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Abstract

The recent inflation episode has been examined in an estimated NK-DSGE model with sticky wages and unemployment. The rise of US price inflation resulted from a combination of a sudden wage rise in 2020, the expansionary monetary policy of 2021, and price-push shocks in the quarters of a global rising on the cost of energy. The projections of the disinflation path indicate that if either prices or wages are further indexed to lagged inflation, wage inflation will be higher and the price disinflation will slow down. Also, a severe tightening of Fed's monetary policy will barely reduce inflation at the cost of higher unemployment.

Keywords: inflation, New Keynesian, indexation, monetary policy
JEL Classification E31, E32, E37, E52.

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1 Introduction

Some time after the global spread of the covid-19 pandemic (declared on March 11th 2020 by the World Health Organization), prices started to rise in many advanced economies. An intense debate has erupted about how to explain this post-covid inflation episode (Ball *et al.*, 2022; Celasun *et al.* 2022; Cerrato and Gitti, 2022; Harding *et al.*, 2022; Shapiro, 2022a and 2022b). The consensus view points at supply-chain disruptions that came across in the aftermath of the pandemic-related business shutdowns, and also at the higher energy costs and commodity prices that resulted from Russia's invasion of Ukraine. Furthermore, some economists argue that fiscal and monetary policies have been inflationary (Bordo and Levy, 2022; Labonte and Weinstock, 2022), and that the labor market resilience has been a consequence of a declining labor force participation (Labonte and Weinstock, 2021). Our paper contributes to this debate using a structural model estimated for the US economy that identifies the sources of the upwards trend of inflation observed in 2021 and 2022.

The model comes from the New Keynesian (NK) tradition of assuming the rational behavior of economic agents facing rigidities on both price setting and wage setting. Similarly to Casares (2010), Galí (2011) and Galí, Smets and Wouters (2012), the wage stickiness results in endogenous fluctuations of the rate of unemployment in ways that characterize the labor market quite differently from what it is commonly assumed in either the Dynamic Stochastic General Equilibrium (DSGE) paradigm with no unemployment (Erceg *et al.*, 2000; Smets and Wouters, 2007) or in extensions that incorporate unemployment in a NK-DSGE model with search and matching frictions *á la* Mortensen and Pissarides (1994).[1]

Involuntary unemployment is obtained in the model as the gap between the labor force and the effective employment. Wages are set in a collective agreement that takes into account the number of jobs demanded by the firms (higher labor demand will push the wage up) and the number of workers supplied by the households (an increase in the labor force will pull the wage down). The presence of nominal rigidities makes wage resetting quite infrequent, creating gaps between the quantity of labor supplied by the households, and the effective labor demand requested by the firms. Our model also features strategic complementarity between price setting and wage setting at the firm level. A firm that operates with a high nominal wage will charge a high price in response to the

[1]Some examples of the New Keynesian literature with Mortensen-Pissarides search frictions and unemployment are Walsh (2005), Krause and Lubik (2007), Trigari (2009), Blanchard and Galí (2010), Christiano, Eichenbaum and Trabandt (2016). and, very recently, Gagliardone and Gertler (2023).

increased marginal cost of production. Meanwhile, if a firm is selling output at a high price, the output sold and labor demanded to produce it will be low which will be pushing down the value of the firm-specific nominal wage.[2]

Another important element for the dynamics of both wage inflation and price inflation is the use of indexation rules when nominal rigidities turn pervasive. The indexation rules used in this model lead to backward-looking components on both the price inflation and the wage inflation Phillips Curves, capturing possible de-anchoring from a constant expected inflation rate and leading to second-round effects from prices/wages indexed to lagged inflation. Regarding the behavior of the central bank, we consider a monetary policy rule in which the nominal interest rate responds to changes in either price inflation or the unemployment rate. Hence, the unobserved output gap, commonly considered in the Taylor (1993)-type rules of NK-DSGE models, is replaced by the unemployment rate in our model.

We estimate the model for the US economy with quarterly observations of time series from the early 1990s to 2022, showing a good fit to the second-moment statistics observed in the data. Then, the causes of the recent high inflation episode are quantitatively estimated through the contributions of the structural shocks to explain the observed upwards trend in US inflation from 2020 to 2022. The estimated impulse-response functions following the three major shocks during this inflation episode (monetary policy, wage push and price push) are displayed and discussed. In addition, we report the projections of the model for the disinflation path over the 2022-2026 period, with alternative patterns of price/wage indexation and the Fed's stance on the inflation surge.

The paper is organized as follows. Section 2 describes the model. Section 3 introduces the estimation strategy and discusses the results observed in the posterior estimates and the second-moment statistics. Section 4 reports the sources of variability that explain the recent US inflation episode, shows the impulse response functions and discusses the Fed's monetary policy actions during the inflation episode. Next, Section 5 forecasts the disinflation paths under different scenarios. Section 6 concludes by reviewing the main findings of the paper.

[2] Galí (2011), and Galí, Smets and Wouters (2012) also introduce unemployment from wage rigidities. However, they assume that households have monopsony power to set wages (as in the seminal paper by Erceg *et al.*, 2000), and ignore the sources of complementarities between price setting and wage setting.

2 The model

In ways that are not typically considered in the standard NK-DSGE literature, this paper is a follow-up of Casares (2010) to bring both endogenous unemployment fluctuations and strategic complementarity between price setting and wage setting decisions.[3] In our model, the presence of nominal wage rigidities explains differences between the firm-level amounts of labor supply and labor demand to bring fluctuations of the unemployment rate around its steady state level. The strategic complementary between firm-specific price and wage setting comes along in an environment of both sticky prices and sticky wages with differentiated Calvo (1983)-type lotteries. The firm-specific price depends on the firm-specific wage and viceversa. Hence, firms with a higher nominal wage will set a higher optimal price because of the transmission of higher wages on the marginal cost of production. This internal relationship can be captured through a (loglinear) relationship between the firm-level optimal price and the relative lagged wages, which for a representative i firm reads as follows[4]

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1 \widetilde{W}_{t-1}(i) \tag{1}$$

where $\widetilde{P}_t^*(i) = \widehat{P}_t^*(i) - \widehat{P}_t$ is the relative price of firm i that sets the optimal price in period t, $\widetilde{P}_t^* = \widehat{P}_t^* - \widehat{P}_t$ is the average optimal price in period t relative to the aggregate price level, $\widetilde{W}_{t-1}(i) = \widehat{W}_{t-1}(i) - \widehat{W}_{t-1}$ is the relative wage of firm i in period $t-1$, and $\tau_1 > 0$ is a positive coefficient to be analytically determined.

Causality also moves in the opposite direction: the relative firm-specific nominal wage, $\widetilde{W}_t^*(i) = \widehat{W}_t^*(i) - \widehat{W}_t$, depends on the current firm-specific relative price, $\widetilde{P}_t(i) = \widehat{P}_t(i) - \widehat{P}_t$. Such conditionality of firm-level nominal wages on firm-level prices is commonly ignored in the DSGE literature, which relies on the sticky-wage framework of Erceg *et al.* (2000) based on the monopsony power of households to set the nominal wage of a differentiated labor service subject to a labor demand constraint. We explore the influence of firm-level prices to wage setting behavior based on the response of labor demand to changes in prices: a firm that charges a high price will have lower demand-determined output produced, and a lower labor demand that will push the nominal wage

[3] In standard NK-DSGE models such as Smets and Wouters (2007), the only source of firm heterogeneity is in the price setting behavior where the variability is due to the history of Calvo-type signals for optimal pricing. Moreover, the optimal price is set taking the market-clearing real wage identically given to all firms.

[4] In our specification, prices are set before wages, thus the state variable for the price set in period t is the nominal wage of the previous period $t-1$.

downwards. This effect is conjectured through a (linear) inverse relationship

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \widetilde{P}_t(i) \tag{2}$$

where $\widetilde{W}_t^* = \widehat{W}_t^* - \widehat{W}_t$ the average labor-clearing nominal wage relative to the aggregate nominal wage. The analytical expression for the elasticity coefficient $\tau_2 > 0$ can be found through some algebra consistent with the structural equations of the model.

This internal complementarity will have significant implications on both price inflation and wage inflation dynamics as the slopes of the respective Phillips Curves are affected by these price-wage interactions. The next two subsections describe the derivation of these price inflation and wage inflation Phillips Curves.

2.1 Price inflation

Following the NK-DSGE tradition, our model assumes that firms operate in monopolistic competition and face a Calvo (1983)-type constant probability of not being able to set the optimal price. The production technology includes capital and labor as variable inputs and there is a fixed cost as in Smets and Wouters (2007). Also as in Smets and Wouters (2007), the firm-level demand elasticity depends on the relative price through the general aggregation scheme of Kimball (1995). For a representative firm i, the loglinearized equation that determines the optimal price is[5]

$$\widehat{P}_t^*(i) = \left(1 - \beta\xi_p\right) E_t^{\xi_p} \sum_{j=0}^{\infty} \beta^j \xi_p^j \left(A\widehat{mc}_{t+j}(i) + \widehat{P}_{t+j} - \sum_{k=1}^{j} x_{t+k}^p \right) \tag{3}$$

where $\beta < 1$ is the discount factor, $0 < \xi_p < 1$ is the probability that firms cannot re-set the price optimally, $E_t^{\xi_p}$ is the rational expectation operator conditional to the lack of future optimal pricing, $A = \frac{1}{(\phi_p - 1)\epsilon + 1}$ is a combination of the Kimball (1995) aggregator curvature ϵ and the value of the parameter ϕ_p that pins down the size of fixed costs of production relative to total output in steady state, $\widehat{mc}_{t+j}(i)$ is the log fluctuation of the real marginal cost around its steady state value in $t+j$, $\widehat{P}_{t+j}$ is the log fluctuation of the aggregate price level in $t+j$ from its trend value, and x_{t+k}^p is the (linearized) indexation factor applied to non-optimal pricing in period $t+k$.

[5]See Smets and Wouters (2007) for the details on the optimizing program of the firm and the related first order conditions. As a standard notation, variables topped with a hat represent the log deviations of the original variable with respect to its steady state level.

If a firm cannot set the price optimally, this is adjusted as determined by the following indexation rule

$$P_t(.) = \left[e^{\varepsilon_t^p}\left(1+\pi_{t-1}^p\right)^{\iota_p}\left(1+\pi^p\right)^{(1-\iota_p)}\right]P_{t-1}(.)$$

which introduces the price indexation factor $X_t^p = e^{\varepsilon_t^p}\left(1+\pi_{t-1}^p\right)^{\iota_p}\left(1+\pi^p\right)^{(1-\iota_p)}$ that combines a response to the value of lagged inflation, π_{t-1}^p, the steady-state (target) inflation, π^p, and a zero-mean price indexation shock, ε_t^p. The parameter $0<\iota_p<1$ accounts for the extent of indexation on lagged inflation as an indicator of the inflation persistence. The linearized approximation for the indexation factor is

$$x_t^p = \iota_p\left(\pi_{t-1}^p - \pi^p\right) + \varepsilon_t^p \tag{4}$$

The aggregate price level (which combines optimal and indexed prices) can be loglinearized to obtain

$$\widehat{P}_t = \left(1-\xi_p\right)\widehat{P}_t^* + \xi_p\widehat{P}_{t-1} + \xi_p x_t^p \tag{5}$$

Using the log approximation to the inflation definition, $\pi_t^p - \pi^p = \widehat{P}_t - \widehat{P}_{t-1}$, in (5) leads to

$$\left(\pi_t^p - \pi^p\right) = \tfrac{\left(1-\xi_p\right)}{\xi_p}\widetilde{P}_t^* + x_t^p$$

As proved at section A of the technical appendix, the previous set of equations can be combined to result in this *hybrid* New Keynesian Phillips Curve for price inflation

$$\pi_t^p - \pi^p = \tfrac{\beta}{1+\beta\iota_p}\left(E_t\pi_{t+1}^p - \pi^p\right) + \tfrac{\iota_p}{1+\beta\iota_p}\left(\pi_{t-1}^p - \pi^p\right) + \kappa_p\widehat{mc}_t + \tfrac{1}{1+\beta\iota_p}\left(\varepsilon_t^p - \beta E_t\varepsilon_{t+1}^p\right) \tag{6}$$

Due to the internal relation between firm-level prices and wages, the slope coefficient, κ_p, is a combination of several deep parameters of the model; ξ_p and ξ_w, characterizing both price and wage rigidities, and parameters from the production technology, firm market power and household preferences:

$$\kappa_p = \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\left(1+\beta\iota_p\right)\left(\left(\phi_p-1\right)\epsilon+1\right)\xi_p\left(1+\Omega\right)}$$

with the auxiliary parameter $\Omega = \frac{\tau_2(1-\alpha)}{\left(\phi_p-1\right)\epsilon+1}\left(1-\frac{\left(1-\beta\xi_p\right)\xi_w}{1-\beta\xi_p\xi_w}\right)$ that includes the coefficient τ_2 measuring the (negative) elasticity of the relative wage on the relative price as introduced in (2). Note that if wages fall substantially when the relative price rises (high τ_2) the slope of the price inflation equation is flatter because the marginal cost of production moves down internally after a price push.

The short-run component of the aggregate real marginal cost, $\widehat{mc}_t$, can be obtained as the difference between the fluctuations of the aggregate real wage, $\widehat{w}_t$, and the marginal product of labor, $\widehat{f}_{n_t}$,

$$\widehat{mc}_t = \widehat{w}_t - \widehat{f}_{n_t} \tag{7}$$

As in Smets and Wouters (2007), there is a Cobb-Douglas production technology with capital, labor, a technology shock and a fixed cost. In log-linear terms, the amount of output produced in the economy, $\widehat{y}_t$, is

$$\widehat{y}_t = \phi_p \left(\alpha \widehat{k}_t + (1-\alpha)\, \widehat{n}_t^d + \varepsilon_t^a \right) \tag{8}$$

with $0 < \alpha < 1$, the parameter $\phi_p = 1 + \frac{\textit{Fixed Cost}}{\textit{Output}}$ pins down the mark-up in steady-state, the effective value of capital is $\widehat{k}_t$ (with variability both at the intensive and extensive margins), labor demand is $\widehat{n}_t^d$, and ε_t^a is an AR(1) technology shock. Thus, the log fluctuation of the marginal product of labor is

$$\widehat{f}_{n_t} = \alpha \left(\widehat{k}_t - \widehat{n}_t^d \right) + \varepsilon_t^a \tag{9}$$

In summary, the price inflation equation (6) features both backward-looking and forward-looking behavior, with the indexation parameter, $0 < \iota_p < 1$, tuning up the inertial component. The endogenous variability is driven by changes in the real marginal cost, $\widehat{mc}_t$, while the exogenous variability is captured by the price indexation shock, ε_t^p.

2.2 Wage inflation

As in Merz (1995), there are identical large households with a continuum of infinitely-lived members.[6] Those who are employed contribute with their labor income to the family-level budget. Some of the household members are unemployed, but they are perfectly insured by the large household and have the same consumption level as the employed members. The intensive margin of labor is a constant number of hours per worker and the variability of labor supply is only driven by changes in the extensive margin (number of workers who are in the labor force). Labor disutility comes from all the household members who are in the labor force independently on whether these household members are actually employed or not.[7] For the utility function, we borrow the specification used

[6] Erceg *et al.* (2000), and most papers that use a NK-DSGE model with sticky wages, assume that each household only supplies a differentiated labor service with monopsony power to set the nominal wage.

[7] In other words, the labor supply disutility when being actually employed is the same as the disutility from being unemployed. This assumption can be considered reasonable in a large household environment that keeps all the

by Galí, Smets and Wouters (2012), that satisfies the desirable property of a balanced-growth path in the steady state,

$$U\left(\widetilde{c}_t, n_t^s(i)\right) = \log \widetilde{c}_t - \chi e^{\varepsilon_t^n} \Theta_t \int_0^1 \frac{n_t^s(i)^{1+\sigma_n}}{1+\sigma_n} di$$

where $\widetilde{c}_t = c_t - h\overline{c}_{t-1}$ is the transformed expression for consumption with external habits measured by the $0 < h < 1$ parameter (note that c_t is household-level current consumption while $\overline{c}_{t-1}$ is the aggregate value of lagged consumption), $n_t^s(i)$ is the number of workers supplied in firm i, ε_t^n is a labor supply preference zero-mean shock, and there is an endogenous preference shifter, Θ_t, that connects labor disutility to consumption as follows

$$\Theta_t \equiv \frac{z_t}{\overline{c}_t - h\overline{c}_{t-1}}$$

where z_t is the trend value of quasi-differenced consumption

$$z_t = z_{t-1}^{1-\upsilon} \left(\overline{c}_t - h\overline{c}_{t-1}\right)^{\upsilon}$$

and the coefficient $0 < \upsilon < 1$ providing the inertial component of trend consumption.

Households choose consumption, capital (both at the extensive margin and its utilization rate), labor supply and bonds seeking to maximize intertemporal utility subject to a standard budget constraint where labor income is obtained from the employed members of the household $\int_0^1 \frac{W_t(i)(1-u_t(i))n_t^s(i)}{P_t} di$. The first order condition on labor supply of type i (workers willing to work at firm i) leads to equalizing the effective real wage to the marginal rate of substitution between consumption and labor

$$\frac{W_t(i)\left(1-u_t(i)\right)}{P_t} = \frac{\chi e^{\varepsilon_t^n} \Theta_t n_t^s(i)^{\sigma_n}}{\widetilde{c}_t^{-1}}$$

which, using the expression for Θ_t introduced above, leads to loglinearized labor supply function

$$\widehat{n}_t^s(i) = \frac{1}{\sigma_n} \left(\widehat{W}_t(i) - \widehat{P}_t - \tfrac{1}{1-u} \left(u_t(i) - u\right) - \varepsilon_t^n - \widehat{z}_t \right) \tag{10}$$

where u is the steady-state rate of unemployment. The loglinear expression for the fluctuations of z is

$$\widehat{z}_t = (1-\upsilon)\, \widehat{z}_{t-1} + \upsilon \left(\tfrac{1}{1-h} \widehat{c}_t - \tfrac{h}{1-h} \widehat{c}_{t-1} \right)$$

members of the family fully insured in case of being unemployed, and having the same level of consumption as the employed members.

with $\overline{h} = \frac{h}{1+\gamma}$ as the growth-adjusted consumption habit formation, and $\gamma > 0$ as the rate of economic growth in steady state.[8]

Labor demand is decided by monopolistically competitive firms. The representative firm faces a demand constraint, with a price elasticity at $\frac{\phi_p}{\phi_p - 1}$ as in Smets and Wouters (2007). Thus, the variable amount of labor demand of firm i is the employment needed to produce the quantity of output demanded at the firm-specific price. This results in the following negative-sign relationship of relative prices, $\widetilde{P}_t(i)$, and relative wages, $\widetilde{W}_t(i)$, on labor demand

$$\widehat{n}_t^d(i) = -\tfrac{1}{\phi_p - 1}\widetilde{P}_t(i) - \alpha \widetilde{W}_t(i) + \widehat{n}_t^d \tag{11}$$

In the vein of Casares (2010), the firm and the household have a collective agreement that sets the value of the firm-specific nominal wage to equalize the planned quantities of labor supply and labor demand. Due to wage rigidity *à la* Calvo (1983), the nominal wage cannot be reset every period to accommodate changes in either labor supply or demand, resulting in a gap between the number of workers supplied by the household and the number of jobs demanded by the firm. The level of firm-level employment is demand-determined, which might be different from the labor force supplied by the households. Let $0 < \xi_w < 1$ be a fixed probability of not being able to update the wage in a given period. If the Calvo-type signal gives the chance to setting a new nominal wage in firm i, its value $\widehat{W}_t^*(i)$ is the one that solves the intertemporal clearing condition:

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \beta^j \xi_w^j \left(\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i) \right) = 0 \tag{12}$$

where $E_t^{\xi_w}$ is the rational expectation operator conditional to the lack of future wage resetting, and the stream of current and expected values of labor supply, $\widehat{n}_{t+j}^s(i)$, and labor demand, $\widehat{n}_{t+j}^d(i)$, are obtained in updated expressions to (10) and (11), respectively. It can be noticed that in the absence of wage rigidity ($\xi_w = 0$), the value of $\widehat{W}_t^*(i)$ delivers contemporaneous clearing $\widehat{n}_t^s(i) = \widehat{n}_t^d(i)$ extended to all firms.[9]

Wage indexation in the labor market is analogous to price indexation in the goods market. Firms that do not receive the Calvo signal to be able to apply equation (12), a fraction $0 < \xi_w < 1$

[8] Symmetric behavior across identical large households results in a household-level consumption equal to aggregate consumption.

