Appendix A Supplementary material

This Supplementary Material contains the following tables and figures to complement the paper "Evaluating recent methods to overcome spatial confounding".

1. Figure A1: Boxplots of the estimated means of β_1 based on 100 simulated datasets for Simulation study 1, Scenario 2 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5, and 0.2.
2. Figure A2: Boxplots of the estimated means of β_1 based on 100 simulated datasets for Simulation study 1, Scenario 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5, and 0.2.
3. Table A1: Average value of mean absolute relative bias (MARB) and mean relative root mean prediction error (MRRMSE) of β_1 based on 100 simulated data sets for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.
4. Table A2: Length of the 95% credible intervals of β_1 for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.
5. Table A3: WAIC values based on 100 simulated data sets for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.
6. Table A4: Average value of mean absolute relative bias (MARB) and mean relative root mean prediction error (MRRMSE) of the relative risks based on 100 simulated data sets for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.
7. Figure A3: Boxplots of the estimated means of β_2 based on 100 simulated datasets for Simulation study 2, Scenario 1 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5, and 0.2.
8. Figure A4: Boxplots of the estimated means of β_2 based on 100 simulated datasets for Simulation study 2, Scenario 2 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5, and 0.2.
9. Figure A5: Boxplots of the estimated means of β_2 based on 100 simulated datasets for Simulation study 2, Scenario 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5, and 0.2.

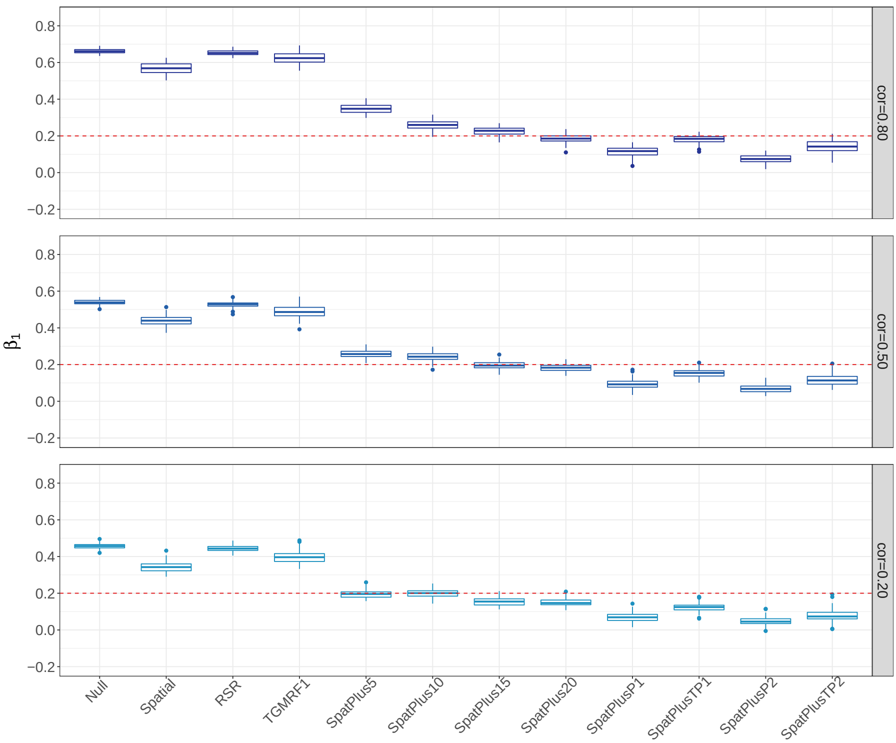

Fig. A1 Boxplots of the estimated means of β_1 based on 100 simulated datasets for Simulation study 1, Scenario 2 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$ (top row), 0.5 (middle row), and 0.2 (bottom row).

