

Chapter 2. Standing Waves for Acoustic Levitation

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1. INTRODUCTION

Sound is a mechanical wave transporting momentum capable of exerting forces on objects as a consequence of the acoustic radiation force [1,2,3,4]. If the forces that act on an object are converging and sufficiently strong, the objects can be suspended and trapped in mid-air [5] (or other propagation media). The trapping of particles using soundwaves is referred to as acoustic levitation, acoustic trapping, acoustic tweezers or acoustophoresis.

Acoustic trapping can be used to hold particles of various materials and sizes. This contrasts with optical trapping in which the trapped objects have to be of micrometric scale and the material should be optically transparent or dielectric [6]. Acoustic trapping is also more efficient in terms of input power to exerted forces [7]. Other types of contactless trapping, such as electrostatic levitation [8] requires a close-loop control system and the range of object materials is also limited, aerodynamic levitation [9] stirs and disturbs the samples. Magnetic levitation is a popular method to hold objects in mid-air [10] however it works only on ferromagnetic materials; diamagnetic materials can also be employed [11,12] but due to the much weaker diamagnetic effect, the required amount of power is only accessible to a couple of research laboratories.

The flexibility of acoustic levitation in mid-air has made it a valuable apparatus for containerless transportation [13,14], pharmaceuticals [15], nano-assemblies [16], the levitation of biological samples [17], little animals [18,19] and food [20]. Acoustic levitation of liquid droplets can be employed to explore novel liquid dynamics [21], measure surface tension [22,23] or rheological properties [24]. Other applications are the formation of suspended ice flakes [25], growing crystals in liquid metals [26], evaporation of solutions [27], investigation of phase transitions [28], the fast crystallization [29] or ionization [30] of samples, and the creation of bubbles for coverings [31]. Samples held with acoustic levitation are not in physical contact with a container, this provides benefits in mass [32] and Raman [33] spectroscopy, e.g. in

algae [34] or blood cells [17] characterization. In general, acoustic levitation is a helpful and adaptable tool in biomaterials research [35], chemistry [36] and enables lab-on-a-drop procedures [37].

Single-axis levitators [38] are the most typical device for producing acoustic traps. They are composed of an acoustic emitter and a reflector opposed to it. A standing wave is generated between the emitter and the reflector. The nodes of the standing wave act as an acoustic traps for spherical particles smaller than half-wavelength and with positive acoustic contrast (i.e. the acoustic impedance of the particle is larger than that of the propagation medium). When the medium is water-based, it is also possible to have particles with negative contrast, in this case, the particles go to the anti-nodes [39]. A simple single-axis levitator is presented in Figure 1 as well as the simulated amplitude field that generates and the forces that exert on a 1mm diameter spherical particle.

