Geometric locus associated with thriedra axonometric projections. Intrinsic curve associated with the ellipse generated.

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Abstract. In previous work on the axonometric perspective, the authors presented some graphic constructions that allowed a single and joint invariant description of the relations between an orthogonal axonometric system, its related orthogonal views, and oblique axonometric systems associated with it. Continuing this work and using only the items drawn on the frame plane, in this communication we start from the three segments, representing trirectangular unitary thriedra, joined in the origin and defining an axonometric perspective. Each is projected onto any direction and the square root of the summa of the squares of these projections is determined. We call this magnitude, orthoedro diagonal whose sides would be formed by the three projections axonometric unit segments. If the diagonal size is built from the origin of coordinates and onto the direction used, this describes a locus here called intrinsic curve associated with the ellipse. When the starting three segments represent an orthogonal axonometric perspective, the intrinsic curve associated with the ellipse is a circle.

Keywords: Axonometric system, Descriptive geometry, Intrinsic curve the ellipse.

1 Introduction

This work is part of a research associated with graphical representation issues. In the article *Main axonometric systemrelated views as tilt of the coordinate plans* [1] orthogonal views related to the orthogonal axonometry is determined. In *New constructions in axonometric fundamentals system* [2] this study was extended, presenting new construction operations perspective from the peculiar arrangement of the ortogonal views. In the communication *Intrinsic relations between the orthogonal axonometric system and Its Associated obliques. Analytical proposal and graphic operations* [3] was pretended to extend this approach, that puts parallel analytical and constructive aspects of axonométric system, from the orthogonal to the oblique, and synthesize as much as possible their algebraic expression and its trace. The intrinsic axonometric triangle was defined and strengthened their geometric properties. projective and metric relations on studied figures were used and new axonometric constructions were developed.

In this communication, a geometric locus here called intrinsic associated curve with the ellipse, defined by the projection of the three segments that concrets an axonometric perspective on any direction contained in the picture plane is presented.

2 Ortogonal and oblique axonometric system

An ortogonal thriedra of reference is chosen $Oxyz$. Taken the vertex O on the straight lines x, y and z respectively, a unit magnitud u, points $\mathbf{I}$, $\mathbf{J}$ and $\mathbf{K}$ are determined. Coordinates of these points are:

$$O=[0,0,0];\ \mathbf{I}=[u,0,0];\ \mathbf{J}=[0,u,0];\ \mathbf{K}=[0,0,u] \tag{1}$$

The normalized equation of the chosen projection plane π_p, that contains the vertex O, is:

$$\pi_p \equiv a_x x + a_y y + a_z z = 0 \tag{2}$$

Where the ortogonal projected direction is $d_\pi \equiv [a_x, a_y, a_z]$, being $\sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = u$.

The ortogonal projection of the points $\mathbf{I}$, $\mathbf{J}$ and $\mathbf{K}$ on the frame plane, determines points I, J and K which coordinates are:

$$\begin{aligned} I &= \left[u^2 - a_x{}^2, -a_x a_y, -a_x a_z\right]/u \\ J &= \left[-a_x a_y, u^2 - a_y{}^2, -a_y a_z\right]/u \\ K &= \left[-a_x a_z, -a_y a_z, u^2 - a_z{}^2\right]/u \end{aligned} \tag{3}$$

If the axis of the reference thriedra are projected $Oxyz$ over the frame plane π_p on whatever direction d_o an oblique axonometric perspective is obtained. This oblique projective direction is $d_o \equiv [a_{xo}, a_{yo}, a_{zo}]$, verifying $\sqrt{a_x a_{xo} + a_y a_{yo} + a_z a_{zo}} = u$.

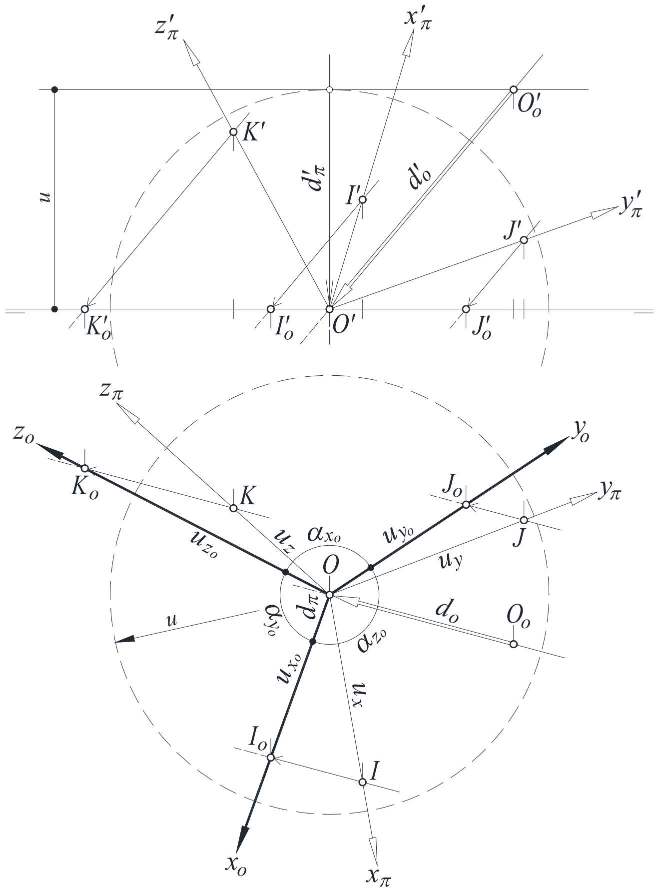

Fig. 1. Axonometric system.

