On the Stability Criteria for Inverter Current Control Loops with LCL Output Filters and Varying Grid Impedance

David Lumbreras, Ernesto L. Barrios, Alfredo Ursúa, Luis Marroyo and Pablo Sanchis
Department of Electrical and Electronic Engineering, Institute of Smart Cities
Public University of Navarra (UPNA)
Campus de Arrosadía
31600 Pamplona, Spain
Email: david.lumbreras@unavarra.es

Acknowledgements
This work has been supported by the Spanish State Research Agency (AEI) and FEDER-UE under grants DPI2013-42853-R, DPI2016-80641-R and by the company Ingeteam Power Technology.

Keywords

Abstract
The use of LC and LCL filters and grid impedance variations are creating new challenges on the controller design for current control loops of photovoltaic and wind turbine inverters. In the design process, stability criteria such as Bode and revised Bode are commonly used. This paper analyses the limitations of Bode and revised Bode criteria to reliably determine stability and proposes a sufficient and necessary stability criterion, based on the Nyquist criterion, but that makes use of the Bode diagram. The proposed criterion, named generalized Bode criterion, is always reliable and helps the controller design. Relative stability in complex control loops is also studied and a relative stability analysis is proposed. Finally, the generalized Bode criterion and the proposed relative stability analysis are illustrated with a practical example in which a PI is designed in order to guarantee stability and achieve relative stability.

Introduction
Due to global warming, environmental concerns and growing consumption of electricity, investigation efforts are increasingly focusing on the renewable-based electricity production systems. Among all, photovoltaic and wind energy are worth to mention, due to their global capacity, current levelized cost of electricity (LCOE) and future perspectives [1]. The increasing share of electricity produced by renewable energies in the electric grid is entailing new challenges and requirements for power electronic converters used in these systems [2].

Considering the injected grid current, severe EMC requirements must be fulfilled. They are forcing power converter manufacturers to use LC and LCL filters instead of first-order filters [3]. Natural response of LC filters have a resonance frequency, and LCL filters include an additional anti-resonance frequency. The frequencies at which these effects happen depend on the values of the filter components. With the increasingly restrictive EMC requirements, bigger filters are needed. As the values of the filter components increase, the resonance and anti-resonance frequencies decrease. These lower resonance frequencies can jeopardize the stability of the injected grid current control loop [4].

One of the main characteristics of photovoltaic systems is their high modularity. It is possible to find them as a single grid-connected converter or as groups of parallelized converters in large power plants. This means that the converter must be designed considering very different scenarios particularly concerning the grid impedance which depends considerably on the characteristics of the grid the converter is connected to [4]. The grid impedance directly influences the resonance and anti-resonance
frequencies of LC and LCL filters. In LC filters, grid impedance transforms the filter into a LCL filter, thus appearing an anti-resonance frequency. In LCL filters, grid impedance is added to the grid-side inductance of the filter. Thus, variations of the grid impedance move the resonance and anti-resonance frequencies making the grid current control loop less stable [5]. Requirements on relative stability can be imposed to overcome this problem.

In order to determine the stability of the current control loop, criteria such as Bode criterion, revised Bode criterion and Nyquist criterion can be used [6]–[8]. Due to its simplicity and applicability to the controller design, Bode criteria are the most commonly used. However, they may not provide reliable stability information about complex control loops [9]. Revised Bode criterion improves the applicability of the Bode criterion but, as it will be shown, it is not always reliable either. On the contrary, Nyquist criterion always provides reliable information. However, the complexity of the Nyquist stability criterion has traditionally made its use in the design of control loops for power converters difficult. The main objective of this paper is to provide controller designers a reliable stability criterion that always ensures stability and helps the controller design task. The proposed criterion, named the generalized Bode criterion, is based on the Nyquist stability criterion but it only needs the use of Bode diagram information. In this way, the stability analysis and the controller design for complex control loops can be easily carried out by means of their Bode representation.

