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OPTIMIZATION OF A VIBRATION BASE PLATE BY MAKING USE OF FINITE ELEMENT MODELING

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Optimization of a vibration base plate by making use of Finite Element Modeling

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Abstract

Modal analysis can be used to determine the vibration characteristics (natural frequencies and corresponding mode shapes) of a structure. In our case, natural frequencies of a PVC base plate. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. Having identified the natural frequency of a base plate, we modified it iteratively, ensured that the mass does not exceed the 10% extra, trying to achieve to maximize the first natural frequency keeping the mass below limit level value.

Keywords: Modal analysis; Natural frequencies; Optimization; Finite element modeling.

1 Introduction

The goal of this project is to design and optimize a plate that is part of a vibrating table. The vibration table is utilized to test the ability of components, devices, equipment, systems, and materials that have to withstand vibrations, shocks, and the like [17]. A platform used in conjunction with a device which delivers controlled vibrations. We will to examine a PVC plate; it is 100 x 75 x 2 mm [1]. First we will calculate the first six natural frequencies of the plate. For this we must find the proper mesh to ensure the most accurate results possible. Having defined the natural frequency of the plate, we are going to modify the plate by adding ribs of different sizes and shapes. Increasingly we ensure that the mass does not exceed the 10% extra. Our goal is to optimize the new plate, considering that the natural frequency is the highest possible keeping the mass below the parameters. Finally we discuss the results and we analyze the relationship between the natural frequency and mass of the plate depending on the type and method number of the ribs.

2 Theoretical framework

2.1 Natural frequency

Natural frequencies are the frequency at which a system naturally vibrates once it is excited. In other words, natural frequency is the number of times in second a system will oscillate between its original position and its displaced position, if there is no external force. If force is applied to a spring-mass system, the system will vibrate at its natural frequency, and the level of vibration will depend on damping inherent in the system. This inherent absorption depends on the geometric and material characteristics of the body, mainly the moment of inertia, i.e. the mass and the way this is distributed around the center of gravity of the body [3].

Where ω_n is the natural angular frequency in free vibration of the system and is equal to:

$$\omega_n = \sqrt{\frac{k}{m}} = 2 \cdot \pi \cdot f_n \quad (1)$$

Here ω_n *(Rad/s)* is the natural angular frequency of a structure, while *k* (N·m) is the stiffness (like the strength of a spring), *m* (kg) is the mass of a particular structure and fn is the natural frequency (Hz) [15].

So can be seen that if the stiffness increases, the natural frequency also increases, and if the mass increases, the natural frequency decreases.

Also must consider resonance, is an operating condition where an excitation frequency is near a natural frequency of the structure. When resonance occurs, the resulting vibration levels can be very high and can cause rapid damage.

2.2 Optimization

Design optimization is a technique that aims to determine the best design or the optimal design. An "optimal design" is one that meets a number of specific requirements, with a minimum cost of certain factors, such as weight, area, volume, effort, etc. Virtually any aspect of your design can be optimized: dimensions (such as thickness), shape (such as fillet radii), placement of supports, cost of fabrication, natural frequency, material property, and so on. In other words, the optimal design is usually the one who manages to "be as effective as possible". In our case, we need to get the highest frequency natural as possible, using the fewest material [3] [5].

2.2.1 Basic definitions optimization

Before describing the optimization procedure is necessary to define the basic terminology used in these types of analyzes, such as, design variables, state variables and objective function [3][4].

Design variables (DV): They are independent quantities that are modified to achieve the optimal design (they are the independent variables of the problem). You need to specify upper and lower limits as "Restrictions" in the design variables. These limits define the range of variation. The DV are usually geometric parameters such as length, thickness, diameter, or the coordinates of the model, and are restricted to positive values.

State variables (SV): they are quantities that determine or constrain the design. Are also known as "dependent variables" and are typically response quantities that are functions of the design variables. Examples of (SV) can be tensions temperatures, the rate of heat flow, the deflections or time. A variable status is not necessarily an amount calculated by ANSYS; virtually any parameter can be defined as a state variable.

Objective function (F): is the dependent variable which tries to minimize. Must be a function of the DV [F = f (DV)], i.e., if the values of the DV change, the value of the function must vary.

