X$ , $W(x) = (W_1(x), \dots, W_P(x))$ and W(x)is the vector of the multiple criteria functions. This definition includes functional that model both inequality averse and inequality seeking preferences and we are only interested in the first ones.

Definition 2. A solution $x'$ is convex efficient if there is a set of weights $\{A_1, \dots, A_p\} \in A_0$ such that

$$\sum_{k \in P} A_k W_k(x) \leq \sum_{k \in P} A_k W_k(x') \quad \forall x \in X \qquad (6)$$

Solutions that are efficient but not convex efficient are called "unsupported" efficient solutions. If the feasible set is convex the set of efficient and convex solutions coincides, so there are no unsupported efficient solutions.

Definition 3. A solution $x'$ is A-efficient if there is a set of weights $\{A_1, \dots, A_p\} \in A$ such that

$$\sum_{k \in P} A_k W_k(x) \leq \sum_{k \in P} A_k W_k(x') \quad \forall x \in X \qquad (7)$$

In this definition a specific subset of the weight simplex, A, is used, and the previous one uses the whole of the weight simplex, $A_0$. Here there are no unsupported solutions.

## 9.4   EXAMPLE

Now we present a numerical example to illustrate the concepts developed above, it is based on a model of treatment for depression. Some characteristics of the depression is that it is very variable from some patients to others, its treating is expensive and a relative high population suffer it.

Depression has three different levels with three different parts in each one. It is shown in the next figure:
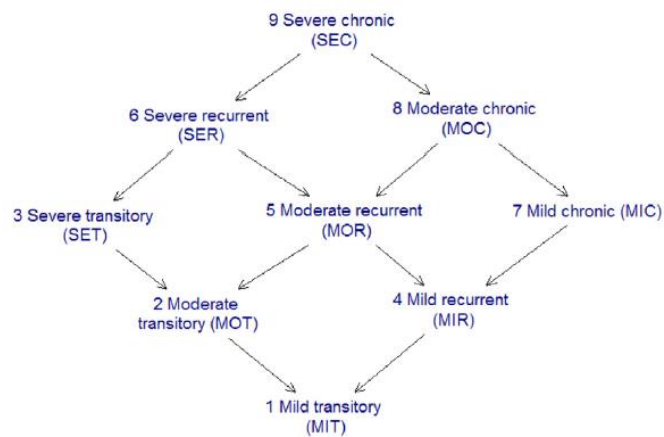
Figure 1.Ordering of depression subgroups.(Alec Morton, "Aversion to health inequalities in healthcare prioritization: A multi-criteria optimization perspective")

### 9.4.1 ANALYSIS OF CONTINUOUS VERSION

We are interested in finding A- efficient solutions to the MCO problem:

$$\max_{x} \sum_{i \in M_1} \Delta_i x_i , \dots , \sum_{i \in M_p} \Delta_i x_i \qquad (8)$$

such that

$$\sum_{i \in M} c_i x_i \leq B \qquad being\ B\ the\ budget \qquad (9)$$

$$\sum_{i \in M_k} x_i = 1 \ \forall\ k\ \in P \qquad (10)$$

$$x \in [0,1]^M \qquad (11)$$

In this formulation, $P = \{1, \dots, P\}$ with P = 9 and $M = \{1, \dots, M\}$ with M = 25 are the index sets for the population groups and alternatives, respectively. $M_1$ through $M_P$ is a partitioning of M into alternatives consumed by each of the population groups. Constraint (9) is the budget constraint; constraint (10) ensures that only one alternative is chosen for each population group and constraint (11) ensures the decision variables lie in [0, 1].

All A-efficient solutions to the continuous MCO resource allocation problem (8) can be generated by starting at one of the heads of the graph and proceeding down the graph selecting increments in such a way that no increment is selected before all the increments superordinate to it in the graph until funds are exhausted.

### 9.4.2   ANALYSIS OF DISCRETE VERSION

Now we have the same problem as before but changing the last constraint to discrete values:

$$\max_{x} \sum_{i \in M_1} \Delta_i x_i , \dots , \sum_{i \in M_p} \Delta_i x_i \qquad (12)$$

such that

$$\sum_{i \in M} c_i x_i \leq B \qquad being\ B\ the\ budget \qquad (13)$$

$$\sum_{i \in M_k} x_i = 1 \ \forall\ k\ \in P \qquad (14)$$

$$x\ \in\ \{0,1\}^M \qquad (15)$$

Solving combinatorial MCO problems is more difficult than solving linear MCO problems. To identify the A-efficient solutions for problem (12) we use a computer code which implements the algorithm described in Argyris et al. (2011). The idea of the algorithm is form a single combinatorial optimization problem with both choices of alternative x and weights $A_k$ variable, and solve multiple times, progressively cutting off solutions after they have been identified.

The results are similar to the results of the analysis of the continuous version of the problem.

### 9.5   CONCLUSIONS

Here we have outlined a formal model of the health-care resource allocation problem, drawing on the welfare economic theory of health. We have shown how the key uncertainties about values which make parameterizing such models problematic can

be easily handled with a multi-criteria formulation. Furthermore we have demonstrated how such formulations might be applied in practice using an example based on the allocation of funds for the treatment of various forms of depression.

## 10 A SINGLE AND TRIPLE-OBJECTIVE MATHEMATICAL PROGRAMMING MODELS FOR ASSIGNMENT OF SERVICES IN A HEALTHCARE INSTITUTION [54]

The assignment of service positions plays a really important role in healthcare institutions. Poorly assigned positions or over-employment may result in increased costs. In hospital departments, the supporting services include financial management, logistics, inventory management, analytic laboratories, etc. An application of operations research model for optimal supporting service jobs allocation in a public hospital is presented. Most of the real problems are formulated as multi-objective mathematical programming models which want to minimize operations costs of a supporting service subject to some constraints. The problem is formulated as a mixed integer program known as the assignment problem. The results of some computational experiments modeled after a real data from a selected Polish hospital are informed.

### 10.1 DATA USED FOR COMPUTATIONS

Data was gathered from a selected Polish public healthcare institution from one month period and they were used for computations. The data included 17 supporting service hospital departments in which there are 74 types of supporting service jobs. Permanent employment is defined as a percentage of permanent post between 25% and 100% according to the size of a job position (par time or full time) for a selected job in a selected department. For instance, it is possible to have two full time permanent employees or four part time permanent employees for two full permanent employments. Supporting service departments consists of 78.5 permanent employments with 192 employees before the optimization. Specific data consists of the average salaries for selected jobs in departments defined as costs of assignment of workers to jobs. Moreover, the average of money paid monthly for services in each department was used. Additional parameters include the number and size of permanent employments in each department for each job defined as partial or full time. Furthermore, the minimum number of permanent employments for each job in each department and the maximal number of positions which can be assigned to a single worker were given (Sawik, 2008b, 2010a, 2010b, 2012a, 2012b; Sawik and Mikulik, 2008a, 2008b).

| Supporting service departments | Number of workers | Number of jobs |
|---|---|---|
| Central Heating Department | 16 | 5 |
| Power Department | 15 | 3 |
| Medical Bottled Gases Department | 6 | 2 |
| Ventilation and Air-condition Department | 8 | 4 |
| Heating and Air-condition Department | 11 | 4 |
| Distribution Department | 6 | 3 |
| Medical Equipment Department | 8 | 4 |
| Technical Department | 11 | 5 |
| Economy Department | 21 | 5 |
| Hospital Pharmacy | 20 | 11 |
| Sterilisation Department | 27 | 5 |
| Material Monitoring Department | 13 | 5 |
| Information Department | 7 | 4 |
| Business Executive Department | 8 | 5 |
| Technical Executive Department | 4 | 4 |
| Law Regulation Department | 7 | 3 |
| Attorneys-at-Law Department | 4 | 2 |
| Number of workers in all department | 192 | 74 |

*Table 6: Number of workers and service jobs in the hospital departments before optimization. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)*

| Supporting service departments | Number of types of permanent employments | Amount of money paid for services [PLN] |
|---|---|---|
| Central Heating Department | 5 | 29,250 |
| Power Department | 3 | 31,050 |
| Medical Bottled Gases Department | 2 | 11,400 |
| Ventilation and Air-condition Department | 4 | 16,650 |
| Heating and Air-condition Department | 4 | 21,200 |
| Distribution Department | 3 | 13,600 |
| Medical Equipment Department | 4 | 17,500 |
| Technical Department | 5 | 20,950 |
| Economy Department | 5 | 31,360 |
| Hospital Pharmacy | 11 | 43,400 |
| Sterilisation Department | 5 | 41,500 |
| Material Monitoring Department | 5 | 27,150 |
| Information Department | 4 | 16,100 |
| Business Executive Department | 5 | 15,450 |
| Technical Executive Department | 4 | 7,150 |
| Law Regulation Department | 7 | 16,100 |
| Attorneys-at-Law Department | 2.5 | 7,950 |
| Money paid for services in all departments | 78.5 | 367,760 |

*Table 7: Number of peranent employments and the máximum amount of money paid for services in the hospital departments before optimization. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)*

## 10.2  PROBLEM FORMULATION

Mathematical programming approach deals with optimization problems of maximizing or minimizing a function of many variables subject to inequality and equality constraints and integrality restrictions on some or all of the variables. Variables can be, for instance, binary but, in this case, the model presented is defined as a mixed integer programming problem.

