Theoretical Modelling and Experimental Verification of the Scattering from a Ferromagnetic Microwire

Iñigo Liberal, Iñigo Ederra, Cristina Gómez-Polo, Alberto Labrador, Jose Ignacio Pérez-Landazábal and Ramón Gonzalo, *Member, IEEE*

***Abstract*—This contribution presents a theoretical modelling of the scattering of ferromagnetic microwires in free-space and inside a rectangular waveguide, providing both an analytical solution and a physical interpretation of the problem. Special attention is devoted to the impact of the microwire radius and its magnetic properties. Theoretical results have been experimentally verified measuring the reflection, absorption and transmission coefficients of a ferromagnetic microwire inside a rectangular waveguide.**

***Index Terms*—Ferromagnetic microwires, Metamaterials, Absorbers.**

I. INTRODUCTION

A glass-coated ferromagnetic microwire is composed of a ferromagnetic conductive core with radius ranging from $2\mu m$ to $50\mu m$ covered with a pyrex coating of one to tens of micrometers thickness [1]. Lately, ferromagnetic microwires have attracted the attention of the research community for their potential applications in the areas of metamaterials (MTMs) [2-9], electromagnetic absorbers [10-14], sensors [15-16] and tunable substrates [17-20].

Ferromagnetic microwires are known to be suitable particles in the design of metamaterials. The main advantage of using such particles lies on their capability to provide double negative behavior with only one single element. On the one hand, ferromagnetic microwires have an electrical response similar to conductive wires, so a negative permittivity can be created by an array of such wires [21]. On the other hand, the negative permeability is an intrinsic feature of ferromagnetic materials, which takes place at frequencies above the Ferromagnetic Resonance (FMR) [22]. Moreover, tunability can be achieved since the frequency where the FMR is centered can be governed by an external dc magnetic field.

In this context, the effective parameters of metamaterials composed of ferromagnetic wires have been theoretically investigated by using Effective Medium Theory (EMT). The main purpose of these studies is focused on studying the possibility of obtaining double negative media [2-6]. In some cases [2-4], these results have been validated numerically with electromagnetic solvers. Moreover, experimental evidence of left-handed transmission has been found for single or several ferromagnetic microwires embedded in a rectangular waveguide [7-9].

Due to the high magnetic losses present at FMR and the dielectric losses produced by a finite conductivity, ferromagnetic microwires are characterized by high absorption coefficients, which have been reported in several contributions [10-14]. Therefore, ferromagnetic microwires are of great value for the design of new Radar Absorbing Materials (RAMs), which could outperform conventional ferrite RAMs in weight, cost and profile.

Ferromagnetic microwires also have potential applications in the design of microwave circuitry. In particular, they have been proposed to develop artificial substrates with tunable permittivity [17-20], with applications in frequency selective surfaces and polarizers. Moreover, they can be employed to substitute bulk ferrite materials as a substrate of microwave components, as it has been made with nanowires subtrates [23] in devices such as circulators [24-25], isolators [26] and phase-shifters [27].

Due to all this active research, a great boost for the design of prototypes of systems employing ferromagnetic microwires is expected in the incoming years. To this end, an accurate knowledge of the analytical response for the scattering from a ferromagnetic microwire is required for two reasons: in the first place, the numerical simulations of structures with micrometer size details is challenging by using the conventional numerical methods that are commonly employed on the design of microwave systems. Therefore, accurate analytical models are required for the design of metamaterials, electromagnetic absorbers, sensors, tunable substrates and microwave components based on ferromagnetic microwires. The scattering from a single microwire is a fundamental piece for these models, which can be employed to analyze more complex structures, such as grids and arrays of wires, and carry out homogenization processes. Secondly, the analysis of the scattering from a ferromagnetic microwire provides physical insight into the interaction between electromagnetic fields and ferromagnetic microwires, contributing with an intuitive vision, which can be helpful for the development of prototypes.

The main objective of this paper is to introduce an analytical model for the scattering of a long ferromagnetic microwire of utility in the design of microwire-based microwave systems. Previous works have employed analytical models to analyze the scattering from short wires under the antenna approximation [17], and have estimated the absorption spectra of long wires [28-29]. This contribution adopts a different formulation giving explicit analytical expressions for the electromagnetic

---

I. Liberal, I. Ederra and R. Gonzalo are with the Antenna Group, Public University of Navarre. Campus Arrosadía, 31006, Navarra, Spain. (e-mails: inigo.liberal@unavarra.es, inigo.ederra@unavarra.es and ramon@unavarra.es)

C. Gómez-Polo, A. Labrador and J.I. Pérez-Landazábal are with the Department of Physics, Public University of Navarre. Campus Arrosadía, 31006, Navarra, Spain.

Manuscript received XXXX, 2010; revised XXX, 2010.

fields, which allows for obtaining fast and accurate predictions of the microwires electromagnetic response, and provides physical insight on the electromagnetic processes that take place inside these microwires.

Moreover, these theoretical analyses are validated with measurements of the scattering of a single ferromagnetic microwire inside a rectangular waveguide. These experiments also have a theoretical value, since experimental verifications of left-handed transmission are typically carried out embedding one or more microwires inside a rectangular waveguide [7-9]. These experiments are significantly influenced by the number and disposition of the microwires in the waveguide, and thus the structure is known to be operating at a mesoscopic level [30], where the results of the EMT approach usually employed to retrieve the effective parameters [1-6] lacks of accuracy. Therefore, the study of the scattering of a ferromagnetic microwire inside a rectangular waveguide provides a correct theoretical framework to analyze such experiments. Furthermore, since a metallic rectangular waveguide is a closed and controlled environment, the knowledge of the response inside a rectangular waveguide allows for using this experiment for the characterization of ferromagnetic microwires. The existing results, e.g. [8], present experimental data for thin ferromagnetic microwires inside a rectangular waveguide. Conversely, this paper presents a general theoretical model, not reported in [8], for the scattering of thin and thick ferromagnetic microwires in free-space and inside a rectangular waveguide. Furthermore, this model is employed to provide physical insight into the electromagnetic processes that take place inside the microwire. This theoretical model will be validated by the results in [8] and our own experimental results for thick microwires.

The paper is organized as follows: firstly, Section II presents the electromagnetic properties of a ferromagnetic microwire and an analytical solution to the scattering of a plane-wave under normal incidence. Secondly, the scattering of a ferromagnetic microwire in the middle of a rectangular waveguide is introduced in Section III. This result is employed to experimentally verify the solution to the scattering problem presented in the previous section. Finally, conclusions and discussion of the results are gathered in Section IV.

II. SCATTERING FROM A FERROMAGNETIC MICROWIRE

A. Electromagnetic Properties of Ferromagnetic Microwires

In this section, a complete description of the properties of the electromagnetic medium that composes the ferromagnetic microwires is provided in order to clarify some relevant aspects for the scattering analysis. As previously introduced, a ferromagnetic microwire is formed by a ferromagnetic conductive core covered by a pyrex coating. Such pyrex coating has a great influence on the magnetic response of the ferromagnetic core due to mechanical stresses, which provoke a uniform gyrotropic permeability. However, the impact of the pyrex coating on the global scattering is negligible due to its inherent low permittivity and null magnetic susceptibility [31].