2.2.2 Optimization Methods

Optimization methods are traditional techniques that strive to minimize a function (objective function) that is subject to restrictions. In the ANSYS program are available two methods of optimization, the subproblem approximation method and first order method.

- The subproblem approximation method is an advanced zero-order method that use approximations (curve fitting) for all dependent variables (SV and the objective function). It is a general method that can be applied effectively to a wide range of engineering problems.
- The method of first order is that which uses the information of the derivative, i.e., the gradients of the dependent variables with respect to the design. The method is very accurate, is more suitable for problems that require high accuracy. However, this method can be computationally expensive.

For both the subproblem approximation and first order methods, the program performs a series of analysis-evaluation-modification cycles. That is, an analysis of the initial design is performed, the results are evaluated against specified design criteria, and the design is modified as necessary. The process is repeated until all specified criteria are met.

Understanding the algorithm used by the program is always useful, particularly for optimization. Below are details of the two optimization techniques in ANSYS.

2.2.2.1 The subproblem approximation method

The subproblem approximation method can be described as an advanced zero-order method in that it requires only the values of the dependent variables, and not their derivatives. There are two concepts that play a key role in the subproblem approximation method: the use of approximations for the objective function and state variables, and the conversion of the constrained optimization problem to an unconstrained problem.

2.2.2.2 First Order Method

Like the subproblem approximation method, the first order method converts the problem to an unconstrained one by adding penalty functions to the objective function. However, unlike the subproblem approximation method, the actual finite element representation is minimized and not an approximation.

The first order method uses gradients of the dependent variables with respect to the design variables. For each iteration, gradient calculations (which may employ a steepest descent or conjugate direction method) are performed in order to determine a search direction, and a line search strategy is adopted to minimize the unconstrained problem.

Thus, each iteration is composed of a number of subiterations that include search direction and gradient computations. That is why one optimization iteration for the first order method performs several analysis loops.

2.2.3 Goal driven optimization

GDO can be used for design optimization in three ways: the Screening approach, the MOGA approach, or the NLPQL approach. The Screening approach is a non-iterative direct sampling method by a quasi-random number generator based on the Hammersley algorithm. The MOGA approach is an iterative Multi-Objective Genetic Algorithm, which can optimize problems with continuous input parameters. NLPQL is a gradient based single objective optimizer which is based on quasi-Newton methods.

MOGA is better for calculating the global optima while NLPQL is a gradient-based algorithm ideally suited for local optimization. So you can start with Screening or MOGA to locate the multiple tentative optima and then refine with NLPQL to zoom in on the individual local maximum or minimum value. Problems with mixed parameter types (i.e., usability, discrete, or scenario parameters with continuous parameters) or discrete problems cannot currently be handled by the MOGA or NLPQL techniques, and in these cases you will only be able to use the Screening technique.

Usually the Screening approach is used for preliminary design, which may lead you to apply the MOGA or NLPQL approaches for more refined optimization results [4].

2.2.3.1 Screening (shifted Hammersley)

The shifted Hammersley method is the sampling strategy used for the Screening process. The conventional Hammersley sampling algorithm is a quasi-random number generator which has very low discrepancy and is used for quasi-Monte Carlo simulations. A low-discrepancy sequence is defined as a sequence of points that approximate the equidistribution in a multi-dimensional cube in an optimal way. In other words, the design space is populated almost uniformly by these sequences and, due to the inherent properties of Monte Carlo sampling, dimensionality is not a problem (i.e., the number of points does not increase exponentially with an increase in the number of input parameters). The conventional Hammersley algorithm is constructed by using the radical inverse function. Any integer n can be represented as a sequence of digits n_0, n_1, n_2, ..., n_m by the following equation:

$$n = n_0 n_1 n_2 n_3 \cdots n_m \quad (2)$$

In general, for a radix R representation, the equation is:

$$n = n_m + n_{m-1} * R + \cdots + n_0 \quad (3)$$

The inverse radical function is defined as the function which generates a fraction in (0, 1) by reversing the order of the digits in **(3)** about the decimal point, as shown below.