[9] Moreover, in the flexible-wage scenario the real wage coincides with the marginal rate of substitution between consumption and labor.

every period, will still have the nominal wage indexed to a combination of lagged price inflation, steady-state price inflation, steady state growth, and a zero-mean wage indexation shock, ε_t^w, as follows

$$W_t(.) = \left[e^{\varepsilon_t^w}\left(1+\pi_{t-1}^p\right)^{\iota_w}(1+\gamma)\left(1+\pi^p\right)^{(1-\iota_w)}\right]W_{t-1}(.)$$

The wage indexation factor $X_t^w = e^{\varepsilon_t^w}\left(1+\pi_{t-1}^p\right)^{\iota_w}(1+\gamma)\left(1+\pi^p\right)^{(1-\iota_w)}$ can be approximated by the linearized expression

$$x_t^w = \iota_w \pi_{t-1}^p + \varepsilon_t^w \tag{13}$$

The loglinearized aggregate nominal wage is $\widehat{W}_t = (1-\xi_w)\widehat{W}_t^* + \xi_w \widehat{W}_{t-1} + \xi_w x_t^w$, and the rate of nominal wage inflation, $(\pi_t^w - \pi^w) = \widehat{W}_t - \widehat{W}_{t-1}$, can be written as a combination of labor-clearing relative wages and the wage indexation factor

$$(\pi_t^w - \pi^w) = \tfrac{(1-\xi_w)}{\xi_w}\widetilde{W}_t^* + x_t^w \tag{14}$$

Finally, the (aggregate) unemployment rate is defined as the excess labor supply through this semi-loglinear expression

$$u_t - u = (1-u)\left(\widehat{n}_t^s - \widehat{n}_t^d\right) \tag{15}$$

where $\widehat{n}_t^s = \int_0^1 \widehat{n}_t^s(i)di$ is the aggregate labor supply. Following the algebra described at section A of the Appendix, the set of equations (10)-(15) can be put together to obtain the wage inflation Phillips Curve

$$\pi_t^w - \pi^w = \beta\left(E_t\pi_{t+1}^w - \pi^w\right) + \iota_w\left(\pi_{t-1}^p - \pi^p\right) - \beta\iota_w\left(\pi_t^p - \pi^p\right) - \kappa_w\left(u_t - u\right) + \left(\varepsilon_t^w - \beta E_t\varepsilon_{t+1}^w\right) \tag{16}$$

that resembles the original Phillips (1958) curve in the inverse relationship between wage inflation and the rate of unemployment. The slope coefficient κ_w is

$$\kappa_w = \frac{(\sigma_n+1)(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\Lambda)(1-u)}$$

with $\Lambda = \frac{\tau_1\beta\xi_w}{(\phi_p-1)(\sigma_n^{-1}+\alpha)}\left(1-\frac{\xi_p(1-\beta\xi_w)}{1-\beta\xi_w\xi_p}\right)$ including the coefficient of the internal price-wage complementarity τ_1. The slope of the wage inflation Phillips Curve, κ_w, depends (inversely) on the parameter measuring wage rigidity, ξ_w, even so it is also determined by the sticky-price probability, ξ_p, and many other structural parameters that play a role in the complementarity between the price and wage setting processes. For example, a greater sensitivity of optimal prices to relative wages (higher τ_1) will reduce the slope of the wage inflation Phillips Curve: if unemployment rises, wages fall in the labor clearing condition and prices fall further in equation (1) in a way that increases output and labor demand to partially compensate the initial increase in unemployment. The wage inflation Phillips Curve (16) is forward looking due to the probability that current wages to be prolonged in the future, but it also takes into account the persistence from lagged inflation that comes from the wage indexation rule.

2.3 Demand-side behavior and the role of public authorities

The intertemporal allocation of consumption is decided by the household, with preferences on consumption habits and labor disutility as in Galí, Smets and Wouters (2012), which result in the hybrid consumption Euler equation

$$\widehat{c}_t = \frac{\overline{h}}{1+\overline{h}}\widehat{c}_{t-1} + \frac{1}{1+\overline{h}}E_t\widehat{c}_{t+1} - \frac{\left(1-\overline{h}\right)}{\left(1+\overline{h}\right)}\left(r_t - r\right)$$

where $\overline{h} = \frac{h}{1+\gamma}$ is the growth-adjusted consumption habit and r_t is the real interest rate that is connected to the nominal interest rate through the Fisher relation

$$\left(r_t - r\right) = \left(R_t - R\right) - E_t\left(\pi^p_{t+1} - \pi^p\right)$$

The investment block is borrowed from Smets and Wouters (2007). The effective value of capital goods, k, is obtained from contributions of both the stock of installed capital, $\overline{k}$, and its utilization intensity, v. There are adjustment costs on investment, i, and the stock of capital is one-period predetermined due to time-to-build requirements. Investment rises with Tobin's q, which depends positively on the expected return on capital goods, r^k, and negatively on the real return on risk-free bonds, r. The capital utilization rate increases when the rental rate of capital goes up because changes in the intensity of the use of capital units are also subject to adjsutment costs. The optimal substitutions between labor and capital are conducted until the ratio of real wage to the real return of capital equalizes the ratio between the marginal product of capital and the marginal product of labor. All these assumptions lead to these loglinearized equations

$$\begin{aligned}
\widehat{i}_t &= \tfrac{1}{1+\beta}\widehat{i}_{t-1} + \tfrac{\beta}{1+\beta}E_t\widehat{i}_{t+1} + \tfrac{1}{(1+\beta)(1+\gamma)^2\varphi}\widehat{q}_t + \varepsilon_t^i \\
\widehat{q}_t &= \tfrac{1-\delta}{1+r^k-\delta}E_t\widehat{q}_{t+1} + \tfrac{r^k}{r^k+1-\delta}E_t\widehat{r}_{t+1}^k - (r_t - r) \\
\widehat{k}_t &= \widehat{\widehat{k}}_{t-1} + \widehat{v}_t \\
\widehat{v}_t &= \tfrac{1-\psi}{\psi}\widehat{r}_t^k \\
\widehat{\widehat{k}}_t &= \tfrac{1-\delta}{1+\gamma}\widehat{\widehat{k}}_{t-1} + \tfrac{\gamma+\delta}{1+\gamma}\widehat{i}_t + (\gamma+\delta)(1+\beta)\gamma\varphi\varepsilon_t^i \\
\widehat{r}_t^k &= \widehat{w}_t - \left(\widehat{k}_t - \widehat{n}_t^d\right)
\end{aligned}$$

where $0 < \delta < 1$ is the rate of capital depreciation, $\varphi > 0$ is the coefficient that captures the curvature of the adjustment costs of investment, and $0 < \psi < 1$ is a parameter that measures the elasticity of changes in the rate of capital utilization for its corresponding adjustment costs function.

The central bank conducts monetary policy through a simple rule that comprises a dual mandate of stabilizing price inflation and the rate of unemployment by setting smoothed movements of the nominal interest rate[10]

$$R_t - R = (1-\mu_R)\left[\mu_\pi\left(\pi_t^p - \pi^p\right) - \mu_u\left(u_t - u\right)\right] + \mu_R\left(R_{t-1} - R\right) + \varepsilon_t^R \tag{18}$$

with $\mu_\pi > 1$ as the response coefficient to inflation (greater than 1.0 to satisfy Taylor's principle), $\mu_u > 0$ as the response coefficient to the observed unemployment rate, $0 < \mu_R < 1$ as the smoothing coefficient on the variations of the nominal interest rate, and the presence of a monetary policy shock, ε_t^R.

Finally, the government exogenously decides how much to spend on purchases of consumption goods, ε_t^g, that are financed through a lump-sum tax to the households or by issuing bonds

$$\varepsilon_t^g = tax_t + (1+r_t)^{-1} b_{t+1} - b_t$$

Combining the household budget constraint with the government budget constraint and using the definition of firm dividends lead to the log-linearized overall resources constraint (goods market

[10]We are not aware of any NK-DSGE model with unemployment that introduces a Taylor-type monetary policy rule with adjustments on the nominal interest rate responding to changes in the unemployment rate. There are papers that only consider the reaction of the nominal interest rate to inflation (Walsh, 2005; Galiardone and Gertler, 2023), others include that also responses to deviations of output from its steady-state level (Trigari, 2009; Christiano *et al.*, 2016), while others incorporate the responses to the output gap obtained as the log difference between current output and the flexible-price, flexible-wage (potential) level of output (Galí *et al.*, 2012).

clearing condition)

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \frac{r^k k}{y}\widehat{v}_t + \frac{g}{y}\varepsilon_t^g \tag{19}$$

which feeds the amount of demand-determined output, $\widehat{y}_t$, from the endogenous contributions of consumption, $\widehat{c}_t$, investment, $\widehat{i}_t$, and changes in capital utilization, $\widehat{v}_t$, and the exogenous component, ε_t^g, that captures autonomous spending shocks.

2.4 Summary and sources of variability

The loglinearized model can be solved for twenty equations that can produce solution paths for the nineteen endogenous aggregate variables of the model: $(\pi_t^p - \pi^p)$, $(\pi_t^w - \pi^w)$, $(u_t - u^n)$, $(R_t - R)$, $(r_t - r)$, $\widehat{y}_t$, $\widehat{c}_t$, $\widehat{z}_t$, $\widehat{w}_t$, $\widehat{mc}_t$, $\widehat{f}_{nt}$, $\widehat{n}_t^s$, $\widehat{n}_t^d$, $\widehat{i}_t$, $\widehat{q}_t$, $\widehat{r}_t^k$, $\widehat{\overline{k}}_t$, $\widehat{v}_t$, and $\widehat{k}_t$.[11]

The variability comes from the presence of the following seven exogenous variables:

AR(1) technology shock, ε_t^a, that identifies changes in the aggregate amount of output produced that are not explained by changes in either labor or effective capital as the variable inputs of the production function (8).

AR(1) monetary policy shock, ε_t^R, that captures the non-systematic component of monetary policy as changes in the nominal interest rate that are not driven by responses of the central bank to stabilize either inflation or unemployment when implementing the monetary policy rule (18).

ARMA(1,1) price indexation shock, ε_t^p, that brings the variability of price inflation from the exogenous component of the price indexation factor, (5). We will refer to ε_t^p as a price-push shock as it captures any change in inflation that is not coming from fluctuations in the real marginal cost. For example, the price mark-up shock in the canonical DSGE model of Smets and Wouters (2007) is observationally equivalent to ε_t^p.

ARMA (1,1) wage indexation shock ε_t^w, which collects the variability of wage inflation driven by the exogenous component of the wage indexation factor, (13). We will refer to ε_t^w as a wage-push shock because it identifies any change in wage inflation that is not caused by fluctuations in the rate of unemployment. For example, the wage mark-up shock of the canonical DSGE model of Smets and Wouters (2007) is observationally equivalent to ε_t^w.

AR(1) fiscal shock, ε_t^g, that explains fluctuations of aggregate demand primarily due to changes in government purchases. Since ε_t^g is the residual term of the overall resources constraint (19), it could

[11] The complete list of equations is available in the on-line Appendix at section C.

also be capturing any autonomous spending shock coming from variations in the current account (not included endogenously in our closed-economy model), or from the autonomous components of consumption or investment. As in Smets and Wouters (2007), there is a positive crossed effect of innovations from the technology shock, ε_t^a, on the spending shock, ε_t^g, as a way to identify possible spillover effects from technological progress on the aggregate demand.

AR(1) investment shock, ε_t^i, which takes the variability of investment caused by factors that are not considered in the determination of Tobin's q in the investment block of the model.

AR(1) labor supply shock, ε_t^n, which identifies changes in the labor force that are not associated to either variations in the real wage or in the amount of household-level consumption (such as population aging or migration flows).

3 Estimation

The NK-DSGE model has been estimated for US quarterly data from 1992 to 2022. The observable series are plotted in Section D of the Appendix, with a detailed explanation of the data sources. Six of the seven series used in the estimation have been retrieved from the FRED database of the Federal Reserve Bank of St. Louis: real GDP (growth rate), price inflation (growth rate of the GDP Implicit Price Deflator), nominal wage inflation (growth rate of the Hourly Compensation for All Employed Persons), real investment (growth rate of real Fixed Private Investment), employment (growth rate of 16+ Employment Level), and the unemployment rate. Series of real GDP, real investment and employment have been obtained in per capita terms dividing by the smoothed series of population published by the US Bureau of Labor Statistics (BLS). As for the seventh series, we have used the shadow interest rate calculated by Wu and Xia (2016), which adjusts the Federal Funds rate to negative values when the Zero Lower Bound is binding, in a way that accounts for unconventional monetary expansions through Fed's asset purchases. The estimation follows a two-step Bayesian procedure using quarterly data from the seven observed series: $\Delta\widehat{y}$, π^p, π^w, u, $\Delta\widehat{i}$, $\Delta\widehat{n}^d$ and R. It should be noted that the slope coefficients of the price and wage inflation Phillips Curves, κ_p in (6) and κ_w in (16), are implicit functions of the undetermined coefficients τ_1 and τ_2. These coefficients were analytically solved through a non-linear four-equation system (see section A.5 in the Appendix) that is introduced as part of the estimation.

A few model parameters have been fixed at calibrated values: the rate of capital depreciation is set at the usual value of $\delta = 0.025$ (2.5% per quarter), the Kimball elasticity is fixed at $\epsilon = 10$ as in Smets and Wouters (2007), and the steady-state ratio of autonomous spending over GDP is set at 0.18 also as in Smets and Wouters (2007). Finally, the coefficient of autocorrelation of the autonomous spending shock has been fixed at a high value of $\rho_g = 0.97$ as estimated in Galí *et al.* (2012), due to observing posterior estimates very close to 1.0 which would make the generating process non-stationary.

Table 1 and 2 report the priors and estimated mean posteriors of each model parameter, together with the 10% and 90% quantiles of the posterior distributions. All the selected priors are consistent with the related NK-DSGE literature. In general, the posterior estimates of the structural parameters are quite standard within the NK-DSGE literature. Hence, the consumption habits coefficient is measured at $h = 0.84$, somewhat higher persistence of consumption habits than the value estimated by Smets and Wouters (2007). The labor supply curvature parameter is estimated at $\sigma_l = 4.74$, consistent with a small value for the Frisch labor supply elasticity as commonly observed in empirical studies. The posterior estimate of the capital share in the production technology at $\alpha = 0.37$ is higher than its prior estimate set at 0.30. The parameter that measures the influence of cyclical consumption on labor disutility comes with a posterior estimate of $\upsilon = 0.78$, markedly higher than the one reported by Galí *et al.* (2012), but still leading to a slight procyclical behavior of the labor supply (cross correlation to output growth at 0.26) due to the small impact of wealth effects. The Calvo-type price and wage rigidities are estimated at $\xi_p = 0.70$ and $\xi_w = 0.82$, respectively, implying nominal rigidities with higher wage stickiness (average duration of nominal wage contracts is estimated at $\frac{1}{1-\xi_w} = \frac{1}{0.18} = 5.56$ quarters or 16.7 months) than price stickiness (average duration of price contracts is estimated at $\frac{1}{1-\xi_p} = \frac{1}{0.3} = 3.33$ quarters or 10 months). Additionally, the estimated firm-level relations for the complementarities between price and wage setting are

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* + 0.0735\widetilde{W}_{t-1}(i)$$

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - 1.6394\widetilde{P}_t(i)$$

which indicate a quite inelastic behavior of the firm-specific optimal prices with respect to the relative wage (this might be explained by the substitution of labor for capital at either the intensive or extensive margin), and a moderate elasticity of the labor-clearing nominal wage to changes in the relative price. The latter comes along because of the high sensitivity of both output produced and labor demand to changes in the relative price (as shown at section A in the Appendix, the price elasticity of labor demand is $\frac{-1}{\phi_p - 1}$, which using the mean posterior estimate at $\phi_p = 1.36$ leads to $\frac{-1}{1.36-1} = -2.78$).

Table 1. Priors and estimated posteriors of the structural parameters.

	Priors			Posteriors		
	Distr	Mean	Std D.	Mean	10%	90%
α, production technology	Beta	0.30	0.05	0.3662	0.3279	0.4055
h, consumption habits	Beta	0.70	0.10	0.8374	0.7667	0.9118
σ_l, hours elasticity	Normal	2.00	1.00	4.7432	3.9162	5.5893
υ, cyclical consumption in labor supply	Beta	0.50	0.20	0.7816	0.6109	0.9633
ψ, capital utilization cost	Beta	0.50	0.10	0.7524	0.6245	0.8799
φ, adjustment costs of investment	Normal	4.00	1.50	5.8997	3.9727	7.7446
ϕ_p, steady-state price mark-up	Normal	1.25	0.12	1.3607	1.2406	1.4805
ξ_p, Calvo sticky prices	Beta	0.50	0.10	0.6961	0.6379	0.7555
ξ_w, Calvo sticky wages	Beta	0.50	0.10	0.8241	0.7932	0.8557
ι_p, price indexation on lagged inflation	Beta	0.50	0.10	0.3925	0.2476	0.5313
ι_w, wage indexation on lagged inflation	Beta	0.50	0.10	0.4920	0.3303	0.6511
μ_R, interest-rate smoothing in MPR	Beta	0.75	0.10	0.8872	0.8642	0.9105
μ_π, inflation coefficient in MPR	Normal	1.50	0.30	1.7022	1.3858	2.0175
μ_u, unemployment coefficient in MPR	Normal	0.50	0.10	0.4053	0.2952	0.5141
γ, steady-state growth	Normal	0.36	0.05	0.3261	0.2711	0.3813
π^p, steady-state price inflation	Gamma	0.47	0.05	0.4853	0.4049	0.5629
u, steady-state unemployment rate	Normal	5.81	0.05	5.8108	5.7282	5.8912
$\beta^{-1} - 1$, steady-state real interest rate	Gamma	0.25	0.10	0.1484	0.0621	0.2317

Table 2. Priors and estimated posteriors of the parameters in the exogenous processes.