Suppose there are $m$ people and $p$ jobs, where $m \neq p$. Each job must be done by at least one person and each person can do at least one job. The cost of person $i$ doing job $k$ is $c_{ik}$. The objective is to assign people to jobs in the right way to minimize the total cost of completing all of the jobs.

The optimal criterion of the problem is to minimize operations costs of a supporting service subject to some specific constraints which represent specific conditions for resource allocation in a hospital. The overall problem is formulated as a modified assignment problem.

| | *Indices* |
|---|---|
| $i$ | Worker, $i \in I = \{1, \ldots, m\}$ |
| $j$ | Supporting service hospital department, $j \in J = \{1, \ldots, n\}$ |
| $k$ | Type of supporting service job, $k \in K = \{1, \ldots, q\}$ |
| | *Input parameters* |
| $c_{ik}$ | Cost of assignment of a worker $i$ to job $k$ (i.e., monthly salary) |
| $C_j$ | Maximal monthly budget for salaries in a department $j$ |
| $e_k$ | Size of permanent (partial or full time) employment for job $k$ (i.e., $e_k = 0.25$ or $0.50$ or $0.75$ or $1.00$) |
| $E_j$ | Maximal number of permanent employments in a department $j$ |
| $h_{jk}$ | Minimal number of permanent employments for job $k$ in a department $j$ |
| $\beta_i$ | Weight of the objective functions $f_i$, $i = 1, 2, 3$ |
| $\gamma$ | Small positive value |
| $f_1^{opt}$ | Ideal solution value of number of workers selected for an assignment to any job in any department |
| $f_2^{opt}$ | Ideal solution value of operational costs of the supporting services |
| $f_3^{opt}$ | Ideal solution value of number of permanent employments for all jobs in all departments |

*Table 8: Notations for mathematical models M1, M2,M3. (Sawik, B. (2013)*
*"A single and triple-objective mathematical programming models for assignment of*
*services in a healthcare institution".)*

## 10.3  OPTIMIZATION MODELS

The problem of optimal assignment is formulated as a single objective (M1,M2) or triple objective mixed integer program (M3), which allows commercially available software (e.g., AMPL/CPLEX; Fourer et al., 1990) to be applied for solving practical instances (Sawik, 2008b, 2010a, 2010b, 2012a, 2012b; Sawik and Mikulik, 2008a, 2008b).

*Variables*

| | |
|---|---|
| $x_{ijk}$ | 1 if worker $i$ is assigned to job $k$ in department $j$, 0 otherwise |
| $y_i$ | 1 if worker $i$ is assigned to any job in any department, 0 otherwise |
| $g_{jk}$ | Number of permanent employments for job $k$ in department $j$ |
| $z$ | Total number of workers assigned to any job in any department |
| $\delta$ | Deviation from the reference solution |

*Table 9: Variables for mathematical models M1, M2, M3. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)*

MODEL M1

The optimal criterion of the single objective mixed integer program M1 is to minimize operational costs of the supporting services.

Minimize:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} c_{ik} \cdot x_{ijk} \qquad (1)$$

Subject to:

$$\sum_{i=1}^{m}\sum_{k=1}^{q} c_{ik} \cdot x_{ijk} \leq C_j , j \in J \quad (2) \qquad \frac{\sum_{j=1}^{n}\sum_{k=1}^{q} x_{ijk}}{\sum_{j=1}^{n} E_j} \leq y_i \leq \sum_{j=1}^{n}\sum_{k=1}^{q} x_{ijk} , i \in I \quad (7)$$

$$\sum_{i=1}^{m} e_k \cdot x_{ijk} \leq E_j , j \in J \quad (3) \qquad x_{ijk} \in \{0,1\}, i \in I, j \in J, k \in K \quad (8)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{q} e_k \cdot x_{ijk} \leq 2 , i \in I \quad (4) \qquad y_i \in \{0,1\}, i \in I \quad (9)$$

$$\sum_{i=1}^{m} x_{ijk} \geq h_{jk}, j \in J, k \in K \quad (5) \qquad\qquad g_{jk} \geq 0, j \in J, k \in K \quad (10)$$

$$g_{jk} \geq h_{jk}, j \in J, k \in K \quad (6)$$

Constraint (2) ensures that the cost of workers assignment to service jobs in each department must be less than or equal to the maximum amount of money paid regularly for services in the department (monthly salaries).

Constraint (3) ensures that the total size of permanent employment (partial or full time) for each job (i.e., 0.25 or 0.50 or 0.75 or 1.00) in each department must be less than or equal to the maximal number of permanent employments in this department.

Constraint (4) ensures that each worker can be assigned to a maximum two full time positions in parallel.

Constraint (5) is responsible for an assignment of workers on at least minimal level requirements, e.g., the number of permanent employments on a selected service jobs.

Constraint (6) is responsible for obtaining only the results which will not lead to solutions without any assignment to some jobs. It compares real and minimal accepted number of permanent employments.

Constraint (7) ensures that worker $i$ is taken ($y_i = 1$) if he gets assignment to any job in any department ($x_{ijk} = 1$ for any $j$ and $k$). It defines the relation between binary decision variables $x_{ijk}$ and $y_i$.

Constraints (8) and (9) define binary decision variables $x_{ijk}$ and $y_i$.

Constraint (10) defines continuous decision variable $g_{jk}$.

MODEL M2

It is the same as Model 1 but replacing constraint 7 for constraint 12, they can be use alternatively.

$$\sum_{j=1}^{n} \sum_{k=1}^{q} x_{ijk} = y_i, i \in I \quad (12)$$

Constraint (12) ensures that each worker can be assigned to at most one job.

MODEL M3

The triple-objective optimization model can be formulated by reference point method. The optimal criteria (13) are to minimize total number of workers selected for an assignment to any job in any department and also to minimize operational costs of the supporting services and finally to minimize the number of permanent employments for all jobs in all departments.

$$\delta + \gamma \left( z + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} c_{ik} \cdot x_{ijk} + \sum_{j=1}^{n}\sum_{k=1}^{q} g_{jk} \right) \qquad (13)$$

Subject to (2) to (10) and

$$\beta_1 \left( z - f_1^{opt} \right) \leq \delta \qquad (14)$$

$$\sum_{i=1}^{m} y_i \leq z \quad (17)$$

$$\beta_2 \left( \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{q} c_{ik} \cdot x_{ijk} - f_2^{opt} \right) \leq \delta \qquad (15) \qquad z \geq 0, integer \ (18)$$

$$\beta_3 \left( \sum_{j=1}^{n}\sum_{k=1}^{q} g_{jk} - f_3^{opt} \right) \leq \delta \qquad (16) \qquad \delta \geq 0 \quad (19)$$

Constraints (14), (15) and (16) define the deviation from the reference solution.

Constraint (17) defines the relation between binary decision variables $y_i$ and $z$.

Constraints (18) and (19) define integer variable z and continuous variable $\delta$.

## 10.4  COMPUTATIONAL RESULTS

In this section, numerical examples and some computational results are presented to illustrate possible applications of the proposed formulations of integer programming of optimal assignment of service positions. Selected problem instances with the examples are modeled on a real data from a Polish hospital.

| Scenario | Operational costs [PLN] | Number of assigned workers | MIP simplex iteration | CPU* | Constraint |
|---|---|---|---|---|---|
| A | 153,251 | 77 | 2 | 10.49 | (7) |
| A | 153,251 | 77 | 0 | 12.25 | (12) |
| B | 209,751 | 108 | 3 | 12.46 | (7) |
| B | 209,751 | 108 | 0 | 12.08 | (12) |
| C | 248,951 | 131 | 3 | 9.17 | (7) |
| C | 248,951 | 131 | 0 | 12.80 | (12) |
| D | 311,651 | 166 | 4 | 7.47 | (7) |
| D | 311,651 | 166 | 0 | 7.47 | (12) |

Note: *CPU seconds for proving optimality on Pentium III, RAM 512MB/CPLEX v.9.1.