$$\begin{aligned} \Phi_R(n) &= 0n_m n_{m-1} n_{m-2} \cdots n_0 \\ &= n_m * R^{-1} + n_{m-1} * R^{-2} + \cdots + n_0 * R^{-(m-1)} \end{aligned} \quad (4)$$

Thus, for a k-dimensional search space, the Hammersley points are given by the following expression:

$$H_k(i) = [i/N, \Phi_{R1}(i), \Phi_{R2}(i), \cdots, \Phi_{Rk-1}(i)] \quad (5)$$

Where i = 0... N indicates the sample points. Now, from the plot of these points, it is seen that the first row (corresponding to the first sample point) of the Hammersley matrix is zero and the last row is not 1. This implies that, for the k-dimensional hypercube, the Hammersley sampler generates a block of points that are skewed more toward the origin of the cube and away from the far edges and faces. To compensate for this bias, a point-shifting process is proposed that shifts all Hammersley points by the amount below:

$$\Delta = \frac{1}{2}N \qquad (6)$$

2.2.3.2 Multi-Objective Genetic Algorithm (MOGA)

The MOGA used in GDO is a hybrid variant of the popular NSGA-II (Non-dominated Sorted Genetic Algorithm-II) based on controlled elitism concepts. Currently, only continuous problems can be solved. The Pareto ranking scheme is done by a fast, non-dominated sorting method that is an order of magnitude faster than traditional Pareto ranking methods. The constraint handling uses the same non-dominance principle as the objectives, thus penalty functions and Lagrange multipliers are not needed. This also ensures that the feasible solutions are always ranked higher than the infeasible solutions.

The first Pareto front solutions are archived in a separate sample set internally and are distinct from the evolving sample set. This ensures minimal disruption of Pareto front patterns already available from earlier iterations. You can control the selection pressure (and, consequently, the elitism of the process) to avoid premature convergence by altering the parameter Percent Pareto.

2.2.3.3 Sequential Quadratic Programming (NLPQL)

NLPQL (Non-linear Programming by Quadratic Lagrangian) is a mathematical optimization algorithm as developed by Klaus Schittkowski. This method solves constrained nonlinear programming problems of the form.

Minimize:

Objective function (F) $f = f(\{x\})$ (7)

Subject to:

Design variables (DV) $g_k(\{x\}) \leq 0, \forall k = 1,2,\ldots,K$ (8)

$h_l(\{x\}) \leq 0, \forall l = 1,2,\ldots,L$ (9)

Where:

State variables (SV) $\{x_L\} \leq \{x\} \leq \{x_U\}$ (10)

It is assumed that objective function and constraints are continuously differentiable. The idea is to generate a sequence of quadratic programming subproblems obtained by a quadratic approximation of the Lagrangian function and a linearization of the constraints. Second order information is updated by a quasi-Newton formula and the method is stabilized by an additional (Armijo) line search.

Newton's iterative method

Before the actual derivation of the NLPQL equations, Newton's iterative method for the solution of nonlinear equation sets is reviewed. Let f(x) be a multivariable function such that it can be expanded about the point x in a Taylor's series.

$$f(x + \Delta x) \approx f(x) + \{\Delta x\}^T\{f'(x)\} + \left(\frac{1}{2}\right)\{\Delta x\}^T[f''(x)]\{\Delta x\} \quad (11)$$

Where, it is assumed that the Taylor series actually models a local area of the function by a quadratic approximation. The objective is to devise an iterative scheme by linearizing the vector **(Eq.11)**. To this end, it is assumed that at the end of the iterative cycle, the **(11)** would be exactly valid. This implies that the first variation of the following expression with respect to Δx must be zero.

$$\phi(\Delta x) = f(x + \Delta x) - \left(f(x) + \{\Delta x\}^T\{f'(x)\} + \left(\frac{1}{2}\right)\{\Delta x\}^T[f''(x)]\{\Delta x\}\right) \quad (12)$$

This implies that:

$$f(x+\Delta x)_{,\Delta x} - (\{f'(x)\} + [f''(x)]\{\Delta x\}) = 0 \quad (13)$$

The first expression indicates the first variation of the converged solution with respect to the increment in the independent variable vector. This gradient is necessarily zero since the converged solution clearly does not depend on the step-length. Thus, **(13)** can be written as the following:

$$\{x_{j+1}\} = \{x_j\} - [f''(x_j)]^{-1}\{f'(x_j)\} \quad (14)$$

Where, the index "j" indicates the iteration **(14)** is thus used in the main quadratic programming scheme.

