ON THE CONVERGENCE OF A MODAL UPDATING ITERATIVE METHOD APPLIED TO A VIBRATING TABLE FOR FOOD TRANSPORTATION

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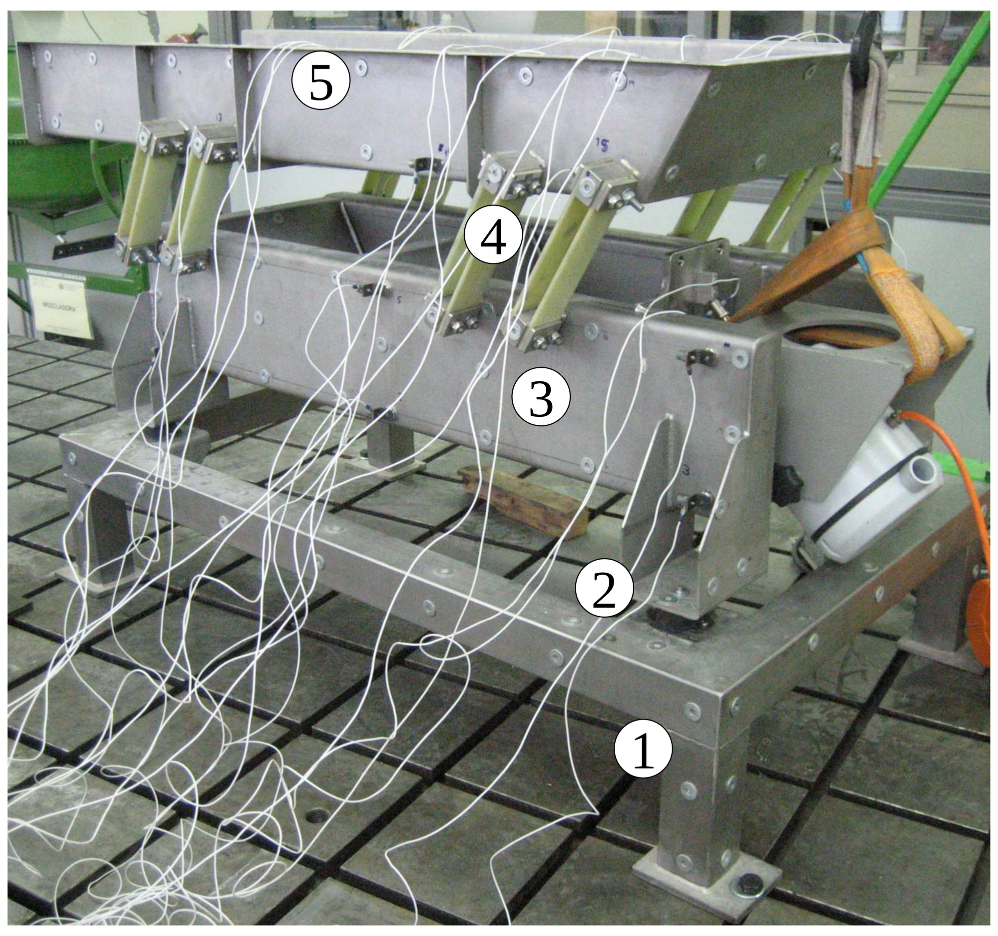
ABSTRACT:

This work focuses on the updating of the parameters of a Vibrating Table Multibody model. Modal Analysis has been done to obtain the experimental **Modal Parameters of the System** (natural frequencies (ω_i), damping ratios (ξ_i) and mode shapes (ϕ_i)), and the unknown **Dynamic Model Parameters (p)** are found through a Newton-Raphson based procedure that fits the Modal Parameters of the dynamic model to those obtained from the Modal Analysis experiment.