*Table 10: Comparison of computational results (models M1 and M2) with alternative constraints. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)*

| Scenario | $f_1^{opt}$ | $f_2^{opt}$ | $f_3^{opt}$ | All variables | Binary variables | Constraints |
|---|---|---|---|---|---|---|
| A | 70 | 150,000 | 75 | 4,448 | 3,188 | 566 |
| B | 110 | 200,000 | 105 | 4,416 | 3,156 | 526 |
| C | 130 | 250,000 | 120 | 4,416 | 3,156 | 526 |
| D | 160 | 300,000 | 155 | 4,404 | 3,144 | 512 |
| $\gamma = 0.01$ | | $\beta_1 = 0.33 \cdot 1000$ | $\beta_2 = 0.34$ | $\beta_3 = 0.33 \cdot 1000$ | | |

*Table 11: The values of parameters for computational experiments and the size of adjusted problema. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)*

| Scenario | $\delta$ | Number of workers | Operational costs [PLN] | Number of permanent employments | MIP simplex iterations | B-&-B nodes | CPU* seconds |
|---|---|---|---|---|---|---|---|
| A | 1,320.00 | 74 | 147,201 | 71.50 | 297 | 6 | 0.265 |
| B | 3,842.51 | 109 | 211,302 | 105.75 | 161 | 0 | 0.202 |
| C | 330.00 | 131 | 248,952 | 121.75 | 433 | 0 | 0.296 |
| D | 1,802.51 | 162 | 305,302 | 159.00 | 310 | 0 | 0.171 |

*Table 12: Comparison of computational results (model M·) with alternative scenarios. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)*

| Supporting service departments | Workers | Permanent employments | Workers | Permanent employments | Workers | Permanent employments | Workers | Permanent employments |
|---|---|---|---|---|---|---|---|---|
| Number of workers and permanent employments in all departments | 74 (A*) | 71.5 (A) | 109 (B*) | 105.75 (B) | 131 (C*) | 121.75 (C) | 162 (D*) | 159 (D) |
| Attorneys-at-Law Department | 2 (A) | 1.5 (A) | 3 (B) | 2.5 (B) | 3 (C) | 2 (C) | 3 (D) | 2.5 (D) |
| Law Regulation Department | 3 (A) | 3 (A) | 5 (B) | 5 (B) | 5 (C) | 4 (C) | 5 (D) | 5 (D) |
| Technical Executive Department | 4 (A) | 3.5 (A) | 4 (B) | 3.5 (B) | 4 (C) | 3.5 (C) | 4 (D) | 3.5 (D) |
| Business Executive Department | 5 (A) | 5 (A) | 6 (B) | 6 (B) | 6 (C) | 6 (C) | 7 (D) | 7 (D) |
| Information Department | 4 (A) | 3.5 (A) | 5 (B) | 4.5 (B) | 5 (C) | 4.5 (C) | 6 (D) | 5.5 (D) |
| Material Monitoring Department | 5 (A) | 5 (A) | 7 (B) | 7 (B) | 8 (C) | 8 (C) | 11 (D) | 11 (D) |
| Sterilisation Department | 5 (A) | 5 (A) | 8 (B) | 8 (B) | 14 (C) | 13.5 (C) | 21 (D) | 21 (D) |
| Hospital Pharmacy | 11 (A) | 10.5 (A) | 15 (B) | 14.5 (B) | 17 (C) | 14.5 (C) | 19 (D) | 18.5 (D) |
| Economy Department | 5 (A) | 5 (A) | 9 (B) | 9 (B) | 14 (C) | 13.5 (C) | 18 (D) | 18 (D) |
| Technical Department | 5 (A) | 5 (A) | 8 (B) | 8 (B) | 8 (C) | 7.5 (C) | 9 (D) | 9 (D) |
| Medical Equipment Department | 4 (A) | 4 (A) | 6 (B) | 5.75 (B) | 6 (C) | 5.75 (C) | 7 (D) | 6.5 (D) |
| Distribution Department | 3 (A) | 3 (A) | 5 (B) | 5 (B) | 5 (C) | 4.5 (C) | 5 (D) | 5 (D) |
| Heating and Air-condition Department | 4 (A) | 4 (A) | 5 (B) | 5 (B) | 7 (C) | 7 (C) | 9 (D) | 9 (D) |
| Ventilation and Air-condition Department | 4 (A) | 3 (A) | 6 (B) | 6 (B) | 6 (C) | 5.5 (C) | 7 (D) | 7 (D) |
| Medical Bottled Gases Department | 2 (A) | 2 (A) | 3 (B) | 3 (B) | 4 (C) | 3.5 (C) | 5 (D) | 5 (D) |
| Power Department | 3 (A) | 3 (A) | 5 (B) | 5 (B) | 8 (C) | 8 (C) | 12 (D) | 12 (D) |
| Central Heating Department | 5 (A) | 4.5 (A) | 9 (B) | 8 (B) | 11 (C) | 10.5 (C) | 14 (D) | 13.5 (D) |

Note: *Scenarios A, B, C and D considered subject to hospital authority requirements.

*Table 13: Assignment of workers in departments according to scenarios. (Sawik, B. (2013) "A single and triple-objective mathematical programming models for assignment of services in a healthcare institution".)*

As it has been recommended by the hospital manager's four different scenarios of the assignment have been implemented. In scenario A, a minimal number of people are employed in each supporting service department so that each type of a job has at least one worker assigned. This rule is implemented in input parameter $h_{jk}$. In scenario B at least two workers were assigned to each job. Scenario C secured the level of supporting service workers. In each department, there are at least two workers assigned to each job, but for some special cases, more than two workers are assigned to each job. Finally, scenario D presents the optimal assignment of workers to jobs with a high service level with all currently employed workers. The results obtained have indicated the problem of over-employment in the hospital.

## 10.5 CONCLUSIONS

This document shows how important and useful are the mathematical programming approaches to optimize the work on a hospital. With the results, it is shown that in almost all the departments the number of hired workers can be reduced. The proposed solution for the single and triple-objective assignments problems can be used in other institutions, not only in healthcare. The results consist of monthly expenses for salaries, the number of workers and the number of permanent employments in all considered departments. Computational time takes only a fraction of a second to find the optimal solution because of the small size of the input data.

## 11  OŚRODEK REHABILITACJI NARZĄDU RUCHU "KRZESZOWICE"

To appreciate the real problems of a hospital we have been in Ośrodek Rehabilitacji Narządu Ruchu "Krzeszowice". It is a hospital in Krzeszowice, a small village near Krakow. There we met a manager from the hospital and he explained to us some problems of it.

This institution has two main different parts: rehabilitation and neurology. They have a lot of treatments for rehabilitation in different places of the hospital. The employees that work at the hospital are doctors, nurses, psychologists and physical therapists. There are also people that work at the administration section and one of them is the person who does the schedules of the employees and the treatments of the patients by hand.

When one patient needs some treatments, the doctor has to prescribe them. And, after that, the person who deals with the scheduling has to find one time period for each patient.

### 11.1  AREAS OF THE HOSPITAL

In the pictures below, we can see some different areas of the hospital.

First of all, we can observe the outside of the hospital which now has one part temporary, it also has its own garden where the patients can rest and be in contact with the environment.



*Figure 2: Hospital of Krzeszowice temporary part (Tornos U. and Villegas R.)*



*Figure 3: Hospital's garden (Tornos U. and Villegas R.)*

Inside the hospital there are some rooms where the physical therapists can treat the patients and others where they can do exercises by themselves.



*Figure 4 Massage room (Tornos U. and Villegas R.)*



*Figure 5: Massage room (Tornos U. and Villegas R.)*



*Figure 6: Patient's Gym (Tornos U. and Villegas R.)*



*Figure 7: Patient's Gym (Tornos U. and Villegas R.)*

They offer treatments like magnetic fields applied to different parts of the body, massages, and hydrothermal therapy. It is possible to do rehabilitation inside of a swimming pool, but we could not take pictures of it.



*Figure 8: Magnetic field cabin (Tornos U. and Villegas R.)*



*Figure 9: Hydrothermal therapy (Tornos U. and Villegas R.)*

In the section of neurology there are some rooms dedicated to speech treatments and a multi-activities room where patients can cook, paint, read…

The hospital also has a place dedicated to investigation.



*Figure 10: Multi-activities room*
*(Tornos U. and Villegas R.)*



*Figure 11: Investigation room*
*(Tornos U. and Villegas R.)*

## 11.2  HOSPITAL'S PATIENTS

In this hospital, there are around 400 patients who are divided in two groups, the ones that are entered called inpatients and the ones that go to the hospital to receive the treatments called outpatients. They can receive the same treatments but there are some important differences between them. For instance, the waiting list is four years for an inpatient and one year and a half for an outpatient. Another important thing is that they do not pay in the same way, inpatients pay an amount of money for being in the hospital but outpatients have to pay for each treatment. That is why, from the management department, they tend to assign the most expensive treatments to outpatients.

## 11.3  ORGANIZATIONAL AND SCHEDULING PROBLEMS

One of the main problems is the way in which the hospital is distributed. As this is a rehabilitation hospital, there are a lot of patients that are disabled and they need the help of a nurse for going to one place to another to receive the treatment. Although it can surprise, this is not taken into account in the scheduling process. The person who works there assigns treatments to periods of time without any knowledge of the patient and this usually creates problems as patients arriving late to the treatments. When this happens some patients lose their citation and then the insurance does not pay to the hospital because the average number of treatments per day is not enough. This also could happen if one patient becomes ill and it is impossible for him to go to the different treatment rooms.

The same person that organizes the schedules often reduces the number of treatments of the patient when there is not enough time for all patients without having any medical knowledge.

Furthermore, it will be better if the scheduling system is done automatically instead of manually.

## 11.4  POSSIBLE SOLUTION

In order to reduce displacements in the hospital one possible solution could be to reorganize the different treatment rooms. By doing a research we will be able to know which treatments are usually done together and to make groups of them. In that case we will have the treatments of each group in the same room or in rooms that will be very close, and this will end with the problem of displacement of disabled people in the hospital. However, this solution is not so easy to carry out; there are some problems that make this difficult.