In Section II, an exhaustive review of the classic stability criteria is carried out. In Section III a new stability criterion, named the generalized Bode criterion, is proposed. Section IV discusses how relative stability must be analyzed in complex control loops, and proposes how to implement this analysis with the Bode plot information. Finally, in Section V, the controller design for the injected grid current control loop of a three-phase photovoltaic 100 KVA inverter is carried out in order to show how the proposed stability criterion together with the relative stability analysis can be applied.

Stability Criteria Review: Bode, Revised Bode and Nyquist

In this Section, the different classic stability criteria are studied and compared. The objective of this review is to obtain a necessary and sufficient condition for ensuring stability. First, the different criteria are explained and then a power converter example is showed, which demonstrates that Bode criteria provide wrong stability information.

Nyquist stability criterion

According to Nyquist stability criterion, a control loop is stable if and only if the difference between the number of counterclockwise encirclements, \( N_{ccw} \), and clockwise encirclements, \( N_{cw} \), of the open-loop Nyquist plot around the \((-1,0)\) point is equal to the number of unstable open-loop poles, \( P \), [6], [7]. This statement is expressed as follows:

\[
N = N_{ccw} - N_{cw} = P.
\]  

(1)

An easy procedure for counting the encirclements around the \((-1,0)\) point is to draw a half line starting in the \((-1,0)\) point with any direction as the green one in Fig. 1 (b). The difference between the number of counterclockwise and clockwise crossings between the Nyquist plot and the half line is the same as the difference between the number of counterclockwise and clockwise encirclements around the \((-1,0)\) point [10]. Poles and zeros on the imaginary axis require special attention because they make Nyquist plot to rotate in clockwise direction with infinite gain. This rotation can also produce encirclements around the \((-1,0)\) point that cannot be seen in the Nyquist diagram, as the dotted line in Fig. 1 (b).

Bode stability criterion

According to Bode stability criterion, positive gain and phase margins in the open-loop transfer function are requested for a control loop to be stable. This criterion only provides reliable stability information about simple control loops with single gain and phase crossover frequencies and where \( N = P = 0 \) [6], [7]. However, in complex control loops, where the open-loop has multiple gain or/and phase crossover frequencies or zeros or poles in the right hand plane (RHP), Bode criterion is not reliable.
Revised Bode stability criterion

The revised Bode criterion was proposed to solve applicability limitations of the Bode criterion. According to this criterion, a closed-loop system is stable if the open-loop does not have poles in the RHP, and the frequency response of the open-loop transfer function has an amplitude ratio of less than unity at all frequencies whose phase is \(-180° \pm n \cdot 360°\), with \(n=0, 1, 2\ldots\) [8]. Once again, this criterion is a sufficient but not necessary requirement since it only considers open-loops in which \(N=P=0\), and not the stability condition of (1).

The Bode and Nyquist plots of the example in Fig. 1 correspond to the open-loop transfer function of an inverter current control loop with a PI controller, an output LCL filter and a 2nd order current filter (with one integrator). Regarding Bode criterion, doubts on how to apply the criterion can appear, since it has two gain margins. If it is considered that all of them must be positive, the Bode criterion will determine that the system is unstable because there is one negative gain margin. Regarding the revised Bode criterion, the closed loop will be also determined as unstable because there is a crossing with \(-180°\) with gain higher than 0 dB (i.e. a negative gain margin). However, the Nyquist criterion determines that the system is stable because the open loop transfer function does not have poles in the RHP \((P=0)\), and the number of encirclements is zero (counting the crossings with the auxiliary half line, \(N_{ccw}=1\) and \(N_{cw}=1\)). Thus, \(N=0\). In consequence, \(N=P\), and the stability condition in (1) is fulfilled.

Fig. 1: (a) Bode plot and (b) Nyquist plot of an open-loop with multiple phase crossover frequencies.