THE SYSTEM: VIBRATING TABLE

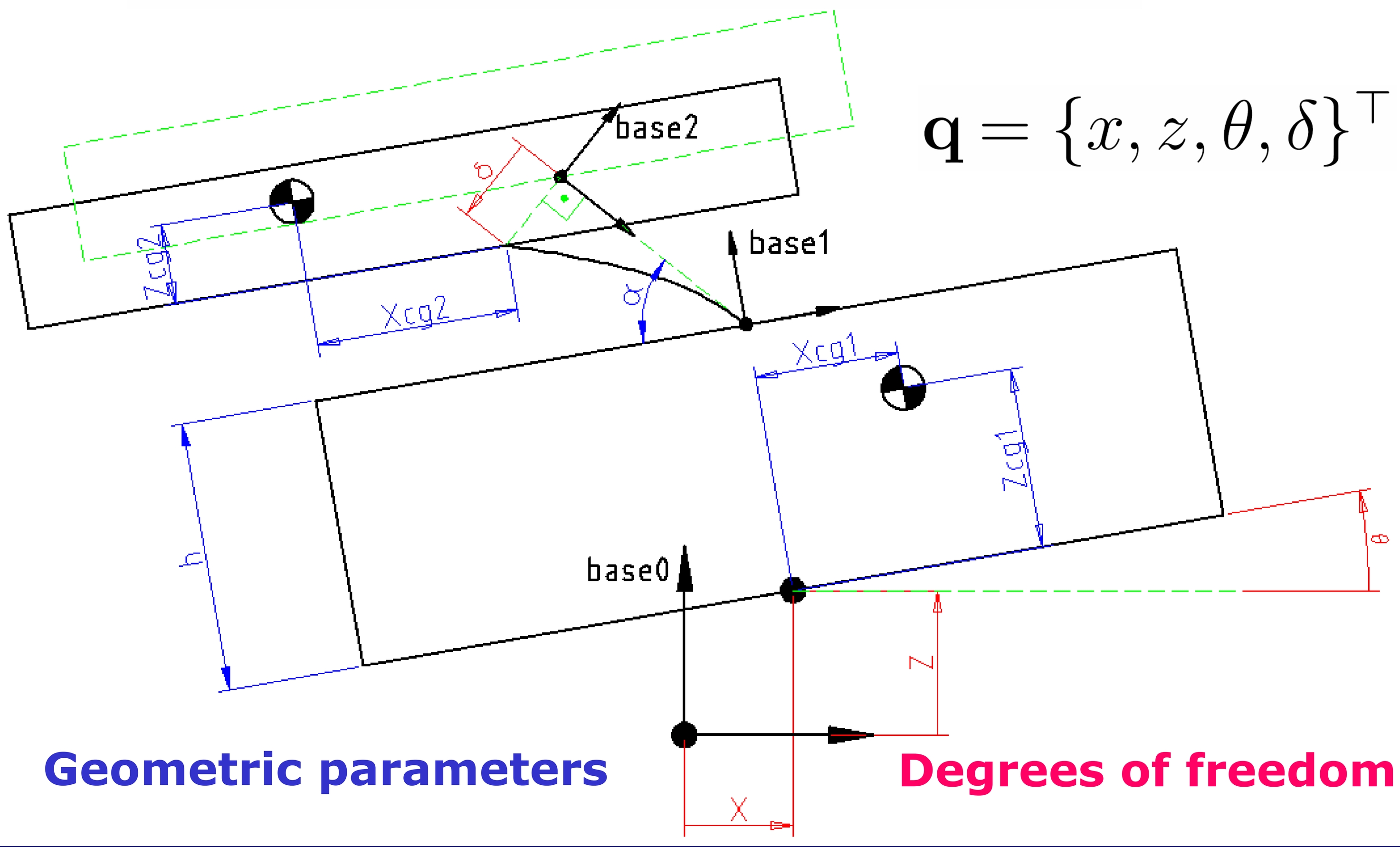


(1) Table (2) Silent-Blocks (3) Frame (4) Beams (5) Plate

THE MULTIBODY MODEL:

$$M(\mathbf{p})\ddot{\mathbf{q}} + C(\mathbf{p})\dot{\mathbf{q}} + K(\mathbf{p})\mathbf{q} = 0$$