Basically there are four main problems. First of all, doctors refuse to choose between different groups of treatments for each patient, they want to make a list of them for each one personalized. The second problem is that the hospital is a place that must be operative all the time, it cannot be closed while a reorganization is been doing. Moreover the hospital has only one room that can be used for making changes provisionally. Thirdly, the distribution of the rooms has to be made under some rules. For instance, the rooms where they provide hydrothermal therapy must be in the ground floor and there are some restrictions about magnetic field machines such as the need of separate cabins. Finally and not less important than the other problems is that some patients could have the feeling that they are treated like objects and could be humiliate for them.

## 11.5  CONCLUSIONS

In this hospital we can see how it is a real problem and how difficult is to deal with it. It will be great for the hospital using an optimization model to find the best distribution of the rooms of the hospital and to improve the scheduling of employees and treatments.

## 12 SZPITAL SPECJALISTYCZNY IM. LUDWIKA RYDYGIERA

The data that we have used for doing the approaches of the model has been gathered from the hospital S*zpital Specjalistyczny im. Ludwika Rydygiera*, which is in Krakow. This hospital is a modern unit that offers medical care to a huge population. The 97.02% of all the admissions of the Hospital are patients from the Malopolska Region and in the last few years the admissions have reach the number of 28000-30000 patients per year.

It offers services in clinics and several surgical processes which are carried out by high qualified personnel. Some of the clinics that this hospital has are Neurological Clinic, Multiple Sclerosis Clinic, Dermatological Clinic, Clinic of Plastic Surgery, Otolaryngology Clinic, Radiotherapy Clinic, Urological Clinic, Clinic of Neonatology, etc…



*Figure 12: Hospital Ludwika Rydygiera (www.szpitalrydygier.pl)*

The hospital provides a lot of services. First of all, it has a lot of different departments, such as department of anesthesiology and intensive care, oncology, obstetrics or radiotherapy, among others.

Then, for having a diagnostic they have a laboratory, with a pathology sub-department. They offer several fields of investigation:

- General analysis
- Clinical chemistry

- Hematology and coagulation

- Immunochemistry

- Microbiology

- Toxicology

- Gynecological cytology

- Blood group serology

They also have some imaging machines which are basically radiology, mammography, ultrasound and computer tomography machines.



*Figure 13 and 14: Medical technology (www.szpitalrydygier.pl)*

The hospital also has a pharmacy were they provided you some basics medicaments.

Moreover, there is a conference room, where employees of the hospital can do seminaries and also people who work in other places can book a period of time for do that.

In this hospital, as in most of hospitals, patients can enter there because of an urgency surgery or because of an elective surgery.

# 13  BRANCH AND BOUND METHOD

There are some approaches that are used to find an optimal solution of the linear integer programming model. Many IP algorithms, as relaxation algorithms, focus on the dual step. It consists on doing an iteration and if an optimal solution is not obtained the relaxation is refined.

Another very important type of relaxation algorithms uses enumerative approach. They are based in the concept of divide. If is too difficult to optimize over one set, maybe the problem can be solved by optimizing over smaller ones and after that putting the results all together. The division of a set is called a partition of it. To do not have any problems it is better to avoid dividing the set in too many subsets. If we established that there are no more subsets needed   we can say that the enumerative approach tree can be pruned at that node or that the subset can be pruned.

The General Branch-and-Bound Algorithm has five important steps: initialization, termination test, problem selection and relaxation, pruning and division.

## 13.1  WEIGHTING AND LEXICOGRAPHIC APPROACH FOR LINEAR AND MIXED INTEGER MULTI-CRITERIA OPTIMIZATION MODELS FORMULATIONS

Mathematical programming approach deals with optimization problems of maximizing or minimizing a function of many variables subject to inequality and equality constraints and integrality (being, containing, or relating to one or more mathematical integers or relating to or concerned with mathematical integrals or integration) restrictions on some or all of the variables (Crescenzi and Kann, 2005; Merris, 2003; Nemhauser and Wolsey, 1999). In particular model equations consist of linear, integer and (representing binary choice) 0-1 variables. Therefore, the optimization models presented in this project are defined as mixed integer or linear programming problems.

The lexicographic optimization generates efficient solutions that can be found by sequential optimization with elimination of the dominating functions. The weighted objective functions also generate various efficient solutions. It provides a complete parametrization of the efficient set for multi-objective mixed integer programs.

An efficient solution to the multi-criteria optimization problem can be found by applying the weighting and lexicographic approach (Ehrgott, 2000; Sawik, 2007b, 2008d, 2009e, 2009g, 2010b, 2013a, 2013b; Steuer, 1986; Wiecek, 2007).

The non-dominated solution set of multi-objective mixed integer, linear or quadratic program models $M$ can be partially determined by the parametrization on $\lambda$ of the following weighted-sum program.

MODEL    $M_\lambda$

Maximization or minimization:        $\sum_{i=1}^{m} \lambda_i f_i$

subject to some specific model constraints, where

$$\lambda_1 > \lambda_2 > \dots > \lambda_m, \quad \lambda_1 + \lambda_2 + \dots + \lambda_m = 1$$

It is well known, however, that the non-dominated solution set of a multi-objective mixed integer or linear or quadratic program such as $M_\lambda$ cannot by fully determined even if the complete parametrization on $\lambda$ is attempted (e.g., Steuer, 1986). To compute unsupported non-dominated solutions, some upper bounds on the objective functions should be added to $M_\lambda$ (e.g., Alves and Climaco, 2007).

Considering the relative importance of the two or the three objective function the multi-criteria mixed integer or linear or quadratic program $M$ can be replaced with $M_\iota$, where $\iota \in 1,2$ in case of two objective functions or $\iota \in 1,2,3$ in case of three objectives, that could be solved subsequently.

MODEL    $M_\iota$ , $\iota = 1,2,3$

Maximization or minimization:          $f_\iota$

Subject to some specific model equations with additional constraints, in which upper or lower bounds are the optimal solution values of all objectives except the one with highest priority ($f_l$) - objective actually optimized:

$$f_l = f_\iota^* ; \quad l < \iota : \iota > 1$$

Where $f_\iota^*$ is the optimal solution value to the mixed integer or linear or quadratic program $M_\iota$ , $\iota = 1,2$ (considering three objective lexicographic problems).

## 14  WEIGHTED-SUM APPROACH TO ASSIGNMENT PERSONNEL IN A HOSPITAL

Some hospitals do not have a good personnel organization. They are organized but not in a good way. Sometimes they could use some computerized optimization models to try to improve it. In this case we are going to show a real case where an optimization model is applied to a real hospital of Poland. We will show what are the data collected from it, the model used and the results obtained as well as the conclusions deducted from them.

We use the weighted sum approach to solve the problem. The objective function is built by adding the goals of the problem, two in the case of bi-criteria and three in the case of triple objective, and multiplying them by a parameter $\beta$. This parameter can take values from 0 to 1, and the closer it is to 1 the more important is the objective related with it.

### 14.1  DATA USED FOR COMPUTATIONS

Data were gathered from S*zpital Specjalistyczny im. Ludwika Rydygiera*. The data include 20 supporting services hospital departments in which there are 88 supporting jobs. Permanent employment is defined as a percent of permanent post between 25% (0.25) to 100% (1.00) according to the size of a job position (part-time or full time) for a selected job in a selected department. Supporting service departments in the hospital consist in total of 214 permanent employments with 221 workers employed before the optimization. Moreover, the maximal amount of money paid monthly for services in each department was used.

Table 16 shows the number of types of supporting services position in departments, the number of permanent employments in departments, the number of employees in each department before optimization and the maximal amount of money monthly paid for services in each department.

| Supporting Services Hospital Departments | Number of types of supporting services position in department | Number of permanent jobs in department | Number of employees in department - before optimization | Maximal amount of money monthly paid for services in department |
|---|---|---|---|---|
| 1.Central Heating Department | 5 | 15.5 | 16 | 29250 |
| 2.Power Department | 3 | 15 | 15 | 31050 |
| 3.Medical Bottled Gases Department | 2 | 6 | 6 | 11400 |
| 4.Ventilation & Air-condition Department | 4 | 8 | 8 | 16650 |
| 5.Heating & Hydraulic Department | 4 | 11 | 11 | 21200 |
| 6.Distribution Department | 3 | 6 | 6 | 13600 |
| 7.Medical Equipment Department | 4 | 6.75 | 8 | 17500 |
| 8.Technical Department | 5 | 11 | 11 | 20950 |
| 9.Economy Department | 5 | 21 | 21 | 31360 |
| 10.Hospital Pharmacy | 11 | 19.5 | 20 | 43400 |
| 11.Sterilization Department | 5 | 27 | 27 | 41500 |
| 12.Stuff Monitoring Department | 5 | 13 | 13 | 27150 |
| 13.Information Department | 4 | 6.5 | 7 | 16100 |
| 14.Business Executive Department | 5 | 8 | 8 | 15450 |
| 15.Technical Executive Department | 4 | 3.5 | 4 | 7150 |
| 16.Law Regulation Department | 3 | 7 | 7 | 16100 |
| 17.Attorneys-at-law Department | 2 | 2,5 | 4 | 7950 |
| 18.Hospital Management Cost Section | 5 | 9 | 9 | 15550 |
| 19.Salary Section | 5 | 6,75 | 9 | 15800 |
| 20.Accounting Section | 4 | 11 | 11 | 26950 |
| TOTAL | 88 | 214 | 221 | 426060 |

*Table 15.Input data used in the model (from Hospital Ludwika Rydygiera)*

## 14.2  FORMULATION OF THE PROBLEM

The model presented in this project is defined as a mixed integer programming problem. Based on the models developed by Bartosz Sawik (Sawik B., 2013c, 2013d). Suppose there are $m$ people and $p$ jobs, where $m \neq p$, and $n$ departments. Each job must be done by at least one person; also, each person can do at least, one job. The cost of person $i$ doing job $k$ is $c_{ik}$. We want to minimize the cost of assignment of people among some jobs and maximize the number of employees and permanent jobs. The objectives are subject to some different constraints which represents conditions for the allocation of personnel in the hospital.