The only stability criterion of the above studied that always provides reliable information is the Nyquist criterion. As it was said before, power electronic engineers normally use Bode criteria for the controller design due to their simplicity. In simple control loops they give the same stability conclusions as Nyquist criterion. However, the resonance and anti-resonance peaks of LC and LCL filters (which are also affected by huge variations of the grid impedance), make the current control loop become more complex. Consequently, Bode criteria can lead to wrong stability conclusions.

Proposed Stability Criterion: Generalized Bode Criterion

As Nyquist criterion is the only of the studied criteria which always provides reliable information, the proposed stability criterion must be based on it. Although Nyquist criterion can be applied in the controller design task, it is difficult to obtain relevant design information from it. Fortunately, there is a close relationship between Bode and Nyquist plots [3], [11]–[13]. Both provide the same information with the only difference that, whereas Bode plots are defined only for positive frequencies, Nyquist plots are defined for both positive and negative frequencies. This is not a problem because the Nyquist plots
of transfer functions with real coefficients are symmetrical with respect to the real axis for positive and negative frequencies [6], [7].

The relationship between Nyquist and Bode plots can be used to count directly, from the Bode diagram, the crossings between the Nyquist plot and the half line starting in the (-1,0) point. It is important to remember that the total number of crossings is the same as the total number of encirclements around the (-1,0) point. In order to make the identification of the crossings possible, the half line is placed starting in the (-1,0) point with the negative real axis direction. Thus, a crossing can be identified now in the Bode diagram when the Bode plot has phase equal to ± n·180° with n=1, 3, 5… and gain higher than 0 dB. Due to the symmetry of the Nyquist plot with respect to the real axis, the number of crossings between the Nyquist plot and the half line at positive frequencies will be the same as at negative frequencies. Thus, the total number of crossings can be obtained multiplying by two the number of crossings at positive frequencies. The crossings, which are now easy to identify in the Bode diagrams, can be of two different types: C+ if the phase is going towards higher values, and C- if the phase is going towards lower values. They correspond respectively to counterclockwise and clockwise encirclements of the Nyquist plot around the (-1,0) point. The expression in (1) is now reformulated with this nomenclature and the following expression is obtained:

\[ 2 \left( C^+ - C^- \right) = P. \] (2)

Examples found in the literature [3], [11]–[13] use (2) to check stability. However, this expression cannot be always used because encirclements can exist at zero frequency as it has already been demonstrated. Bode plots do not show this information because zero frequency is not represented. Thus, it is necessary to study them in advance. These encirclements at zero frequency, named from now on C0, depend on the following characteristics of the system under study:

- The number of poles and zeros at the origin in the open loop system transfer function.
- The gain of the system at frequency 0+
- The phase of the system at frequency 0+

Whereas zeros at the origin are not common in power converter control loops, poles at the origin can be widely found due to PI or PID controllers or high order current filters. As a consequence, only transfer functions with poles at the origin will be considered in this paper. The general expression of this type of open-loop transfer functions is:

\[ F(s) = \frac{a_0 s^n + a_{n-1} s^{n-1} + \ldots + a_0}{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0} \] (3)

where \((m+n)>n\). Fig. 2 summarizes different examples of the studied open-loop transfer functions and their crossings at zero frequency. In the examples, little importance has been given to the frequency intervals \((-\infty, 0)\) and \((0^+, \infty)\) because Bode diagram provides information about crossings in those frequency ranges. As the studied transfer functions have more poles than zeros, for infinite frequencies, the Nyquist plot ends at the origin of the complex plane.