Conversely, both the permittivity and permeability of the core have a great influence on the scattering. On the one hand, the permittivity is dominated by its electrical conductivity, which is equivalent to a very large negative imaginary part [32]:

$$\varepsilon = \varepsilon_o - j\frac{\sigma}{w} \tag{1}$$

On the other hand, the permeability is characterized by a gyrotropic response. Thus, if the DC magnetic bias is oriented along the Z axis, the permeability tensor will be given by

$$\bar{\bar{\mu}} = \begin{bmatrix} \mu & jk & 0 \\ -jk & \mu & 0 \\ 0 & 0 & \mu_o \end{bmatrix} \tag{2}$$

The values of μ and k follow a Lorentz-type behaviour, which can be obtained from [32]

$$\mu = \mu_o \left(1 + X_p - jX_s\right) \tag{3}$$

$$k = \mu_o \left(K_p - jK_s\right) \tag{4}$$

with

$$X_p = \frac{w_o w_m \left(w_o^2 - w^2\right) + w_o w_m w^2 \alpha^2}{\left[w_o^2 - w^2\left(1+\alpha^2\right)\right]^2 + 4w_o^2 w^2 \alpha^2} \tag{5}$$

$$X_s = \frac{w w_m \alpha \left(w_o^2 + w^2\left(1+\alpha^2\right)\right)}{\left[w_o^2 - w^2\left(1+\alpha^2\right)\right]^2 + 4w_o^2 w^2 \alpha^2} \tag{6}$$

$$K_p = \frac{w w_m \left(w_o^2 - w^2\left(1+\alpha^2\right)\right)}{\left[w_o^2 - w^2\left(1+\alpha^2\right)\right]^2 + 4w_o^2 w^2 \alpha^2} \tag{7}$$

$$K_s = \frac{2 w_o w_m w^2 \alpha}{\left[w_o^2 - w^2\left(1+\alpha^2\right)\right]^2 + 4w_o^2 w^2 \alpha^2} \tag{8}$$

where $w_m = \mu_o \gamma M_s$ stands for the resonance frequency at the saturation limit, γ is the gyromagnetic ratio, M_s is the saturation magnetization, and $w_o = \mu_o \gamma H_o$ is the Larmor resonance frequency. Moreover, α is a dimensionless damping factor taking into account magnetic losses.

When a plane-wave is propagating in such media there are two different responses depending on the wave polarization. In the first place, if the magnetic field is polarized parallel to the dc magnetic bias (Z-axis), the field will be unaffected by the magnetization of the medium, as shown in (2). On the contrary, if the magnetic field is transverse to the bias, it will be affected by the magnetization leading to the so-called extraordinary waves [32], characterized by a component of the magnetic field in the direction of propagation. Since in the former case the medium is equivalent to a conventional dielectric, the latter case will be considered here.

The effective permeability and the propagation constant for the extaordinary waves supported by the medium are respectively given by

$$\mu_e = \frac{\mu^2 - k^2}{\mu} \tag{9}$$

$$\beta_e^2 = w^2 \varepsilon \mu_e \tag{10}$$

Fig. 1 shows the real and imaginary part of the effective permeability of the ferromagnetic microwires that will be

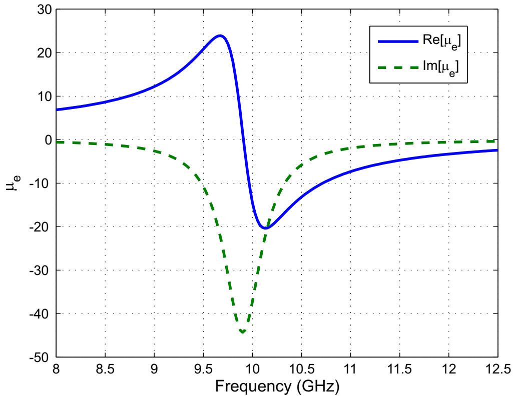


Fig. 1. Real and imaginary part of the effective permeability for the considered ferromagnetic microwires.

employed in the measurements. Such microwires have a composition of $(Co_{0.94}Fe_{0.06})_{75}Si_{12.5}B_{12.5}$ and are characterized by $\alpha = 0.02$, $\gamma = 1.33 \cdot 10^{11} T^{-1}s^{-1}$, $\mu_o M_s = 0.55T$, $H_o = 213kA/m$ and a conductivity of $\sigma = 6.5 \cdot 10^4 S/m$. In Fig. 1 can be seen that the effective permeability is characterized by high magnetic losses, i.e. a large imaginary part with a maximum at the FMR point. Moreover, the electromagnetic field is able to propagate at frequencies below the FMR, where the real part of the effective permeability is positive. However, these propagating fields will be strongly attenuated due to the magnetic losses and the conductivity of the medium. On the contrary, at frequencies above FMR only exponentially decaying evanescent waves will be present due to a negative real part of the effective permeability.

B. Analytical solution of the scattering

Consider the case when a plane-wave propagating along the X-axis and with the electric field polarized along the Z-axis impinges on a ferromagnetic microwire with their axis and the dc magnetic bias also oriented along the Z-axis (see Fig. 2). The analytical solution of the scattering of a plane wave on an infinitely long ferromagnetic microwire resembles to the scattering on a very thin ferrite post [33]. Main differences arise from the complex part of the permittivity, produced by the conductivity and the effects that appear when the penetration depth is comparable to the cylinder radius. Due to the cylindrical symmetry of the problem, the latter can be reduced to a scalar problem in the XY plane, and, as many other scattering problems, it can be solved by expanding the incident, scattered and transmitted-into-the-cylinder fields as a series of cylindrical Bessel functions and by solving the boundary conditions.

Particularly, for an incident plane wave propagating along the X axis, the fields will be given by

$$E_z^{inc}(r,\phi) = E_o \sum_{n=-\infty}^{\infty} j^n J_n(\beta r) e^{-jn\phi} \quad (11)$$

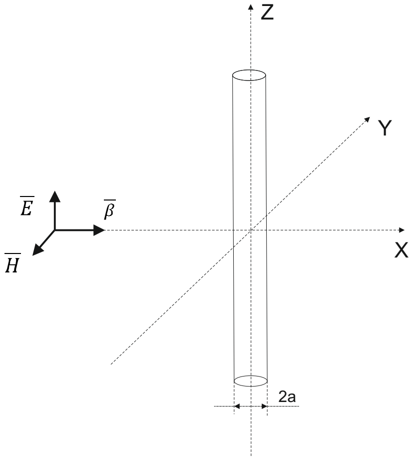


Fig. 2. Geometry of the problem with relevant polarizations.

$$H_r^{inc}(r,\phi) = \frac{E_o}{w\mu_o r} \sum_{n=-\infty}^{\infty} n \cdot j^n J_n(\beta r) e^{-jn\phi} \quad (12)$$

$$H_\phi^{inc}(r,\phi) = \frac{-j\beta E_o}{w\mu_o} \sum_{n=-\infty}^{\infty} j^n J_n'(\beta r) e^{-jn\phi} \quad (13)$$

where $\beta = w\sqrt{\mu_o \varepsilon_o}$ stands for the propagation constant in free-space and $J_n(-)$ for a Bessel function of the first kind and order n.