	Priors			Posteriors		
	Distr	Mean	Std D.	Mean	10%	90%
σ_{η^a}, std. dev. innov. technology shock	Invgamma	0.10	2.00	0.5226	0.4624	0.5839
σ_{η^p}, std. dev. innov. price-push. shock	Invgamma	0.10	2.00	0.5062	0.2949	0.7234
σ_{η^w}, std. dev. innov. wage-push shock	Invgamma	0.10	2.00	1.1617	1.0123	1.3030
σ_{η^n}, std. dev. innov. labor supply shock	Invgamma	0.10	2.00	1.9225	1.6080	2.2426
σ_{η^R}, std. dev. innov. mon. policy shock	Invgamma	0.10	2.00	0.1166	0.1026	0.1306
σ_{η^g}, std. dev. innov. fiscal shock	Invgamma	0.10	2.00	5.0600	4.4631	5.6339
σ_{η^i}, std. dev. innov. investment shock	Invgamma	0.10	2.00	0.4931	0.4064	0.5794
ρ_a, autocorrelation technology shock	Beta	0.50	0.15	0.9127	0.8402	0.9870
ρ_p, autocorrelation price-push shock	Beta	0.50	0.15	0.9069	0.8406	0.9740
μ_p, moving average of price-push shock	Beta	0.50	0.15	0.3261	0.1798	0.4726
ρ_w, autocorrelation wage-push. shock	Beta	0.50	0.15	0.3879	0.1067	0.6570
μ_w, moving average of wage-push shock	Beta	0.50	0.15	0.4870	0.2451	0.7230
ρ_n, autocorrelation labor supply shock	Beta	0.50	0.15	0.9760	0.9580	0.9952
ρ_R, autocorrelation mon. policy shock	Beta	0.50	0.15	0.4344	0.3293	0.5383
ρ_{ga}, cross effect fiscal and tech. shock	Normal	0.50	0.25	0.5086	0.1002	0.9153
ρ_i, autocorrelation investment shock	Beta	0.50	0.15	0.4931	0.4064	0.5794

The estimated values of the weight of lagged inflation in the determination of the price indexation factor is $\iota_p = 0.40$, and that for wage indexation is slightly higher at $\iota_w = 0.50$. Thus, prices that are adjusted following the indexation rule rely slightly more on the (constant) steady state rate of inflation than in the observed value of lagged inflation, while nominal wages do it evenly among the two contributors. In the price inflation Phillips Curve (6), the forward-looking component (attached to expected next-period inflation) is estimated at $\frac{\beta}{1+\beta\iota_p} = 0.78$ while the backward-looking component (attached to lagged inflation) is at $\frac{\iota_p}{1+\beta\iota_p} = 0.22$. Therefore, the expectations on future real marginal costs are substantially more influential that their past realizations.

The monetary policy rule (18) estimates the fluctuations of the nominal interest rate with a high coefficient of the interest rate smoothing, $\mu_R = 0.89$, and a response to price inflation estimated at $\mu_{\pi^p} = 1.70$, slightly higher to the 1.5 value implied by the original Taylor (1993)'s prescription. The macroeconomic indicator that the central bank uses to measure the business cycle momentum is the deviation of the rate of unemployment with respect to its steady state value, $u_t - u$. The unemployment rate can be observed in the labor market and real-time series are commonly published by official agencies. Other NK-DSGE model introduce the output gap or log deviations of output from trend that cannot be observed by the central banks and must be pinned down through modelling methods. The prior estimate for the coefficient on unemployment fluctuations was set at 0.5 and the estimated value is some little below at $\mu_u = 0.41$.

As for the parameters that shape the generating processes of the seven shocks of the model, the Bayesian posterior estimates provide a very long persistence of labor supply, technology and price push shocks (see Table 2). The estimated standard deviations of the innovations to the shocks are scaled up or down depending on the way the enter the structural equations. The values obtain lead to both an overall and relative macroeconomic volatility on the endogenous variables of the model similar to that observed in US data. The comparison of the standard deviations of some of these model-based variables with those obtained from US data corroborates the similarities (see Table 3).

Table 3. Second-moment statistics.

	$\triangle\widehat{y}_t$	$u_t - u$	$\pi_t^p - \pi^p$	$\pi_t^w - \pi^w$	$R_t - R$
U.S. data, 1992:1-2022:4					
Std. deviation, %	1.2171	1.7712	0.3502	1.0133	0.6502
Corr. with $\triangle\widehat{y}_t$	1.0000	-0.2261	0.2544	-0.3894	0.0743
Corr. with $u_t - u$	-0.2261	1.0000	-0.3015	-0.0515	-0.4833
Autocorrelation	-0.2044	0.8483	0.6896	-0.2152	0.9795
Estimated model					
Std. deviation, %	1.2919	1.8121	0.4779	1.2094	0.5790
Corr. with $\triangle\widehat{y}_t$	1.0000	-0.3359	0.0631	0.3055	-0.1580
Corr. with $u_t - u$	-0.3359	1.0000	-0.3315	-0.2871	-0.2617
Autocorrelation	0.2701	0.8551	0.7773	0.0683	0.9611

Table 3 labels: $\triangle\widehat{y}_t$ is output growth, $u_t - u$ is the deviation of the unemployment rate with respect to its steady state value, $\pi_t^p - \pi^p$ is the deviation of the rate of price inflation with respect to its steady state value, $\pi_t^w - \pi^w$ is the deviation of the rate of nominal wage inflation with respect to its steady state value, and $R_t - R$ is the deviation of the nominal interest rate with respect to its steady state value.

Table 3 reports a selection of second-moment statistics obtained in the simulations of the estimated model, and a comparison to those corresponding statistics observed in US quarterly data. This table also includes two alternative measures of cyclical cross correlations: one on the rate of growth of real (GDP) output (goods market indicator), and another one relative to the rate of unemployment (labor market indicator). The model provides quite a good fit to US data, with similar numbers obtained for the standard deviations (volatility) of the selection of reported variables. Perhaps, it could be mentioned that the price inflation volatility in the model (0.48%) is higher than in the data (0.35%), although the only episode of high inflation has been seen in the very last quarters of the 30-year sample. The cross correlations of the rate of unemployment with output growth, price inflation, wage inflation, and the nominal interest rate are all negative and between -0.5 and 0 in both the model and the US data. In the cross correlation to output growth, the model delivers a countercyclical behavior of both the unemployment rate (also observed in the data) and the nominal interest rate (in the data the sign of the correlation turns to mildly positive). Both consumption and investment respond with falls to higher interest rates in NK-DSGE model which makes the growth of demand-determined output shape a negative co-movement with the nominal interest rate. As for the endogenous persistence, Table 3 shows a negative autocorrelation of output growth (due to the dramatic one-period shifts of US real GDP in the quarter of the pandemic and its subsequent one) and also a negative autocorrelation of nominal wage inflation. The latter corresponds to large quarter-to-quarter swings in the quarterly series of Hourly Compensation to US workers (see the time series in Section D of the Appendix). The model reports low but positive coefficients of autocorrelation in these two variables. Price inflation is moderately persistent (with autocorrelation at around 0.7), the rate of unemployment has longer inertia (with autocorrelation near 0.85) and the nominal interest rate is the most persistent variable with a autocorrelation coefficient at 0.97. The model-based and US data numbers for these three autocorrelations are very similar (see Table 3).

3.1 Impulse-response functions

The transmission mechanism from a single shock on the endogenous variables can be examined in the impulse-response function analysis. Figure 1 shows the estimated responses after a price-push shock (blue line), a wage-push shock (red line) and an interest-rate shock (yellow line).[12] The size of the shock is normalized at one estimated standard deviation of the innovation to each shock.

The rise of inflation following a price-push shock leads to an economic contraction because both consumption and investment fall. The central bank raises the interest rates when plugging the observed inflation in the monetary policy rule. As firms shrink the amount of production, the demand less labor and the effective level of employment falls. Meanwhile, there is a mild decrease in the labor force (caused by lower real wages). The rate of unemployment moves up by approximately two-thirds of the cut in labor demand. Wage inflation initially goes slightly down due to the increase in the unemployment rate, but it swings to the positive side in the following quarter because of the indexation of wages on lagged inflation. These responses to the price-push shock are very similar to the ones obtained after an oil price shock in the NK model with unemployment of Gagliardone and Gertler (2023).

The effects of the wage-push shock on output are two-sided. On the one hand, higher wage inflation will raise labor supply (real wages go due to price stickiness) and the increase in the amount of labor force can have an expansionary effect of output. On the other hand, an increase in costs of production will bring higher prices set by firms and the central bank will announce interest rate hikes that will reduce the aggregate demand and the amount of output produced. As illustrated in Figure 1, the first effect dominates at the time of the shock but it is soon overcome by the second effect bringing net contractionary effects on output. Price inflation remains at higher values than the steady-state rate due to the inertial component on the price indexation rules. The unemployment rate climbs as a combined result of lower employment and a rise of the labor force.

Finally, Figure 1 depicts the effects of a monetary (interst-rate) shock entering the policy rule (18). The real interest rate rises (due to price stickiness) and both consumption and investment fall describing u-shaped patterns explained by the sluggishness in these responses generated by having consumption habits and adjustment costs on investment. The quantitative implications for output, price inflation and unemployment are significant: around a 10 basis point rise in the quarterly interest rate is recessionary with a maximum decrease of output at 0.36% of its steady-state level, and a reduction of inflation in by nearly 10 basis points, and a higher rate of unemployment by

[12]Price-push, wage-push, and monetary policy shocks concentrate a very large portion of the contributions to explain the recent US excess inflation (see subsection 4.1). This is the reason why these 3 shocks have been selected to be examined in the impulse-response analysis.

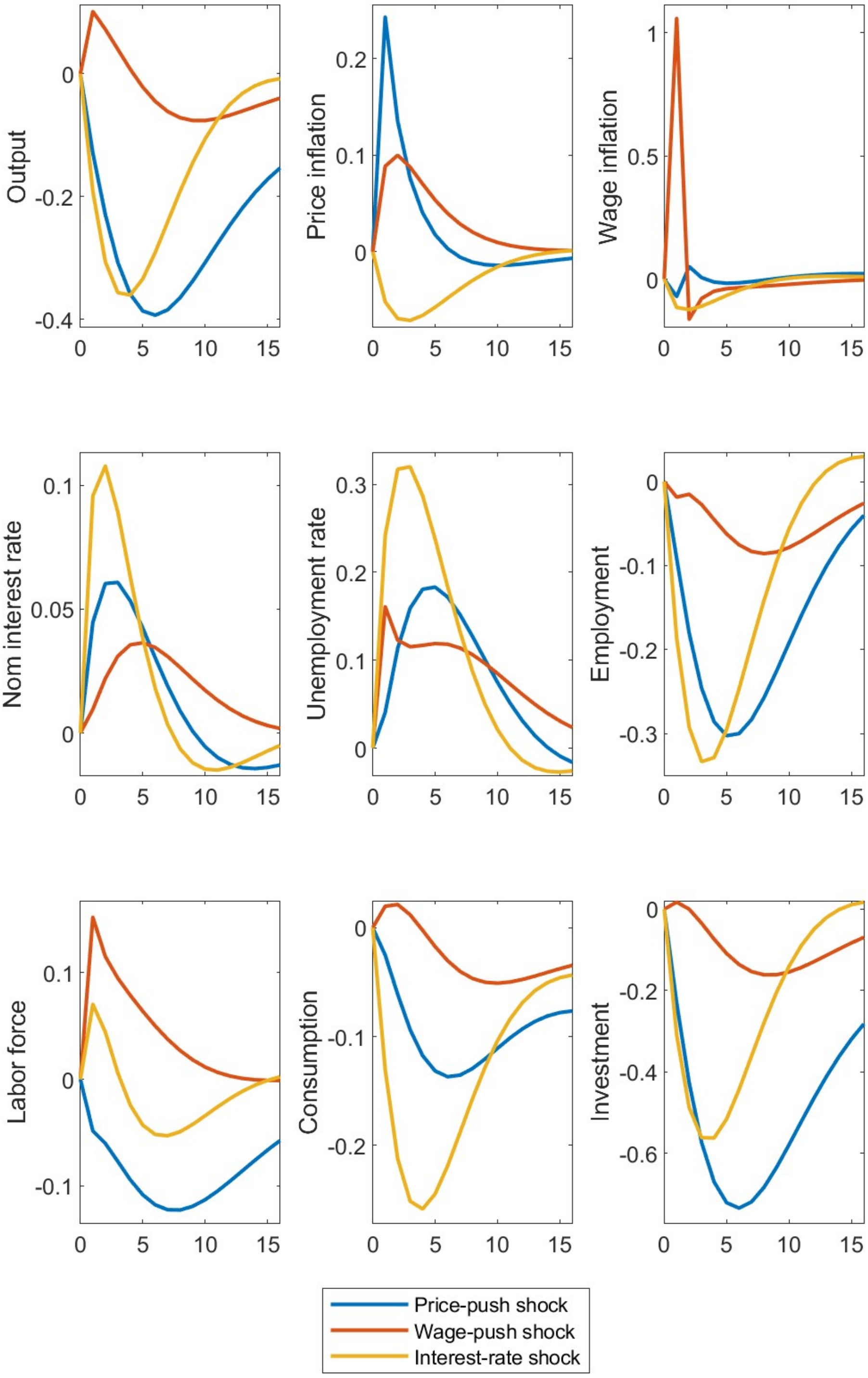

Figure 1: Estimated impulse-response functions

32 basis points beyond the steady-state rate. Wage inflation goes down as a combination of a lower labor demand and the indexation of wages on a series of decreasing values of lagged price inflation. The labor force swings from countercyclical at the time of the shock (due to welfare effects) to procyclical (due to lower real wages) a few quarters after the shock. The variability on labor supply is much smaller than that on labor demand (employment). So, the reduction of employment becomes the main driver to explain the higher unemployment rate.

4 The recent US inflation episode

This section takes our estimated model to examine the determinants of the variability of US price inflation observed in the quarters that followed the covid-19 pandemic.

4.1 Shock decomposition

Figure 2 shows the variance decomposition of price inflation, wage inflation, the unemployment rate and the nominal interest rate among the seven shocks generating variability in the model. The analysis is focused on the last part of the sample, covering from the first quarter of 2019 to the last available observation in the fourth quarter of 2022. The colored vertical bars of Figure 2 display the quarterly contributions of the technology shock ε_t^a (red), the monetary policy shock ε_t^R (green), the price push shock ε_t^p (yellow), the wage push shock ε_t^w (dark blue), the autonomous spending shock ε_t^g (pink), the labor supply shock ε_t^n (orange), and the investment shock ε_t^i (light blue).

Technology shocks have a minor role in the recent US inflation episode. As showed at panel (a) in Figure 2, they are only inflationary at the beginning of 2022 (2022:1-2022:2), when total factor productivity might be lower due to a tight labor market with unemployment at all-time low rates. They account only for an accumulated 0.14% increase of inflation that only takes 9.15% of the high inflation episode (see Table 4). The influence of technology shocks on unemployment during the pandemic swings from positive in 2020-21 to negative in 2022, which indicates that total factor productivity trended down in the quarters following the lockdown enforced in April-May 2022. This lower productivity may reflect the supply-chain disruptions that took place once in those quarters (Celasun *et al.*, 2022, Shaphiro, 2022a).

Monetary policy shocks are estimated in the model to be inflationary throughout the whole subsample period from 2020:3 to 2022:2 (see the green vertical bars in panel (a) of Figure 2). As reported in Table 4, the accumulated contribution of the monetary policy shocks to inflation is 3.43% (with 43 basis points averaged per quarter) which represents a 27.23% of the total excess inflation. Another way of getting the same conclusion is by looking at the estimated monetary policy shocks in the panel (d) of the nominal interest rate. All the green bars, except that of last quarter of 2022, are on the negative side of the diagram, which indicates that the estimated residuals of the monetary policy rule (18) are negative and therefore taking action as expansionary shocks. Indeed, the series of the nominal interest rate only reaches the steady-state value in the last observation, with an estimated policy shock of small value but with a positive sign. These results indicate that the end of the monetary stimulus got too late and that some of the inflation rise was propagated by the low interest rates set by the Fed in 2020 and 2021, as recently argued by Bordo and Levy (2022), Labonte and Weinstock (2022), and Levin and Nelson (2023).[13]

The combination of business shutdowns and overloaded work in some basic sectors (health, food, transportation) resulted in a large increase of nominal wages during the pandemic; the average quarterly wage inflation rate peaked at 6.5% in 2020:2 (the wage inflation series is available in Section 5 of the Appendix). The shock decomposition in the estimated model identifies wage push shocks as the main contributor for the big rise of wage inflation at the beginning of the pandemic (see the blue bars at panel (b) of Figure 2). Such rise in the cost of labor increased the real wage and the real marginal cost, which would have increased the rate of inflation as indicated in the Phillips Curve (6). The estimated shock decomposition of the model shows this effect in the contributions of wage push shocks for price inflation (see the blue bars in panel (a) of Figure 2), which precisely switch from negative to positive (and high) in 2020:3, and remain on the positive side until the beginning of 2022. The quarterly contributions to the rise of inflation represents a 19.21% of all the increase in inflation (see Table 4), concentrated in the first half of the inflation episode (2020 and 2021).

Price push shocks capture the changes in inflation that are not explained by variations in either the real wage or the marginal product of labor. They might identify the effects of higher mark-ups (Smets and Wouters, 2007) or energy costs (Gagliardone and Gertler, 2023), as the exogenous variability entering the price inflation Phillips Curve. The oil price moved down during the first wave

[13]In the Spring of 2021, Fed officials insisted that they had not yet started discussions about phasing out the program and had not even begun to "talk about talking about it" (Levin and Nelson, 2022). The FOMC began tapering the amount of open-market asset purchases in November 2021, and the program ended in March 2022.