The oblique projection of the points $\mathbf{I}$, $\mathbf{J}$ and $\mathbf{K}$ are I_o, J_o and K_o. Its co-ordinates are noted as:

$$
\begin{aligned}
I_o &= \left[u^2 - a_x a_{xo}, -a_x a_{yo}, -a_x a_{zo}\right] / u \\
J_o &= \left[-a_y a_{xo}, u^2 - a_y a_{yo}, -a_y a_{oz}\right] / u \\
K_o &= \left[-a_z a_{xo}, -a_z a_{yo}, u^2 - a_z a_{zo}\right] / u
\end{aligned}
\tag{4}
$$

Axonometric oblique scales can be written as:

$$
\begin{aligned}
\left|\overline{OI_o}\right| &= u_{xo} = \sqrt{u^2 + a_x{}^2 e_o{}^2 - 2a_x a_{xo}} \\
\left|\overline{OJ_o}\right| &= u_{yo} = \sqrt{u^2 + a_y{}^2 e_o{}^2 - 2a_y a_{yo}} \\
\left|\overline{OK_o}\right| &= u_{xo} = \sqrt{u^2 + a_z{}^2 e_o{}^2 - 2a_z a_{zo}}
\end{aligned}
\tag{5}
$$

Next relation between axonometric scales defined the fundamental longitude:

$$
l = \sqrt{u_{xo}{}^2 + u_{yo}{}^2 + u_{zo}{}^2} = u\sqrt{e_o{}^2 + 1} \tag{6}
$$

Here, it has been noted some geometric elements that forms the axonometric view (figure 1.). To broaden this info, see the paper Gimena et al. 2015 [3].

3 Diagonal magnitude

In this section, we start from three segments, which defines the axonometric perspective. Obtained from the joint in the origin, of the coordinates of the points I_o, J_o and K_o. Each of the segments are projected on whatever direction b, contained in the frame plane and the square root is determined from the addition of the square exponential of these projections. The direction is $b \equiv \left[b_x, b_y, b_z\right]/u$, satisfying $\sqrt{b_x{}^2 + b_y{}^2 + b_z{}^2} = u$.

The projection of the axonometric unit segments can be noted as:

$$
\begin{aligned}
\overline{OB_{xo}} &= I_o \cdot b = \left[b_x u^2 - a_x\left(a_{xo}b_x + a_{yo}b_y + a_{zo}b_z\right)\right]/u^2 = b_{xo} \\
\overline{OB_{yo}} &= J_o \cdot b = \left[b_y u^2 - a_y\left(a_{xo}b_x + a_{yo}b_y + a_{zo}b_z\right)\right]/u^2 = b_{yo} \\
\overline{OB_{zo}} &= K_o \cdot b = \left[b_z u^2 - a_z\left(a_{xo}b_x + a_{yo}b_y + a_{zo}b_z\right)\right]/u^2 = b_{zo}
\end{aligned}
\tag{7}
$$

Here, we called this magnitude, orthohedral diagonal which axis would be formed by the projections of the three axonometric unit segments. From the origin coordinates and on the used direction the orthohedral diagonal is constructed and annotated as:

$$
\overline{OB_o} = \sqrt{b_{xo}{}^2 + b_{yo}{}^2 + b_{zo}{}^2} = b_o \tag{8}
$$

In next figure 2 are plotted: the projection direction, the axonometric unit segments projections and the orthohedral diagonal associated.
Whatever projected direction can be expressed in function of two principal directions which in this paper are, first $b(\pi/2)$ perpendicular to d_π and d_o, secondly $b(0)$ perpendicular to the former direction.