As for the scattered field, Hankel functions of the second kind, $H_n^{(2)}(-)$, are chosen to represent outgoing waves

$$E_z^s(r,\phi) = E_o \sum_{n=-\infty}^{\infty} a_n^s H_n^{(2)}(\beta r) e^{-jn\phi} \quad (14)$$

$$H_r^s(r,\phi) = E_o \frac{1}{w\mu_o r} \sum_{n=-\infty}^{\infty} n \cdot a_n^s H_n^{(2)}(\beta r) e^{-jn\phi} \quad (15)$$

$$H_\phi^s(r,\phi) = E_o \frac{-j\beta}{w\mu_o} \sum_{n=-\infty}^{\infty} a_n^s H_n^{\prime(2)}(\beta r) e^{-jn\phi} \quad (16)$$

Finally, the transmitted field inside the microwire is represented by Bessel functions of the first kind

$$E_z^t(r,\phi) = E_o \sum_{n=-\infty}^{\infty} a_n J_n(\beta_e r) e^{-jn\phi} \quad (17)$$

$$H_r^t(r,\phi) = E_o \frac{1}{w(\mu^2 - k^2)} \left[\frac{\mu}{r} \sum_{n=-\infty}^{\infty} n \cdot a_n J_n(\beta_e r) e^{-jn\phi} - k\beta_e \sum_{n=-\infty}^{\infty} a_n J_n'(\beta_e r) e^{-jn\phi} \right] \quad (18)$$

$$H_\phi^t(r,\phi) = E_o \frac{j}{w(\mu^2 - k^2)} \left[\frac{k}{r} \sum_{n=-\infty}^{\infty} n \cdot a_n J_n(\beta_e r) e^{-jn\phi} \right.$$

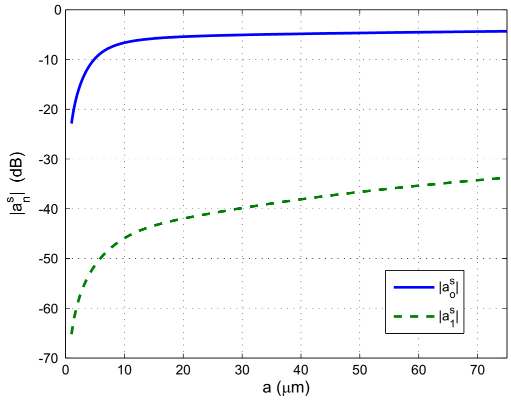


Fig. 3. Magnitude of the scattering coefficients a_o^s and a_1^s at 12.5GHz for the considered ferromagnetic microwires with radius ranging from $1\mu m$ to $75\mu m$.

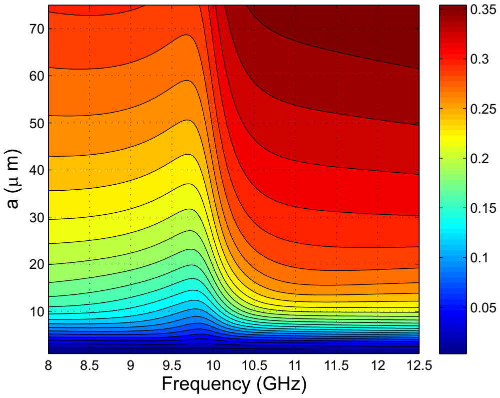


Fig. 4. Magnitude of the scattering field coefficient, a_o^s, as a function of frequency for the considered ferromagnetic microwires with radius ranging from $1\mu m$ to $75\mu m$

$$-\mu\beta_e \sum_{n=-\infty}^{\infty} a_n J_n'(\beta_e r) e^{-jn\phi}\Bigg] \tag{19}$$

The unknowns of the problem are the coefficients of the scattered field, a_n^s, and transmitted field into the cylinder, a_n, which can be found by solving the boundary conditions, i.e. the continuity of the tangential electric and magnetic fields at the surface of the microwire, leading to

$$a_n^s = j^n \frac{J_n'(\beta a) J_n(\beta_e a) - D_n J_n(\beta a)}{D_n H_n^{(2)}(\beta a) - H_n'^{(2)}(\beta a) J_n(\beta_e a)} \tag{20}$$

$$a_n = j^n \frac{H_n^{(2)}(\beta a) J_n'(\beta a) - J_n(\beta a) H_n'^{(2)}(\beta a)}{D_n H_n^{(2)}(\beta a) - H_n'^{(2)}(\beta a) J_n(\beta_e a)} \tag{21}$$

where a is equal to the microwire radius and the quantity D_n is given by

$$D_n = \frac{\beta_e \mu_o}{\beta \mu_e} \left[J_n'(\beta_e a) - \frac{k \cdot n}{a \mu \beta_e} J_n(\beta_e a) \right] \tag{22}$$

C. Physical interpretation

Substituting (20-22) in (11-19) allows for computing the electric and magnetic fields inside and outside the microwire, and therefore they constitute a complete and rigorous solution to the problem. However, typical radius range from $2\mu m$ to $50\mu m$, which are equivalent to lengths from 0.000083λ to 0.0021λ at the highest studied frequency, 12.5GHz. Thus, the microwires can be considered electrically thin over the whole operational bandwidth. In fact, the numerical evaluation of the coefficients a_o^s and a_1^s for radius ranging from $1\mu m$ to $75\mu m$ presented in Fig. 3 reveals that only the $n = 0$ term of the series will be significant for all the studied radii.

Consequently, the scattered electrical field will be equal to $a_o^s E_o H_o^{(2)}(\beta r)$, and thus the scattering coefficient a_o^s is sufficient to describe the scattered field. It is worth remarking that in this situation the scattered field and the field transmitted into the microwire have perfect cylindrical symmetry, and thus the scattered field by a ferromagnetic microwire will be independent of the azimuthal angle-of-arrival (AOA) of the incident plane-wave.

Fig. 4 represents the magnitude the a_o^s coefficient as a function of the frequency and the radius for a $(Co_{0.94}Fe_{0.06})_{75}Si_{12.5}B_{12.5}$ microwire with the parameters previously defined. Although all the considered radii are small compared to the wavelength, the microwire radius has a great impact on the scattered field. In fact, two different behaviours can be observed depending on the length of the penetration depth compared to the microwire radius. For the sake of clarity, the penetration depth, defined as the inverse of the imaginary part of the propagation constant, $\delta = 1/Im\left[\beta_e\right]$, is presented in Fig. 5. As shown, the penetration depth ranges between $2\mu m$ and $10\mu m$ along the studied bandwidth, with a minimum at the FMR frequency, where the material losses are maximal. It is worth remarking there are other usual definitions for the penetration depth that do not take into account the magnetic response [23]. Thus, the frequency-domain behaviour of the penetration depth does not display a minimum at the FMR frequency, but decreases monotonically. However, since the magnetic response contributes as well to the field confinement inside the microwire, the used definition gives more physical insight into the scattering behaviour of the microwires.