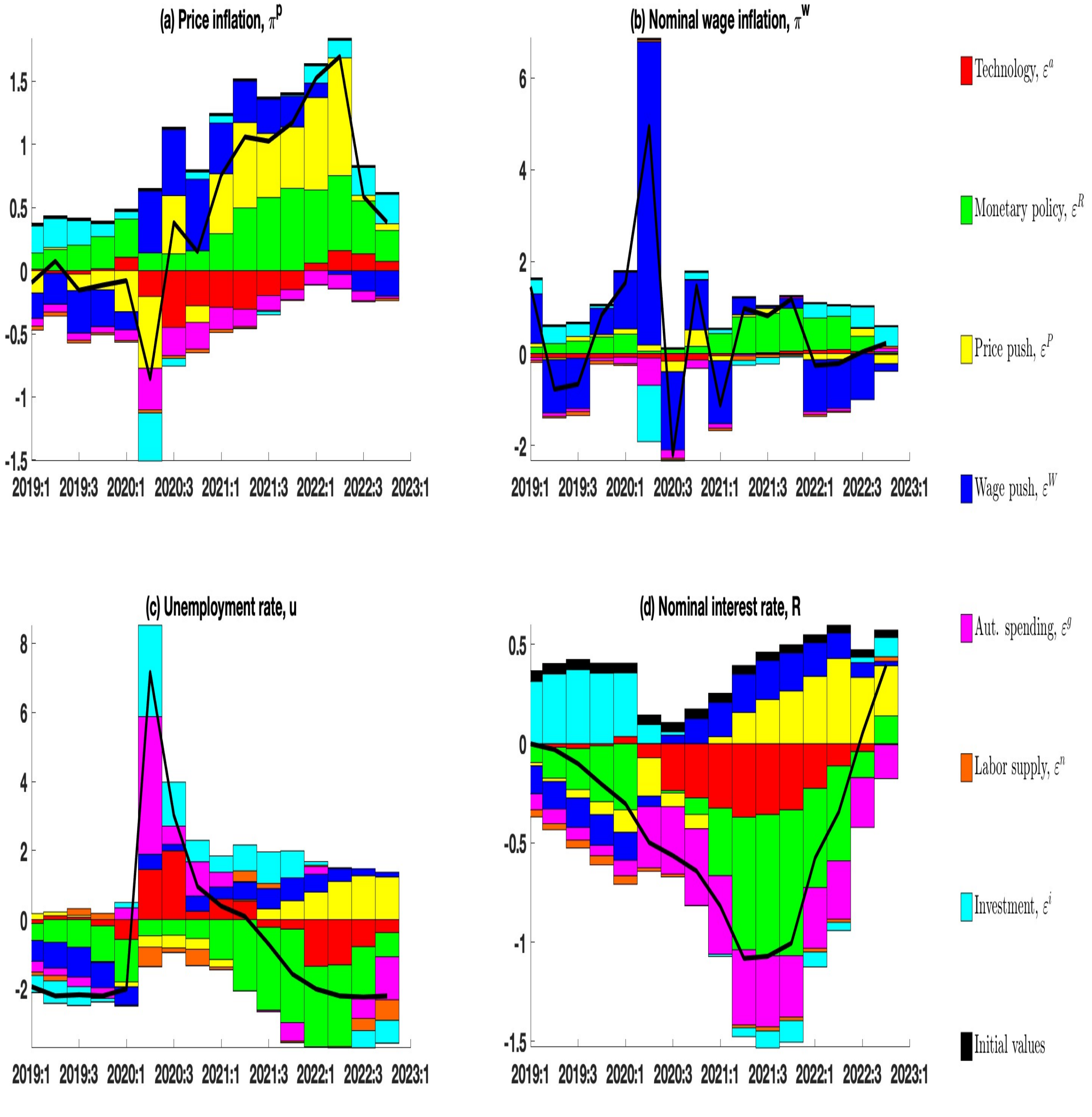

Figure 2: Estimated shock decomposition (2019:1 to 2023:2)

of the pandemic (due to a plummeting demand), but it began a dramatic rise in the second half of 2021 (higher demand when pandemic-related socioeconomic restrictions are lifted), and especially in 2022 due to Russia's invasion of Ukraine (severe reduction on the supply side). The contributions of the estimated price-push shocks of the model reflects the evolution of the oil price in international markets (see yellow bars of the price inflation panel (a) in Figure 2). These contributions are particularly large at the end of 2021 and in the first half of 2022 with a marginal effect averaged at around 63 basis points in each quarter (4.40% higher inflation in accumulated). The contribution of the price push shocks represents a 39.40% of all the excess inflation episode, being the largest share among the seven shocks of the model.

Table 4. Shocks explaining the US accelerating inflation episode (2020:3 to 2022:2).

	Technology ε_t^a	Mon. Policy ε_t^R	Price-push ε_t^p	Wage-push ε_t^w	Investment ε_t^i
Timing					
Periods	22:1 & 22:2	20:3 to 22:2	20:3 & 21:1-22:2	20:3 to 22:1	20:4, 21:1 & 21:4 to 22:2
# of quarters	2	8	7	7	5
Contributions					
Total	0.2920	3.4348	4.4011	2.4516	0.3995
Quarterly	0.1460	0.4345	0.6287	0.3065	0.0799
Overall, %	9.15%	27.23%	39.40%	19.21%	5.01%

Table 4 labels: ε_t^a is the technology shock, ε_t^R is the monetary policy shock, ε_t^p is the shock in the price indexation rule, ε_t^w is the shock in the wage indexation rule, and ε_t^i is the shock in the adjustment costs of investment function.

The investment shock explains around 40% of the recession and the rise of unemployment that was observed in the US at the quarter of the covid-19 lockdown (2020:2). However, the influence of the investment shock on the inflation episode is very small (see Figure 2). In the last quarter of 2020 and the first quarter of 2021, the investment shock is slightly inflationary as the spending on capital goods bounces back during the economic reactivation. Then, investment cools down and turns weakly deflationary in 2021:2 and 2021:3. In the last 3 quarters of the inflationary period

(2021: 4 to 2022:2), the investment shock is again expansionary and it brings some demand-side inflationary pressure. The overall contribution is rather small as it only accumulates to 40 basis points of higher inflation, which accounts for just a 5% of the total excess inflation.

The autonomous spending shocks are estimated to be contractionary during the covid-19 pandemic, with a very large contribution to explain the latest soaring of the unemployment rate (see pink bars at panel (a) in Figure 2) and also on the negative output growth (observed in the data but not documented in Figure 2). However, these demand-side shocks are not behind the increase on price inflation or wage inflation (pink bars are below the zero line at panels (a) and (b) of Figure 2). This result does not go in line with some recent papers claiming that demand-side and fiscal shocks had a substantial role to explain the post-covid inflation (Shaphiro, 2022a; Labonte and Weinstock, 2022). Note that on our model the spending shock ε^g collects all the aggregate demand variability that generates the increase in public spending and crowds out consumption and investment.

Finally, the labor supply shocks have a mild contribution in explaining the evolution of the unemployment rate (see orange bars in the unemployment rate panel (c) of Figure 2). The US has experience a decreasing trend of the labor force from the 2000s decade and the model captures such declining labor supply with a positive estimate of the labor supply shock, ε^n. As labor supply shrinks, some negative impact on output is accompanied by higher wages and a lower rate of unemployment. The estimated shock decomposition shows a very small contribution of this labor supply shock; a reduction of the unemployment rate in 2022:4, and some increase in nominal wage inflation in the quarters of 2022. There is no estimated effect of the labor supply shock on higher inflation.

4.2 The stance of Fed's monetary policy

The analysis of the shock decomposition has shown that the monetary policy actions decided by the Fed during the quarters of the covid-19 pandemic have had a significant contribution to explain the acceleration of inflation from the third quarter of 2020 to the second quarter of 2022.

For a deeper analysis, we have built three indicators of the stance of Fed's monetary policy:

i) the deviations between the model-based prescription that results from the implementation of the estimated Taylor-type monetary policy rule (18) and the *shadow* value of the Federal Funds rate obtained using the Wu and Xia (2016) method. The Wu-Xia shadow rate includes negative interest rate to take into account the monetary stimulus from open-market purchases of assets by the Fed in times when the official rates are very close to 0%.[14]

ii) the estimated interest-rate shocks obtained from the model-based residuals of the monetary policy rule (18), which may reflect the leaning of the monetary policy stance to tightening (a positive interest rate shock) or easing (a negative interest rate shock).

iii) the real interest rate that lingers the adjustments of the nominal interest rate to expected inflation. In a NK-DSGE model, the changes in the real interest rates are crucial to explain fluctuations of the endogenous components of aggregate demand. High real interest rates characterize contractionary monetary policies while a low real interest rate (often a negative number due to a close-to-0% nominal interest rate and a positive expected inflation) is indicative of a expansionary monetary policy.

The top diagram of Figure 3 displays the model-based nominal interest rate provided by the estimated Taylor-type monetary policy rule (18) in comparison to the Wu-Xia (2016) shadow Federal Funds Rate during the 2019-2022 period. Numbers are obtained as annualized percentage rates and plotted in Figure 3 as deviations with respect to the steady-state rate (sample mean for the 1992 to 2022 period). In 2019, both lines report similar values. However, during the pandemic lockdown (second quarter of 2020) the Fed lowered the interest rates but not as strongly as recommended by the estimated interest-rate rule with unemployment (heavily influenced by a rate of unemployment that spiked to 13% in 2020:2). By the second half of 2020 and early 2021, the model-based rule recommends little variations around the -2% line but the Fed shadow rates keep falling to lower values with a minimum around -4% (400 basis points below the long-run rate) by the middle of 2021. In 2022, the uphill move of the Fed shadow interest rate is steeper than the one suggested by the model-based rule, illustrating the fast and aggressive Fed's actions to restore monetary neutrality. The last observation (last quarter of 2022) shows convergence of both series at values slightly above zero, showing that monetary policy is no longer expansionary and it responds coherently to the targets of stabilizing inflation and unemployment as the US economy enters the disinflation path.

The estimated monetary shocks in the central diagram of Figure 3 corroborate the pandemic blip in 2020:2 with a positive interest-rate shock, followed by a sequence of negative interest rate

[14] The Wu-Xia shadow Federal Funds rate captures the effects of the expansion on the monetary base ("quantitative easing") that characterized the Fed's actions during the Great Recession that followed the 2008 financial crisis and in the aftermath of the covid-19 pandemic.

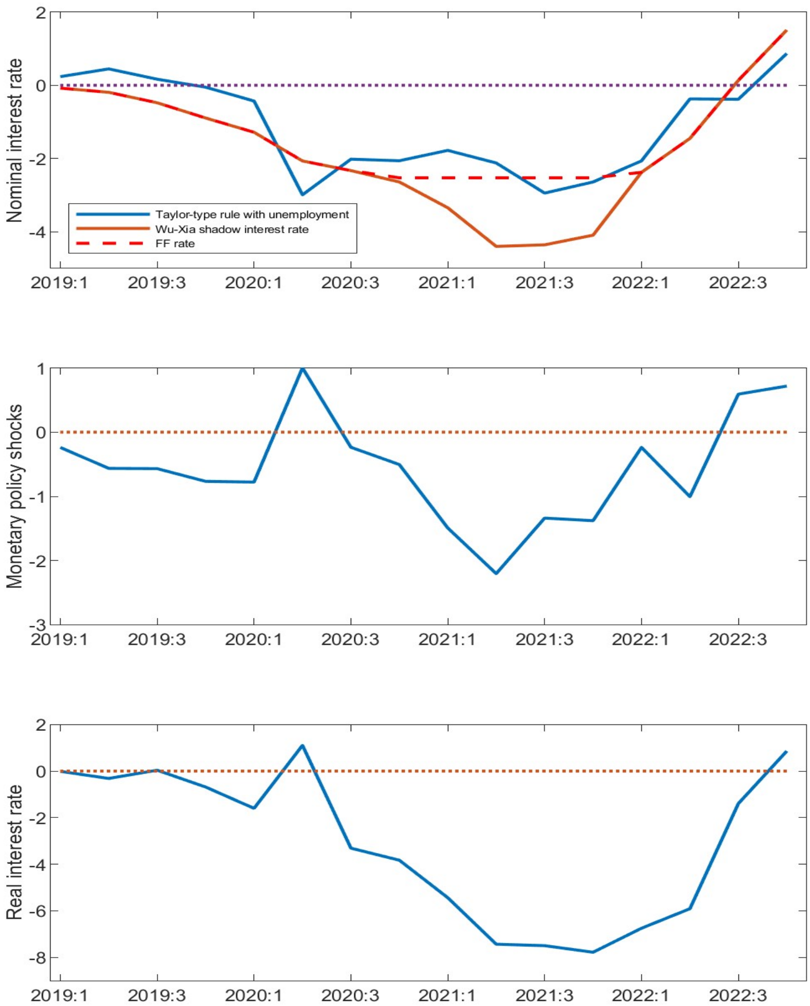

Figure 3: US monetary policy (2019:1 to 2022:4)

shocks that turn quite significant along the four quarters of 2021. The second quarter of 2021 reports the largest expansionary interest rate shock at -2.1% (210 basis points below the stabilizing interest rate).

In the bottom diagram of Figure 3, the real interest rate draws a U-shape swing during the years of the inflation episode. The pandemic lockdown pushes up the value of $r_t - r$ above the zero line, but the combination of lower nominal interest rates and accelerating prices feeding inflation expectations move the series deeper to the negative zone. In the second half of 2021 the estimated real interest rate is about 8% below the steady state rate (2% below the steady-state rate in quarterly terms). Since the steady-state interest rate is very close to 0%, our estimates indicate a very negative real interest rate close to -8% which accounts for a Fed's stance of monetary expansions. Again, the numerous interest rate hikes announced by the Fed in 2022 moved rapidly the real interest rate to the neutral range close to 0% by the end of 2022.

5 Projections for the disinflation period

The rapid rise of US price inflation after the covid-19 pandemic had not been seen since the high-inflation episode of the 70s. Economic agents might have adjusted upwards their inflation expectations. Figure 4 shows some evidence on this matter by plotting data on the evolution of the expected rate of inflation.

The Survey Research Center at the University of Michigan conducts the Survey of Consumers since 1946. Each month consumers answer a survey of 50 questions, including one asking what percentage they expect prices to go up/down during the next twelve months. Figure 4 shows the quarterly averages from 2019 to 2023. As for firms' inflation expectations, Figure 4 shows the one-year inflation expectation from the "Business Inflation Expectations" survey published by the Federal Reserve Bank of Atlanta.[15] The firms' inflation expectations was very stable around the 2% target of the Fed in que quarters prior to the covid-19 pandemic, while consumers expectations were close to 3%. Since the beginning of 2021, both series of expected inflation increased considerably describing an upwards trend that somehow lagged the observed series of inflation. The peak values of expected one-year inflation are reported by the middle of 2022, with consumer expectations

[15] The geographical coverage of this survey is the Six District of the Federal Reserve System that includes the States of Alabama, Florida, Georgia, and portions of Louisiana, Mississippi, and Tennessee.

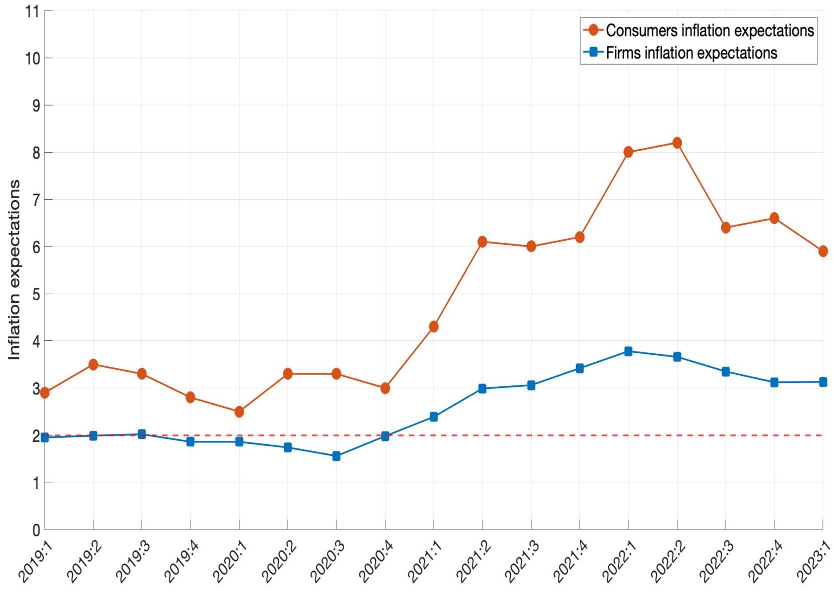

Figure 4: US inflation expectations (2019:1 to 2023:1)

around 8% and firms expected inflation near 4%. Therefore, this evidence suggests that during the post-covid inflation episode the expectations have de-anchored from the 2% Fed target. In the second half of 2022, the values of expected inflation shown in Figure 4 stop rising and initiate a soft landing towards the 2% target (still uncertain in early 2023 at the time of writing this paper).[16]

The estimated model has been projected for sixteen quarters (four years) after the inflation peak observed in the second quarter of 2022. For the design of this forecasting exercise, it is considered that i) the initial values of the state variables are set at the estimates provided by the model, and ii) all the innovations of the shocks are assumed to be zero. Therefore, the projected series are intended to capture an scenario in which the exogenous variability ceases and the endogenous variables follow their returning paths to their corresponding steady-state levels. It should be therefore understood as a neutral scenario in which no further shocks would hit the economy during the disinflation period.

[16]It was precisely on the second half of 2022 when the Fed announced the largest interest-rate hikes accompanied by FOMC statements of a strong commitment to inflation targeting. For example, in the FOMC statement that followed the 75 basis points increase of the official rates, on November 2^{nd}, 2022, said "The Committee anticipates that ongoing increases in the target range will be appropriate in order to attain a stance of monetary policy that is sufficiently restrictive to return inflation to 2 percent over time".

Moreover, the projected series are computed through the lens of the deep structural parameters estimated for the entire sample period, 1993:1 to 2022:2. Since the recent inflation episode might have changed the value of some of these parameters, alternative scenarios regarding the indexation patterns of the private sector and the behavior of the central bank are also examined.

Figure 5 shows the model-based projected evolution of Year-over-Year (YoY) price inflation, YoY wage inflation, the unemployment rate and the annualized nominal interest rate during the disinflation period.[17] Dotted lines indicate the steady-state values. The inertial components of the Phillips Curves for price inflation (6), and for wage inflation (16), are governed by the indexation weights on lagged inflation, which are estimated at $\iota_p = 0.40$ and $\iota_w = 0.49$, respectively. The possible de-anchoring of inflation expectations discussed some paragraphs above may explain increasing values for these indexation parameters during the post-covid inflation episode.[18] Subsequently, three indexation patterns are examined: higher price indexation responding to lagged inflation ($\iota'_p = 0.80$), higher wage indexation on lagged inflation ($\iota'_w = 0.80$), and both higher price and wage indexation on observed inflation ($\iota'_p = \iota'_w = 0.80$).