When \(\alpha=0\) (i.e. without poles at origin), \(C_0\) depends on the exact value of \(a_0/b_0\), which determines the gain and phase of the Nyquist or Bode plot at frequency 0°. Fig 2 (a) shows an example of these systems. For all other values of \(\alpha\), the number of poles at the origin and the sign of \(a_0/b_0\) determine \(C_0\). When studying the value of \(C_0\), the following statements must be taken into account:

- The number of poles at the origin indicates the rotation between frequencies 0° and 0°. For each pole at the origin, the Nyquist plot rotates 180° in clockwise direction. In addition, each pole at the origin makes a phase contribution of -90° at frequency 0°.
- The sign of \(a_0/b_0\) influences the phase at frequency 0°. If \(a_0/b_0\) is negative, there is an extra contribution of 180° at frequency 0°. If it is positive, there is none extra contribution.

An approximation of the phase at frequency 0° can be calculated applying the previous statements. With the approximation of the phase at frequency 0° and the rotation between frequencies 0° and 0°, \(C_0\) can be easily obtained. However, the calculated phase at frequency 0° is an approximate value because the
Nyquist or Bode plot starts tangentially from it. In fact, the Bode plot can start tangentially from above or below the calculated value. As it can be seen in Fig. 2 (c) and Fig. 2 (d), this fact implies special attention when the calculated phase at frequency 0° is \( \pm n \cdot 180° \), with \( n = 1, 3, 5 \ldots \) since it changes \( C_0 \). In the rest of cases, it does not affect \( C_0 \). The example in Fig. 1 corresponds to the same system as the one shown in Fig. 2 (d).

With the presented analysis, it is demonstrated that \( C_0 \) is not always 0. The proposed stability criterion, named “Generalized Bode criterion”, can be used to determine the stability for any control loop with more poles than zeros and without zeros at the origin. The Generalized Bode criterion is expressed as follows:

\[
2 \cdot \left( C^+ - C^- \right) + C_0 = P
\]

where \( P \) is the number of poles of the open-loop in the RHP, \( C^+ \) is the number of crossings counted in Bode plot when the phase tends towards higher values, \( C^- \) is the number of crossings counted in Bode plot when the phase tends towards lower values and \( C_0 \) is the number of encirclements between frequencies 0° and 0°. It is necessary to remember that a crossing is counted in Bode when the Bode plot has phase equal to \( \pm n \cdot 180° \) with \( n = 1, 3, 5 \ldots \) and gain higher than 0 dB. Table I summarizes the values of \( C_0 \) depending on the number of poles at the origin, the gain of the system at frequency 0° and the phase of the system at frequency 0°. These values have been obtained by inspection for systems with more poles than zeros, with no zeros at the origin and a maximum of three poles at origin.

The example presented in Fig. 1 can be now analyzed with the proposed stability criterion. The open loop transfer function has 2 poles at the origin as it was said before. Analyzing the Bode plot at low frequencies, the starting Bode phase is lower than 180°. Thus, according to Table I, \( C_0 = -2 \). As the open-loop transfer function has no poles in the RHP, \( P = 0 \). From the Bode diagram, \( C^+ = 1 \) and \( C^- = 0 \). Therefore, (4) is satisfied and the test correctly indicates that the closed loop system is stable as the Nyquist criterion indicates.
Relative Stability in complex control loops

With the proposed stability criterion, system stability is guaranteed but relative stability is unknown. For simple control loops, gain and phase margins are widely used as indicators of relative stability [6], [7]. However, in so doing, only two single frequencies of the Nyquist plot are being considered. There are systems with high gain and phase margins whose Nyquist plot between the gain and the phase crossover frequencies gets close to the (-1,0) point. In consequence, parameter variations can move the Nyquist plot of these systems changing the number of encirclements around the (-1,0) and making the system unstable. In addition, there are also systems with multiple gain and phase margins. In such cases, designers have often not very clear whether all of them must be considered or not. Control loops with LCL filters are examples of both type of systems. Thus, other indicators of relative stability must be used in complex control loops.