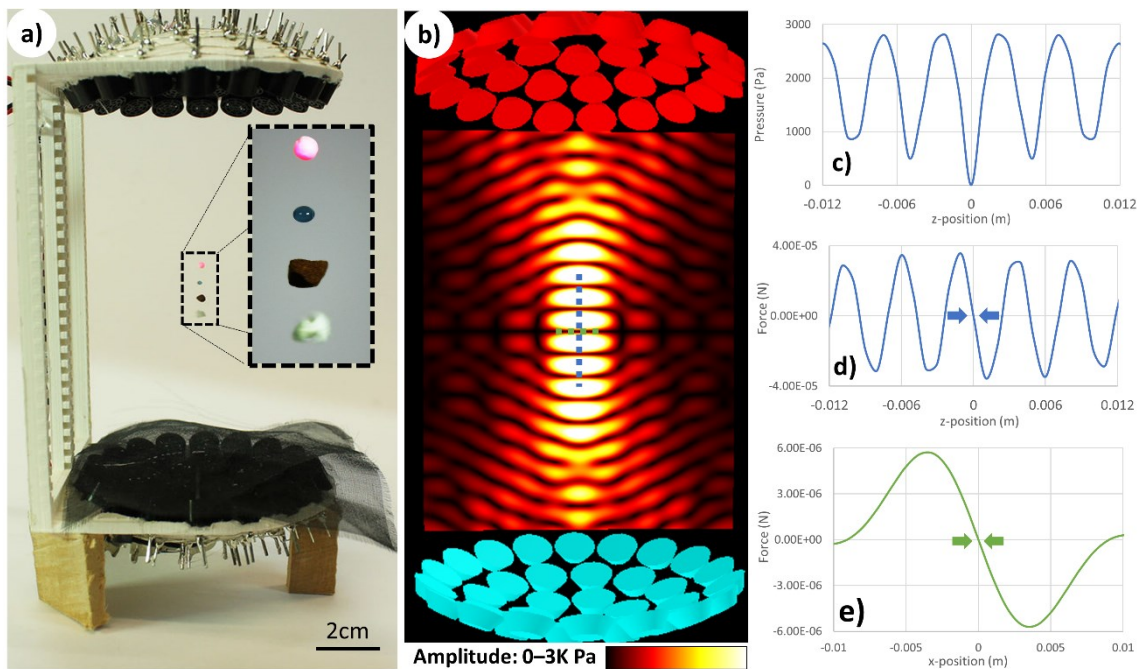


Figure 1. a) TinyLev, a single-axis levitator, trapping plastic, water, soil and paper. b) the amplitude pressure field generate by the levitator. c) the amplitude pressure distribution across the vertical axis z. d) the force acting on a particle of 1mm diameter depending on its position along the z-axis and (e) x-axis. Note that both forces are converging towards the center.

Standing waves are the most common method to trap particles given its trapping strength but present some limitations such as particle size (it has to be smaller than half-wavelength) and spherical shape. Some of these limitations have been overcome in the past years. Particles larger than half-wavelength can be levitated close to an emitter using near-field levitation [40,41]. Also, special types of acoustic vortices can trap objects larger than the wavelength in the far-field [42,43] however they require high-

power and only very light objects have been trapped. Non-spherical particles can be steadily trapped using the acoustic lock technique [44] but extra control on the emission of the fields is needed.

Apart from standing-waves, acoustic beams emitted from a single-sided device are capable of trapping particles in three dimensions [45,46,7,47], these beams are also referred to as tractor beams since they can attract particles towards the source, i.e. generate negative forces along the propagation direction of the beam. However, complex equipment or high-power is required for generating functional tractor beams. Consequently, standing-wave trapping remains as the most common method to trap samples in mid-air. A more detailed description of the acoustic levitation methods can be found in these reviews [48,49].

We have covered the trapping of one or a series of particle along the static nodes of a standing-wave, but it is also possible to generate multiple standing waves with nodes at user specified positions. Multi-emitter arrays arranged in an opposed configuration enable the trapping of multiple particles that can be moved independently, these traps are named Holographic Acoustic Tweezers [50].

In the second section of the chapter, we will review the main models employed to calculate the acoustic field generated by acoustic levitators and how to determine the exerted forces on the objects that are contained in these fields. In the third section, the most common configurations used for standing-waves levitators are reviewed.

2. PRINCIPLE GOVERNING EQUATIONS

In this section, we describe the most common models used to calculate the generated acoustic field by an acoustic emitter, and the forces that this field exerts on objects. When the models operate in the frequency domain at a single-frequency, the acoustic pressure (p) is represented as a complex scalar field: the magnitude represents the amplitude in pascals and the argument is the phase in radians. When the models operate in the time-domain, the pressure is a real scalar field. The particle velocity of the medium (\mathbf{v}) is a vector field that also appears in the calculations, it can be obtained from the model or as the gradient of the pressure.

Other terms that regularly appear in the models are: total, incident and scattered fields. The incident field is the acoustic field generated by the emitters into the free-space, i.e.

no reflectors or obstacles that affect the emitted waves are in the domain. The total field is the pressure distribution that exists in the space when all the elements that affect the wave are considered, in theory the total field is what gets measured in the experiments. The total, incident and scattered fields are linear both for pressure and velocity under the assumptions of most of the models. Consequently, total = incident + scattered, by having two of the fields the other field can be obtained, some models calculate just the incident field whereas other models determine the scattered or directly the total field.

2.1. Generated Acoustic Field

Free-field piston source.