The baseline projection indicates a gradual return to the 2% target, with values around 6% by the middle of 2023, around 4% in 2024 and touching down to 2% by the end of 2024 (see blue solid lines in Figure 5). The unemployment rate rises continuously throughout 2023 and 2024 to reach a maximum value around 6.5% at the beginning of 2024 (a higher rate than the one forecasted by Ball *et al.*, 2022). However, on the cases where we consider price/wage indexation patterns largely based on lagged inflation, the strong second-round effects feed the inflation dynamics, pushing the peak to slightly higher levels and delaying the return to the steady-state target rates. The forecasting of wage inflation comes with a peak value in the second quarter of 2023 (around 5% in the baseline case) and then has a rapid decline to the vicinity of the steady-state wage inflation rate in a less persistent pattern than the one predicted for price inflation. The nominal interest rate set by the estimated monetary policy rule (18) reaches the peak value at the middle of 2023. This highest (*terminal*)

[17] In a way that resembles the typical observations reported by the media, the projections of YoY price inflation and YoY wage inflation are obtained as the accumulated over the last four quarters, $\sum_{k=0}^{3} \pi^p_{t-k}$ and $\sum_{k=0}^{3} \pi^w_{t-k}$, respectively. The nominal interest rate has been multiplied by four, $4R_t$, to express it in annualized terms.

[18] The "sticky" component of one-year price inflation reported by the Federal Reserve Bank of Atlanta (2022) can also signal possible de-anchoring of inflation expectations. This inflation component was stable and close to 2% from 2012 to 2020 (including just a slight decline in the months of the pandemic lockdown). However, from January of 2021 to November of 2022 "sticky" inflation rose from 1.7% to 6.6%, moving away from the 2% target of the Fed.

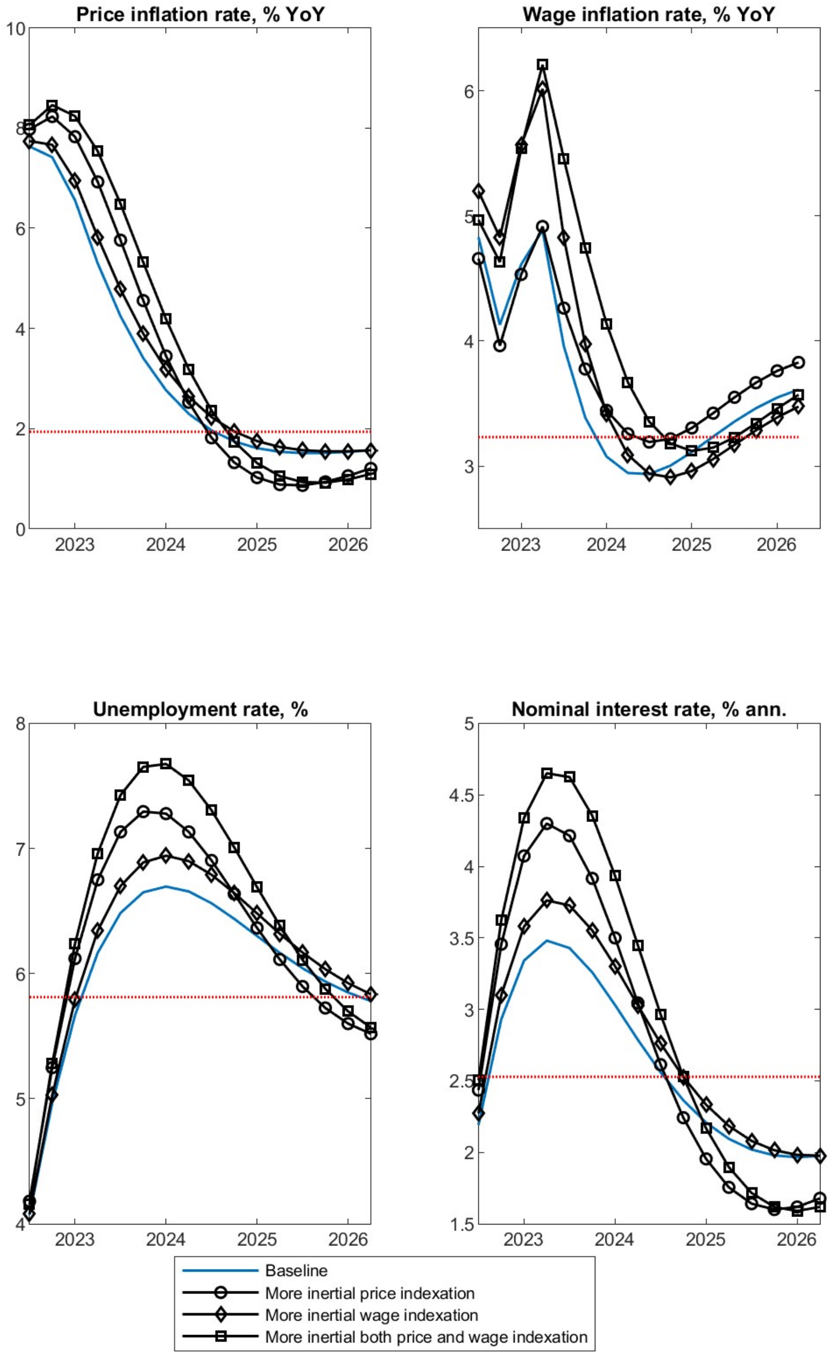

Figure 5: Projections for the US disinflation path (2022-26) with different price-wage inertial components

value varies with the degree of price/wage indexation, ranging from 3.5% in the baseline case to 4.65% when both price and wage indexation rely more on lagged inflation. The projected responses of unemployment capture the contractionary effects of the interest rate hikes that result when the Fed behavior corresponds to that of the estimated monetary policy rule (18). The unemployment peak comes later than the interest rate peak, with a delay of 2-3 quarters to be predicted by the beginning of 2024. The scenarios with more inertial indexation project deeper economic downturns and higher unemployment rates (see Figure 5), with the unemployment peak reaching 7.75% in the case of larger second-round effects from more de-anchored price and wage indexation patterns ($\iota_p' = \iota_w' = 0.80$). By 2026, the unemployment forecasts converge to the long-run rate estimated by the model (5.8% as the sample mean observed between 1992 and 2022).

Figure 6 provides the projections for the same variables at alternative designs of the monetary policy rule (18). The dramatic rise of inflation after the covid pandemic might explain a greater concern of the Fed for stabilizing inflation. The estimated coefficients of the rule (18) for the 1993:1 to 2022:2 sample period are $\mu_\pi = 1.70$, $\mu_u = 0.41$and $\mu_R = 0.89$. After some preliminary checks of the policy responses, we have raised the estimated coefficient on stabilizing inflation by a factor of either 2 ($\mu_\pi' = 1.70 \times 2 = 3.4$) or 3 ($\mu_\pi'' = 1.70 \times 3 = 5.1$). The results displayed in Figure 6 show little effects on the price disinflation path, while for wage inflation the downwards phase is faster if the monetary tightening is more aggressive. The nominal interest rate reaches a higher ceiling value with $\mu_\pi' = 3.4$,and even higher with $\mu_\pi'' = 5.1$, but the inflation rates are just slightly lower than those with the conventional policy at $\mu_\pi = 1.70$. Note that the stabilizing role of monetary policy is based on a demand-determined transmission mechanism that cannot cope with the supply-side determinants of the current inflation episode. There is some fall in the real marginal cost as a combination of higher productivity (due to lower employment) and a lower real wage. But the disinflation path barely changes since the quantitative effects of lower real marginal costs on inflations are small (the estimated slope of the price inflation Phillips Curve is $\kappa_p = 0.0189$). Nevertheless the quantitative effects of the different Fed stances towards inflation are noticeable in the labor market where unemployment rises rapidly to reach rates close to the 8% in 2024 and remain substantially higher than the long-run rates in 2025. Hence, a very aggressive sequence of interest rate hikes comes along with an unfavorable trade-off between lower inflation and a higher unemployment rate. Comparing the baseline policy to that of a very high inflation targeting ($\mu_\pi'' = 5.1$), the interest rate hikes would peak at 6% instead of at 3.25% with an average

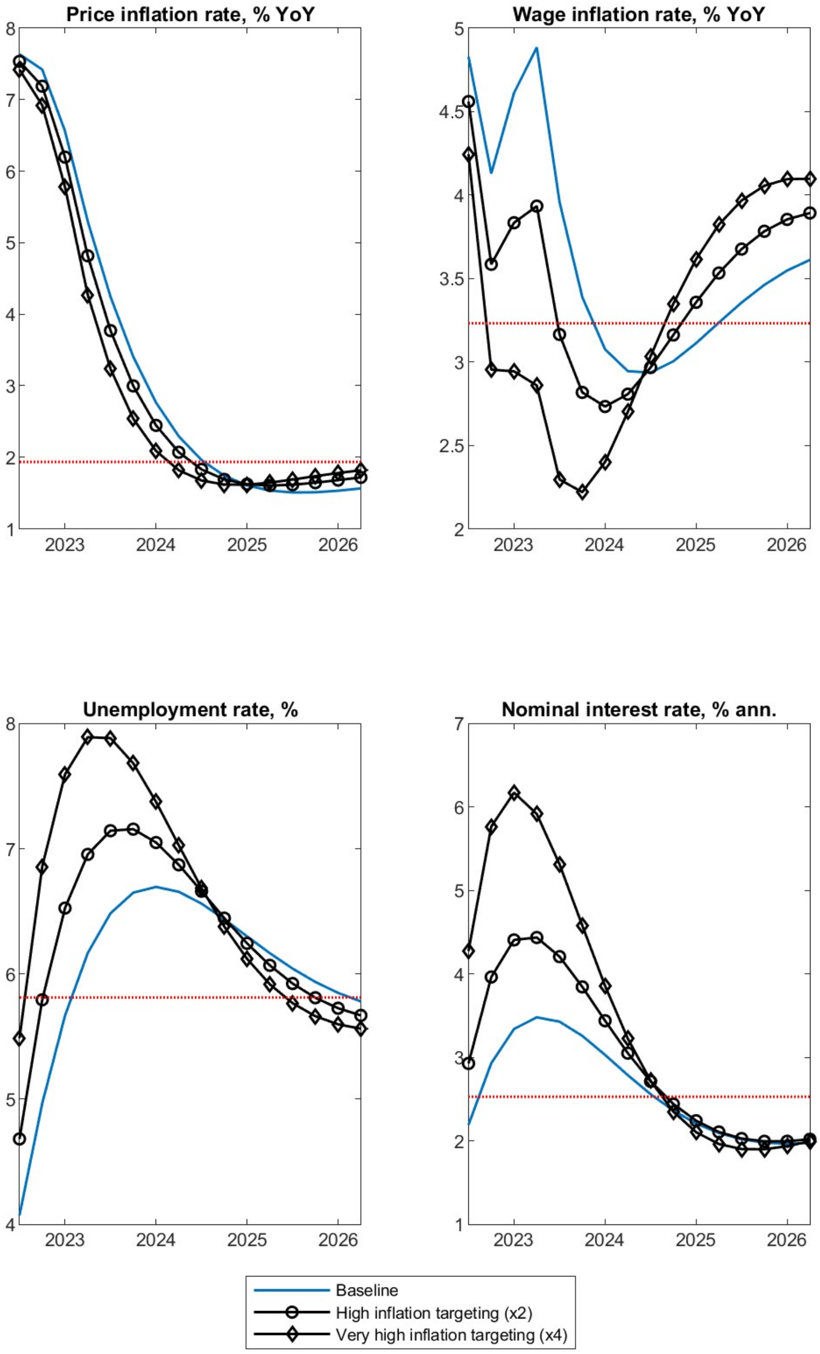

Figure 6: Projections for the US disinflation path (2022-26) with different Fed's preference for inflation targeting

gain from reducing inflation at 31 basis points per quarter, and a maximum reduction of 100 basis points in 2023:3. Meanwhile, the cost of higher unemployment is measured in the projections at an average increase 57 basis points per quarter with a maximum of 193 higher basis points in the rate of unemployment predicted for 2023:2. The sacrifice ratio takes a value $\frac{\Delta u}{-\Delta \pi^p} \simeq 2$ indicating a costly trade-off between unemployment and price inflation if monetary policy is more aggressive on fighting inflation. This result goes in line with Del Negro *et al.* (2022) who argue, using a DSGE model, that a severe tightening of monetary policy may not be successful to shorten the disinflation path because the inflationary episode has not been caused by shocks on the demand-side of the economy.[19]

6 Conclusions

The recent post-covid inflation surge has been examined in a NK-DSGE model that incorporates unemployment fluctuations. Wages are set in a collective agreement between households-workers and firms, subject to Calvo-type rigidities that result in unemployment fluctuations as gaps between amounts of jobs supplied and demanded. An inverse relationship between nominal wages and unemployment drives the dynamics of wage inflation in a way that resembles the original Phillips (1958) curve. The monetary policy rule captures the systematic behavior of the central bank when adjusting the nominal interest rate to jointly stabilize price inflation and the rate of unemployment, including an smoothing component on changes of the policy rate. The model is estimated for US aggregate quarterly fluctuations from 1992 to 2022, replicating most of the statistics of volatility, cross correlation and persistence observed in the data.

The estimated model is used to examine the episode of post-covid inflation surge (2020-2022). We found that the inflation acceleration has been triggered by a mix of three factors. The exogenous component in the price inflation Phillips Curve explains 39% of the inflation surge from the second quarter of 2021 onwards. These price-push shocks can identify the higher cost of energy as they correspond to the quarters of a rapid increase in the price of oil. The model estimates large and expansionary monetary shocks in 2021 to be the second largest contributor to the inflation acceleration in the US, which explain a 27% of the observed fluctuations. The inflationary effects of

[19] Also following this idea, Korinek and Stiglitz (2022) suggest a mix of targeted fiscal policies instead of the blunt that the current monetary policy towards inflation may entail to the economy.

Fed's monetary stimulus in 2021 (through both open-market purchases of assets and close-to-zero official rates) are also captured as large residuals estimated in the stabilizing interest-rate rule and a negative real interest rate that reached values around an annualized -8% at the end of 2021. Finally, the labor market tightness during the pandemic lockdown made the cost of labor increase dramatically in the second and third quarter of 2020 which accounted responsible for a 19% of the observed higher inflation variability, concentrated during the first part of the episode.

According to the model-based projections, the disinflation path may run for approximately 3 years to land back to the 2% Fed's inflation target. However, this path of lower inflation rates may be substantially affected by the behavior of firms and households when adjusting prices and wages. If the indexation patterns are highly based on lagged inflation, the dynamics of price and wage inflation become more persistent forcing the Fed to keep up with interest rate hikes and driving the economy to a more severe contraction with higher rates of unemployment. The analysis also examines alternative scenarios of the Fed stance to stabilize inflation. Our simulations show that a more aggressive inflation targeting may have little effects on the reduction of price inflation at the cost of increasing the unemployment rate.

References

[1] **Ball, M. Laurence, Daniel Leigh and Prachi Mishra.** 2022. "Understanding U.S. Inflation During the COVID Era", *International Money Fund*, Working Paper 22/208.

[2] **Blanchard, Olivier, and Jordi Galí**. 2010. "Labor markets and monetary policy: a New Keynesian Model with unemployment", *American Economic Journal: Macroeconomics* 2: 1-30.

[3] **Bordo, Michael D., and Mickey D. Levy.** 2022. "The Fed's Delayed Exits from Monetary Ease: The Fed Rarely Learns from History", *Shadow Open Market Committee*, February 11.

[4] **Calvo, Guillermo A.** 1983. "Staggered prices in a utility-maximizing framework", *Journal of Monetary Economics* 12 (3): 383–398.

[5] **Casares, Miguel.** 2010. "Unemployment as excess supply of labor: Implications for wage and price inflation", *Journal of Monetary Economics* 57 (2): 233–243.

[6] **Celasun, Oya, Niels-Jakob H Hansen, Aiko Mineshima, Mariano Spector and Jing Zhou.** 2022. "Supply Bottlenecks: Where, Why, How Much, and What Next", *IMF Working paper.*

[7] **Cerrato, Andrea, and Giulia Gitti.** 2022. "Inflation since covid: Demand or supply", Available at SSRN 4193594.

[8] **Christiano, Lawrence J., Martin S. Eichenbaum and Mathias Trabandt**. 2016. "Unemployment and business cycles", *Econometrica* 84 (4): 1523-1569.

[9] **Del Negro, Marco, Aidan Gleich, Donggyu Lee, Ramya Nallamotu and Sikata Sengupta.** 2022. "The New York Fed DSGE Model Forecast", *Liberty Street Economics* (blog), Federal Reserve Bank of New York, December 16. https://libertystreeteconomics.newyorkfed.org/2022/12/the-new-york-fed-dsge-model-forecast-december-2022 .

[10] **Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin.** 2000. "Optimal monetary policy with staggered wage and price contracts", *Journal of Monetary Economics* 46: 281–313.

[11] **Federal Reserve Bank of Atlanta.** 2022. "Inflation Project: Sticky-price CPI". https://www.atlantafed.org/research/inflationproject/stickyprice (accessed December 23, 2022).

[12] **Gagliardone, Luca, and Mark Gertler**. 2023. "Oil Prices, Monetary Policy and Inflation Surges", working paper.

[13] **Galí, Jordi.** 2011. "The Return of the Wage Phillips Curve", *Journal of the European Economic Association* 9: 436–461.

[14] **Galí, Jordi, Frank Smets and Rafael Wouters.** 2012. "Unemployment in an Estimated New Keynesian Model", *NBER Macroeconomics Annual 2011*, University of Chicago Press: 329–360.

[15] **Harding, Martín, Jesper Lindé and Mathias Trabandt.** 2022. "Understanding post-covid inflation dynamics", *Bank of Canada Working Paper* 2022-50.

[16] **Kimball, Miles S**. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model", *Journal of Money, Credit and Banking* 27 (4, Part 2): 1241-1277.

[17] **Korinek, Anton, and Joseph E. Stiglitz**. 2022. "Macroeconomic Stabilization for a Post-Pandemic World: Revising the Fiscal-Monetary Policy Mix and Correcting Macroeconomic Externalities", Hutchins Center on Fiscal & Monetary Policy at Brookings, Working Paper 78.

[18] **Krause, Michael U., and Thomas A. Lubik**. 2007. "The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions", *Journal of Monetary Economics* 54: 706-727.

[19] **Labonte, Marc, and Lida R. Weinstock.** 2021. "Labor Market Tightness and the Economic Recovery, Part 2", *Congressional Research Service* Insight IN11771.

[20] **Labonte, Marc, and Lida R. Weinstock.** 2022. "Inflation in the U.S. Economy: Causes and Policy Options", *Congressional Research Service*, R47273.

[21] **Levin, Andrew T., and William N. Nelson.** 2023. The Federal Reserve's Balance Sheet: Costs to Taxpayers of Quantitative Easing. Monetary Policy, Policy Briefs, Mercatus Center. George Mason University.

[22] **Merz, M.** 1995. "Search in the labor market and the real business cycle", *Journal of Monetary Economics* 36 (2), 269-300.