A robust indicator of relative stability is the maximum value of the closed loop sensitivity function, named from now on \( M_s \). The inverse of this value is the minimum distance from the (-1,0) point to the Nyquist plot [6], [9], [14]. A limit in the value of \( M_s \) implies a forbidden area in the complex plane. As shown in Fig. 4 (a), this forbidden area is a circle of radius \( 1/M_s \) centered in the (-1,0) point. The Nyquist plot of the system open loop transfer function for all frequencies cannot enter the forbidden circle if the considered relative stability is desired. This indicator is more robust than gain and phase margins since all frequencies are analyzed. There exists a relationship between \( M_s \) and gain and phase margins [14].

Typical restrictions of \( M_s \) are between 2 and 1.4 which correspond to minimum phase margins of 29° and 41° respectively [14].

As the proposed stability criterion uses the Bode diagram, the restrictions in the value of \( M_s \) must be analyzed in the Bode diagram in order to be useful. The belonging of a point of the system to the forbidden region can be determined by means of the circle equation. However, this condition cannot be easily expressed in a Bode diagram. In this work, in order to make the analysis of the relative stability easier in the controller design procedure, the following limits of gain and phase are proposed:

\[
G_{\text{MAX}}(\text{dB}) = 20 \cdot \log_{10} \left( 1 + \frac{1}{M_s} \right) \quad ; \quad G_{\text{MIN}}(\text{dB}) = 20 \cdot \log_{10} \left( 1 - \frac{1}{M_s} \right)
\]  

(5)

\[
\phi_{\text{MAX}}(\circ) = 180^\circ + \sin^{-1} \left( \frac{1}{M_s} \right) \quad ; \quad \phi_{\text{MIN}}(\circ) = 180^\circ - \sin^{-1} \left( \frac{1}{M_s} \right)
\]  

(6)
These limits are easily expressed by means of forbidden regions in the Bode diagram as it is shown in Fig. 3 (b). When the open loop transfer function has, for a single frequency, both gain and phase inside these areas in the Bode diagram, relative stability is not achieved. As it can be seen in Fig. 3 (a), although all the points that belong to the circle have gain and phase inside the limits proposed in (5) and (6), there are points outside the circle that fulfill the proposed limits as well. Thus, it can be concluded that the proposed forbidden regions in the Bode diagrams are more restrictive than the forbidden circle in the Nyquist diagram, ensuring this way relative stability.

**Case study: PI Controller Design**

Once the Generalized Bode criterion and the relative stability condition are proposed, their application is demonstrated by means of the PI controller design procedure for the output current control loop of the photovoltaic inverter shown in Fig. 4.

In Fig. 5 the output-current control loop of one of the phases is represented. The digitalization effect approximation is taken from [15]. The current filter is a first order filter. The transfer function of the LCL filter is obtained with the relationship between $\frac{1}{M_{cl}}$ and $\frac{1}{S_{cl}}$, assuming that current is measured in $\frac{E}{2}$ [15]. For the controller design, 5 different scenarios regarding the power of the substation transformer (and therefore the grid impedance) are considered. They take into account different locations in which the inverter can be installed.

**Fig. 3. Proposed forbidden areas used to ensure relative stability in (a) Nyquist diagram and (b) Bode diagram**

**Fig. 4: Studied inverter with LCL filter and its characteristics**
In order to have a first design, an initial PI is calculated for a phase margin of 30° and bandwidth of 500 Hz for the higher grid impedance (is the worst design scenario as later will be demonstrated). The open-loop Bode plots for this first PI design and the 5 different studied scenarios are given in Fig. 6 (a).

Stability is now studied with the proposed stability criterion. For all scenarios, the open-loop transfer function has one pole at the origin and starting Bode phase equal to -90° (i.e. 270°). Thus, according to Table I, \( C_0 = 0 \). As the open-loop does not have poles in the RHP in any scenario (\( P = 0 \)), the closed loop will be stable if the number of positive and negative crossings counted in Bode is the same, i.e. \( C^+ = C^- \).