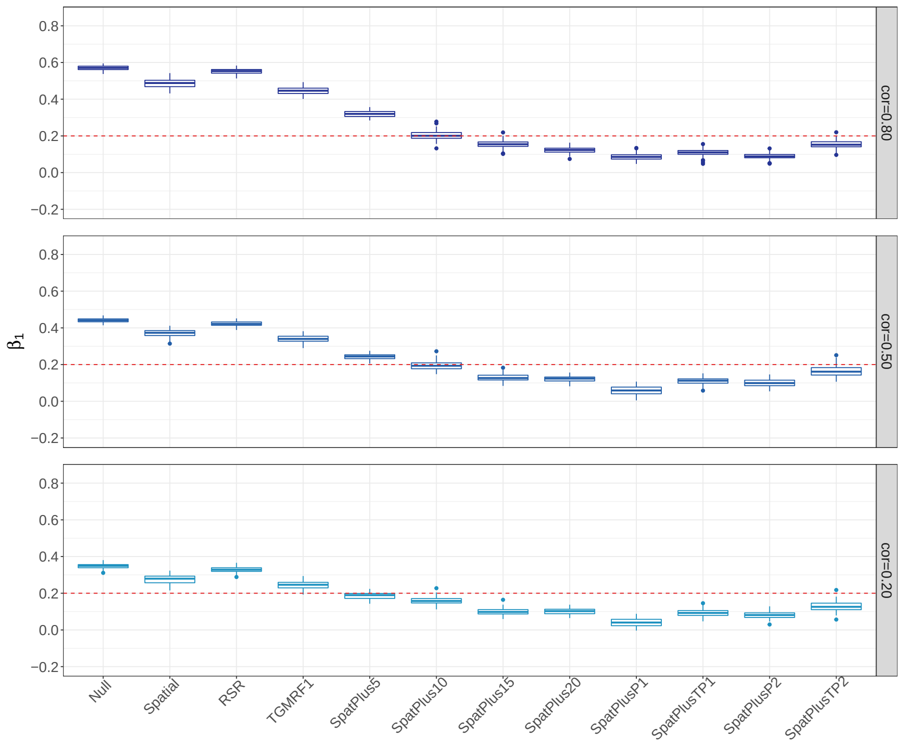

Fig. A2 Boxplots of the estimated means of β_1 based on 100 simulated datasets for Simulation study 1, Scenario 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$ (top row), 0.5 (middle row), and 0.2 (bottom row).

Table A1 Average value of mean absolute relative bias (MARB) and mean relative root mean prediction error (MRRMSE) of β_1 based on 100 simulated data sets for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.

		cor=0.80		**cor=0.50**		**cor=0.20**	
	Model	**MARB**	**MRRMSE**	**MARB**	**MRRMSE**	**MARB**	**MRRMSE**
Scenario 1	Null	1.2755	1.2783	0.6387	0.6425	0.1763	0.1906
	Spatial	1.3014	1.3080	0.6199	0.6326	0.1495	0.2099
	RSR	1.2845	1.2873	0.6387	0.6428	0.1714	0.1864
	TGRMF1	1.2229	1.2266	0.6916	0.6957	0.2301	0.2451
	SpatPlus5	0.4188	0.4324	0.0302	0.1017	0.3203	0.3370
	SpatPlus10	0.1422	0.1855	0.0399	0.1100	0.2786	0.2985
	SpatPlus15	0.0694	0.1306	0.2819	0.2987	0.4857	0.4965
	SpatPlus20	0.2572	0.2775	0.3056	0.3214	0.4613	0.4720
	SpatPlusP1	0.4115	0.4245	0.5070	0.5170	0.6186	0.6272
	SpatPlusTP1	0.2106	0.2340	0.3252	0.3384	0.4728	0.4823
	SpatPlusP2	0.4129	0.4240	0.5278	0.5374	0.6375	0.6463
	SpatPlusTP2	0.0990	0.1496	0.3293	0.3462	0.5024	0.5161
Scenario 2	Null	2.3062	2.3069	1.6992	1.7004	1.2820	1.2838
	Spatial	1.8413	1.8477	1.1947	1.2020	0.7198	0.7344
	RSR	2.2638	2.2647	1.6364	1.6382	1.2218	1.2246
	TGRMF1	2.1223	2.1278	1.4376	1.4476	0.9825	0.9956
	SpatPlus5	0.7342	0.7448	0.2908	0.3078	0.0231	0.1087
	SpatPlus10	0.2896	0.3214	0.2146	0.2480	0.0005	0.1208
	SpatPlus15	0.1286	0.1736	0.0208	0.1162	0.2269	0.2527
	SpatPlus20	0.0742	0.1387	0.0882	0.1384	0.2524	0.2736
	SpatPlusP1	0.4297	0.4476	0.5353	0.5491	0.6549	0.6665
	SpatPlusTP1	0.0944	0.1442	0.2271	0.2542	0.3801	0.3967
	SpatPlusP2	0.6247	0.6328	0.6542	0.6631	0.7654	0.7736
	SpatPlusTP2	0.2886	0.3325	0.4186	0.4481	0.6123	0.6342
Scenario 3	Null	1.8531	1.8541	1.2042	1.2055	0.7380	0.7411
	Spatial	1.4332	1.4384	0.8592	0.8667	0.3846	0.4034
	RSR	1.7601	1.7615	1.1158	1.1178	0.6421	0.6467
	TGRMF1	1.2304	1.2352	0.6984	0.7059	0.2248	0.2493
	SpatPlus5	0.6002	0.6066	0.2129	0.2291	0.0630	0.1049
	SpatPlus10	0.0098	0.1186	0.0236	0.1227	0.2035	0.2287
	SpatPlus15	0.2274	0.2485	0.3499	0.3620	0.5056	0.5140
	SpatPlus20	0.3898	0.3985	0.3924	0.4009	0.5008	0.5078
	SpatPlusP1	0.5643	0.5720	0.7107	0.7194	0.7986	0.8060
	SpatPlusTP1	0.4572	0.4681	0.4465	0.4566	0.5433	0.5520
	SpatPlusP2	0.5550	0.5606	0.4987	0.5090	0.5927	0.5992
	SpatPlusTP2	0.2370	0.2622	0.1763	0.2308	0.3560	0.3778