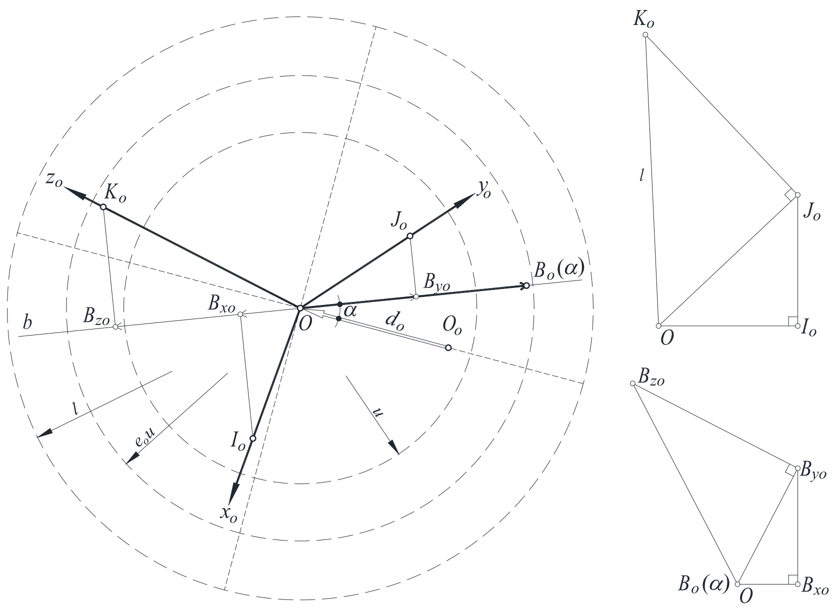

Fig. 2. Direction of projection and diagonal magnitud.

The projection direction in function of these ones chosen, can be expressed analitically as:

$$b \equiv b(\alpha) = b(0)\cos\alpha + b(\pi/2)\sin\alpha \tag{10}$$

Also, the diagonal magnitude can be annotated as:

$$\overline{OB_o} = \overline{OB_o(\alpha)} = l_1 = u\sqrt{e_o{}^2\cos^2\alpha + \sin^2\alpha} \tag{11}$$

When the three starting segments represents an orthogonal axonometric perspective, the diagonal magnitude fits u .

4 Intrinsic curve associated to the ellipse

Once the diagonal magnitude is constructed from the origin of coordinates, this measure describes a geometric locus here noted as intrinsic curve associated to the ellipse. In Fig. 3 this geometric locus is represented.

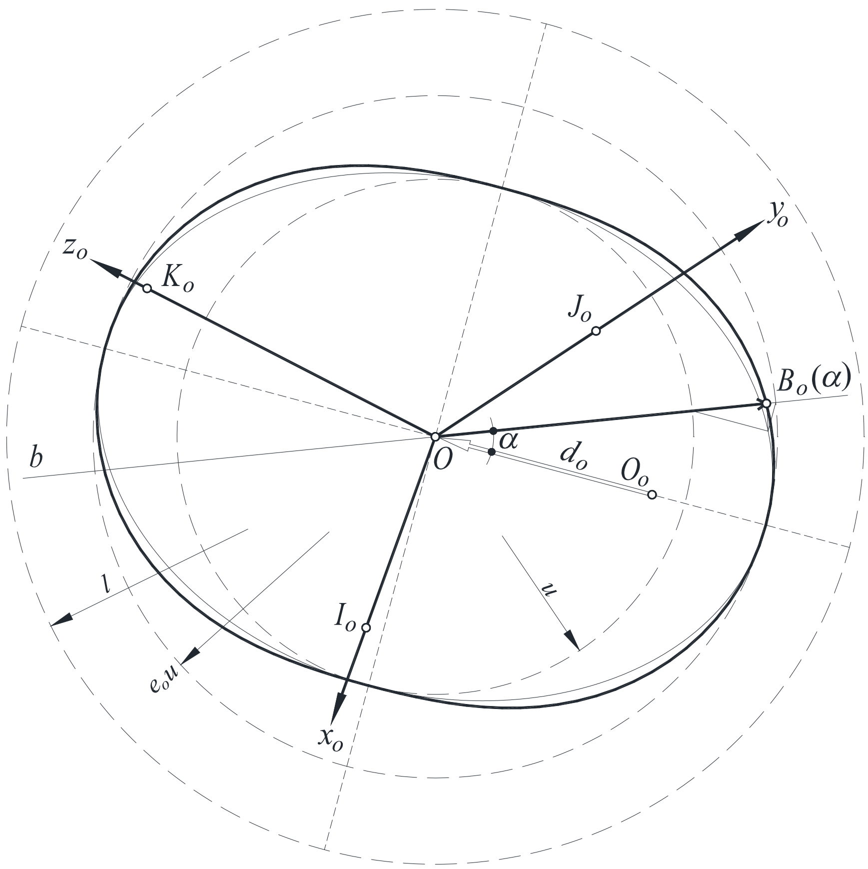

Fig. 3. Intrinsic curve associated to the ellipse.

When the three starting segments represents an orthogonal axonometric perspective, the intrinsic curve associated to the ellipse is a circle.

4 Conclusions

It has been defined diagonal magnitude as the length of the diagonal segment of an orthohedral whose sides are formed by projections of the three unitary segments over an axonometric unit coplanar to their directions. Intrinsic curve to the ellipse has been defined as the geometric locus describing the diagonal magnitude if it is built from the origin and its associated directions. When the starting three segments represent an axonometric orthogonal perspective, the intrinsic curve associated with the ellipse is a circle.

References

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