When the microwire radius is considerably larger than the penetration depth, the fields inside the microwire are constrained to the surface, due to the high attenuation produced by the magnetic losses and electric conductivity. This is evidenced in Fig. 6a, which shows the field inside a microwire of radius $45\mu m$ for frequencies ranging from 8GHz to 12.5GHz.

Moreover, the scattered field presents a strong dependency on the frequency due to the variation of the material properties. Particularly, when the microwire radius is larger than the penetration depth there are two different responses corresponding to the frequency ranges where $Re\left[\mu_e\right] > 0$ and $Re\left[\mu_e\right] < 0$, respectively. To clarify this fact, Fig. 7b represents the magnitude of the scattering coefficient as a function of frequency for a microwire of radius $45\mu m$. At frequencies below the FMR, $Re\left[\mu_e\right] > 0$, the fields inside the microwire behaves similarly

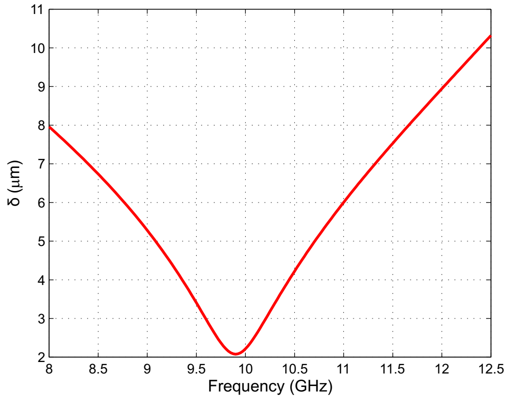
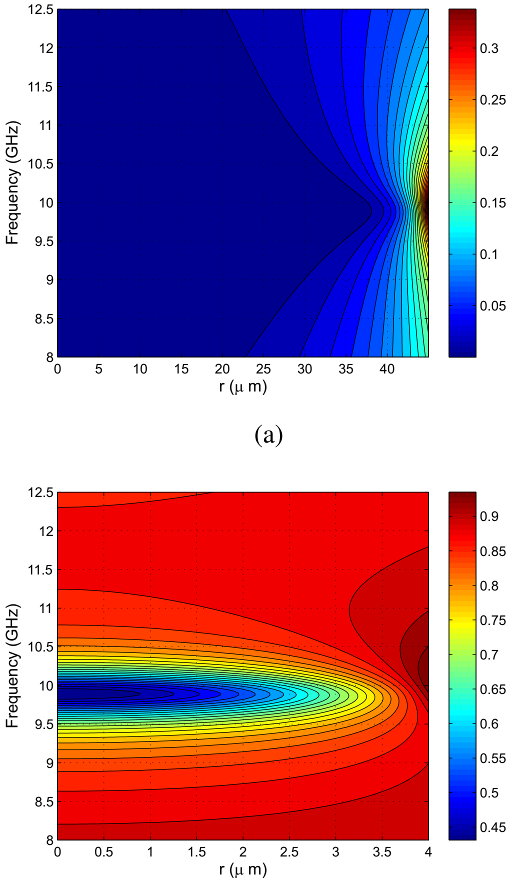


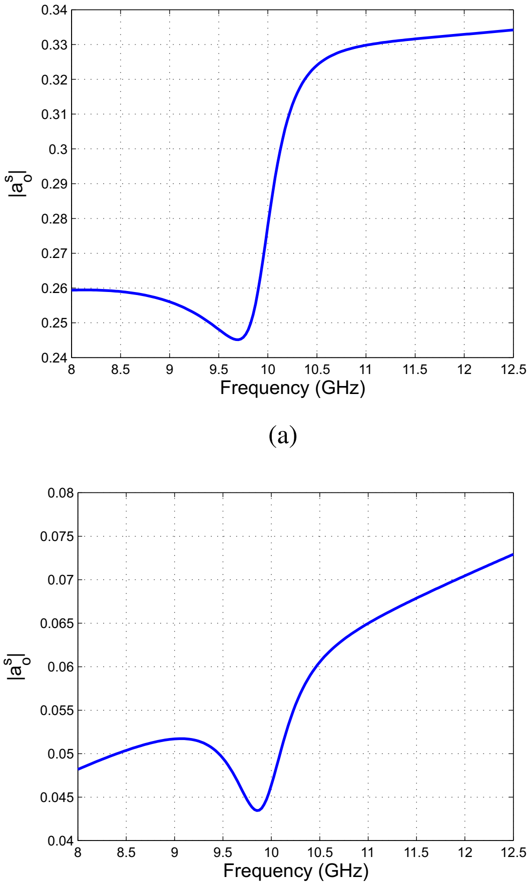
Fig. 5. Penetration depth as a function of frequency for the considered ferromagnetic microwires.



(a)

(b)

Fig. 6. Magnitude of the electric field as a function of frequency inside the considered microwires for (a) $45\mu m$, and (b) $4\mu m$ radius.



(a)

(b)

Fig. 7. Magnitude of the scattering field coefficient, a_o^s, as a function of frequency for the considered microwires for (a) $45\mu m$, and (b) $4\mu m$ radius.

as a lossy dielectric cylinder. Consequently, increasing μ_e decreases the scattered field, since the medium impedance becomes closer to the freespace impedance as the impact of the large permittivity produced by the finite conductivity is compensated. On the other hand, at frequencies above FMR, $Re\left[\mu_e\right] < 0$. Recall from Section II.A that the inside fields are exponentially decaying evanescent waves, resulting in a larger scattered field than in the $Re\left[\mu_e\right] > 0$ case.

A different situation is presented when the penetration depth is comparable to the microwire radius. As it can be inferred from Fig. 6b, which shows the electric field inside a microwire of radius $4\mu m$ for frequencies ranging from 8 to 12.5GHz, the fields are no longer superficial and they are able to fully penetrate the microwire except at the FMR frequency, where the propagation constant, β_e, is purely imaginary and the medium losses are maximal. Therefore, since both the exponentially decaying evanescent waves and the propagating waves attenuated by the medium losses are able to fully penetrate the microwire, there are no great differences in the scattered field when $Re\left[\mu_e\right] > 0$ and when $Re\left[\mu_e\right] < 0$. This is confirmed in Fig. 7b, where the magnitude of the scattering coefficient as a function of frequency for a microwire of radius $4\mu m$ is represented. This figure illustrates that there are no longer two differents areas of scattered field, and such field increases as the electrical size of the microwire is augmented. However, a minimum of scattering is obtained at the frequency of FMR due to the low penetration of the fields in the microwire.

III. FERROMAGNETIC MICROWIRE INSIDE A WAVEGUIDE

The experimental verification of the scattering of an object requires the use of a complex setup including an anechoic chamber. However, one way to simplify this experiment is to reduce it to a closed environment, i.e. a metallic rectangular waveguide. Furthermore, the problem of the scattering of a ferromagnetic microwire inside a waveguide is of great interest since it gives a correct theoretical base to analyze the experimental evidences of left-handed transmission [7-9]. Moreover, this analysis provides the theoretical framework to the development of microwave components based on ferromagnetic microwires embedded in rectangular waveguides.