$$\mathbf{q} = \{x, z, \theta, \delta\}^\top$$



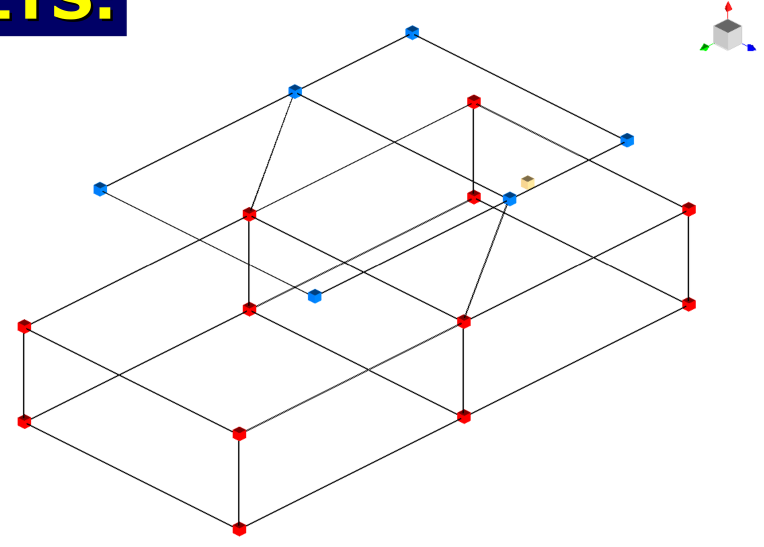
MODELIZATION:

- **Linear model:** Dynamic equations are linearized.
- **2D Model:** Movement and excitation are plane.
- **Rigid Bodies:** The tests validate this hypothesis measuring very small relative movements between different points of the same body.
- **Translational joint between Frame and Plate:** The high stiffness of the beams in the axial direction prevents relative rotation and axial translation. This hypothesis is also validated through experimental data.

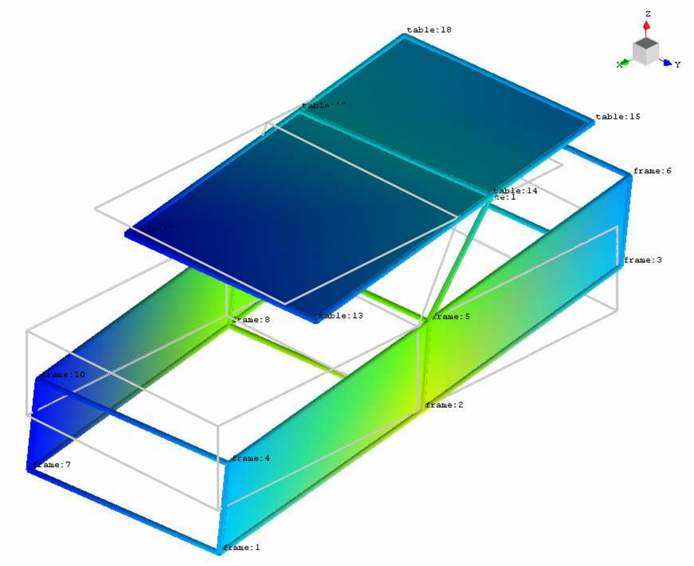
TEST DESCRIPTION AND RESULTS:

- **18 accel. x 2 directions:** red and blue points in X and Z directions.
- **Electrodynamic excitation:** in the yellow isolated point.
- **Frequency Response Functions:** Real and Complex Natural Frequencies and Modes are calculated from FRF-s to fit the experimental data.
  - 3rd Mode violates Rigid Body hypothesis when constrained to be Real.
  - The rest of the modes are very similar in Real and Complex models.
  - The model will be updated to fit these mode shapes.
- **Experimental Natural Frequencies:** The model will be updated to fit these natural frequencies.

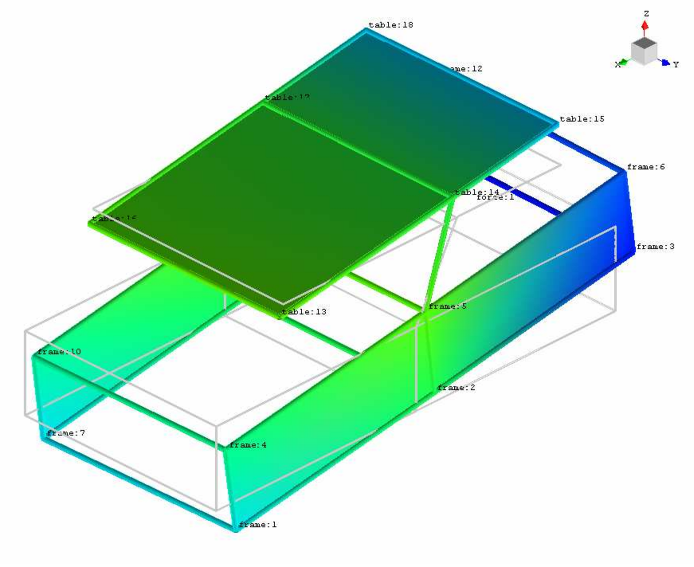
$$\omega_m = (4.48, 7.75, 10.80, 23.03)^\top Hz$$