This is one of the simplest models to calculate an incident field generated by one or multiple emitters which are shaped or can be approximated as circular radiating pistons. The model is only valid for the far-field of the emitters, this is usually not a problem since levitation occurs in the far-field. In general, this method is not suitable for calculating models with complex reflecting geometry or particles which are larger than half-wavelength since the scattered field would have a significant magnitude. Simple planar reflectors can be approximated by mirroring the emitters and adding an attenuation coefficient depending on the material. On the other hand, this method is fast and can run in real time for hundreds of emitters [51].

The complex acoustic pressure p at point \mathbf{r} due to a piston source emitting at a single frequency can be modelled as [52]:

$$p(\mathbf{r}) = P_0 V \frac{D_f(\theta)}{d} e^{i(\varphi + kd)}$$

Where P_0 is a constant that defines the transducer output efficiency and V is the excitation signal peak-to-peak amplitude. D_f is a far-field directivity function that depends on the angle θ between the piston normal and the the point \mathbf{r} . The directivity function of a piston source can be expressed as $D_f = 2J_1(ka \sin \theta) / ka \sin \theta$, where J_1 is a first order Bessel function of the first kind and a is the radius of the piston [53]. This directivity function can be simplified as $D_f = \text{sinc}(ka \sin \theta)$. Other geometries such as square or line transducers can be approximated with other directivity functions. The term $1/d$ accounts for divergence, where d is the propagation distance in free

space. $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. φ is the emitting phase of the source.

The total acoustic field (P) generated by N transducers is the addition of the individual fields, i.e. $P = \sum_{j=1}^N p_j$. To characterize a transducer, the constant (P_0) and the piston radius (a) are needed; the constants for different transducers can be found in the supplementary information of [54]. For instance, the commonly used MA40S4S (Murata Electronics, Japan) have these values: $a=4.5\text{mm}$ and $P_0 = 0.17 \frac{\text{Pascal} \times \text{Meter}}{\text{Volt}}$

Scattered Field form a Particle using Spherical Harmonics

If the particle is larger than half-wavelength, the generated scatered field would influence the simulations, thus modelling only the incident field may not be sufficient. A way of obtaining the scattered field is using spherical harmonics. Here, we explain the overall method but complete solutions are described in [55,42,56].

The linearity of the acoustic fields allows us to represent the incident p_{in} and scattered p_{sc} fields as series of spherical harmonics $Y_n^m(\Omega)$ (where $\Omega = (\theta, \varphi)$ is the solid angle in spherical coordinates), with modal amplitudes A_n^m and T_n^m . The incident field can be expressed as:

$$p_{in}(\mathbf{r}) = p_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n j_n(kr) A_n^m Y_n^m(\Omega).$$

And the scattered field as:

$$p_{sc}(\mathbf{r}) = p_0 \sum_{n=0}^{\infty} \sum_{m=-n}^n h_n^{(1)}(kr) T_n^m A_n^m Y_n^m(\Omega).$$

Where j_n ($h_n^{(1)}$) is a Hankel function. The scattering coefficients T_n^m characterise the properties of the scattering particle: they depend on the particle shape, the boundary conditions and the internal properties of the particle as well as on the frequency. For a rigid sphere [57] of radius a , the scattering coefficient is given by $T_n^m = -j_n'(ka)/h_n^{(1)'}(ka)\delta_{nm}$, where $'$ denotes differentiation and δ_{nm} is the Kronecker's delta. Other coefficients can be obtained for non-solid particles of different shapes.

Using the orthogonality property of the spherical harmonics, the expansion coefficients A_n^m (often called beam-shape coefficients) can be obtained as:

$$p_0 A_n^m = \frac{1}{j_n(kR)} \iint_{\Omega} p_{in}(R, \Omega) Y_n^{m*}(\Omega) d\Omega,$$

where R is the radius of the spherical volume in which the incident field p_{in} propagates; this volume contains the trapped object.