The electromagnetic problem of a ferrite cylinder inside a waveguide has been thoroughly studied due to its multiple applications in microwave components such as polarizers and circulators [34-35]. In fact, a rigurous solution to this problem was introduced in [36]. Although corrected [37] and commented [38], the approach introduced there is a correct and general solution to the problem. However, such solution is tedious and complex so a different approach will be followed here leading to much simpler, but also restricted, solution. In particular, the solution will be limited to an electrically small ferromagnetic microwire positioned on the middle of the waveguide.

Fig. 8 shows the geometry of the problem. A TE_{10} mode is assumed to be propagating along a metallic rectangular waveguide, in which a ferromagnetic microwire is placed with the conductive core connected to the waveguide walls. Since there is no variation of the fields along the Z axis the problem is reduced to a two-dimensional problem in the XY plane. Therefore, the incident field will be given by

$$E_z^{inc}(x,y) = E_o cos\left(\frac{\pi y}{d}\right) e^{-j\beta_g x} \tag{23}$$

where $\beta_g^2 = \beta^2 - \left(\frac{\pi}{d}\right)^2$ stands for the propagation constant inside the waveguide and d for the waveguide width. It is worth noting that the incident field to the microwire can be decomposed as the sum of two different plane-waves [39]. Recall from the previous section that the scattering from a ferromagnetic microwire is independent of the AOA of the incident plane-wave, thus the scattered field when the incident field is the TE_{10} mode will be equal to the field scattered by a plane wave.

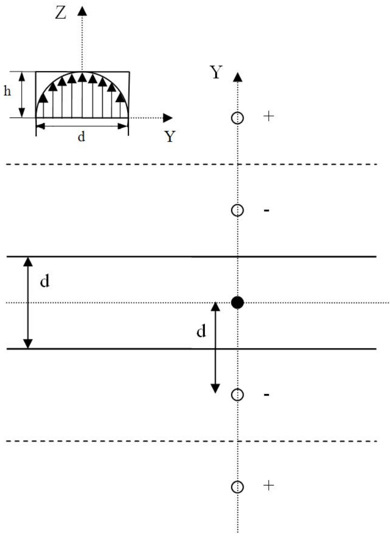


Fig. 8. Ferromagnetic microwire in the middle of a rectangular waveguide and images created by the metallic walls.

Furthermore, the impact of the metallic walls of the waveguide must also be considered. Therefore, the total scattered field will be equal to the addition of the alternating positive and negative images created by the walls of the waveguide (see Fig. 8).

$$E^s(x,y) = a_o^s E_{loc} \sum_{n=-\infty}^{\infty} \left[H_o^{(2)}(\beta\sqrt{x^2-(y-n\cdot 2d)^2}) - H_o^{(2)}(\beta\sqrt{x^2-(y-d-n\cdot 2d)^2}) \right] \tag{24}$$

where E_{loc} stands for the local field in the surface of the microwire, given by the addition of the incident field and the field generated by the images

$$E_{loc} = \frac{E_o}{1-a_o^s G_w} \tag{25}$$

where the waveguide interaction parameter G_w has been defined

$$G_w = \sum_{n\neq 0} H_o^{(2)}(\beta\cdot|n2d|) - \sum_{n=-\infty}^{\infty} H_o^{(2)}(\beta\cdot|n2d-d|) \tag{26}$$

Therefore, the expression of the local field can be included in (24) and the total scattered field can be evaluated. As an expression based on a series of Hankel function is quite complex, it is convenient to simplify it using the Poisson summation rule [40]

$$\sum_{n=-\infty}^{\infty} H_o^{(2)}(\beta\sqrt{x^2-(y-n\cdot d)^2}) = \frac{2}{d}\frac{e^{-j\beta|x|}}{\beta} + \frac{4j}{d}\sum_{m=1}^{\infty} cos(\frac{2\pi m y}{d})\frac{e^{-\sqrt{(\frac{2\pi m}{d})^2-\beta^2}|x|}}{\sqrt{(\frac{2\pi m}{d})^2-\beta^2}} \tag{27}$$

The expression introduced above has a clear physical meaning. The first term is a propagating wave excited by the mean current of the grid. On the other hand, the second term is a series of Floquet modes [40]. After a lengthy but straightforward calculation, substituing (27) in both series of (24), the first term of both series is canceled out since no TEM mode is propagating on a rectangular waveguide, and the rest of the termsare combined giving rise to the modes of the waveguide. Subsequently, the total scattered field will be given by

$$E^s = E_o \frac{a_o^s}{1-a_o^s G_w}\frac{4j}{d}\sum_{m\,odd} cos\left(m\frac{\pi y}{d}\right)\frac{e^{-\gamma_m|x|}}{\gamma_m} \tag{28}$$

where γ_m is the propagation constant for each mode which is given by

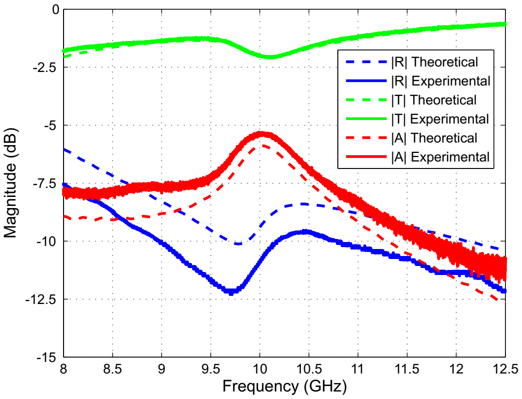


Fig. 9. Comparison between theoretical and experimental reflection, transmission and absorption coefficients for the considered ferromagnetic microwires of radius $45\mu m$ inside a WR-90 waveguide.

$$\gamma_m = \sqrt{(\frac{\pi m}{d})^2 - \beta^2} \tag{29}$$

It is worth mentioning that since the microwire is positioned at the center of the waveguide only the TE_{mo} with m being an odd number will be excited. Once the scattered field is known, it is possible to compute the reflection, transmission and absorption coefficients. Firstly, the reflected field will be equal to the scattered field, and thus the reflection coefficient for each mode will be given by

$$R_m = \left| \frac{a_o^s}{1 - a_o^s G_w} \cdot \frac{4j}{\gamma_m d} \right|^2 \quad m = 1, 3.... \tag{30}$$

Secondly, the transmitted field will be equal to the addition of the incident and scattered field, and thus the transmission coefficient will be given by

$$T_m = \left| \delta_{1m} + \frac{a_o^s}{1 - a_o^s G_w} \cdot \frac{4j}{\gamma_m d} \right|^2 \quad m = 1, 3.... \tag{31}$$

where δ_{1m} stands for the Kronecker delta. Finally, the absorption coefficient can be computed to satisfy the energy conservation

$$A = 1 - \sum_m T_m - \sum_m R_m \tag{32}$$

Fig. 9 depicts the theoretical reflection, transmission and absorption coefficients for a $(Co_{0.94}Fe_{0.06})_{75}Si_{12.5}B_{12.5}$ ferromagnetic microwire of radius $45\mu m$ inside a WR-90 waveguide (22.86mm x 10.16mm). Since the reflected field is produced by the scattering of the microwire, the reflection coefficient resembles the scattering coefficient presenting a minimum below the FMR frequency.