In all scenarios of Fig. 6 (a), the phase crosses the -180° after the resonance peak. As the gain is higher than 0 dB at the crossing frequency, a negative crossing is counted. As there are no positive crossings, with this PI controller all the closed loop systems will be unstable. Thus, the first PI design needs to be changed in order to assure stability. Studying the different Bode plots, it can be seen that it is not possible to have a positive crossing without having a negative one that cancels it. Therefore, in order to avoid the negative crossing and to make the control loop stable with a PI controller, the only option is to make the gain at the frequencies that cross -180° lower than 0 dB. As \( T_n \) has little influence at high frequencies, \( K_p \) is the parameter to be modified. As shown in Fig. 6 (b), \( K_p \) is reduced until the -180° crossing frequencies have gain below 0 dB for all scenarios.

Once stability is ensured, relative stability must be analyzed. In this example, \( Ms = 2 \) is considered. As it can be seen in Fig. 6 (b), the problematic frequencies for stability are also problematic for relative stability because they have gain and phase at the same time inside the forbidden areas. Thus, either their gain or their phase has to be changed. For this purpose, the gain is reduced below \( G_{MIN} \) by reducing the \( K_p \) value, obtaining the results showed in Fig. 7 (a).

Although the resonance frequencies around the -180° crossings are no longer a problem, frequencies around the first 0 dB crossing for the higher grid impedance scenarios are still problematic for relative stability. In fact, the highest grid impedance is the one with more points that have gain and phase inside the forbidden areas. This is the reason why the highest grid impedance is the worst design scenario. The gain or phase of these problematic frequencies have to be changed. The phase is chosen here in order to avoid the deterioration of the relative stability at the resonance frequencies if gain is increased or to do the system slower if gain is decreased. In order to change the phase at these frequencies, modifying the \( T_n \) value is the best option. Thus, \( T_n \) is increased until there are no points inside the forbidden areas for all the different scenarios as shown in Fig. 7 (b). This way, the desired relative stability is finally achieved and the final PI design is obtained.

Although a broad range of grid impedance values have been considered, variations of the power converter components (i.e. \( L_1, L_2 \) and \( C \) values variations due tolerances and/or aging) have not been studied in detail. With the considered relative stability, they will not alter the studied system stability. However, if it is considered that these variations must be studied, the worst scenario of the filter values will be identified and the same methodology to ensure relative stability will be used.
It has been shown that thanks to the proposed Generalized Bode criterion and to the proposed relative stability analysis, the PI design procedure can be carried out in a reliable way by means of only using Bode diagram information even when complex systems are studied.

Finally, it is also interesting to highlight that, thanks to the proposed stability criterion, the use of extended rules of thumb in the controller design procedure can be avoided. A common rule of thumb used in practical design procedures when a non-damped resonant system is considered, is to decrease the resonant peak below 0 dB [16]–[18] to ensure no C-crossings. It can be done by means of passive or active damping [16], or also reducing the gain of the whole system by means of the controller parameters [17], [18]. All of them have negative consequences as the increase of the power loss or the implementation complexity or the decrease of the system bandwidth. However, as shown in Fig. 7 (b), in the studied example it is demonstrated that it is not necessary to decrease the resonant peak below 0 dB to ensure stability and achieve relative stability. This way, thanks to the proposed stability criterion, there is no need for passive or active damping and a high bandwidth can be achieved.
Conclusions

In this paper, classic stability criteria are analyzed. A stability criterion named Generalized Bode criterion is proposed. It is always reliable, and solves the applicability problems of Nyquist criterion in the controller design procedure. It is also demonstrated that the maximum value of the sensitivity function is a more robust relative stability constraint than gain and phase margins. Thus, a relative stability analysis based on the sensitivity function is proposed. The final example shows an easy-to-use PI controller design procedure for the output current control loop of a 100 kVA photovoltaic inverter based on the proposed stability criterion and the proposed relative stability analysis. The example also demonstrates that it is not necessary to decrease the resonant peak below 0 dB to achieve stability, which is commonly used as a rule of thumb in the controller design procedure.

References