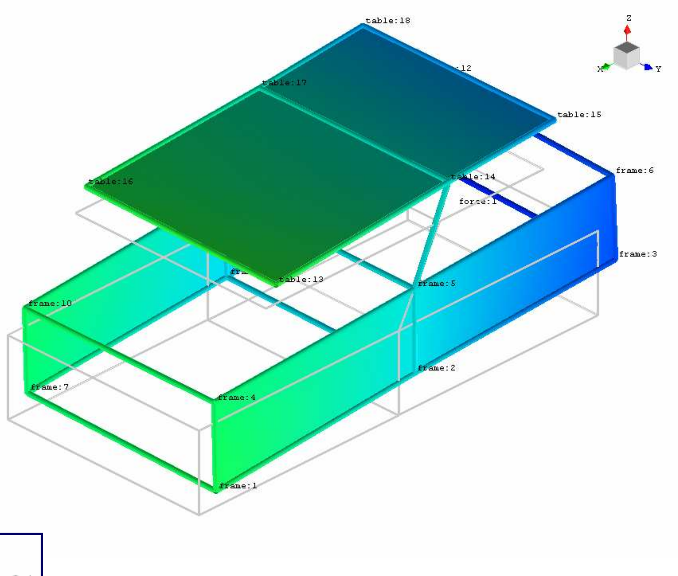
Accelerometers' location



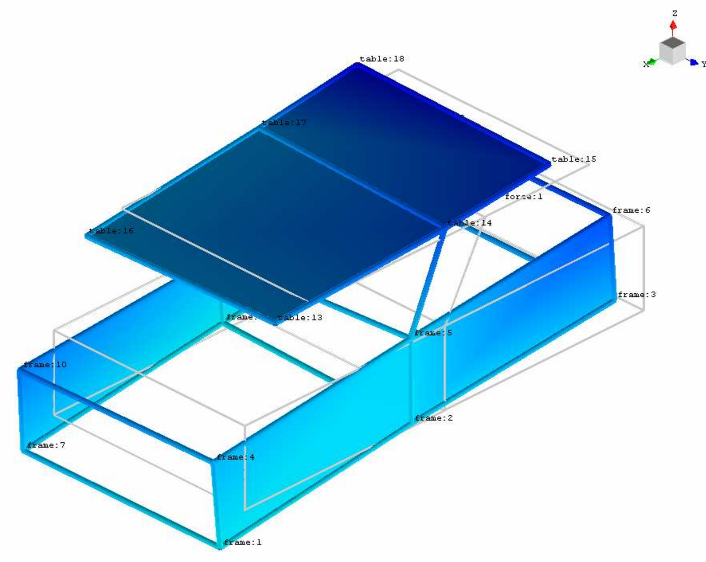
3rd Real Mode



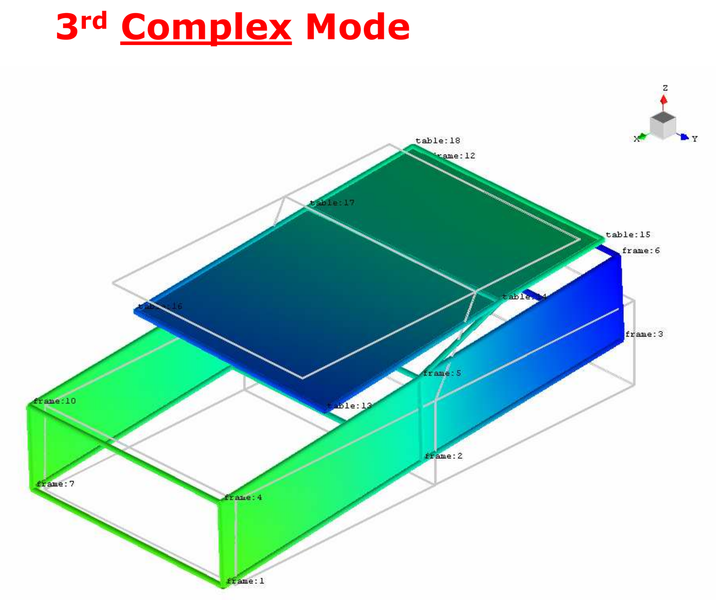
3rd Complex Mode



1st Real Mode



2nd Real Mode



4th Real Mode

COORD. TRANSFORMATION:

- **Two coordinate systems:** measurements are done as absolute displacements, while in the multibody model depends on the coords. **q**.
- **Relation:** writing the displacement of a point of the frame (d_1) or the plate (d_2), in terms of the relative coords. **q**, and linearizing leads to:

$$D_1 = \begin{bmatrix} 1 & 0 & -r_z & 0 \\ 0 & 1 & r_x & 0 \end{bmatrix} \quad D_2 = \begin{bmatrix} 1 & 0 & -r_z & -\sin(\alpha) \\ 0 & 1 & r_x & -\cos(\alpha) \end{bmatrix}$$

r represents point's position at rest.

- Measured displacements $\hat{d}$ can be written as:

$$\hat{\mathbf{d}} = \begin{bmatrix} D_{1,1} \\ D_{1,2} \\ \vdots \\ D_{2,17} \\ D_{2,18} \end{bmatrix} \mathbf{q} = \mathbf{D}\mathbf{q}$$

For a mode in the **q** coordinates system, the pseudo-inverse of **D** is used.

$$\phi_q = (\mathbf{D}^\top\mathbf{D})^{-1}\mathbf{D}^\top\phi_a$$

MODEL UPDATING:

- **Vector φ collects freq. and modes:**

$$\varphi = (\omega_1, \phi_1^\top, \ldots, \omega_4, \phi_4^\top)^\top$$

- **Function "*eigs(p)*"** calculates the natural frequencies and mode shapes of the model.

$$\varphi = eigs(\mathbf{p})$$

- **Objective:** find the parameter vector **p** that minimizes the difference between measured φ_m and theoretical results. φ

$$F(\varphi_m, \mathbf{p}) = (\varphi_m - \varphi)^\top(\varphi_m - \varphi)$$

- **Updating algorithm:** the iterative Newton-Raphson algorithm, using the pseudo-inverse of **S** (the jacobian of *eigs(**p**)* w.r.t. **p**)

$$\mathbf{p}_{i+1} = \mathbf{p}_i + (S_i^\top S_i)^{-1} S_i^\top(\varphi_m - \varphi_i)$$

NOTES ON THE ALGORITHM:

- **Iteration Starting Point:** has to be sufficiently good to avoid diverging.
- **Mode Normalization:** the experimental and theoretical modes have to be normalized the same way in order to be compared.
- **Mode Pairing:** natural frequencies have to be compared with corresponding modes. To ensure correspondence, modes are paired according to the *Modal Assurance Criterion* (MAC). This way, frequencies are compared if their modes are similar according to MAC.
- **Hypotheses validation:** matrix **D** serves for hypotheses validation. Small cosines between $\mathbf{D}\phi_q$ and ϕ_a ensure that ϕ_a lies nearly in the image of ϕ_q and therefore Rigid Body and Translational Joint hypotheses hold.
- **Updating Parameters:** geometrical, mass and, beam and silent block stiffness are used.

UPDATING RESULTS:

$$\omega = (4.58, 7.73, 10.77, 23.03)^\top Hz$$

- **Updated Frequencies:** the frequencies are successfully updated since the model has 14 parameters to update only 4 frequency values.
- **Updated Modes: MAC**

Values near 1 represent good agreement between measured and theoretical results. 0 represents orthogonality between modes.

$$MAC = \begin{pmatrix} 0.96 & 0.10 & 0.01 & 0.07 \\ 0.00 & 0.69 & 0.08 & 0.04 \\ 0.03 & 0.21 & 0.56 & 0.05 \\ 0.00 & 0.01 & 0.35 & 0.84 \end{pmatrix}$$

The identity matrix would be the ideal MAC

CONCLUSIONS:

- A Modal Testing and Analysis has been performed to a Vibrating Table for Food Transportation in order to fit a Multibody model which would predict the behaviour of the system.
- A Coordinates Transformation Matrix has been built in order to establish the kinematic relations between the accelerometers' displacements and the Multibody coordinates **q**. Moreover, this matrix has been useful to validate the Rigid Body and Relative Motion hypotheses.
- A Complex updating procedure could make 2nd and 3rd Mode Updating better, together with a proper silent-blocks and beams damping model.