**Indices**

| | |
|---|---|
| i | Worker $i \, \epsilon \, M = \{1 \dots m\}$ |
| j | Supporting service hospital department $j \, \epsilon \, N = \{1 \dots n\}$ |
| k | Type of supporting service job $k \, \epsilon \, P = \{1 \dots p\}$ |

**Input parameters**

| | |
|---|---|
| $d_j$ | Minimal labor cost in department $j$. |
| $c_{ik}$ | Cost of assignment of a worker $i$ to job $k$ (monthly salary) |
| $C_j$ | Maximal monthly budget for salaries in a department $j$ |
| $CB$ | Maximal total budget |
| $f_k$ | Minimal size of permanent employment for job $k$ |
| $e_{ik}$ | Size of permanent employment for job $k$ and employee $i$. |
| $E_j$ | Maximal number of permanent employments in a department $j$ |
| $EP$ | Maximal number of permanent employments. |
| $W$ | Maximal number of employees |
| $h_{jk}$ | Maximal number of permanent employments in department $j$ and job $k$ |
| $\beta_i$ | Weight of the objective functions where $i = 1,2,3$. |

*Table 16.Indices and input parameters used in the optimization model*

## 14.3  OPTIMIZATION MODEL

The problems of assignment are going to be handled in two different ways. Firstly, we solve the problem as a bi-objective mixed integer program and using the weighted-sum approach, MODEL 1, and right after we use a triple-objective model with the same approach, MODEL 2. For solving that we have used the CPLEX 9.1 computer

program, and all the results have been obtained thanks to the aid of Professor Bartosz Sawik PhD.

| **Decision variables** | |
| --- | --- |
| $x_{ijk}$ | 1 if worker $i$ is assigned to job $k$ in department $j$, 0 otherwise. |
| $y_i$ | 1 if worker $i$ is assigned to any job in any department, 0 otherwise. |
| $g_k$ | Number of permanent employments for a job $k$. |

*Table 17.Decision variables used in the optimization model*

### 14.3.1 MODEL 1.BI-OBJECTIVE MODEL

This is a bi-objective model which tries to optimize the allocation of personnel of a Polish hospital. As the weighted-sum approach is the one that we use, we are going to have two different weights for both objectives.

The objective is to minimize operational costs of supporting services and maximizing the total number of employees:

Maximize:

$$\beta_1 \cdot \sum_{i \in M} y_i - \beta_2 \cdot \sum_{i \in M} \sum_{j \in N} \sum_{k \in P} c_{ik} \, x_{ijk} \qquad (1)$$

It is important to take into account that the objectives are in different scales so to compare them it is necessary to multiply the value of $\beta_1$ to 1000. In that way the values of both objectives are going to be in the same range.

Subject to

$$\sum_{i \in M} \sum_{k \in P} c_{ik} \, x_{ijk} \le C_j \qquad \forall j \in N \qquad (2)$$

$$\sum_{i \in M} \sum_{k \in P} e_{ik} \, x_{ijk} \le E_j \qquad \forall j \in N \qquad (3)$$

$$\sum_{i \in M} \sum_{j \in N} \sum_{k \in P} d_j \, x_{ijk} \le CB \qquad \forall i \in M, \forall j \in N, \forall k \in P \qquad (4)$$

$$\sum_{i \in M} \sum_{j \in N} \sum_{k \in P} f_k \, x_{ijk} \le EP \qquad \forall i \in M, \forall j \in N, \forall k \in P \qquad (5)$$

$$\sum_{i \in M} x_{ijk} \leq h_{jk} \qquad \forall j \in N, \forall k \in P \qquad (6)$$

$$\sum_{j \in N} \sum_{k \in P} x_{ijk} \geq y_i \qquad \forall i \in M \qquad (7)$$

$$x_{ijk} \in \{0,1\} \qquad \forall i \in M, \forall j \in N, \forall k \in P \qquad (8)$$

$$y_i \in \{0,1\} \qquad \forall i \in M \qquad (9)$$

Constraint (2) says that the total cost of assignment of all the employees of a department $j$ must be less than or equal the maximal monthly budget for salaries in that department.

Constraint (3) ensures that the total number of permanent employments in department $j$ must be less than or equal the maximum number of permanent employments in that department.

Constraint (4) assures that the minimal labor cost of all the departments is less than or equal the maximal total budget.

Constraint (5) guarantees that the minimal number of permanent employments is less than or equal the given maximum number of permanent employments.

Constraint (6) assumes that the assignment of workers to selected permanent jobs type is not more than maximal requirements.

Constraint (7) shows the relation of both variables $x_{ijk}$ and $y_i$ which assures that at least one employee must be working in one job of one department of the hospital.

Constraint (8) and (9) assures that $x_{ijk}$ and $y_i$ are binary decision variables.

### 14.3.2  MODEL 2.TRIPLE-OBJECTIVE MODEL

The second model is a triple objective mixed integer program. As the weighted-sum approach is the one that we use, we are going to have three different weights for both objectives.

The objective is to minimize operational costs of supporting services and maximizing the total number of employees and maximizing the total number of permanent employments:

Maximize:

$$\beta_1 \cdot \sum_{i \in M} y_i - \beta_2 \cdot \sum_{i \in M} \sum_{j \in N} \sum_{k \in P} c_{ik}\, x_{ijk} + \beta_3 \cdot \sum_{k \in P} g_k \qquad (10)$$

For this model we are not going to rewrite all the constraints, we are only going to write the ones that are different from MODEL 1.

All the constraints of MODEL 1 are used in this model too. Now we add another three constraints for this triple-objective model:

$$g_k \leq h_{jk} \qquad\qquad \forall j \in N, \forall k \in P \qquad (11)$$

$$\sum_{i \in M} \sum_{j \in N} \frac{x_{ijk}}{f_k} \geq g_k \qquad \forall i \in M, \forall j \in N, \forall k \in P \qquad (12)$$

$$g_k \geq 0, \forall k \in P \qquad (13)$$

Constraint (11) assures that the number of permanent employment of a job $k$ is less than or equal to the maximal number of permanent employments of that job of department $j$.

Constraint (12) shows the relation between $x_{ijk}$ and $g_k$.

Constraint (13) defines $g_k$ as an integer value greater than 0.

## 14.4 COMPUTATIONAL RESULTS

14.4.1 MODEL 1

First of all we solve the bi-objective optimization model for which we are going to change the values of the weights of the objective function.

We are going to have 9 different cases where we obtain these values:

| $\beta_1$ | $\beta_2$ | Total number of employees | Cost of assignment (PLN) |
|---|---|---|---|
| 0 | 1 | 0 | 0 |
| 100 | 0,9 | 145 | 279500 |
| 200 | 0,8 | 161 | 310400 |
| 300 | 0,7 | 165 | 318100 |
| 500 | 0,5 | 170 | 327700 |
| 700 | 0,3 | 177 | 341200 |
| 800 | 0,2 | 183 | 352800 |
| 900 | 0,1 | 207 | 399070 |
| 1000 | 0 | 220 | 424310 |

*Table 18.Results obtained in the bi-objective model*

In table 18 we can see how the number of employees and the cost of assignment increase with $\beta_1$. We can also notice that when $\beta_1$ is 0, we only have one objective function, the cost assignment function, and therefore the number of workers hired is zero because there is not any constraint to limit the number of workers. On the opposite side, when $\beta_2$ is 0 we are maximizing the total number of workers so we obtain that only one worker is going to be fired.