The incident and scattered field in the far field ($r \rightarrow \infty$), can be expressed as:

$$p_{in} \approx \frac{p_0}{kr} \sum_{n=0}^{\infty} \sum_{m=-n}^n \sin(kr - n\pi/2) A_n^m Y_n^m(\Omega);$$

$$p_{sc} \approx \frac{p_0}{ikr} e^{ikr} \sum_{n=0}^{\infty} \sum_{m=-n}^n i^{-n} A_n^m T_n^m Y_n^m(\Omega).$$

This method requires the calculation of the incident field, for instance using the previously described free-field piston model. The analytical scattered field generated by the particle can be added to the incident field to get the total field. This model cannot capture the reflections caused by reflectors of complex shapes or other objects affecting the field apart from the levitated object.

Numerical Methods

Numerical methods are preferred to calculate the acoustic field when the domain to be simulated contains complex geometry, for instance hollow tubes of different lengths [46,58], a levitated particle of complex shape [44], or a curved reflector [59]. In general, these methods take more time to execute than the analytic methods and are less precise but are the only feasible alternative when there are complex geometries in the domain.

One of the simplest numerical methods are the Finite Differences Time Domain (FDTD) simulations in which the domain is divided into a staged grid for pressure and velocity [60]. More general Finite Elements modelling is also possible [61]. If the propagation is on liquids then it is sufficient to use a scalar pressure and velocity vector, but on solids a complete simulations requires the use of stress tensors [62].

The most employed numerical method on the literature is the boundary elements from the COMSOL package. It allows for irregular meshes of different densities thus giving a good compromise between accuracy and execution time.

If the trapped particle is smaller than half-wavelength, it could be possible to simulate the field without the particle inside the levitator. The obtained field could be used to

calculate forces, independently from the type and position of the particle. However, for large particles and resonant levitators, a simulation with the particle inside is recommended.

2.2. Acoustic Radiation Force

The acoustic field will exert a radiation force on the particles contained inside the field. There are other effects such as thermal or viscous contributions [63] but here we focus on the radiation force since it is the dominant force for particles that are large in comparison to the viscous layer [1].

Two methods are presented, one is the Gork'ov potential which is a simple method to calculate the force acting on a particle. The Gork'ov potential only requires the incident field to determine the forces acting on a particle but it assumes that the particle is much smaller than the wavelength, spherical and rigid. On the other hand, the flux integral is a more general method to calculate the force acting on an arbitrary-shaped object of any size, it is a more complex approach since it requires the calculation of the total field and an integration over the surface of a sphere that encloses the trapped object.

Gork'ov potential

Gork'ov derived a simplification of the forces acting on a particle when it is in an acoustic field [3]. A modern and more detailed derivation of the Gork'ov potential was described by Bruus [1]. To calculate the force exerted on a sphere significantly smaller than the wavelength due to a complex pressure field, the negative gradient of the Gork'ov potential can be used $\mathbf{F} = -\nabla U$ where the potential U can be defined in terms of the incident pressure:

$$U = 2K_1(|p|^2) - 2K_2(|p_x|^2 + |p_y|^2 + |p_z|^2)$$

$$K_1 = \frac{1}{4}V \left(\frac{1}{c_0^2 \rho_0} - \frac{1}{c_s^2 \rho_s} \right)$$

$$K_2 = \frac{3}{4}V \left(\frac{\rho_0 - \rho_s}{\omega^2 \rho_0 (\rho_0 + 2\rho_s)} \right)$$

where V is the volume of the spherical particle, ω is the frequency of the emitted waves, ρ is the density and c is the speed of sound (subscripts 0 and s referring to the host medium and the particle material respectively). p is the complex pressure and p_x, p_y, p_z are its spatial derivatives over x, y and z . U can also be expressed as a function of the

pressure (p) and the velocity (\mathbf{v}) but since the velocity can be obtained as the gradient of the pressure [1], we consider this expression more compact. In any case, the fields should be calculated without the particle in the model.

Radiation Flux Integral.