Moreover, a maximum is obtained above the FMR frequency where the microwire core has plasma characteristics. However, the reflection coefficient does not grow monotonically as the scattering coefficient does due to the influence of the waveguide. That is to say, although the scattering coefficient becomes larger as the frequency increases, the separation between the microwire and its images is electrically larger and therefore the overall reflection coefficient is reduced. Furthermore, the frequency-domain behaviour of the absorption coefficient is characterized by a maximum between the FMR frequency and the frequency with the largest negative real permeability, justified by the high magnetic losses and the amount of field inside the microwire.

This result has been tested with the measurements shown in Fig. 9. The amorphous glass-coated microwires where obtained by the Taylor–Ulitovsky technique and kindly supplied by Prof. M. Vzquez (Instituto de Ciencia de Materiales de Madrid, CSIC, Spain). It can be concluded that there is a very good agreement between the theoretical and experimental results.

As was anticipated in the scattering description, the situation is different when the penetration depth is similar to the wire radius. Fig. 10 depicts the theoretical reflection, transmission and absorption coefficients for a $(Co_{0.94}Fe_{0.06})_{75}Si_{12.5}B_{12.5}$ ferromagnetic microwire with radius $4\mu m$ inside a WR-90 waveguide. As it was expected from the scattering analysis, the reflection coefficient follows a descendent fashion similarly to the response of a metallic cylinder embbeded in a waveguide. As it is represented in Fig. 6b, the fields are able to fully penetrate the microwire except at the FMR frequency, where the magnetic losses are maximal. Therefore, a minimum of the reflection and absorption coefficients, and therefore a maximal transmission, is found at the FMR frequency, corresponding to the frequency where the fields are unable to penetrate in the microwire. This peak of transmission has been identified as an evidence of left-handed transmission in several works [7-9]. However, here it is shown that such band can be attributed to the scattering response of the ferromagnetic microwire. In conclusion, the rigorous extraction of the constitutive parameters through a homogenization procedure would be required to discern if this peak is a consequence of left-handed transmission or not.

Furthermore, it is worth mentioning that the absorption coefficient is larger than the reflection coefficient over the whole studied bandwidth, making ferromagnetic microwires with a radius comparable with the penetration depth strong candidates to the development of wideband electromagnetic absorbers.

In this case the results presented in [8] will be employed to experimentally verify the theoretical model. The microwires employed in [8] are glass coated amorphous microwires with a general composition $Co_{72.5}Si_{12.5}B_{15}$. Such microwires have a radius of $2.5\mu m$ and a conductivity $\sigma = 6.7 \cdot 10^5 S/m$. Their magnetic response is also a gyrotropic tensor whose characteristics are given by $\alpha = 0.02$, $\gamma = 1.93 \cdot 10^{11} T^{-1} s^{-1}$, $\mu_o M_s = 0.55T$ and $H_o = 200kA/m$. Fig. 11 represents the theoretical reflection, transmission and absorption coefficients when such microwires are embedded in a WR-62 waveguide (15.8mm x 7.9mm). A reasonable agreement is found between theoretical and experimental results (see Fig. 2, [8]).

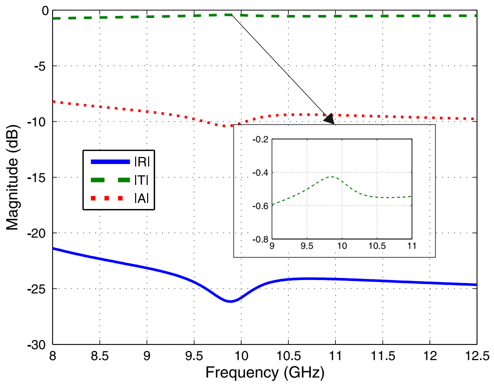


Fig. 10. Theoretical reflection, transmission and absorption coefficients for the considered ferromagnetic microwires of radius $4\mu m$ inside a WR-90 waveguide.

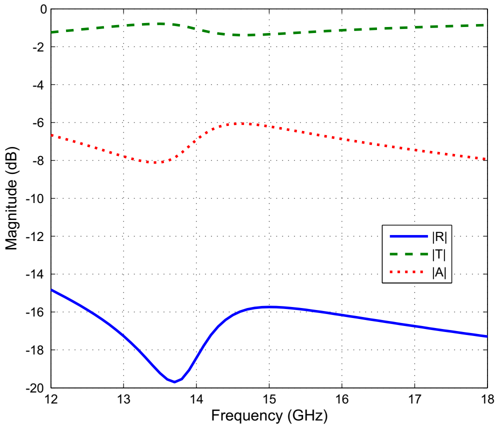


Fig. 11. Theoretical transmission, reflection and absorption coefficients for a ferromagnetic microwire $Co_{72.5}Si_{12.5}B_{15}$ with $2\mu m$ radius inside a WR-62 waveguide.

IV. CONCLUSION

This paper has introduced a theoretical study on the scattering from a ferromagnetic microwire in free-space and inside a rectangular waveguide. Consequently the analytical solution and a physical interpretation of the problem have been presented. Furthermore, theoretical results have been experimentally verified with measurements of the reflection, transmission and absorption coefficients for a $(Co_{0.94}Fe_{0.06})_{75}Si_{12.5}B_{12.5}$ ferromagnetic microwire with $45\mu m$ radius embedded in a WR-90 waveguide and a $Co_{72.5}Si_{12.5}B_{15}$ ferromagnetic microwire with $2.5\mu m$ radius embedded in a WR-62 waveguide.

Particularly, it has been shown that the scattering from the ferromagnetic microwire presents two different responses depending on the relation between the penetration depth and the microwire radius. On the one hand, when the radius is larger than the penetration depth the fields inside the microwire are merely superficial. Therefore, when $Re[\mu_e] > 0$ the scattering is similar to a lossy dielectric and it is reduced as the magnitude of $Re[\mu_e]$ is increased. Conversely, when $Re[\mu_e] < 0$ the microwire has a plasma response leading to a significantly larger scattering. Moreover, absorption coefficients as high as -5dB frequencies close to FMR are achieved with these microwires when they are embedded in a WR-90 waveguide.

On the other hand, when the penetration depth is comparable to the microwire radius the fields are able to fully propagate along the microwire and thus there is no such difference in the magnitude of the scattered field depending of the sign of $Re[\mu_e]$. However, a minimum of scattering and absorption, and thus a maximum of transmission, takes place at FMR due to the small fields inside the microwire. While, this transmission peak has been identified as an evidence of left-handed transmission in several works, this contibution shows that it can explained by the impact of the skin effect on the scattering response of a ferromagnetic microwire. Therefore, the nature of this transmission peak is to be studied with homogeneization procedures.

Finally, the absorption coefficient is larger than the reflection coefficient along the whole studied bandwidth, making ferromagnetic microwires strong candidates to develop wideband electromagnetic absorbers.

ACKNOWLEDGMENT

This work was supported by the Spanish Ministry of Science and Innovation Project Nos. TEC2009-11995 and CSD2008-00066.

REFERENCES

[1] A. Zhukov, M. Vazquez, J. Velazquez, A. Hernando and V. Larin. “Magnetic Properties of Fe-based Glass-Coated Microwires”. Journal of Magnetism and Magnetic Materials, Vol. 170, No. 3, pp. 323-330, 1997.