[23] **Mortensen, Dale T. and Christopher A. Pissarides** 1994. "Job creation and job destruction in the theory of unemployment", *Review of Economic Studies* 61 (3): 397-415

[24] **Phillips, A.W.** 1958. "The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957". *Economica* 25: 283–299.

[25] **Shapiro, Adam Hale.** 2022a. "How Much do Supply and Demand Drive Inflation?", *Federal Reserve Bank of San Francisco*, Economic Letter 15.

[26] **Shapiro, Adam Hale.** 2022b. "A Simple Framework to Monitor Inflation", *Federal Reserve Bank of San Francisco*, Working Paper 2020-29.

[27] **Smets, Frank, and Rafael Wouters.** 2007. "Shocks and frictions in us business cycles: A Bayesian DSGE approach", *American Economic Review* 97 (3): 586–606.

[28] **Taylor, John B.** 1993. "Discretion versus policy rules in practice", *Carnegie-Rochester Conference Series on Public Policy* 39: 195-214.

[29] **Trigari, Antonella**. 2009. "Equilibrium unemployment, job flows and inflation dynamics", *Journal of Money, Credit and Banking* 41: 1-33.

[30] **Walsh, Carl E**. 2005. "Labor market search, sticky prices, and interest rate policies", *Review of Economic Dynamics* 8: 829-849.

[31] **Wu, Jing Cynthia, and Fan Dora Xia.** 2016. "Measuring the macroeconomic impact of monetary policy at the zero lower bound", *Journal of Money, Credit and Banking* 48 (2-3): 253–291.

Appendix

Table of contents:

A. On the derivation of model equations

A.1. Household optimizing program

The household budget constraint in period t expressed in terms of the consumption bundle is

$$\int_0^1 \frac{W_t(i)(1-u_t(i))n_t^s(i)}{P_t}di + r_t^k v_t \overline{k}_{t-1} - tax_t = c_t + i_t + a(v_t)\overline{k}_{t-1} + (1+r_t)^{-1} b_t - b_{t-1} + d_t. \qquad \text{(A1)}$$

where the total capital value is obtained as the product of the stock of capital, $\overline{k}$, multiplied by the utilitzation rate, v. Since the capital stock is predetermined, the income obtained from the capital rentals in period t is $r_t^k v_t \overline{k}_{t-1}$ where r_t^k is the real rental rate. Capital accumulation incorporates a constant rate of depreciation per quarter, δ, and an adjustment cost function as the one used in Smets and Wouters (2007), which result in this equation for the law of motion of capital

$$\overline{k}_t = (1-\delta)\,\overline{k}_{t-1} + e^{\varepsilon_t^i}\left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] i_t, \qquad \text{(A2)}$$

with the properties of the investment adjustment cost function in the detrended steady-state $S(1) = S'(1) = 0$ and the curvature parameter at $S''(1) = \varphi_k > 0$. There is also an AR(1) shock, ε_t^i, that provides exogenous investment variability.

The optimizing program of the household consists of maximizing intertemporal utility with the preferences assumed in Galí, Smets and Wouters (2013)

$$E_t \sum_{j=0}^{\infty} \beta^j \left(e^{\varepsilon_t^c} \log(c_{t+j} - h\overline{c}_{t+j-1}) - \chi e^{\varepsilon_t^n} \Theta_{t+j} \int_0^1 \frac{n_{t+j}^s(i)^{1+\sigma_n}}{1+\sigma_n} di \right)$$

subject to the budget constraint and the capital accumulation constraint considered both in the current period and in expected future periods.

A.2. Firm optimizing program

The representative firm will choose $P_t(i)$ to maximize the expected stream of real dividends conditional to the lack of future optimal pricing

$$E_t^{\xi_p} \sum_{j=0}^{\infty} \beta^j \xi_p^j \left[\left(\frac{P_t(i)\Pi_{t,t+j}^p}{P_{t+j}} \right) - mc_{t+j}(i) \right] y_{t+j}(i),$$

where $\Pi_{t,t+j}^p$ is the price indexation factor all of them between periods t and $t+j$

$$\Pi_{t,t+j}^p = \prod_{k=0}^{j} \left[e^{\varepsilon_{t+1+k}^p} \left(1 + \pi_{t+k}^p\right)^{\iota_p} (1+\pi^p)^{(1-\iota_p)} \right]$$

The optimal choices of the firm must be subject to the expected schedule of demand constraints governed by a Kimball aggregator on relative prices, $G'^{-1}\left(\frac{P_t(i)\Pi^p_{t,t+j}}{P^c_{t+j}}\right)$,

$$y_{t+j}(i) = G'^{-1}\left(\frac{P_t(i)\Pi^p_{t,t+j}}{P^c_{t+j}}\right) y_{t+j}, \text{ for } j = 0, 1, 2, ...$$

A.3. Firm-level labor demand depending on relative prices and wages

The loglinearized production function at firm level is

$$\widehat{y}_t(i) = \phi_p\left(\alpha\widehat{k}_t(i) + (1-\alpha)\widehat{n}^d_t(i) + \varepsilon^a_t\right)$$

which determines the log of firm-specific capital demand

$$\widehat{k}_t(i) = \frac{1}{\alpha}\left(\frac{1}{\phi_p}\widehat{y}_t(i) - (1-\alpha)\,\widehat{n}^d_t(i) - \varepsilon^a_t\right).$$

Using the equation for optimal subsitutions across factors of production at the firm level,

$$\widehat{n}^d_t(i) = \widehat{k}_t(i) + \widehat{r}^k_t - \widehat{W}_t(i) + \widehat{P}_t.$$

Substituting the value of $\widehat{k}_t(i)$ from the last expression in the labor demand equation and rearranging terms results in

$$\widehat{n}^d_t(i) - \widehat{r}^k_t + \widehat{W}_t(i) - \widehat{P}_t = \frac{1}{\alpha}\left(\frac{1}{\phi_p}\widehat{y}_t(i) - (1-\alpha)\,\widehat{n}^d_t(i) - \varepsilon^a_t\right).$$

As discussed in Woodford (2003, p. 168), the Kimball (1995) scheme for the aggregation of goods –also used in the Smets and Wouters (2007)'s model–, yields a log approximation of demand-determined relative output that is inversely related to the relative price,

$$\widehat{y}_t(i) = \widehat{y}_t - \tfrac{\phi_p}{\phi_p-1}\widetilde{P}_t(i),$$

where $\frac{\phi_p}{\phi_p-1}$ defines the elasticity of demand and the relative price is $\widetilde{P}_t(i) = \log P_t(i) - \log P_t = \log P_t(i) - \int_0^1 \log P_t(i)di$. Inserting this demand function to substitute out $y_t(i)$ in the labor demand function yields

$$\widehat{n}^d_t(i) - \widehat{r}^k_t + \widehat{W}_t(i) - \widehat{P}_t = \frac{1}{\alpha}\left(\frac{1}{\phi_p}\left(\widehat{y}_t - \frac{\phi_p}{\phi_p-1}\widetilde{P}_t(i)\right) - (1-\alpha)\,\widehat{n}^d_t(i) - \varepsilon^a_t\right).$$

which simplifies to

$$\frac{1}{\alpha}\widehat{n}_t^d(i) - \widehat{r}_t^k + \widehat{W}_t(i) - \widehat{P}_t = \frac{1}{\alpha}\left(\left(\frac{1}{\phi_p}y_t - \frac{1}{\phi_p - 1}\widetilde{P}_t(i)\right) - \varepsilon_t^a\right).$$

Subtracting the analogous expression evaluated at the aggregate level, $\frac{1}{\alpha}\widehat{n}_t^d - \widehat{r}_t^k + \widehat{W}_t - \widehat{P}_t = \frac{1}{\alpha}\left(\frac{1}{\phi_p}\widehat{y}_t - \varepsilon_t^a\right)$, gives

$$\frac{1}{\alpha}\left(\widehat{n}_t^d(i) - \widehat{n}_t^d\right) + \widetilde{W}_t(i) = \frac{1}{\alpha}\left(-\frac{1}{\phi_p - 1}\widetilde{P}_t(i)\right).$$

with the relative nominal wage as $\widetilde{W}_t(i) = \widehat{W}_t(i) - \widehat{W}_t$. Finally, some rearranging can be done to reach the expression for firm-specific labor demand

$$\widehat{n}_t^d(i) = -\alpha\widetilde{W}_t(i) - \frac{1}{\phi_p - 1}\widetilde{P}_t(i) + \widehat{n}_t^d.$$

A.4. Price inflation Phillips Curve with price-wage complementarity

The first order condition on the optimal price set by firm i in period t is

$$\widehat{P}_t^*(i) = \left(1 - \beta\xi_p\right) E_t^{\xi_p}\sum_{j=0}^{\infty}\beta^j\xi_p^j\left(A\widehat{mc}_{t+j}(i) + \widehat{P}_{t+j} - \sum_{k=1}^{j}x_{t+k}^p\right)$$

where the real marginal cost is a combination of the unit costs of labor and capital with a diminishing effect from the technology shock as follows

$$\widehat{mc}_{t+j}(i) = (1-\alpha)\left(\widehat{W}_{t+j}(i) - \widehat{P}_{t+j}\right) + \alpha\widehat{r}_{t+j}^k - \varepsilon_{t+j}^a$$

Rewritting the above expression in aggregate units and noticing the common terms gives

$$\widehat{mc}_{t+j}(i) = \widehat{mc}_{t+j} + (1-\alpha)\widetilde{W}_{t+j}(i)$$

with the aggregate real marginal cost is $\widehat{mc}_{t+j} = \int_0^1 \widehat{mc}_{t+j}(i)di$ and the relative nominal wage is $\widetilde{W}_{t+j}(i) = \widehat{W}_{t+j}(i) - \widehat{W}_{t+j}$. Inserting the last expression

$$\widehat{P}_t^*(i) = \left(1 - \beta\xi_p\right) E_t^{\xi_p}\sum_{j=0}^{\infty}\beta^j\xi_p^j\left(A\widehat{mc}_{t+j} + A(1-\alpha)\widetilde{W}_{t+j}(i) + \widehat{P}_{t+j} - \sum_{k=1}^{j}x_{t+k}^p\right)$$

Subtracting the aggregate price level on both sides

$$\widetilde{P}_t^*(i) = \left(1 - \beta\xi_p\right) E_t^{\xi_p}\sum_{j=0}^{\infty}\beta^j\xi_p^j\left(A\widehat{mc}_{t+j} + A(1-\alpha)\widetilde{W}_{t+j}(i) + \widehat{P}_{t+j} - \widehat{P}_t - \sum_{k=1}^{j}x_{t+k}^p\right)$$

with relative optimal price $\widetilde{P}_t^*(i) = \widehat{P}_t^*(i) - \widehat{P}_t$. Noticing that $\widehat{P}_{t+j} - \widehat{P}_t = \sum_{k=1}^{j}\left(\pi_{t+k}^p - \pi^p\right)$, the stream of future expected rates of inflation can be introduced in the expression of the optimal price to obtain

$$\widetilde{P}_t^*(i) = \left(1-\beta\xi_p\right) E_t^{\xi_p}\sum_{j=0}^{\infty}\beta^j\xi_p^j\left(A\widehat{mc}_{t+j} + A\left(1-\alpha\right)\widetilde{W}_{t+j}(i)\right) + E_t^{\xi_p}\sum_{j=i}^{\infty}\beta^j\xi_p^j\left(\left(\pi_{t+j}^p - \pi^p\right) - x_{t+k}^p\right) \tag{A1}$$

The internal relationship between firm-level prices and nominal wage is $\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1\widetilde{W}_{t-1}(i)$, with $\tau_1 > 0$ to be pinned down through the undetermined coefficient method. Since the term $E_t^{\xi_p}\sum_{j=0}^{\infty}\beta^j\xi_p^j\widetilde{W}_{t+j}(i)$ is included in the optimal price function, we will look for its link to the state variable $\widetilde{W}_{t-1}(i)$. The Calvo probability for wage rigidity, ξ_w, determines the distribution of relative wages between the labor-clearing agreed formulation and those following the indexation rule which for wage setting in period t results in

$$\widetilde{W}_t(i) = \left(1-\xi_w\right)\widetilde{W}_t^*(i) + \xi_w\left(\widetilde{W}_{t-1}(i) + x_t^w - \pi_t^w\right).$$

with x_t^w denotes the linearized wage indexation factor in period t. If firm i receives the Calvo signal to have an agreement between managers and workers to set the labor-clearing wage, that will be inversely related to the relative optimal price in accordance with the internal complementarity equation

$$\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2\widetilde{P}_t^*(i),$$

that can be inserted in the equation for $\widetilde{W}_t(i)$ to obtain

$$\widetilde{W}_t(i) = \left(1-\xi_w\right)\left(\widetilde{W}_t^* - \tau_2\widetilde{P}_t^*(i)\right) + \xi_w\left(\widetilde{W}_{t-1}(i) + x_t^w - \pi_t^w\right).$$

Recalling that $\widetilde{W}_t^* = \frac{\xi_w}{1-\xi_w}\left(\pi_t^w - x_t^w\right)$ from Calvo-type sticky wages, and cancelling terms, we transform the previous expression into

$$\widetilde{W}_t(i) = \xi_w\widetilde{W}_{t-1}(i) - \tau_2\left(1-\xi_w\right)\widetilde{P}_t^*(i).$$

Replicating the procedure one period ahead for $E_t^{\xi_p}\widetilde{W}_{t+1}(i)$, we have the distribution between the two possible outcomes of wage setting

$$E_t^{\xi_p}\widetilde{W}_{t+1}(i) = \left(1-\xi_w\right)E_t^{\xi_p}\widetilde{W}_{t+1}^*(i) + \xi_w\left(\widetilde{W}_t(i) + E_t x_{t+1}^w - E_t\left(\pi_{t+1}^w - \pi^w\right)\right).$$

where using $\widetilde{W}_{t+1}^*(i) = \widetilde{W}_{t+1}^* - \tau_2 \widetilde{P}_{t+1}(i)$ conditional on no-optimal pricing in $t+1$ and optimal pricing in t yields

$$E_t^{\xi_p} \widetilde{W}_{t+1}^*(i) = \widetilde{W}_{t+1}^* - \tau_2 \left(\widetilde{P}_t^*(i) + E_t x_{t+1}^p - E_t \left(\pi_{t+1}^p - \pi^p \right) \right),$$

Also, $\widetilde{W}_{t+1}^* = \frac{\xi_w}{1-\xi_w} \left(\left(\pi_{t+1}^w - \pi^w \right) - E_t x_{t+1}^w \right)$ from the aggregation scheme and the expression for $\widetilde{W}_t(i)$ derived above

$$\begin{aligned} E_t^{\xi_p} \widetilde{W}_{t+1}(i) &= (1-\xi_w) \left[\tfrac{\xi_w}{1-\xi_w} \left(\left(\pi_{t+1}^w - \pi^w \right) - E_t x_{t+1}^w \right) - \tau_2 \left(\widetilde{P}_t^*(i) + E_t x_{t+1}^p - E_t \left(\pi_{t+1}^p - \pi^p \right) \right) \right] \\ &\quad + \xi_w \left(\xi_w \widetilde{W}_{t-1}(i) - \tau_2 (1-\xi_w) \widetilde{P}_t^*(i) + E_t x_{t+1}^w - E_t \left(\pi_{t+1}^w - \pi^w \right) \right) \end{aligned}$$

which simplifies to

$$E_t^{\xi_p} \widetilde{W}_{t+1}(i) = \xi_w^2 \widetilde{W}_{t-1}(i) - \tau_2 \left(1-\xi_w^2\right) \widetilde{P}_t^*(i) + \tau_2 (1-\xi_w) \left(E_t \left(\pi_{t+1}^p - \pi^p \right) - E_t x_{t+1}^p \right)$$

A generalization for a $t+j$ future period gives the following expression

$$E_t^{\xi_p} \widetilde{W}_{t+j}(i) = \xi_w^{j+1} \widetilde{W}_{t-1}(i) - \tau_2 \left(1-\xi_w^{j+1}\right) \widetilde{P}_t^*(i) + \tau_2 E_t \sum_{k=1}^{j} \left(1-\xi_w^{j-k+1}\right) \left(\left(\pi_{t+k}^p - \pi^p \right) - x_{t+k}^p \right).$$

which can be used to compute the expected sum of the stream of conditional relative wages becomes

$$\begin{aligned} E_t^{\xi_p} \sum_{j=0}^{\infty} \beta^j \xi_p^j \widetilde{W}_{t+j}(i) &= \tfrac{\xi_w}{1-\beta\xi_w\xi_p} \widetilde{W}_{t-1}(i) - \tau_2 \left(\tfrac{1}{1-\beta\xi_p} - \tfrac{\xi_w}{1-\beta\xi_w\xi_p} \right) \widetilde{P}_t^*(i) \\ &\quad + \tau_2 \left(\tfrac{1}{1-\beta\xi_p} - \tfrac{\xi_w}{1-\beta\xi_w\xi_p} \right) E_t \sum_{j=1}^{\infty} \beta^j \xi_p^j \left(\left(\pi_{t+j}^p - \pi^p \right) - x_{t+j}^p \right). \quad \text{(A2)} \end{aligned}$$

Substituting (A2) in (A1), and inserting the definition of $A = \frac{1}{(\phi_p-1)\epsilon+1}$, yields the loglinear formulation of the relative optimal price (where

$$\begin{aligned} (1+\Omega) \widetilde{P}_t^*(i) &= \tfrac{(1-\alpha)\left(1-\beta\xi_p\right)\xi_w}{\left((\phi_p-1)\epsilon+1\right)\left(1-\beta\xi_w\xi_p\right)} \widetilde{W}_{t-1}(i) + A \left(1-\beta\xi_p\right) E_t \sum_{j=0}^{\infty} \beta^j \xi_p^j \widehat{mc}_{t+j} \\ &\quad + (1+\Omega) E_t \sum_{j=1}^{\infty} \beta^j \xi_p^j \left(\left(\pi_{t+j}^p - \pi^p \right) - x_{t+j}^p \right), \end{aligned}$$

where

$$\Omega = \frac{\tau_2(1-\alpha)}{\left(\phi_p-1\right)\epsilon+1} \left(1 - \frac{\left(1-\beta\xi_p\right)\xi_w}{1-\beta\xi_p\xi_w} \right)$$

The result obtained in the equation for $\widetilde{P}_t^*(i)$ validates the loglinear relationship $\widetilde{P}_t^*(i) = \widetilde{P}_t^* + \tau_1 \widetilde{W}_{t-1}(i)$ with τ_1 given by

$$\tau_1 = \frac{(1-\alpha)\left(1-\beta\xi_p\right)\xi_w}{\left(\left(\phi_p-1\right)\epsilon+1\right)\left(1-\beta\xi_w\xi_p\right)(1+\Omega)}$$

and implies the following dynamic equation for average optimal prices

$$\widetilde{P}_t^* = \frac{\left(1-\overline{\beta}\xi_p\right)}{\left(\left(\phi_p-1\right)\epsilon+1\right)(1+\Omega)} E_t \sum_{j=0}^{\infty} \beta^j \xi_p^j \widehat{mc}_{t+j} + E_t \sum_{j=1}^{\infty} \beta^j \xi_p^j \left(\left(\pi_{t+j}^p - \pi^p\right) - x_{t+j}^p\right). \tag{A3}$$