The radiation force acting on an object can be obtained by the integration of the momentum fluxes over an enclosing sphere. If the total field is given in the frequency domain, the second-order approximation of the radiation force can be expressed as [64]:

$$\mathbf{F} = \iint_S \left\{ \left(\frac{1}{4}\rho|\mathbf{v}|^2 - \frac{1}{4\rho c^2}|p|^2 \right) \mathbf{n} - \frac{1}{2}\rho\text{Re}[\mathbf{v}^*(\mathbf{v} \cdot \mathbf{n})] \right\} dS,$$

where p and \mathbf{v} are the complex pressure and particle velocity due to an acoustic field with time dependence $\exp(-i\omega t)$, S is a surface, \mathbf{n} is the normal to the surface enclosing the system, and $*$ is the complex conjugation.

If the pressure and velocity fields are in the time domain the expression is as follows [61,56,65]:

$$\mathbf{F} = \iint_S -\frac{1}{2\rho c^2}\langle p^2 \rangle + \frac{1}{2}\rho\langle v^2 \rangle - \rho\langle (\mathbf{v}\mathbf{n}) \cdot \mathbf{v} \rangle d\mathbf{a},$$

With $\langle \rangle$ representing the time average.

3. Levitator Geometries

The most employed configuration for acoustic levitators is the single-axis levitator [38] which can be divided into two main categories. On the one hand, a standing-wave can be generated by an emitter and an opposed reflector; the shape, distance and material of the reflector has a fundamental role on the efficiency. On the other hand, two opposed emitters can be employed to add extra acoustic power and versatility [66].

Another classification attends to the resonant nature of the levitator, there are non-resonant [66] and resonant [67] levitators. These division is more like a continuous spectrum in which the levitators have a certain degree of resonance. Resonant devices are more efficient (i.e. larger trapping forces per input power) but are harder to tune due to changes in atmospheric temperature, humidity and barometric pressure. Non-resonant levitators are simpler to use since they do not need tuning but are not as efficient. In general, a low ratio of distance between the opposed elements and their emission

aperture leads to a highly resonant device, whereas levitators with small radiating surfaces and large separations will be less resonant.

Langevin horns are a common design for the emitters, they are devices made of piezoelectric disks clamped between a backing material and a resonating horn [68]. They can operate at high-voltages (typically 100-1000 V) and generate high acoustic pressures with a single emitter; however, they have some disadvantages. It is difficult to tune Langevin horns to a specific resonant frequency, Weber *et al* [66] built dozens of horns to obtain two horns with a sufficiently close resonant frequency. Also, the high-voltage required to operate the horns could be dangerous. Furthermore, Langevin horns typically heat up over time and shift their resonant frequency. One alternative is to employ small ultrasonic transducers used predominantly in distance ranging applications. They are capable of outputting enough power to obtain levitation of samples of up to 7 g/cm^3 [54]. To summarize, the emitters used for levitation are either Langevin horns or off-the-shelf ranging transducers.

An example of different standing-wave levitator configurations is shown in Figure 2, each of the arrangements is described with more detail in the following subsections.