With all these results we draw some graphics to see easily what is happening:
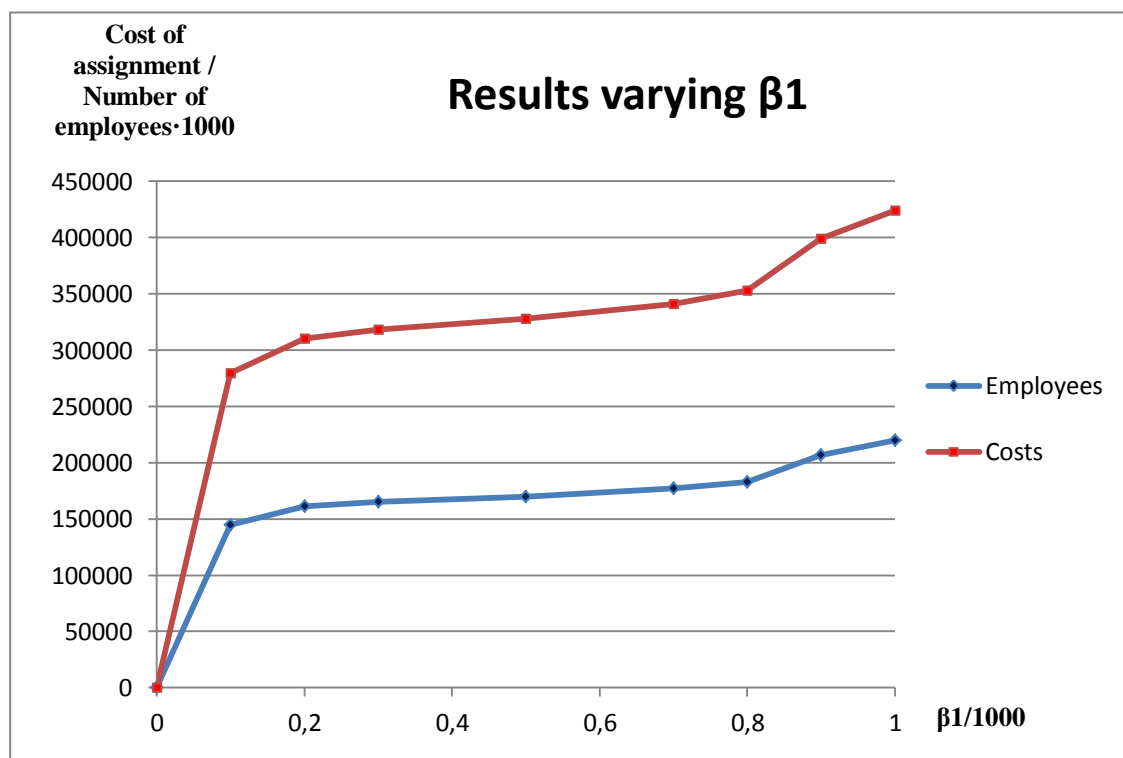


*Figure 15.Evolution of costs and employees with the variation of $\beta_1$(Bi-objective model).*

In the figure 15 we can see that as $\beta_1$ increases the number of employees increases too and the total budget grows too. This result is the one that we could expect because if we give more importance to the objective one, we are optimizing more the number of employees than the costs of assignment.
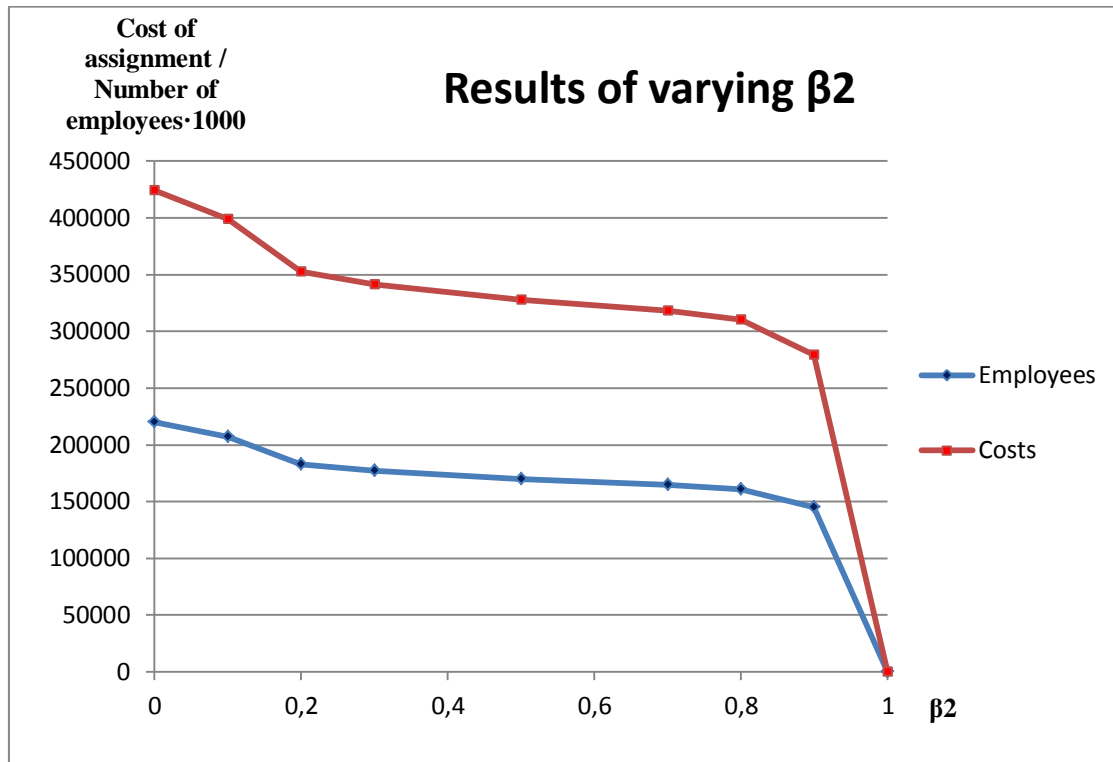


*Figure 16.Evolution of costs and employees with the variation of $\beta_2$.(Bi-objective model).*

In this other graphic we see the impact of $\beta_2$ , as it increases the total number of workers and the total budget decreases, as we expected. We are giving more importance to the second objective, that is why the total budget decreases and the number of employees too.
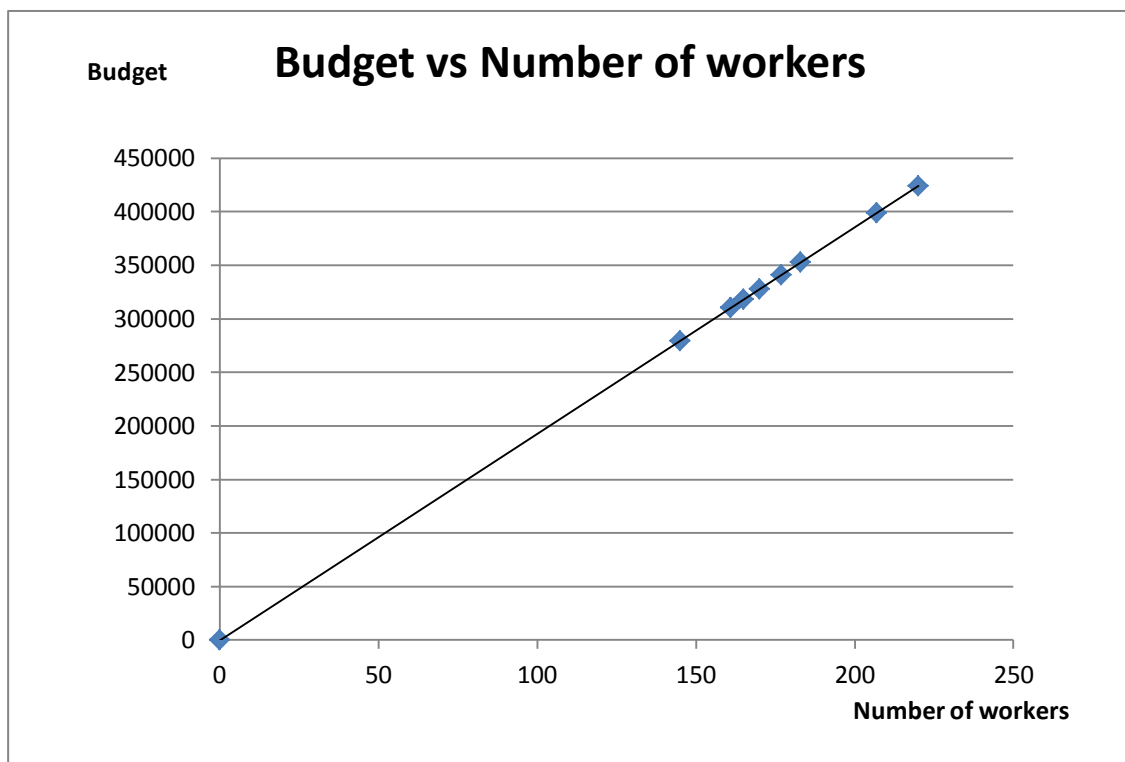
*Figure 17.Relation between total budget and total number of workers (Bi-objective model).*

In this last graphic we see that there is a linear relation between total budget and the number of workers. It is logical because it shows that the relation between workers and their salaries is proportional. If more workers are hired, more money is needed to pay them.