The definition of the aggregate price level, combined with that of the rate of price inflation and the distribution between optimal and indexed prices results into this relationship between price inflation, optimal prices, lagged inflation and the price indexation shock

$$\left(\pi_t^p - \pi^p\right) = \frac{\left(1-\xi_p\right)}{\xi_p} \widetilde{P}_t^* + x_t^p \tag{A4}$$

where the equation (A3) that determines the relative optimal price can be plugged in to obtain

$$\left(\pi_t^p - \pi^p\right) = \frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\left(\left(\phi_p-1\right)\epsilon+1\right)(1+\Omega)\xi_p} E_t \sum_{j=0}^{\infty} \beta^j \xi_p^j \widehat{mc}_{t+j} + \frac{\left(1-\xi_p\right)}{\xi_p} E_t \sum_{j=1}^{\infty} \beta^j \xi_p^j \left(\left(\pi_{t+j}^p - \pi^p\right) - x_{t+j}^p\right) + x_t^p$$

Writing down the update to $\overline{\beta}\xi_p E_t \left(\pi_{t+1}^p - \pi^p\right)$ and making the difference

$$\begin{aligned}
\left(\pi_t^p - \pi^p\right) - \beta\xi_p E_t \left(\pi_{t+1}^p - \pi^p\right) &= \frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\left(\left(\phi_p-1\right)\epsilon+1\right)(1+\Omega)\xi_p} E_t \sum_{j=0}^{\infty} \beta^j \xi_p^j \widehat{mc}_{t+j} \\
&+ \frac{\left(1-\xi_p\right)}{\xi_p} E_t \sum_{j=1}^{\infty} \beta^j \xi_p^j \left(\left(\pi_{t+j}^p - \pi^p\right) - E_t x_{t+j}^p\right) + x_t^p \\
- \left(\frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\left(\left(\phi_p-1\right)\epsilon+1\right)(1+\Omega)\xi_p} E_t \sum_{j=1}^{\infty} \beta^j \xi_p^j \widehat{mc}_{t+j} \right. &+ \left. \frac{\left(1-\xi_p\right)}{\xi_p} E_t \sum_{j=2}^{\infty} \beta^j \xi_p^j \left(\left(\pi_{t+j}^p - \pi^p\right) - x_{t+j}^p\right) + \beta\xi_p E_t x_{t+1}^p \right)
\end{aligned}$$

which simplifies substantially to

$$\left(\pi_t^p - \pi^p\right) = \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\left(\left(\phi_p-1\right)\epsilon+1\right)(1+\Omega)\xi_p} \widehat{mc}_t + \beta E_t \left(\pi_{t+1}^p - \pi\right) + x_t^p - \beta E_t x_{t+1}^p$$

The (linearized) price indexation factor used in the model is

$$x_t^p = \iota_p \left(\pi_{t-1}^p - \pi\right) + \varepsilon_t^p$$

that can be inserted in the inflation equation to obtain

$$\left(\pi_t^p - \pi^p\right) = \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\left(\left(\phi_p-1\right)\epsilon+1\right)(1+\Omega)\xi_p} \widehat{mc}_t + \beta E_t \left(\pi_{t+1}^p - \pi\right) + \iota_p \left(\pi_{t-1}^p - \pi\right) + \varepsilon_t^p - \beta \left(\iota_p \left(\pi_t^p - \pi\right) + E_t \varepsilon_{t+1}^p\right)$$

Putting terms involving $\left(\pi_t^p - \pi\right)$ together on the left hand side solves for the expression for the price inflation New Keynesian Phillips Curve:

$$\left(\pi_t^p - \pi^p\right) = \frac{\beta}{1+\beta\iota_p} E_t \left(\pi_{t+1}^p - \pi\right) + \frac{\iota_p}{1+\beta\iota_p} \left(\pi_{t-1}^p - \pi\right) + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{(1+\beta\iota_p)\left(\left(\phi_p-1\right)\epsilon+1\right)(1+\Omega)\xi_p} \widehat{mc}_t + \frac{1}{1+\beta\iota_p} \left(\varepsilon_t^p - \beta\varepsilon_{t+1}^p\right)$$

A.5. Wage inflation Phillips Curve with price-wage complementarity

The nominal wage agreed between workers and manager of firm i is determined as follows

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \beta^j \xi_w^j \left(\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i) \right) = 0 \tag{A5}$$

where labor supply and labor demand schedules in terms of relative wages and prices are

$$\widehat{n}_t^s(i) = \frac{1}{\sigma_n} \left(\widetilde{W}_t(i) - \tfrac{1}{1-u} \left(u_t(i) - u_t \right) \right) + \widehat{n}_t^s$$

and

$$\widehat{n}_t^d(i) = -\alpha \widetilde{W}_t(i) - \tfrac{1}{\phi_p - 1} \widetilde{P}_t(i) + \widehat{n}_t^d$$

After computing the difference is $\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i)$ with the labor supply and demand in $t+j$, we obtain

$$\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i) = \frac{1}{\sigma_n} \left(-\tfrac{1}{1-u} \left(u_{t+j}(i) - u_{t+j} \right) \right) + \left(\frac{1}{\sigma_n} + \alpha \right) \widetilde{W}_{t+j}(i) + \tfrac{1}{\phi_p - 1} \widetilde{P}_{t+j}(i) + \widehat{n}_{t+j}^s - \widehat{n}_{t+j}^d$$

The unemployment rate is proportional to the aggregate-level labor gap, $u_{t+j} - u^n = (1 - u^n) \left(\widehat{n}_{t+j}^s - \widehat{n}_{t+j}^d \right)$ which can be used in the last expression to reach

$$\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i) = \frac{1}{\sigma_n} \left(-\tfrac{1}{1-u} \left(u_{t+j}(i) - u_{t+j} \right) \right) + \left(\frac{1}{\sigma_n} + \alpha \right) \widetilde{W}_{t+j}(i) + \tfrac{1}{\phi_p - 1} \widetilde{P}_{t+j}(i) + \tfrac{1}{1-u^n} \left(u_{t+j} - u \right)$$

Using $u_{t+j}(i) - u_t = (u_{t+j}(i) - u) - (u_{t+j} - u)$ transforms the previous expression as follows

$$\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i) = \frac{1}{\sigma_n} \left(-\tfrac{1}{1-u} \left(u_{t+j}(i) - u \right) \right) + \left(\frac{1}{\sigma_n} + \alpha \right) \widetilde{W}_{t+j}(i) + \tfrac{1}{\phi_p - 1} \widetilde{P}_{t+j}(i) + \tfrac{\frac{1}{\sigma_n} + 1}{1-u} \left(u_{t+j} - u \right)$$

The firm-specific rate of unemployment is defined by the firm-level labor gap

$$u_{t+j}(i) - u = (1 - u) \left(\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i) \right)$$

which can be inserted in the previous expression leading to

$$\widehat{n}_{t+j}^s(i) - \widehat{n}_{t+j}^d(i) = \tfrac{\left(\sigma_n^{-1} + \alpha\right)}{\left(\sigma_n^{-1} + 1\right)} \widetilde{W}_{t+j}(i) + \tfrac{1}{\left(\sigma_n^{-1} + 1\right)\left(\phi_p - 1\right)} \widetilde{P}_{t+j}(i) + \tfrac{1}{1-u} \left(u_{t+j} - u \right) \tag{A6}$$

Plugging (A6) in the labor-clearing condition that determines the agreed nominal wage, $W_t^*(i)$, yields

$$E_t^{\xi_w} \sum_{j=0}^{\infty} \beta^j \xi_w^j \left(\tfrac{\left(\sigma_n^{-1} + \alpha\right)}{\left(\sigma_n^{-1} + 1\right)} \widetilde{W}_{t+j}(i) + \tfrac{1}{\left(\sigma_n^{-1} + 1\right)\left(\phi_p - 1\right)} \widetilde{P}_{t+j}(i) + \tfrac{1}{1-u} \left(u_{t+j} - u \right) \right) = 0$$

where relative wages, conditional to the lack of future resetting on the labor-clearing rule, are determined by this expression

$$E_t^{\xi_w}\widetilde{W}_{t+j}(i)=\widetilde{W}_t^*(i)-E_t\sum_{k=1}^{j}\left(\pi_{t+k}^w-x_{t+k}^w\right)$$

being x_{t+k}^w the (linearized) wage indexation factor in period $t+k$. Combining the last two equation, we get

$$\frac{\left(\sigma_n^{-1}+\alpha\right)}{(1-\beta\xi_w)\left(\sigma_n^{-1}+1\right)}\widetilde{W}_t^*(i)=E_t^{\xi_w}\sum_{j=0}^{\infty}\beta^j\xi_w^j\left(-\frac{1}{\left(\sigma_n^{-1}+1\right)\left(\phi_p-1\right)}\widetilde{P}_{t+j}(i)-\frac{1}{1-u}\left(u_{t+j}-u\right)\right)$$
$$+\frac{\left(\sigma_n^{-1}+\alpha\right)}{(1-\beta\xi_w)\left(\sigma_n^{-1}+1\right)}E_t\sum_{j=1}^{\infty}\beta^j\xi_w^j\left(\pi_{t+j}^w-x_{t+j}^w\right)$$

which is equivalent to

$$\widetilde{W}_t^*(i)=\frac{(1-\beta\xi_w)}{\left(\sigma_n^{-1}+\alpha\right)}E_t^{\xi_w}\sum_{j=0}^{\infty}\beta^j\xi_w^j\left(-\frac{1}{\left(\phi_p-1\right)}\widetilde{P}_{t+j}(i)-\frac{\left(\sigma_n^{-1}+1\right)}{(1-u)}\left(u_{t+j}-u\right)\right)+E_t\sum_{j=1}^{\infty}\beta^j\xi_w^j\left(\pi_{t+j}^w-x_{t+j}^w\right) \tag{A7}$$

The relative collectively-agreed wage depends on relative prices due to the internal price-wage compementarities at the firm level, $\widetilde{W}_t^*(i)=\widetilde{W}_t^*-\tau_2\widetilde{P}_t(i)$. If these relative prices were set optimally they would be having some variability depending on the relative wages of agreed in period $t-1$ as captured by the loglinear internal relationship, $\widetilde{P}_t^*(i)=\widetilde{P}_t^*+\tau_1\widetilde{W}_{t-1}(i)$.

Next, we do some algebra to find an expression for the expected stream of relative prices $\widetilde{P}_{t+j}(i)$, included in (A7), that relates it to the current choice of $\widetilde{W}_t^*(i)$.

For the case of having $j=1$, the distribution of prices between optimal and indexed according to the Calvo probabilities gives the following conditional probability

$$E_t^{\xi_w}\widetilde{P}_{t+1}(i)=\xi_p\left(\widetilde{P}_t(i)+E_tx_{t+1}^p-E_t\left(\pi_{t+1}^p-\pi^p\right)\right)+\left(1-\xi_p\right)E_t^{\xi_w}\widetilde{P}_{t+1}^*(i),$$

where x_{t+1}^p is the linearized price indexation factor in $t+1$. The conditional expectation of next-period relative prices obtained from the internal generating process is $E_t^{\xi_w}\widetilde{P}_{t+1}^*(i)=E_t\widetilde{P}_{t+1}^*+\tau_1\widetilde{W}_t^*(i)$ which inserted in the previous expression yields

$$E_t^{\xi_w}\widetilde{P}_{t+1}(i)=\xi_p\left(\widetilde{P}_t(i)+E_tx_{t+1}^p-E_t\left(\pi_{t+1}^p-\pi^p\right)\right)+\left(1-\xi_p\right)\left(E_t\widetilde{P}_{t+1}^*+\tau_1\widetilde{W}_t^*(i)\right).$$

The aggregation across prices implies $\widetilde{P}_{t+1}^*=\frac{\xi_p}{1-\xi_p}\left(\pi_{t+1}-x_{t+1}^p\right)$ that can also be used in the right-hand side of the last expression to obtain the simpler equation

$$E_t^{\xi_w}\widetilde{P}_{t+1}(i)=\xi_p\widetilde{P}_t(i)+\tau_1\left(1-\xi_p\right)\widetilde{W}_t^*(i). \tag{A8}$$

Replicating the steps to obtain an equation for $E_t^{\xi_w}\widetilde{P}_{t+2}(i)$, we initially get

$$E_t^{\xi_w}\widetilde{P}_{t+2}(i) = \xi_p\left(E_t^{\xi_w}\widetilde{P}_{t+1}(i) + E_t x_{t+2}^p - E_t\left(\pi_{t+2}^p - \pi^p\right)\right) + \left(1-\xi_p\right)E_t^{\xi_w}\widetilde{P}_{t+2}^*(i),$$

and then using (A8) for $E_t^{\xi_w}\widetilde{P}_{t+1}(i)$ leads to

$$E_t^{\xi_w}\widetilde{P}_{t+2}(i) = \xi_p\left(\xi_p\widetilde{P}_t(i) + \tau_1\left(1-\xi_p\right)\widetilde{W}_t^*(i) + E_t x_{t+2}^p - E_t\left(\pi_{t+2}^p - \pi^p\right)\right) + \left(1-\xi_p\right)E_t^{\xi_w}\widetilde{P}_{t+2}^*(i).$$

The expected optimal price in period $t+2$ conditional on the lack of wage resetting is $E_t^{\xi_w}\widetilde{P}_{t+2}^*(i) = E_t\widetilde{P}_{t+2}^* + \tau_1 E_t^{\xi_w}\widetilde{W}_{t+1}(i) = E_t\widetilde{P}_{t+2}^* + \tau_1\left(\widetilde{W}_t^*(i) + E_t x_{t+1}^w - E_t\pi_{t+1}^w\right)$, which is susbtituted in the last term of the previous expression to obtain

$$\begin{aligned} E_t^{\xi_w}\widetilde{P}_{t+2}(i) = {} & \xi_p\left(\xi_p\widetilde{P}_t(i) + \tau_1\left(1-\xi_p\right)\widetilde{W}_t^*(i) + E_t x_{t+2}^p - E_t\left(\pi_{t+2}^p - \pi^p\right)\right) \\ & + \left(1-\xi_p\right)\left(E_t\widetilde{P}_{t+2}^* + \tau_1\left(\widetilde{W}_t^*(i) + E_t x_{t+1}^w - E_t\left(\pi_{t+1}^w - \pi^w\right)\right)\right), \end{aligned}$$

where using $E_t\widetilde{P}_{t+2}^* = \frac{\xi_p}{1-\xi_p}\left(E_t\left(\pi_{t+2} - \pi\right) - E_t x_{t+2}^p\right)$ simplifies to

$$E_t^{\xi_w}\widetilde{P}_{t+2}(i) = \xi_p^2\widetilde{P}_t(i) + \tau_1\left(1-\xi_p^2\right)\widetilde{W}_t^*(i) - \tau_1\left(1-\xi_p\right)\left(E_t\left(\pi_{t+1}^w - \pi^w\right) - E_t x_{t+1}^w\right). \quad \text{(A9)}$$

A generalization of (A8) and (A9) results in the following rule:

$$E_t^{\xi_w}\widetilde{P}_{t+j}(i) = \xi_p^j\widetilde{P}_t(i) + \tau_1\left(1-\xi_p^j\right)\widetilde{W}_t^*(i) - \tau_1 E_t\sum_{k=1}^{j-1}\left(1-\xi_p^{j-k}\right)\left(\left(\pi_{t+k}^w - \pi^w\right) - x_{t+k}^w\right),$$

implying the following expected sum of discounted relative prices:

$$\begin{aligned} E_t^{\xi_w}\sum_{j=0}^{\infty}\beta^j\xi_w^j\widetilde{P}_{t+j}(i) = {} & \frac{1}{1-\beta\xi_w\xi_p}\widetilde{P}_t(i) \\ & + \tau_1\left(\frac{\beta\xi_w}{1-\beta\xi_w} - \frac{\beta\xi_w\xi_p}{1-\beta\xi_w\xi_p}\right)\left(\widetilde{W}_t^*(i) - E_t\sum_{j=1}^{\infty}\beta^j\xi_w^j\left(\left(\pi_{t+j}^w - \pi^w\right) - x_{t+j}^w\right)\right). \quad \text{(A10)} \end{aligned}$$

Inserting (A10) in equation (A7) that determines the evolution of the firm-level agreed nominal wage, we obtain:

$$\begin{aligned} \left(1+\Lambda\right)\widetilde{W}_t^*(i) = {} & -\frac{\left(1-\beta\xi_w\right)}{\left(\sigma_n^{-1}+\alpha\right)\left(\phi_p-1\right)\left(1-\beta\xi_w\xi_p\right)}\widetilde{P}_t(i) - \frac{\left(1-\beta\xi_w\right)\left(1+\sigma_n^{-1}\right)}{\left(\sigma_n^{-1}+\alpha\right)\left(1-u\right)}E_t\sum_{j=0}^{\infty}\beta^j\xi_w^j\left(u_{t+j}-u\right) \quad \text{(A11)} \\ & + \left(1+\Lambda\right)E_t\sum_{j=1}^{\infty}\beta^j\xi_w^j\left(\left(\pi_{t+j}^w - \pi^w\right) - x_{t+j}^w\right), \end{aligned}$$

with

$$\Lambda = \frac{\tau_1\beta\xi_w}{\left(\sigma_n^{-1}+\alpha\right)\left(\phi_p-1\right)}\left(1 - \frac{\xi_p\left(1-\beta\xi_w\right)}{1-\beta\xi_w\xi_p}\right)$$

Equation (A11) proves right the proposed linear relation $\widetilde{W}_t^*(i) = \widetilde{W}_t^* - \tau_2 \widetilde{P}_t(i)$, with the following analytical solution for τ_2

$$\tau_2 = \frac{(1-\beta\xi_w)}{(\sigma_n^{-1}+\alpha)\left(\phi_p - 1\right)(1-\beta\xi_w\xi_p)(1+\Lambda)},$$

and the following expression for the aggregate relative wage set in period t

$$\widetilde{W}_t^* = -\tfrac{(1-\beta\xi_w)\left(1+\sigma_n^{-1}\right)}{\left(\sigma_n^{-1}+\alpha\right)(1-u^n)(1+\Lambda)} E_t \sum_{j=0}^{\infty} \beta^j \xi_w^j (u_{t+j} - u) + E_t \sum_{j=1}^{\infty} \beta^j \xi_w^j \left(\left(\pi_{t+j}^w - \pi^w\right) - x_{t+j}^w\right) \quad \text{(A12)}$$