3 Problem statement

3.1 Maximization of the first natural frequency of the vibration plate

As mentioned in the introduction, the goal of this project is to design a plate that is part of a vibrating table to calculate the first six natural frequencies. We will study a PVC plate with dimensions it is 100 x 75 x 2 mm. First we are going to determine the properties of the base plate. To do this, we will enter in engineering data and we will create a new material. We will to name the new material (PVC), and we will add properties that need to define the material, the physical properties (density) and Isotropic elasticity properties (Young's modulus and Poisson).

Property	Value	Unit
Density	1025	kg m^-3
Isotropic Elasticity		
Young's Modulus	2,473E+09	Pa
Poisson's Ratio	0,3	

Table 1: Properties of the base plate [1]

The first steps in our finite element analysis are: draw our figure, enter the material properties and finally introduce a suitable mesh. We take special care in this last step because too many cells may result in long solver runs, and too few may lead to inaccurate results. So we will use a mesh sufficiently small to get more exact results possible.

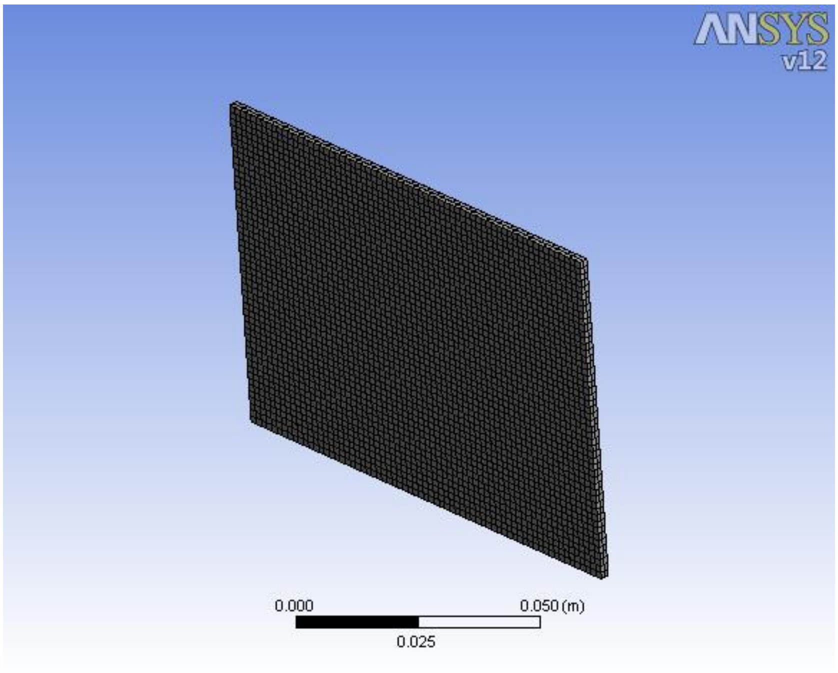

Figure 1: base plate mesh

The purpose is getting the more adequate mesh for our base plate. Let us calculate the natural frequencies with a mesh that we believe appropriate for the size of our plate (0.0015mm elements size with 6700 elements). After obtaining the natural frequencies of the plate, and the number of elements we have divided the plate by the mesh, we will reduce the size of the mesh (0.0012 mm size element size with 10584 elements). With this new mesh size, we again obtain these results. In the following table are the data collected:

	Frequency [Hz] with 0.0015mm size element	Frequency [Hz] with 0.0012 mm size element	Error (%)
1	265,21	265,2	0,0038
2	318,58	318,58	0
3	591,07	591,07	0
4	626,95	626,95	0,0013
5	766,77	766,76	0
6	927,03	927,03	0
Nº of elements	6700	10584	

Table 2: Data from the natural frequencies and number of elements of the two mesh sizes and the error between them.