Finally we also show the total number of workers assigned to each department while $\beta$'s values are changing:

| | 0/1* | 100/0.9 | 200/0.8 | 300/0.7 | 500/0.5 | 700/0.3 | 800/0.2 | 900/0.1 | 1000/0 | Before optimization |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 8 | 10 | 10 | 10 | 10 | 12 | 14 | 16 | 16 |
| 2 | 0 | 8 | 10 | 10 | 10 | 11 | 11 | 13 | 15 | 15 |
| 3 | 0 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 4 | 0 | 7 | 7 | 7 | 7 | 7 | 7 | 8 | 8 | 8 |
| 5 | 0 | 8 | 8 | 8 | 8 | 8 | 8 | 10 | 11 | 11 |
| 6 | 0 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 0 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 8 | 0 | 8 | 10 | 10 | 10 | 10 | 10 | 10 | 11 | 11 |
| 9 | 0 | 11 | 13 | 13 | 14 | 16 | 18 | 20 | 21 | 21 |
| 10 | 0 | 13 | 15 | 17 | 18 | 18 | 19 | 19 | 20 | 20 |
| 11 | 0 | 15 | 18 | 20 | 22 | 24 | 25 | 26 | 26 | 27 |
| 12 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 14 | 13 | 13 |
| 13 | 0 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 7 | 7 |
| 14 | 0 | 6 | 7 | 7 | 7 | 7 | 7 | 7 | 8 | 8 |
| 15 | 0 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 16 | 0 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 7 | 7 |
| 17 | 0 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 18 | 0 | 7 | 8 | 8 | 8 | 8 | 8 | 8 | 9 | 9 |
| 19 | 0 | 6 | 6 | 6 | 6 | 8 | 8 | 8 | 9 | 9 |
| 20 | 0 | 6 | 7 | 7 | 8 | 8 | 8 | 10 | 11 | 11 |

Note: * $\beta_1/\beta_2$

*Table 19. Number of workers of each department for each values of $\beta_1$ and $\beta_2$ (Bi-objective model).*

In the 20 departments we can see that by increasing the value of $\beta_1$ and reducing the value of $\beta_2$ the number of workers increases.

### 14.4.2 MODEL 2

After MODEL 1 we solve this model which is a triple-objective model. The values of the weights are going to change as we can see in the next table which shows the results obtained too:

| $\beta_1$ | $\beta_2$ | $\beta_3$ | Total number of employees | Cost of assignment (PLN) | Number of permanent employments |
|---|---|---|---|---|---|
| 0 | 0 | 1000 | 221 | 426060 | 214 |
| 100 | 0,2 | 700 | 207 | 399070 | 201 |
| 300 | 0,3 | 400 | 170 | 327700 | 165 |
| 300 | 0,4 | 300 | 170 | 327700 | 165 |
| 400 | 0,3 | 300 | 170 | 327700 | 165 |
| 100 | 0,7 | 200 | 165 | 318100 | 160 |
| 700 | 0,2 | 100 | 177 | 341200 | 172 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 1000 | 0 | 0 | 221 | 426060 | 214 |

*Table 20. Results obtained in the triple-objective model*

In this model we can see that when the values of $\beta_i$ are between 0.3 or 0.4 (300 or 400 in case of the first beta and the last one) the results obtained are the same. We can also notice that when $\beta_2$ has a big value, the number of employees, permanent jobs and cost of assignment is quite reduced and if it has a small value all these three parameters increase.
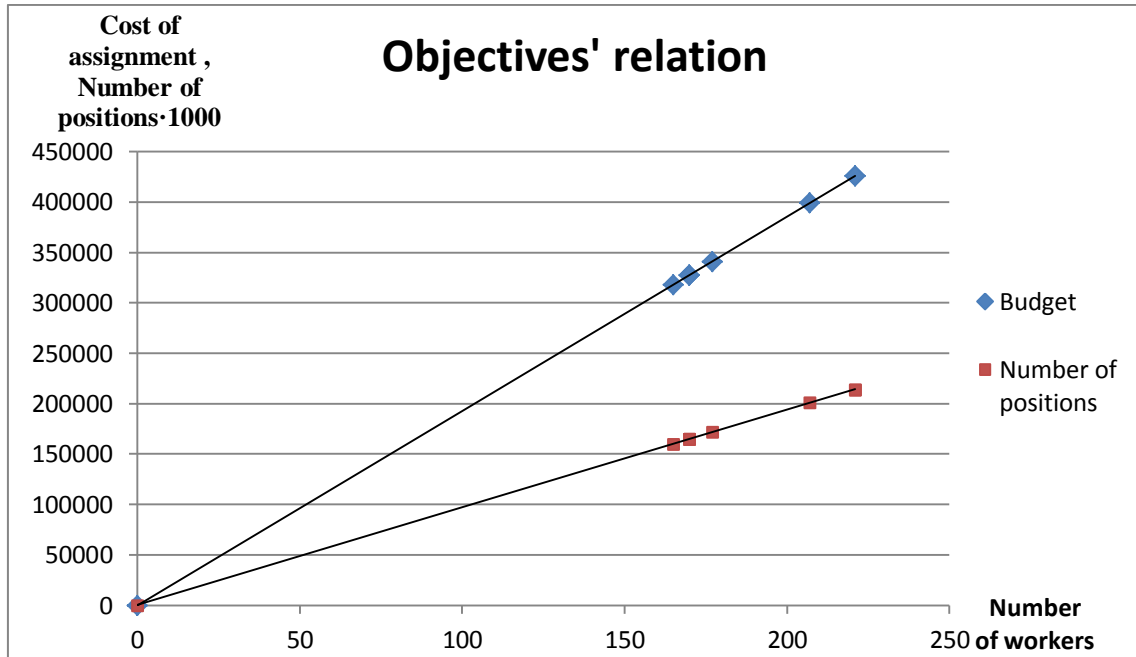


*Figure 18.Objectives' relation in the triple objective model*

By looking at this graph we are able to see that the relation between the number of positions and total budget with the number of employees is a linear relation. Logically if we increase the number of workers, the number of permanent positions and the total budget increase as well.

In this last graphic we show the total budget, the number of employees and the number of permanent employments for each case of different *betas*:
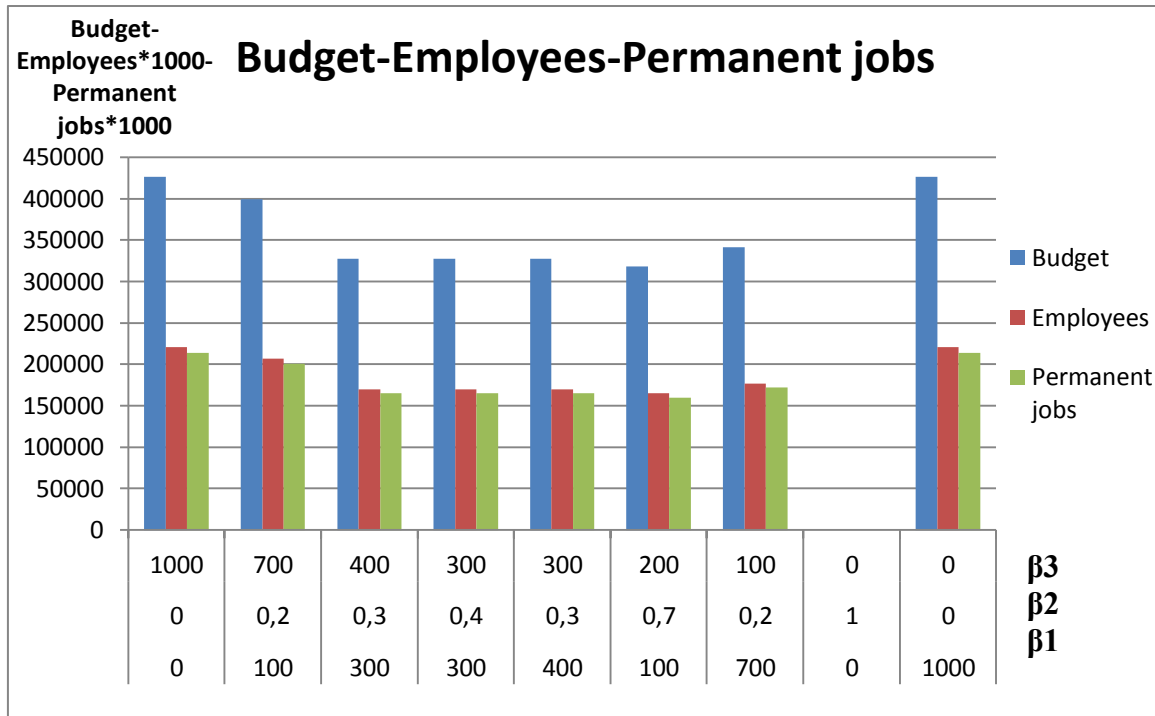
*Figure 19.Total budgets, number of employees and number of permanent Jobs for different values of betas in triple objective model.*

In the figure 19 we can see the same as in the table 20 but more clearly. We see that when the values of *betas* are similar (400,0.3,300; 300,0.4,300 and 300,0.3,400) the values of budget, number of employees and permanent jobs do not change. In addittion, when the value of $\beta_2$ is 0.7 we have the lowest values for budget, number of employees and permanent jobs. We can also see, as in the bi-objective model, that when $\beta_2$ is 1 all the values are 0 due to the fact that we do not have any constraint for the number of employees or the number of permanent employments. Moreover, when $\beta_1 = 1000$ or $\beta_3 = 1000$ the results are more or less the same with the maximum number of employees and permanent jobs and the maximal total budget too.