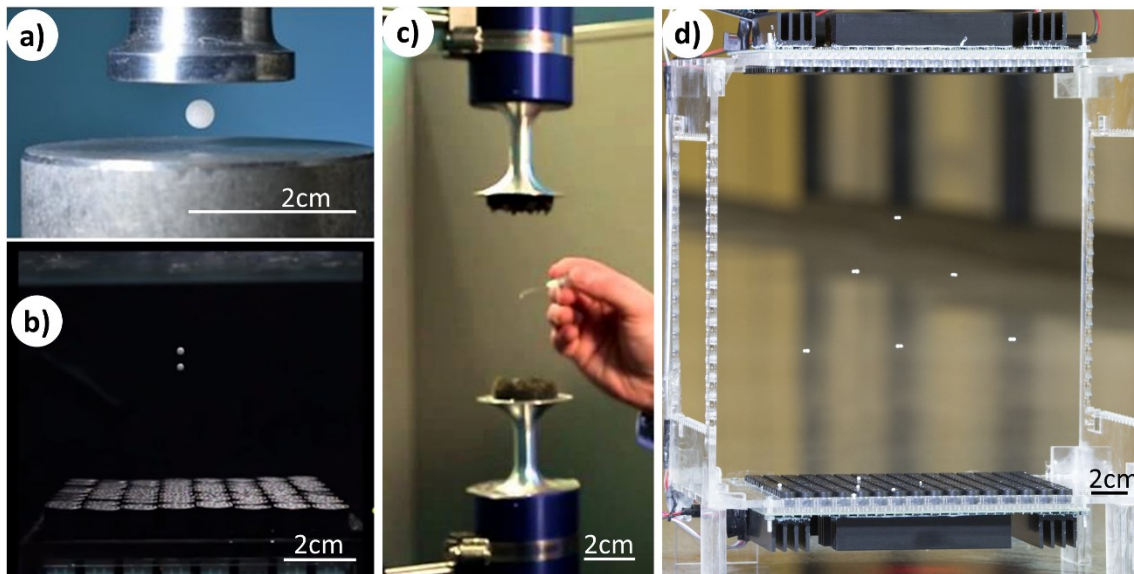


Figure 2. a) Emitter on top and reflector on the bottom, Figure extracted from [48]. b) phased-array on the bottom and reflector on top. c) two opposed emitters, Figure extract from [66] d) two opposed phased-arrays.

Single-emitter and a reflector.

In this configuration, an emitter generates an acoustic wave that reflects on a solid element placed in front of it at a certain distance. The reflected wave adds to the emitted wave and create a standing wave.

Most of the times, the emitter is a high-power Langevin transducer [68]. An emitter with large diameter of the radiator leads to more stable levitation [69,70,71], therefore Langevin horns for acoustic levitation usually have special designs with a large diameter of the radiation surface (i.e. horn shape).

Considerable research has been conducted on the shape and size of the reflector. A concave reflector leads to stronger acoustic traps than the common planar reflector [59,70,71]. Using a concave emitter increased significantly the efficiency of the levitators by locally concentrating the acoustic energy [67]. To improve the adaptability of emitter-reflector levitators, a morphing reflector made of water or elastic materials was employed [72,73]. It is also possible to enclose the levitator with a tube to improve the performance [74].

When the levitator is resonant, a change in temperature can detune the levitator and reduce the trapping strength [75]. Similarly, introducing large samples in the levitator can shift the resonant frequency [76] and create instabilities [77]. Also, non-linear behaviors such as second harmonic generation can reduce the trapping force [78].

Two opposed emitters.

Instead of using a passive reflector to create the standing wave, another possibility is to use two opposed emitters. This allows to put more direct acoustic power and also to move the samples along the axis by changing the phase [38]. Given the extra acoustic power and phase adjustment these systems can be used to create non-resonant levitators which are in general more versatile [66]. Using this approach, a levitator is more robust to external conditions (e.g. temperature, humidity or barometric pressure), being able to operate at temperatures ranging from -40 to $+40$ °C and requiring less calibration of the separation between the emitters.

Phased-array and a reflector

Phased arrays are an assortment of transducers that transmit or receive using user defined phase or time delays. They are commonly used in radar [79] or sonar [80] given their ability to dynamically steer and shape the beam. Phased-arrays are employed also in standing-wave levitators [7,54,51] for their capability of refocusing and shifting the standing waves, thus moving the trapped particles dynamically. However, the trapped

particles cannot be moved perpendicularly to the reflector, they can only be moved in a plane parallel to it.

The traps can be moved without displacing the levitator by adjusting the phase of the emitters. A common configuration is to use a phased-array opposed to a parallel reflector. The principle of operation is the same as the emitter-reflector but this time the emitted focal point can be moved dynamically; it is also possible to create multiple focal points. These focal points reflect on the opposed reflector and create standing waves. This has been shown for manipulating multiple samples and merging them in mid-air [13,82,83].

Two opposed phased-arrays.

Using two-opposed phased-arrays is possible to create standing waves that also shift their nodes perpendicularly to the arrays. Firstly, it was shown that with two-opposed arrays 3D positioning of one particle was possible [81], and then that the individual positioning of multiple particles can be achieved [50]. An iterative backpropagation (IB) algorithm is employed to generate multiple functional traps using arbitrary arrangements of transducers [50]. Two opposed 256-emitter phased-arrays are manipulating individually 6 millimetric particles in Figure 3.