Table A2 Length of the 95% credible intervals of β_1 for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.

	Model	**cor=0.80**	**cor=0.50**	**cor=0.20**
Scenario 1	Null	0.0628	0.0659	0.0686
	Spatial	0.1776	0.2343	0.2626
	RSR	0.0629	0.0672	0.0699
	TGRMF1	0.1220	0.1769	0.1983
	SpatPlus5	0.1485	0.1606	0.1692
	SpatPlus10	0.1726	0.1568	0.1588
	SpatPlus15	0.1614	0.1519	0.1510
	SpatPlus20	0.1577	0.1420	0.1413
	SpatPlusP1	0.1518	0.1423	0.1404
	SpatPlusTP1	0.1612	0.1490	0.1480
	SpatPlusP2	0.0810	0.0837	0.0857
	SpatPlusTP2	0.1028	0.1052	0.1108
Scenario 2	Null	0.0567	0.0584	0.0603
	Spatial	0.3289	0.3548	0.3738
	RSR	0.0598	0.0628	0.0648
	TGRMF1	0.3269	0.3704	0.3902
	SpatPlus5	0.2470	0.2470	0.2483
	SpatPlus10	0.2752	0.2527	0.2461
	SpatPlus15	0.2375	0.2208	0.2161
	SpatPlus20	0.2243	0.2051	0.2009
	SpatPlusP1	0.2117	0.1990	0.1955
	SpatPlusTP1	0.2352	0.2176	0.2131
	SpatPlusP2	0.0920	0.0901	0.0919
	SpatPlusTP2	0.1542	0.1470	0.1492
Scenario 3	Null	0.0487	0.0513	0.0531
	Spatial	0.3496	0.3646	0.3799
	RSR	0.0494	0.0524	0.0546
	TGRMF1	0.3105	0.3153	0.3265
	SpatPlus5	0.2434	0.2415	0.2444
	SpatPlus10	0.2815	0.2532	0.2479
	SpatPlus15	0.2501	0.2327	0.2264
	SpatPlus20	0.2228	0.2090	0.2070
	SpatPlusP1	0.2068	0.1973	0.1944
	SpatPlusTP1	0.2396	0.2282	0.2225
	SpatPlusP2	0.0665	0.0704	0.0727
	SpatPlusTP2	0.1025	0.1106	0.1160

Table A3 WAIC based on 100 simulated data sets for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.

