Sobolev inner product as a solution of inverse Darboux transformation.

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The Darboux transformation, with remarkable applications in various areas of mathematics and physics, initially arises as a tool in the field of integrable systems. The Darboux transformation consists in the factorization of a non-negative self-adjoint operator as a product of two other ones, and subsequently, the commutation of both factors providing a new non-negative self-adjoint operator.

When we apply this factorization to Jacobi matrices, canonical representations of self-adjoint operators, a Darboux transformation is equivalent to a Christoffel modification of the underline orthogonality measure.However, inverse Darboux corresponds to the so called Geronimus transformation which divides the measure by a degree one polynomial and may add a mass point at the zero of the polynomial.

For the unitary case, Darboux transformation of CMV matrices requires a Laurent polynomial modification of degree two to transform the unitary matrix into an hermitian one.We find strong similarities between Darboux for Jacobi and CMV as well as, some differences. The main difference concerns inverse Darboux, which for the CMV case presents spurious solutions.

In this talk we expose a survey of Darboux transformations for Jacobi and CMV matrices, relating them with orthogonal polynomials on the real line, orthogonal polynomials on the unit circle and integrable systems, and also highlighting their similarities and differences. We will show the interest of the above mentioned spurious solutions of inverse Darboux for CMV matrices: they are related to Sobolev type inner products.

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