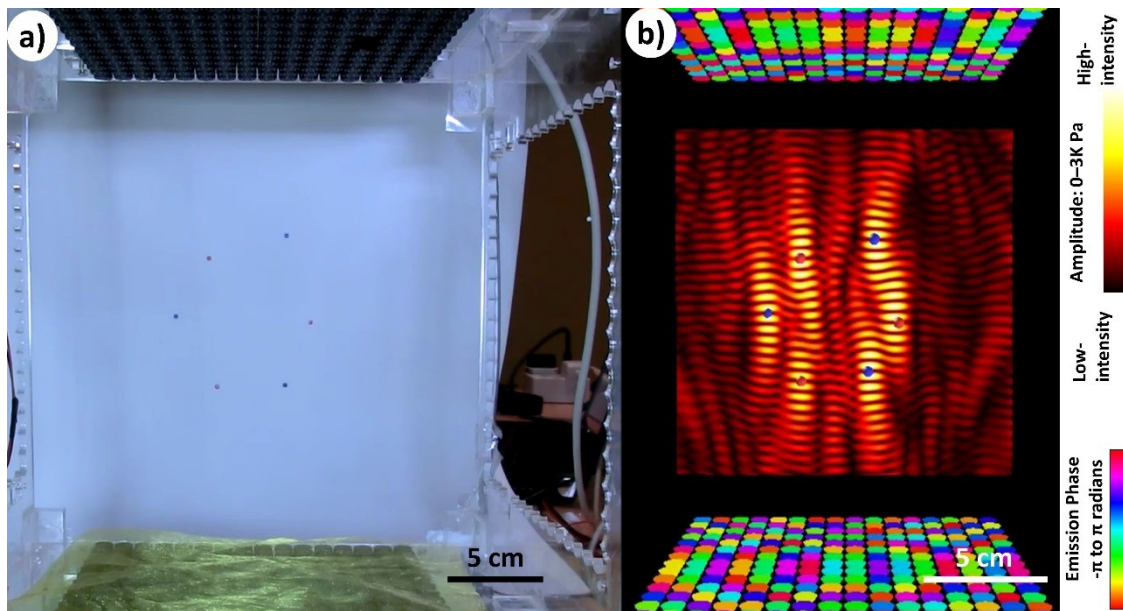


Figure 3. a) Two opposed arrays made of 16x16 ultrasonic transducers operating at 40 kHz are trapping 6 spherical particles made of Styrofoam. b) the simulated amplitude pressure field as well as the emission phases of the transducers.

When designing standing-wave levitators made of phased-arrays it must be taken into account that the maximum trapping forces are achieved with emitting arrays that satisfy

Nyquist sampling [50] (i.e. the emitters are half-wavelength in size) and an emission phase discretisation of $\pi/8$ radians [51]. That is, for standing-wave trapping applications there is no increase in the trapping strength when having emitters smaller than half-wavelength or creating electronics that support phase resolutions of more than 16 divisions per period.

Other geometries

There are other geometries apart from the ones presented here: four orthogonal emitters [84] or arrays [85], emitters arranged in a heptagon [86], or in a circle [87,88,89]. Also, instead of using phased-arrays, the required phase modulations can be achieved using metamaterials [46,58,39] or a mix of metamaterials and phased-arrays [90]. Although this method has less complexity (regarding hardware and cost) it only enables static levitation or restricted movement.

2.3. Conclusion

An acoustic standing wave with enough pressure amplitude will trap particles smaller than half-wavelength and of positive acoustic contrast (the particle has more acoustic impedance than the medium). This is the basic principle behind acoustic levitation and enables multiple applications in contactless manipulation of samples for spectroscopic analysis of materials, amorphous crystallization of solutions or characterization of liquid properties.

Multiple levitator configurations exist to generate standing waves of high-amplitude. Single-axis levitators made of an emitter and an opposed reflector are a simple way to hold samples in mid-air. More complex devices based on phased-arrays allow the manipulation of multiple particles independently and thus enable more complex protocols such as mixing of samples. Despite the advances on acoustic trapping with vortices or tractor beams, standing waves remain as the main method to trap particles in mid-air.

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