		cor=0.80	**cor=0.50**	**cor=0.20**
	Model	**WAIC**	**WAIC**	**WAIC**
Scenario 1	Null	511.1920	570.7955	603.5141
	Spatial	478.3038	481.9702	483.7083
	RSR	478.2911	481.9354	483.6587
	TGRMF1	469.1397	477.0736	479.0910
	SpatPlus5	467.5615	477.8921	482.1455
	SpatPlus10	468.1339	476.7807	482.0877
	SpatPlus15	467.8642	475.6478	480.7339
	SpatPlus20	468.5505	476.1541	481.3502
	SpatPlusP1	469.7314	478.7731	483.4565
	SpatPlusTP1	467.7620	476.4177	481.5442
	SpatPlusP2	512.7174	525.2011	538.8787
	SpatPlusTP2	512.7815	525.8824	539.8822
Scenario 2	Null	909.9282	1132.7212	1169.1424
	Spatial	479.9236	476.5743	477.5975
	RSR	479.8296	476.4710	477.4913
	TGRMF1	477.6394	477.4624	478.2152
	SpatPlus5	476.1727	474.9490	476.6245
	SpatPlus10	475.3869	474.8285	476.6761
	SpatPlus15	474.7964	474.5057	476.4105
	SpatPlus20	474.4604	473.8010	475.8573
	SpatPlusP1	474.7412	475.4856	477.2628
	SpatPlusTP1	474.6742	474.9620	476.7513
	SpatPlusP2	547.6223	590.7540	597.2066
	SpatPlusTP2	551.8037	592.1867	597.5115
Scenario 3	Null	1994.5542	1882.3481	1881.4471
	Spatial	488.7525	491.5679	493.3131
	RSR	488.6512	491.4571	493.1944
	TGRMF1	489.1019	491.3708	493.0371
	SpatPlus5	487.9618	491.2215	493.2403
	SpatPlus10	488.1447	490.5906	492.7840
	SpatPlus15	487.9912	490.4640	492.6054
	SpatPlus20	487.7714	490.4205	492.5961
	SpatPlusP1	488.2181	491.2471	493.1762
	SpatPlusTP1	488.2595	490.6967	492.7421
	SpatPlusP2	525.0075	553.3336	567.9812
	SpatPlusTP2	528.1840	556.9639	570.4615

Table A4 Average value of mean absolute relative bias (MARB) and mean relative root mean prediction error (MRRMSE) of the relative risks based on 100 simulated data sets for Simulation Study 1, Scenarios 1, 2 and 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$, 0.5 and 0.2.

		cor=0.80		**cor=0.50**		**cor=0.20**	
	Model	**MARB**	**MRRMSE**	**MARB**	**MRRMSE**	**MARB**	**MRRMSE**
Scenario 1	Null	0.1506	0.1574	0.2280	0.2321	0.2622	0.2661
	Spatial	0.0934	0.1373	0.0918	0.1561	0.0905	0.1636
	RSR	0.0934	0.1373	0.0918	0.1561	0.0905	0.1636
	TGRMF1	0.0843	0.1309	0.0899	0.1550	0.0883	0.1636
	SpatPlus5	0.0708	0.1397	0.0833	0.1559	0.0873	0.1636
	SpatPlus10	0.0676	0.1525	0.0793	0.1538	0.0861	0.1616
	SpatPlus15	0.0606	0.1527	0.0752	0.1567	0.0836	0.1634
	SpatPlus20	0.0586	0.1540	0.0780	0.1577	0.0872	0.1644
	SpatPlusP1	0.0571	0.1558	0.0774	0.1637	0.0880	0.1687
	SpatPlusTP1	0.0591	0.1539	0.0776	0.1592	0.0868	0.1651
	SpatPlusP2	0.1078	0.1672	0.1268	0.1836	0.1404	0.1963
	SpatPlusTP2	0.1150	0.1666	0.1307	0.1830	0.1417	0.1952
Scenario 2	Null	0.3893	0.3928	0.4491	0.4512	0.4736	0.4759
	Spatial	0.0795	0.1744	0.0634	0.1682	0.0645	0.1705
	RSR	0.0795	0.1744	0.0634	0.1682	0.0645	0.1705
	TGRMF1	0.0731	0.1782	0.0571	0.1744	0.0563	0.1762
	SpatPlus5	0.0696	0.1751	0.0591	0.1697	0.0620	0.1713
	SpatPlus10	0.0649	0.1791	0.0601	0.1709	0.0639	0.1716
	SpatPlus15	0.0644	0.1777	0.0573	0.1709	0.0610	0.1716
	SpatPlus20	0.0597	0.1778	0.0555	0.1702	0.0611	0.1711
	SpatPlusP1	0.0580	0.1792	0.0554	0.1754	0.0605	0.1751
	SpatPlusTP1	0.0606	0.1790	0.0568	0.1732	0.0614	0.1733
	SpatPlusP2	0.1100	0.1986	0.1370	0.2122	0.1454	0.2205
	SpatPlusTP2	0.1134	0.1993	0.1376	0.2120	0.1457	0.2203
Scenario 3	Null	0.6622	0.6653	0.6858	0.6881	0.7195	0.7217
	Spatial	0.0514	0.1581	0.0549	0.1610	0.0562	0.1620
	RSR	0.0514	0.1581	0.0549	0.1610	0.0562	0.1620
	TGRMF1	0.0497	0.1640	0.0529	0.1667	0.0536	0.1680
	SpatPlus5	0.0491	0.1579	0.0537	0.1607	0.0558	0.1616
	SpatPlus10	0.0409	0.1619	0.0488	0.1616	0.0526	0.1614
	SpatPlus15	0.0399	0.1622	0.0479	0.1621	0.0525	0.1621
	SpatPlus20	0.0407	0.1623	0.0478	0.1619	0.0532	0.1624
	SpatPlusP1	0.0387	0.1624	0.0472	0.1631	0.0525	0.1631
	SpatPlusTP1	0.0410	0.1632	0.0482	0.1627	0.0531	0.1627
	SpatPlusP2	0.0904	0.1611	0.1025	0.1703	0.1141	0.1791
	SpatPlusTP2	0.0963	0.1613	0.1081	0.1713	0.1167	0.1795