[2] J. Chen, D. Tang, B. Zhang, Y. Yang, M. Lu, H. Lu, F. Lu and W. Xu. “Left-handed Materials Made of Dilute Ferromagnetic Wire Arrays with Gyrotropic Tensors”. Journal of Applied Physics, Vol. 102, No. 2, pp. 023106, 2007.

[3] S. Liu, W. Chen, J. Du, Z. Lin, S. Chui and C. Chan. “Manipulating Negative-Refractive Behavior with a Magnetic Field”. Physical Review Letters, Vol. 101, No. 15, pp. 157407, 2008.

[4] J. Jin, S. Liu, Z. Lin and S. Chui. “Effective-Medium Theory for Anisotropic Magnetic Metamaterials”. Physical Review B - Condensed Matter and Materials Physics, Vol. 80, No. 11, pp. 115101, 2009.

[5] A. Ivanov, V. Galkin, V. Ivanov, D. Petrov, K. Rozanov, A. Shalygin and S. Starostenko. “Metamaterials Fabricated of Amorphous Ferromagnetic Microwires: Negative Microwave Permeability”. Solid State Phenomena, Vol. 152-153, pp. 333-336, 2009.

[6] A. Ivanov, A. Shalygin, V. Galkin, A. Vedyayev and V. Ivanov. “Metamaterials from Amorphous Ferromagnetic Microwires: interaction between microwires”. Solid State Phenomena, Vol. 152-153, pp. 357-360, 2009.

[7] H. Garcia-Miquel, J. Carbonell, V. Boria and J. Sanchez-Dehesa. “Experimental Evidence of Left Handed Transmission Through Arrays of Ferromagnetic Microwires”. Applied Physics Letters, Vol. 94, No. 5, pp. 054103, 2009.

[8] J. Carbonell, H. Garcia-Miquel and J. Sanchez-Dehesa. “Double Negative Metamaterials Based on Ferromagnetic Microwires”. Physical Review B, Vol. 81, No. 2, pp. 024401, 2010.

[9] A. Labrador, C. Gomez-Polo, J. Perez-Landazabal, V. Zablotskii, I. Ederra, R. Gonzalo, G. Badini-Confalonieri and M. Vázquez. “Magneto-tuneable Left-Handed FeSiB Ferromagnetic Microwires”. Optic Letters, Vol. 35, No. 13, pp. 2161-2163, 2010.

[10] S. Baranov. “Use of a Microconductor with Natural Ferromagnetic Resonance for Radio-Absorbing Materials”. Technical Physics Letters, Vol. 24, No. 7, pp. 549-550, 1998.

[11] S. Baranov, M. Yamaguchi, K. Garcia and M. Vazquez. “Dimensional Absorption High-Frequency Properties of the Cast Glass Coated Microwires”. Surface Engineering and Applied Electrochemistry, Vol. 44, No. 6, pp. 425-427, 2008.

[12] H. Garcia-Miquel, M. Esbri, J. Andres, J. Garcia, J. Garcia-Beneytez and M. Vázquez. “Power Absorption and Ferromagnetic Resonance in Co-rich Metallic Glasses”. IEEE Transactions on Magnetics, Vol. 37, No. 1, pp. 561-564, 2001.

[13] M. Vazquez, G. Badini, K. Pirota, J. Torrejon, A. Zhukov, A. Torcunov, H. Putzner, M. Rohn, A. Merlo, B. Marquard and T. Meydan. “Applications of Amorphous Microwires in Sensing Technologies ”. Vol. 25, No. 1-4, pp. 441-446, 2007.

[14] P. Marin, D. Cortina, and A. Hernando. “Electromagnetic Wave Absorbing Material Based on Magnetic Microwires”. IEEE Transactions on Magnetics, Vol. 44, No. 11, pp. 3934-3937, 2008.

[15] J. Torrejon, K. Pirota, G. Badini-Confalonieri and M. Vazquez. “Optimization of Magnetoelastic Temperature Sensor Based on Multilayer Microwires ”. Sensor Letters, Vol. 7, No. 3, pp. 236-239, 2009.

[16] L. Panina, D. Makhnovskiy and K. Mohri. “Magnetoimpedance in Amorphous Wires and Multifunctional Applications: From Sensors to Tunable Artificial Microwave Materials”. Journal of Magnetism and Magnetic Materials, Vol. 272-276, No. 2, pp. 1452-1459, 2004.

[17] D. Makhnovskiy and L. Panina. “Field Dependent Permittivity of Composite Materials Containing Ferromagnetic Wires”. Journal of Applied Physics, Vol. 93, No. 7, pp. 4120-4129, 2003.

[18] D. Makhnovskiy, L. Panina, C. Garcia, A. Zhukov and J. Gonzalez. “Experimental Demonstration of Tunable Scattering Spectra at Microwave Frequencies in Composite Media Containing CoFeCrSiB Glass-Coated Amorphous Ferromagnetic Wires and Comparison with Theory”. Physical Review B, Vol. 74, No. 6, pp. 064205, 2006.

[19] O. Reynet, A. Adenot, S. Deprot, O. Acher and M. Latrach. “Effect of the Magnetic Properties of the Inclusions on the High-Frequency Dielectric Response of Diluted Composites”. Physiscal Review B, Vol. 66, No. 9, pp. 094412, 2002.

[20] H. Peng, F. Qin, M. Phan, J. Tang, L. Panina, M. Ipatov, V. Zhukova, A. Zhukov and J. Gonzalez. “Co-based magnetic microwire and field-tunable multifunctional macro-composites ”. Journal of Non-Crystalline Solids, Vol. 355, No. 24-27, pp.1380-1386, 2009.

[21] J. Pendry, A. Holden, W. Stewart and I. Youngs. “Extremely Low Frequency Plasmons in Metallic Mesostructures”. Physical Review Letters, Vol. 76, No. 25, pp. 4773-4776, 1996.

[22] A. Gurevich and G. Melkov. “Magnetization Oscillation and Waves”. CRC Press, 2006.

[23] J. Spiegel, J. De La Torre, M. Darques, L. Piraux and I. Huynen. “Permittivity Model for Ferromagnetic Nanowired Substrates”. IEEE Microwave and Wireless Components Letters, Vol. 17, No. 7, pp. 492-494, 2007.

[24] M. Darques, J. Spiegel, J. De la Torre Medina, I. Huynen and L. Piraux. “Ferromagnetic Nanowire-Loaded Membranes for Microwave Electronics”. Journal of Magnetism and Magnetic Materials, Vol. 321, No. 14, pp. 2055-2065, 2009.

[25] J. Spiegel and I. Huynen, “Microwave Properties of Ferromagnetic Nanowires and Applications to Tunable Devices”. Diffusion and Defect Data Pt.B: Solid State Phenomena, Vol. 152-153, pp. 389-393, 2009.

[26] B. Kuanr, V. Veerakumar, R. Marson, S. Mishra and R. Camley. “Non-reciprocal microwave devices based on magnetic nanowires”. Applied Physics Letters, Vol. 94, No. 20, pp. 202505, 2009.