The aggregation across nominal wages separating between collectively-agreed wages and indexed wages brings this rate for nominal wage inflation

$$\left(\pi_t^w - \pi^w\right) = \tfrac{(1-\xi_w)}{\xi_w} \widetilde{W}_t^* + x_t^w$$

where inserting (A12) leads to the wage inflation equation

$$\left(\pi_t^w - \pi^w\right) = -\tfrac{(1-\beta\xi_w)\left(1+\sigma_n^{-1}\right)(1-\xi_w)}{\left(\sigma_n^{-1}+\alpha\right)(1-u)(1+\Lambda)\xi_w} E_t \sum_{j=0}^{\infty} \beta^j \xi_w^j (u_{t+j} - u) + \tfrac{(1-\xi_w)}{\xi_w} E_t \sum_{j=1}^{\infty} \beta^j \xi_w^j \left(\left(\pi_{t+j}^w - \pi^w\right) - x_{t+j}^w\right) + x_t^w$$

Writing down $\beta\xi_w E_t\left(\pi_{t+1}^w - \pi^w\right)$ and doing the difference $\left(\pi_t^w - \pi^w\right) - \beta\xi_w E_t\left(\pi_{t+1}^w - \pi^w\right)$

$$\begin{aligned}
\left(\pi_t^w - \pi^w\right) - \beta\xi_w E_t\left(\pi_{t+1}^w - \pi^w\right) &= -\tfrac{(1-\beta\xi_w)\left(1+\sigma_n^{-1}\right)(1-\xi_w)}{\left(\sigma_n^{-1}+\alpha\right)(1-u)(1+\Lambda)\xi_w} E_t \sum_{j=0}^{\infty} \beta^j \xi_w^j (u_{t+j} - u) \\
&+ \tfrac{(1-\xi_w)}{\xi_w} E_t \sum_{j=1}^{\infty} \beta^j \xi_w^j \left(\left(\pi_{t+j}^w - \pi^w\right) - x_{t+j}^w\right) + x_t^w \\
&- \left(\begin{array}{c} -\tfrac{(1-\beta\xi_w)\left(1+\sigma_n^{-1}\right)(1-\xi_w)}{\left(\sigma_n^{-1}+\alpha\right)(1-u)(1+\Lambda)\xi_w} E_t \sum_{j=1}^{\infty} \beta^j \xi_w^j (u_{t+j} - u) \\ + \tfrac{(1-\xi_w)}{\xi_w} E_t \sum_{j=2}^{\infty} \beta^j \xi_w^j \left(\left(\pi_{t+j}^w - \pi^w\right) - x_{t+j}^w\right) + \beta\xi_w x_{t+1}^w \end{array} \right)
\end{aligned}$$

which after cancelling terms becomes

$$\begin{aligned}
\left(\pi_t^w - \pi^w\right) - \beta\xi_w E_t\left(\pi_{t+1}^w - \pi^w\right) &= -\tfrac{(1-\beta\xi_w)(1+1/\sigma_l)(1-\xi_w)}{\left(\sigma_l^{-1}+\alpha\right)(1-u)(1+\Lambda)\xi_w} (u_t - u) \\
&+ \tfrac{(1-\xi_w)}{\xi_w} \beta E_t \xi_w \left(\left(\pi_{t+1}^w - \pi^w\right) - x_{t+1}^w\right) + x_t^w - \beta\xi_w E_t x_{t+1}^w
\end{aligned}$$

and further simplifications to yield

$$\left(\pi_t^w - \pi^w\right) = \beta E_t\left(\pi_{t+1}^w - \pi^w\right) - \tfrac{\left(1+\sigma_n^{-1}\right)(1-\beta\xi_w)(1-\xi_w)}{\left(\sigma_n^{-1}+\alpha\right)(1-u)(1+\Lambda)\xi_w} (u_t - u) + x_t^w - \beta E_t x_{t+1}^w$$

The (linearized) indexation factor for nominal wages is

$$x_t^w = \iota_w\left(\pi_{t-1}^p - \pi\right) + \varepsilon_t^w$$

that can be used for x_t^w and the update to x_{t+1}^w in the previous equation to obtain the wage inflation New Keynesian Phillips Curve

$$(\pi_t^w - \pi^w) = \beta E_t \left(\pi_{t+1}^w - \pi^w\right) + \iota_w \left(\pi_{t-1}^p - \pi\right) - \beta \iota_w \left(\pi_t^p - \pi\right) - \frac{\left(1+\sigma_n^{-1}\right)(1-\beta\xi_w)(1-\xi_w)}{\left(\sigma_n^{-1}+\alpha\right)(1-u)(1+\Lambda)\xi_w} (u_t - u) + \varepsilon_t^w - \beta E_t \varepsilon_{t+1}^w$$

A.6. Slopes of the Phillips Curves

The marginal response of the optimal price to the relative lagged wage is registered in the loglinear equation $\widehat{P}_t^*(i) = \widehat{P}_t^* + \tau_1 \left(\widehat{W}_{t-1}(i) - \widehat{W}_{t-1}\right)$, where the value of the coefficient τ_1 is found to be

$$\tau_1 = \frac{(1-\alpha)\left(1-\beta\xi_p\right)\xi_w}{\left(\left(\phi_p - 1\right)\epsilon + 1\right)\left(1-\beta\xi_p\xi_w\right)(1+\Omega)} \tag{A13}$$

including the auxiliary coefficient, Ω, defined as follows

$$\Omega = \frac{\tau_2(1-\alpha)}{\left(\left(\phi_p - 1\right)\epsilon + 1\right)} \left(1 - \frac{\left(1-\beta\xi_p\right)\xi_w}{1-\beta\xi_p\xi_w}\right) \tag{A14}$$

Likewise, the firm-level (labor-clearing) wage varies depending on the relative selling price of the good produced in that firm as indicated in the loglinear equation $\widehat{W_t^*}(i) = \widehat{W_t^*} - \tau_2 \left(\widehat{P}_t(i) - \widehat{P}_t\right)$, where following the derivation steps described in this Appendix the value of the coefficient τ_2 is found to be

$$\tau_2 = \frac{(1-\beta\xi_w)}{\left(\phi_p - 1\right)\left(\sigma_n^{-1} + \alpha\right)\left(1-\beta\xi_w\xi_p\right)(1+\Lambda)} \tag{A15}$$

and an auxiliary coefficient, Λ, has been used with the following definition

$$\Lambda = \frac{\tau_1 \beta \xi_w}{\left(\sigma_n^{-1} + \alpha\right)\left(\phi_p - 1\right)} \left(1 - \frac{\xi_p (1-\beta\xi_w)}{1-\beta\xi_w\xi_p}\right) \tag{A16}$$

Equations (A13) to (A16) can be solved in a non-linear system that determines the numerical values of τ_1 , τ_2 , Ω and Λ, given the values of the deep parameters of the model β, ξ_p, ξ_w, ϕ_p, ϵ, σ_n and α.

The slope of the price inflation Phillips Curve is

$$\kappa_p = \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{(1+\beta\iota_p)\left(\left(\phi_p - 1\right)\epsilon + 1\right)(1+\Omega)\xi_p}$$

which depends on the Calvo sticky-price probability, ξ_p , but also in all the other model parameters.

The slope of the wage inflation Phillips Curve is

$$\kappa_w = \frac{\left(1+\sigma_n^{-1}\right)\left(1-\beta\xi_w\right)\left(1-\xi_w\right)}{\left(\sigma_n^{-1}+\alpha\right)\left(1-u\right)\left(1+\Lambda\right)\xi_w}$$

which depends on the Calvo sticky-price probability, ξ_p, but also in all the other model parameters.

For example, a higher Calvo sticky-wage probability, ξ_w, reduces κ_w for a lower value in the slope coefficient of wage inflation (less aggregate wage inflation variability), and also increases κ_p for a higher value in the slope coefficient of price inflation (more aggregate price inflation variability). This complementary effect is caused by the firm-level response of optimal prices to relative wages; the value of τ_1 rises with a lower κ_w, optimal prices turn more sensitive to the relative wage and inflation volatility is higher.

Analytical solution

The following non-linear 4-equation system can be solved to determine the numerical values of τ_1, τ_2, Θ and Λ:

$$\begin{aligned}
\tau_1 &= \frac{(1-\alpha)\left(1-\beta\xi_p\right)\xi_w}{((\phi_p-1)\epsilon+1)\left(1-\beta\xi_p\xi_w\right)(1+\Omega)} \\
\Omega &= \frac{\tau_2(1-\alpha)}{(\phi_p-1)\epsilon+1}\left(1-\frac{\left(1-\beta\xi_p\right)\xi_w}{1-\beta\xi_p\xi_w}\right) \\
\tau_2 &= \frac{(1-\beta\xi_w)}{\left(\phi_p-1\right)\left(\sigma_n^{-1}+\alpha\right)\left(1-\beta\xi_w\xi_p\right)(1+\Lambda)} \\
\Lambda &= \frac{\tau_1\beta\xi_w}{\left(\phi_p-1\right)\left(\sigma_n^{-1}+\alpha\right)}\left(1-\frac{\xi_p\left(1-\beta\xi_w\right)}{1-\beta\xi_w\xi_p}\right)
\end{aligned}$$

Define the auxiliary combination of parameters

$$\begin{aligned}
a_1 &= \frac{\xi_p\left(1-\beta\xi_w\right)}{1-\beta\xi_w\xi_p} \\
a_2 &= \frac{\beta\xi_w}{\left(\phi_p-1\right)\left(\sigma_n^{-1}+\alpha\right)} \\
a_3 &= \frac{(1-\beta\xi_w)}{\left(\phi_p-1\right)\left(\sigma_n^{-1}+\alpha\right)\left(1-\beta\xi_w\xi_p\right)} \\
a_4 &= \frac{\xi_w\left(1-\beta\xi_p\right)}{1-\beta\xi_w\xi_p} \\
a_5 &= \frac{(1-\alpha)}{(\phi_p-1)\epsilon+1}
\end{aligned}$$

so that the analytical expression for τ_2 can be written as

$$\tau_2 = \frac{a_3}{\left(1 + \frac{a_5 a_4 a_2 (1-a_1)}{(1+\tau_2 a_5 (1-a_4))}\right)}$$

Using the Scientific Workplace algorithm, the analytical solution for τ_2 is:

$$\tau_2 = \left\{ \frac{1}{2a_5 - 2a_4 a_5} \left(\begin{array}{c} a_3 a_5 + \\ \sqrt{\begin{array}{c} a_3^2 a_5^2 + 2a_3 a_5 + a_2^2 a_4^2 a_5^2 + a_3^2 a_4^2 a_5^2 - 2a_3^2 a_4 a_5^2 \\ +2a_2 a_4 a_5 - 2a_3 a_4 a_5 + a_1^2 a_2^2 a_4^2 a_5^2 \\ +2a_2 a_3 a_4^2 a_5^2 - 2a_1 a_2 a_4 a_5 - 2a_1 a_2^2 a_4^2 a_5^2 - 2a_2 a_3 a_4 a_5^2 - 2a_1 a_2 a_3 a_4^2 a_5^2 \\ +2a_1 a_2 a_3 a_4 a_5^2 + 1 \end{array}} \\ -a_2 a_4 a_5 - a_3 a_4 a_5 + a_1 a_2 a_4 a_5 - 1 \end{array} \right) \right\}$$

which can be plugged in the following expression to obtain the analytical solution of Ω

$$\Omega = \tau_2 a_5 (1 - a_4)$$

The corresponding solution for τ_1 given that of Ω is found as follows

$$\tau_1 = \frac{a_5 a_4}{1 + \Omega}$$

and, finally, Λ has its analytical solution in the next form that comprises the analytical solution of τ_1 found above

$$\Lambda = \tau_1 a_2 (1 - a_1)$$

B. Endogenous and exogenous variables

List of endogenous variables (19):

- Price inflation rate: $\pi_t^p - \pi^p$
- Wage inflation rate: $\pi_t^w - \pi^w$
- Unemployment rate: $u_t - u^n$
- Nominal interest rate: $R_t - R$
- Real interest rate: $r_t - r$
- Output: $\widehat{y}_t$
- Consumption: $\widehat{c}_t$
- Labor supply shifter: $\widehat{Z}_t$
- Real wage: $\widehat{w}_t$
- Real marginal cost: $\widehat{mc}_t$
- Marginal product of labor: $\widehat{f}_{n_t}$
- Labor supply (labor force): $\widehat{n}_t^s$
- Labor demand (employment): $\widehat{n}_t^d$
- Investment: $\widehat{i}_t$
- Tobin's q: $\widehat{q}_t$
- Rental rate of capital: $\widehat{r}_t^k$
- Stock of capital: $\widehat{\overline{k}}_t$
- Capital utilization rate: $\widehat{v}_t$
- Effective capital: $\widehat{k}_t$

List of exogenous variables (7):

- AR(1) technology shock $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ with $\eta_t^a \sim N(0, \sigma_{\eta^a}^2)$
- AR(1) monetary policy shock $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$ with $\eta_t^R \sim N(0, \sigma_{\eta^R}^2)$
- ARMA(1,1) price-push shock $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$ with $\eta_t^p \sim N(0, \sigma_{\eta^p}^2)$
- ARMA(1,1) wage-push shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$ with $\eta_t^w \sim N(0, \sigma_{\eta^w}^2)$
- AR(1) autonomous spending shock $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \rho_{ga} \eta_t^a + \eta_t^g$ with $\eta_t^g \sim N(0, \sigma_{\eta^g}^2)$
- AR(1) labor supply shock $\varepsilon_t^n = \rho_n \varepsilon_{t-1}^n + \eta_t^n$ with $\eta_t^n \sim N(0, \sigma_n^2)$
- AR(1) investment shock $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ with $\eta_t^i \sim N(0, \sigma_i^2)$

C. Set of semi-loglinearized dynamic equations

$$\pi_t^p - \pi^p = \frac{\beta}{1+\beta\iota_p}\left(E_t\pi_{t+1}^p - \pi^p\right) + \frac{\iota_p}{1+\beta\iota_p}\left(\pi_{t-1}^p - \pi^p\right) + \kappa_p\widehat{mc}_t + \frac{1-\iota_p}{1+\beta\iota_p}\left(\varepsilon_t^p - \beta E_t\varepsilon_{t+1}^p\right)$$

$$\widehat{mc}_t = \widehat{w}_t - \widehat{f}_{n_t}$$

$$\widehat{f}_{n_t} = \alpha\left(\widehat{k}_t - \widehat{n}_t^d\right) + \varepsilon_t^a$$

$$\widehat{y}_t = \phi_p\left(\alpha\widehat{k}_t + (1-\alpha)\,\widehat{n}_t^d + \varepsilon_t^a\right)$$

$$\pi_t^w - \pi^w = \beta\left(E_t\pi_{t+1}^w - \pi^w\right) + \iota_w\left(\pi_{t-1}^p - \pi^p\right) - \beta\iota_w\left(\pi_t^p - \pi^p\right) - \kappa_w\left(u_t - u^n\right) + \left(\varepsilon_t^w - \beta E_t\varepsilon_{t+1}^w\right)$$

$$u_t - u^n = (1-u^n)\left(\widehat{n}_t^s - \widehat{n}_t^d\right)$$

$$\widehat{n}_t^s = \frac{1}{\sigma_n}\left(\widehat{w}_t - (1-u^n)^{-1}(u_t - u^n) - \varepsilon_t^n - \widehat{z}_t\right)$$

$$\widehat{z}_t = (1-\upsilon)\,\widehat{z}_{t-1} + \upsilon\left(\frac{1}{1-\overline{h}}\widehat{c}_t - \frac{\overline{h}}{1-\overline{h}}\widehat{c}_{t-1}\right)$$

$$\widehat{c}_t = \frac{\overline{h}}{1+\overline{h}}\widehat{c}_{t-1} + \frac{1}{1+\overline{h}}E_t\widehat{c}_{t+1} - \frac{\left(1-\overline{h}\right)}{\left(1+\overline{h}\right)}(r_t - r)$$

$$(r_t - r) = (R_t - R) - \left(E_t\pi_{t+1}^p - \pi^p\right)$$

$$R_t - R = (1-\mu_R)\left[\mu_\pi\left(\pi_t^p - \pi^p\right) - \mu_u\left(u_t - u^n\right)\right] + \mu_R\left(R_{t-1} - R\right) + \varepsilon_t^R$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \frac{r^k k}{y}\widehat{\upsilon}_t + \frac{g}{y}\varepsilon_t^g$$

$$\widehat{w}_t = \widehat{w}_{t-1} + \left(\pi_t^w - \pi^w\right) - \left(\pi_t^p - \pi^p\right)$$

$$\widehat{i}_t = \frac{1}{1+\beta}\widehat{i}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{i}_{t+1} + \frac{1}{(1+\beta)(1+\gamma)^2\varphi}\widehat{q}_t + \varepsilon_t^i$$

$$\widehat{q}_t = \frac{1-\delta}{1+r^k-\delta}E_t\widehat{q}_{t+1} + \frac{r^k}{r^k+1-\delta}E_t\widehat{r}_{t+1}^k - (r_t - r)$$

$$\widehat{k}_t = \widehat{\overline{k}}_{t-1} + \widehat{\upsilon}_t$$

$$\widehat{\upsilon}_t = \frac{1-\psi}{\psi}\widehat{r}_t^k$$

$$\widehat{\overline{k}}_t = \frac{1-\delta}{1+\gamma}\widehat{\overline{k}}_{t-1} + \frac{\gamma+\delta}{1+\gamma}\widehat{i}_t + (\gamma+\delta)(1+\beta)\,\gamma\varphi\varepsilon_t^i$$

$$\widehat{r}_t^k = \widehat{w}_t - \left(\widehat{k}_t - \widehat{n}_t^d\right)$$

D. Data sources

We provide the sources for the seven observed variables used in the estimation. Population series are used to normalize quantity variables.

Real GDP growth: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate (FRED, GDPC1), logarithmic first difference on the per-capita time series.

Price inflation: Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Seasonally Adjusted (FRED, GDPDEF), logarithmic first difference.

Nominal wage inflation Nonfarm Business Sector: Hourly Compensation for All Workers, Index 2012=100, Seasonally Adjusted (FRED, COMPNFB), logarithmic first difference on the per-capita time series.

Unemployment: Unemployment Rate, Percent, Seasonally Adjusted (FRED, UNRATE).

Real investment growth: Fixed private investment, billions of dollars, seasonally adjusted annual rate (FRED, FPI), logarithmic first difference.

Employment growth: Employment level, thousands of persons, seasonally adjusted (FRED, CE16OV), logarithmic first difference on the per-capita time series.

Nominal interest rates: Wu-Xia Shadow Federal Funds Rate (Federal Reserve Bank of Atlanta).

Population: Smoothed population level for population control adjustments, 1990-2017, seasonally adjusted, Bureau of Labor Statistics. Data from 2017-2022, Population Level, thousands of persons, not seasonally adjusted (FRED, CNP16OV).

E. Time series

US quarterly series (1992:1 to 2022:4) used as observable variables in the model estimation