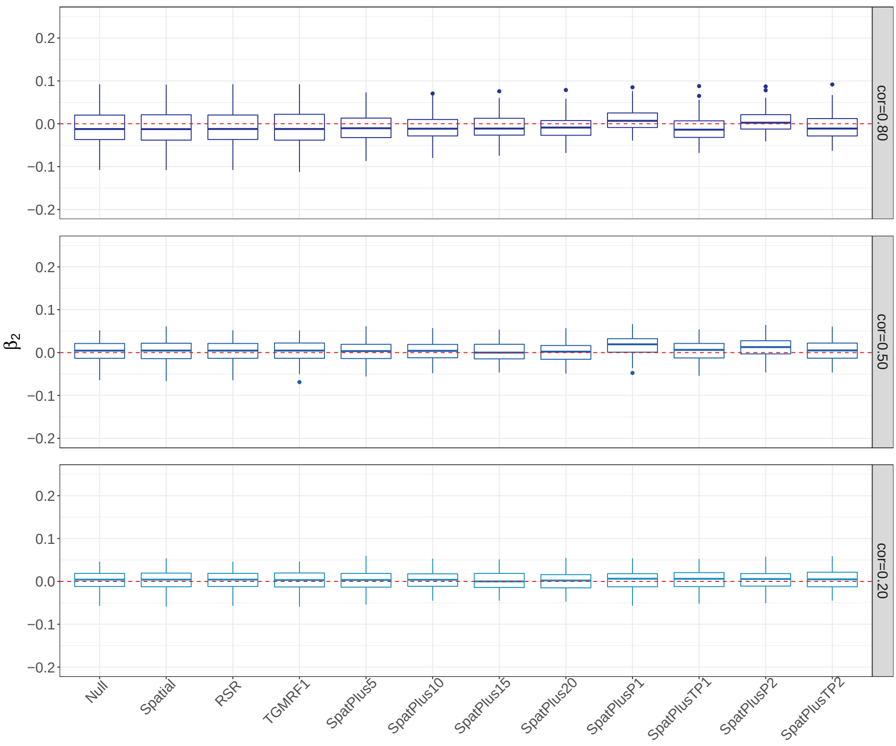

Fig. A3 Boxplots of the estimated means of β_2 based on 100 simulated datasets for Simulation study 2, Scenario 1 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$ (top row), 0.5 (middle row), and 0.2 (bottom row).