[27] J. De la Torre Medina, J. Spiegel, M. Darques, L. Piraux and I. Huynen. “Differential phase shift in nonreciprocal microstrip lines on magnetic nanowired substrates”. Applied Physics Letters, Vol. 96, No. 7, pp. 072508, 2010.

[28] S. Lofland, H. Garcia-Miquel, M. Vazquez and S. Bhagat. “Microwave Magnetoabsorption in Glass-Coated Amorphous Microwires with Radii close to Skin-Depth”. Journal of Applied Physics, Vol. 92, No. 4, pp. 2058, 2002.

[29] S. Samaddar. “Scattering of plane electromagnetic waves by radially inhomogeneous infinite cylinders”. Applied Science Research, Vol. 10, pp. 385, 1963.

[30] P. Ikonen, E. Saenz, R. Gonzalo, C. Simovski and S. Tretyakov. “Mesoscopic Effective Material Parameters for Thin Layers Modeled as Single and Double Grids of Interacting Loaded Wires ”. Metamaterials, Vol. 1, No. 2, pp. 89-105, 2007.

[31] R. David. “Handbook of Chemistry and Physics”. CRC Press, 2001.

[32] D. Pozar. “Microwave engineering” . Wiley & Sons, 2005.

[33] W. Eggiman. “Scattering of a Plane-Wave on a Ferrite Cylinder at Normal Incidence”. IRE Transactions on Microwave Theory and Techniques, Vol. 8, No. 4, pp. 440-445, 1960.

[34] P. Epstein and A. Berk. “Ferrite Post In a Rectangular Waveguide”. Journal of Applied Physics, Vol. 27, pp. 1328-1335, 1956.

[35] T. Yoshida, M. Umeno and S. Miki. “Propagation Characteristics of a Rectangular Waveguide Containig a Cylindrical Rod of Magnetized Ferrite”. IEEE Transactions of Microwave Theory and Techniques, Vol. 20, No. 11, pp. 739-743, 1972.

[36] N. Okamoto, I. Nishkioka and Y. Nakanishi. “Scattering by a Ferrimagnetic Cylinder in a Rectangular Waveguide”. IEEE Transactions on Microwave Theory and Techniques, Vol. 19, No. 6, pp. 521-527, 1971.

[37] N. Okamoto and Y. Nakanishi. “Correction to “Scattering by a Ferrimagnetic Circular Cylinder in a Rectangular Waveguide””. IEEE Transactions of Microwave Theory and Techniques, Vol. 20, No. 11, pp. 782-783, 1972.

[38] P. Bhartia. “Coments on “Scattering by a Ferrimagnetic Circular Cylinder in a Rectangular Waveguide””. IEEE Transactions on Microwave Theory and Techniques, Vol. 22, No. 11, pp. 975, 1974.

[39] P. Hannan and M. Balfour. “Simulation of a Phased-Array Antennas in Waveguides”. IEEE Transactions on Antennas and Propagation, Vol. 13, No. 3, pp. 342-353, 1965.

[40] S. Tretyakov. “Analytical Modeling in Applied Electromagnetics”. Artech House, 2003.

Iñigo Liberal was born on October 31, 1985 in Pamplona, Navarra, Spain. In 2009 he received the M. Sc. Degree in telecommunication engineering from the Public University of Navarra (UPNA), Pamplona, Spain. He was the recipient of a scholarship to carry out his M. Sc. Thesis at the International Research Center for Telecommunications and Radar (IRCTR) at the Delft University of Technology, The Netherlands.

Since August 2009 he has been with the Antenna Group at the Public University of Navarra (UPNA), where he is currently working towards his Ph.D. His main research interests are in the field of microwave applications of magnetic materials, the design of low-profile metamaterial-inspired antennas and UWB antenna systems.

Iñigo Ederra was born in Isaba, Navarra, Spain in 1972. He received the Ingeniero de Telecomunicación and Ph.D. degrees from the Universidad Pública de Navarra, Pamplona, Spain, in 1996 and 2004, respectively.

In 1997, he joined the Microwave and Millimetre Wave Group, Universidad Pública de Navarra. From 1999 to 2000 he was with the European Space Research and Technology Centre (ESTEC), ESA, Noordwijk, The Netherlands, where he was working on Electromagnetic Bandgap materials and their applications in the field of antennas. Since 2001 he is with the Antenna Group, Universidad Pública de Navarra. From June to October 2002 he was visitor scientist at the Rutherford Appleton Laboratory, Chilton, Didcot, UK, participating in the Startiger project.

He was co-recipient of the LAPC 2006 and IWAT 2007 best paper award. His research interests are in the field of Electromagnetic Bandgap materials and Metamaterials and their applications in microwave and millimetre wave components and antennas.

Cristina Gómez-Polo was born in Madrid, Spain. She received the M.Sc. and the Ph.D. degrees in Physics (Material Science) from the Complutense Universtity, Madrid, Spain, in 1988 and 1992, respectively. In 1995 she joined the Public University of Navarre as lecturer.

Her research activity is mainly focused on the magnetic properties and applications of nanostructured magnetic materials and include the study of amorphous, nanocrystalline and nanoparticle systems.

Alberto Labrador was born in Pamplona, Navarra, Spain on June 4, 1981. He received the M.Sc. degree in telecommunication engineering from Public University of Navarra (UPNA), Pamplona, Spain, in 2008.

Since December 2008 until July 2010, he was working at Public University of Navarra (UPNA) in Physics Department in Physical properties and applications of materials Group. His research was focused on the analysis of ferromagnetic microwires properties and the applications of these microwires in metamaterials development. To carry out this research he received a scholarship from Gobierno de Navarra.

Jose Ignacio Pérez-Landazábal received the Ph.D. degree in solid state physics in 1995 from Bask Country University in Spain. He is currently Associate Professor in the Physics Department of the Public University of Navarra.

His current research interests include Ferromagnetic Shape Memory alloys and magnetic properties of nanoparticles. He is interested in all experimental aspects related to phase transformations and its influence on their magnetic and themo-mechanical properties of materials.

Ramón Gonzalo (S95) was born on July 15, 1972 in Logroõ, La Rioja. He received the M.Sc. and the Ph. D. degree in Ingeniero de Telecomunicación both with honors, from the Public University of Navarra (UPNa), Spain.

Since October 1995 he is with the Antennas Group at the Electrical and Electronic Engineering Department in UPNa where he currently is Associate Professor. From September 1997 to December 1998, he was joined as Research Fellow to the Antenna Section in ESA-ESTEC where he was involved in the modelling and design of electromagnetic crystal devices at microwave and millimetre wave frequencies. He has been involved in more than 25 research project, European, and National level acting as co-ordinator in several of them. In particular he has been co-ordinator of 5 projects funded by the European Space Agency, 2 projects in the framework of the European Commission and several of them funded by the Spanish Minister of Science. He has more than 50 journal publications in peer review magazines and 100 conference papers related to his research lines. From January 2006 untill April 2008 he was acting as sub-director of the Engineering Faculty and from April 2008 he is acting as Head of Electrical and Electronic Engineering Department.

His current area of research is in the field of Terahertz technologies, subsystems and devices, Electromagnetic Band Gap technology with emphasis on space antenna applications, design of waveguide transmission lines and corrugated horn antennas.