## 14.5  CONCLUSIONS

In this models we have seen how the weighted-sum approach functions, how varying the values of the weights of the objective function can make a lot of changes in the results obtained. In the bi-objective model it is more easily to see the variations of values due to the changes of weights but in the triple-objective is not so easy to conclude some solutions.

## 15 CONCLUSIONS

Health care is a really important issue in the society, that is why it is too important to have health care institutions well developped and organized. All along this project we have seen that there are a lot of problems in hospitals such as delays in the Emergency Rooms, low bed occupancy levels, bad allocation of treatments or disagreement of the personnel with the schedule, as well as nurse rostering problems.

All over the years these problems have been treated with solutions carried out by manually processes. Nowadays, to make this easier we can take advantage of computer programs which use optimization models to improve health care institutions' work.

Therefore, to realize the allocation of personnel in a hospital it is a good idea to use an optimization model. One posible method is the branch and bound method with the weighted sum approach which here tries to fix the assignment of services in a Polish hospital. It is possible to solve the problem with different objective functions and the results must be studied and analyzed deeply.

## APPENDIX A: SCRIPTS OF OPTIMIZATION MODELS (WEIGHTED SUM APPROACH)

### BI-OBJECTIVE MODEL

#MEDICAL SERVICES OPTIMIZATION MODEL ©Bartosz Sawik PhD

#-------------------------Sets-------------------------

param m:=221;                                *# Number of employees i*

set M:=1..m;                                 *# Set of employees*

param n:=20;                                 *# Number of departments j*

set N:=1..n;                               *# Set of hospital's departments*

param p:=88;                                 *# Number of jobs k*

set P:=1..p;                                 *# Set of jobs*

#-------------------------Parameters-------------------------

param d{j in N};                *# Minimal labor cost in department j.*

param c{i in M, k in P};        *# Cost of assignment of a worker i to job k (monthly salary)*

param C{j in N};                *# Maximal monthly budget for salaries in a department j*

param CB;                       *# Maximal total budget*

param f{k in P};                *# Minimal size of permanent employment for job k*

param e{i in M, k in P};        *# Size of permanent employment for job k and employee i.*

param E{j in N};                *# Maximal number of permanent employments in a department j*

param EP;                       *# Maximal number of permanent employments.*

param W;                        *# Maximal number of employees*

param h{j in N, k in P};        *# Maximal number of permanent employments in department j and job k*

param beta1:=0.8*1000;          *# 0 .1 .2 .5 .8 .9 1 #weight*

param beta2:=0.2;               *# 0 .1 .2 .5 .8 .9 1 #weight*

```
#------------------------Variables------------------------
```

var x{i in M, j in N, k in P} binary;　　*# 1 if worker i is assigned to job k in department j, 0 otherwise.*

var y{i in M} binary;　　*# 1 if worker i is assigned to any job in any department, 0 otherwise.*

```
#------------------------Objective------------------------
```

#OBJECTIVE FUNCTION#

maximize　FC: beta1 * sum{i in M}y[i] - beta2 * sum{i in M, j in N, k in P}c[i,k]*x[i,j,k];

```
#------------------------Constraints------------------------
```

subject to Budget{j in N}: sum{i in M, k in P}c[i,k]*x[i,j,k]<=C[j]; *# Maximal budget for each department j*

subject to Posts{j in N}: sum{i in M, k in P}e[j,k]*x[i,j,k]<=E[j]; *# Maximal number of permanent positions in each department j*

subject to Budget1: sum{i in M, j in N, k in P}d[j]*x[i,j,k]<=CB; *# Maximal - total budget for all departments*

subject to Posts1: sum{i in M, j in N, k in P}f[k]*x[i,j,k]<=EP; *# Maximal - total number of positions in all departments*

subject to WPosts{k in P}: sum{i in M, j in N}x[i,j,k]<=sum{j in N}h[j,k]; *# Maximal requirements for positions*

```
#------------------------Binary variable constraint------------------------
```

subject to OGR_y{i in M}:sum{j in N, k in P}x[i,j,k]>=y[i]; *# Relation between variables*

end;

## TRIPLE OBJECTIVE MODEL

#MEDICAL SERVICES OPTIMIZATION MODEL ©Bartosz Sawik PhD

```
#------------------------Sets------------------------
```

param m:=221;　　　　*# Number of employees i*

set M:=1..m;　　　　*# Set of employees*

param n:=20;　　　　*# Number of departments j*

set N:=1..n;　　　　*# Set of hospital's departments*

param p:=88;　　　　*# Number of jobs k*

```
set P:=1..p;                              # Set of jobs

#------------------------Parameters------------------------
```

param d{j in N};                 *# Minimal labor cost in department j.*

param c{i in M, k in P};         *# Cost of assignment of a worker i to job k (monthly salary)*

param C{j in N};                 *# Maximal monthly budget for salaries in a department j*

param CB;                        *# Maximal total budget*

param f{k in P};                 *# Minimal size of permanent employment for job k*

param e{i in M, k in P};         *# Size of permanent employment for job k and employee i.*

param E{j in N};                 *# Maximal number of permanent employments in a department j*

param EP;                        *# Maximal number of permanent employments.*

param W;                         *# Maximal number of employees*

param h{j in N, k in P};         *# Maximal number of permanent employments in department j and job k*

param beta1:=0.3*1000;           *# 0 .1 .2 .3 .4 .7 1 #weight*

param beta2:=0.3;                *# 0 .1 .2 .3 .4 .7 1 #weight*

param beta3:=0.4*1000;           *# 0 .1 .2 .3 .4 .7 1 #weight*

```
-------------------------Variables------------------------
```

var x{i in M, j in N, k in P} binary;        *# 1 if worker i is assigned to job k in department j, 0 otherwise.*

var y{i in M} binary;            *# 1 if worker i is assigned to any job in any department, 0 otherwise.*

*var g{k in P}>=0;*              *# Number of permanent employments for a job k, integer variable.*

```
#-------------------------Objective------------------------

#OBJECTIVE FUNCTION#
```

maximize  FC: beta1 * sum{i in M}y[i] - beta2 * sum{i in M, j in N, k in P}c[i,k]*x[i,j,k] + beta3 * sum{k in P}g[k];

```
#-----------------------Constraints------------------------
```

subject to Budget{j in N}: sum{i in M, k in P}c[i,k]*x[i,j,k]<=C[j]; *# Maximal budget for each department j*

subject to Posts{j in N}: sum{i in M, k in P}e[j,k]*x[i,j,k]<=E[j]; *# Maximal number of permanent positions in each department j*

subject to Budget1: sum{i in M, j in N, k in P}d[j]*x[i,j,k]<=CB; *# Maximal - total budget for all departments*

subject to Posts1: sum{i in M, j in N, k in P}f[k]*x[i,j,k]<=EP; *# Maximal - total number of positions in all departments*

subject to WPosts{k in P}: sum{i in M, j in N}x[i,j,k]<=sum{j in N}h[j,k]; *# Maximal requirements for positions*

subject to WPosts_g{j in N, k in P}: g[k]<=h[j,k]; *# Maximal requirements for positions*

subject to OGR_g{k in P}: sum{i in M, j in N}x[i,j,k]/f[k]>=g[k]; *# Relation between variables*

```
#-----------------------Binary variable constraint------------------------
```

subject to OGR_y{i in M}:sum{j in N, k in P}x[i,j,k]>=y[i]; *# Relation between variables*

end;

## APPENDIX B: OPTIMIZATION MODELS' SOLVING CHARACTERISTICS

### BI-OBJECTIVE MODEL ($\beta_1 = 500, \beta_2 = 0.5$)

389181 variables, all binary

351 constraints, all linear; 1949441 nonzeros

1 linear objective; 389181 nonzeros.


CPLEX 9.1.0: mipdisplay=3

timing=1

Dual steepest-edge pricing selected.

Clique table members: 66

MIP emphasis: balance optimality and feasibility

Root relaxation solution time =    0.14 sec.

Times (seconds):

Input =  0.904

Solve =  12.465

Output = 0.343

CPLEX 9.1.0: optimal integer solution; objective

221 MIP simplex iterations


389181 variables, all binary

351 constraints, all linear; 1949441 nonzeros

1 linear objective; 389181 nonzeros.


### TRIPLE-OBJECTIVE MODEL ($\beta_1 = 300, \beta_2 = 0.4, \beta_3 = 300$)

Presolve eliminates 1760 constraints.

Adjusted problem:

389269 variables:

      389181 binary variables

      88 linear variables

352 constraints, all linear; 2338489 nonzeros

1 linear objective; 389269 nonzeros.

CPLEX 9.1.0: mipdisplay=3timing=1 Dual steepest-edge pricing selected. Clique table members: 66MIP emphasis: balance optimality and feasibility Root relaxation solution time =    0.27 sec.

Times (seconds): Input =  1.06 Solve =  844.383 Output = 0.374 CPLEX 9.1.0: optimal integer solution within mipgap or absmipgap; objective

16903 MIP simplex iterations

13510 branch-and-bound nodes


Presolve eliminates 1760 constraints.

Adjusted problem:

389269 variables:

      389181 binary variables

      88 linear variables

352 constraints, all linear; 2338489 nonzeros

1 linear objective; 389269 nonzeros.

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