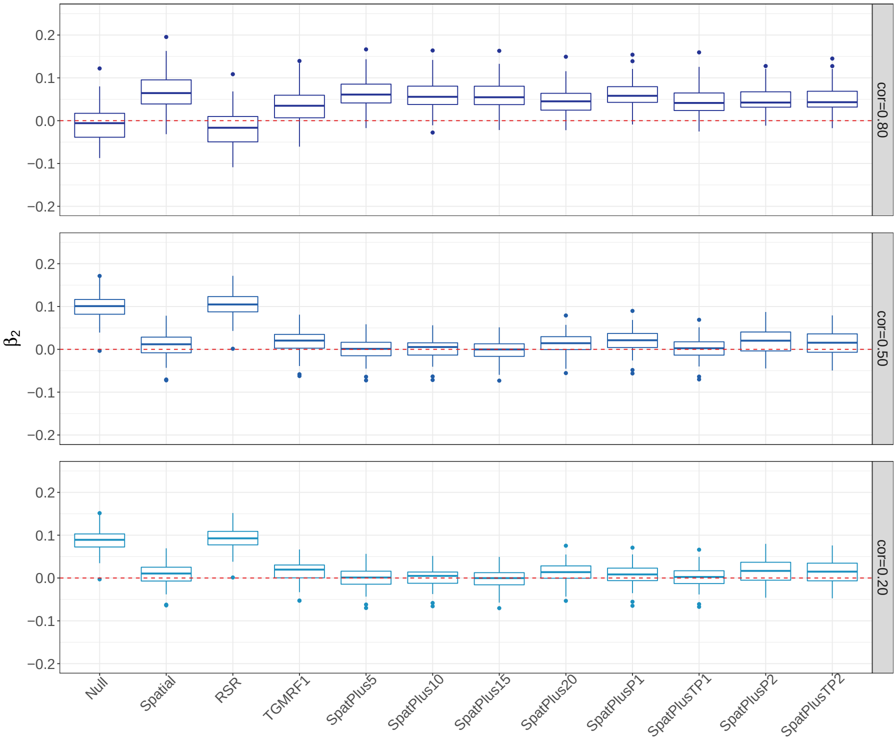

Fig. A4 Boxplots of the estimated means of β_2 based on 100 simulated datasets for Simulation study 2, Scenario 2 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$ (top row), 0.5 (middle row), and 0.2 (bottom row).

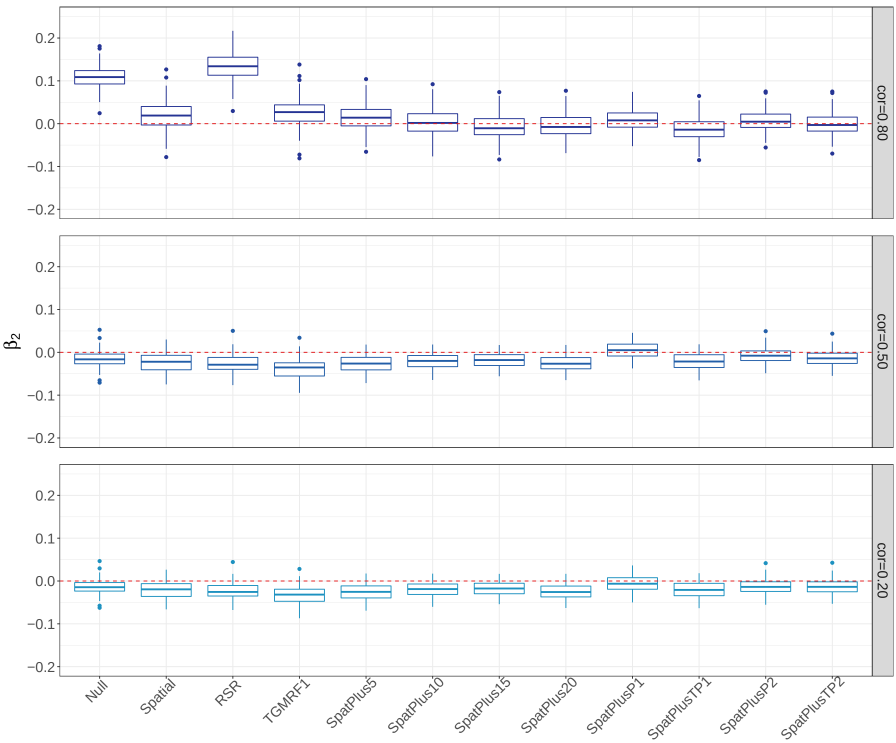

Fig. A5 Boxplots of the estimated means of β_2 based on 100 simulated datasets for Simulation study 2, Scenario 3 and $cor(\boldsymbol{X}_1, \boldsymbol{X}_2) = 0.8$ (top row), 0.5 (middle row), and 0.2 (bottom row).

Appendix B TGMRF

This appendix contains the details about the marginal distributions $\boldsymbol{F} = (F_1, F_2, ..., F_n)^{'}$ chosen for $\boldsymbol{r} = (r_1, r_2, ..., r_n)^{'}$ and the correlation matrix $\boldsymbol{\Omega}$ that determines the spatial dependence structure in TGMRF model (Prates et al, 2015).

Any continuous distribution can be chosen as a marginal distribution of the relative risks. For instance, in this work a gamma distribution is chosen. The covariates can be incorporated either into the shape or scale parameter leading to two different gamma marginal distributions. If the covariates are included in the scale parameter,

$$r_i \sim \Gamma(1/\upsilon, \upsilon \exp(\boldsymbol{X}_{i,\cdot}\, \boldsymbol{\beta}))$$

where $\upsilon > 0$ and $\boldsymbol{X}_{i,\cdot}$ is the ith row of the observed covariates matrix $\boldsymbol{X}$. In contrast, if the covariates are included in the shape parameter,

$$r_i \sim \Gamma(\exp(\boldsymbol{X}_{i,\cdot}\, \boldsymbol{\beta})/\upsilon, \upsilon).$$

Here, we consider the following priors: $\upsilon \sim \Gamma(0.01, 0.01)$ and $\beta_i \sim N(0, \tau = 0.001)$, $j = 1, \ldots, p$. As initial values for the MCMC algorithm we chose $\upsilon = 1$ and $\beta_1 = \beta_2 = ... = \beta_p = 0$.

An equivalent expression of the TGMRF method introduced in (7) is

$$\boldsymbol{r} \sim TGMRF(\boldsymbol{F}, \boldsymbol{Q}_*) \tag{B1}$$

which is obtained replacing the correlation matrix $\boldsymbol{\Omega}$ that determines the spatial dependence structure in the Gaussian copula by a precision matrix $\boldsymbol{Q}_*$. The precision matrix $\boldsymbol{Q}_*$ must lead to a valid correlation matrix so that $\boldsymbol{\Omega} = \boldsymbol{Q}_*^{-1}$.

The precision matrix of the Gaussian copula, $\boldsymbol{Q}_*$, is based on the precision matrix $\boldsymbol{D} - \rho\boldsymbol{M}$ of a proper conditional autorregresive (CAR) distribution. In this case, $\boldsymbol{D}$ is a diagonal matrix where the diagonal entries d_{ii} are equal to the number of neighbours of the ith area and $\boldsymbol{M}$ is a neighbourhood matrix with non-diagonal elements $m_{ij} = 1$ if areas i and j are neighbours and 0 otherwise. Note that $(\boldsymbol{D} - \rho\boldsymbol{M})^{-1}$ is not a correlation matrix as its diagonal elements are not equal to one. Therefore, since copulas are scale invariant, $\boldsymbol{Q}_*$ is scaled as follows

$$\boldsymbol{Q}_* = \boldsymbol{\Lambda}^{1/2}(\boldsymbol{D} - \rho\boldsymbol{M})\boldsymbol{\Lambda}^{1/2},$$

where $\boldsymbol{\Lambda} = diag(\lambda_1^2, \lambda_2^2, ..., \lambda_n^2)$ and λ_i^2 is the ith diagonal element of $(\boldsymbol{D} - \rho\boldsymbol{M})^{-1}$ for $i = 1, 2, ..., n$. Then $\boldsymbol{\Omega} = \boldsymbol{Q}_*^{-1}$ is a correlation matrix and (B1) is an equivalent way of expressing (7). Here ρ follows a standard uniform distribution.