As we see in the table, we obtained different results at some frequencies, while in others obtain exactly the same results. The best results obtained in some frequencies as we see in the error, is minimal. Therefore we will work with this mesh bigger size (0.0015mm size element), because the calculations are performed faster and the difference in results is minimal.

Therefore the mesh used for our calculations will have 0.0015mm size element and the following statics properties:

Nodes	37791
Elements	6700
Mesh Metric	None

Table 3: Mesh statics properties of the base plate.

As already mentioned, any physical system can vibrate. The frequencies at which vibration occurs naturally, and its mode shapes are characteristic of the system, and can be determined analytically using modal analysis. So this analysis determines the modes of vibration, the fundamental forms and natural frequencies. Therefore by applying modal analysis on the finite element model in Ansys, we will calculate the first six natural frequencies of our base plate.

The modal analysis resulted in the following values:

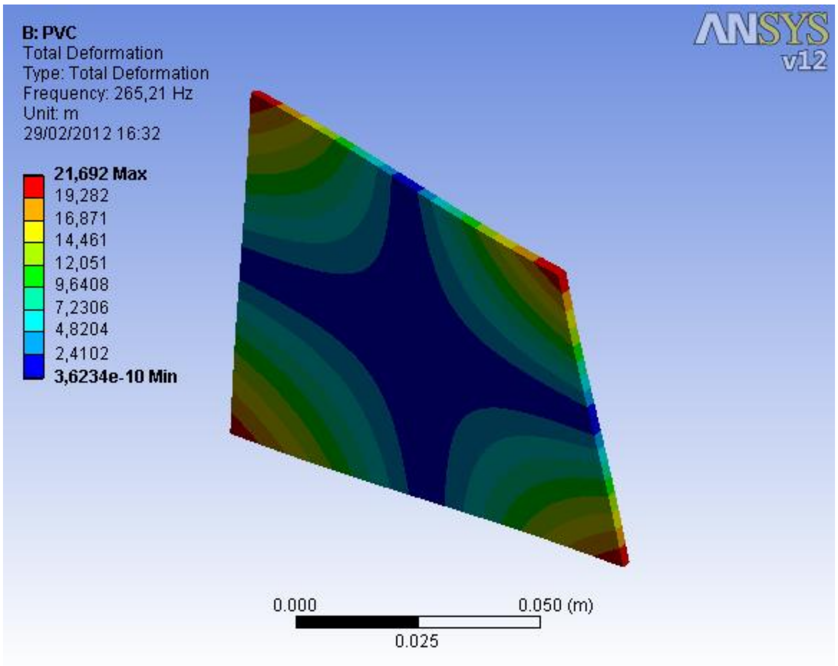

Figure 2: First torsion mode at 265, 21 Hz

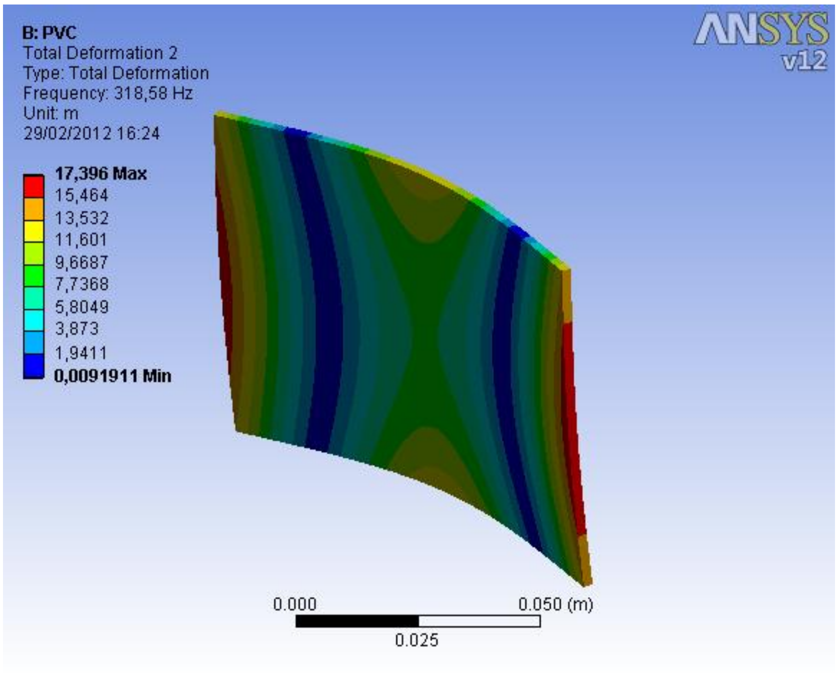

Figure 3: First bending mode at 318, 58 Hz

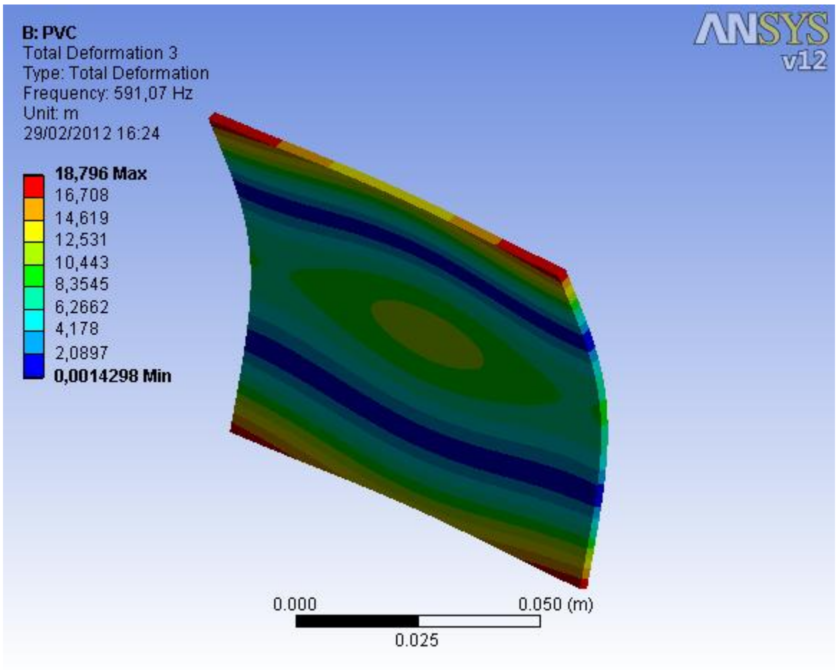

Figure 4: Bending at 591, 07 Hz

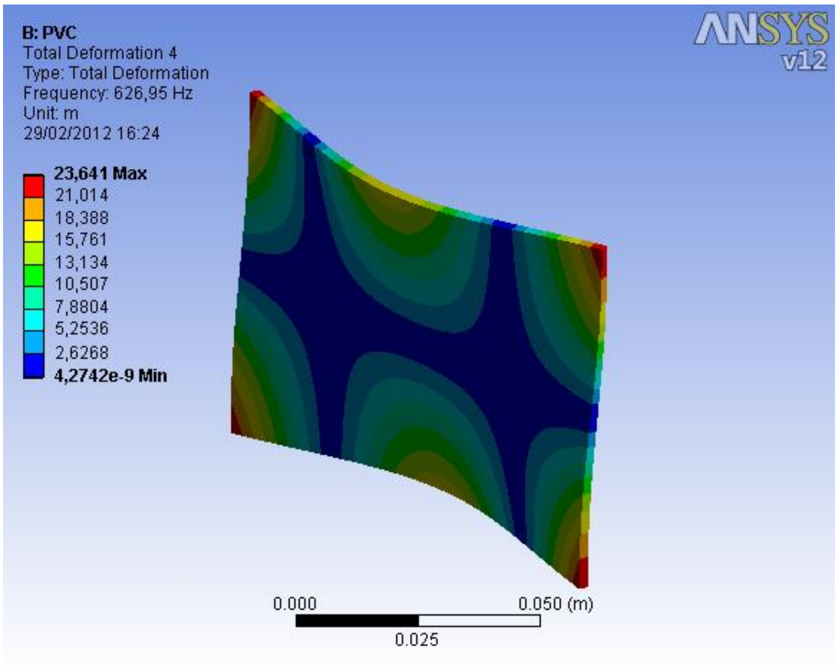

Figure 5: Second torsion mode at 626, 95 Hz

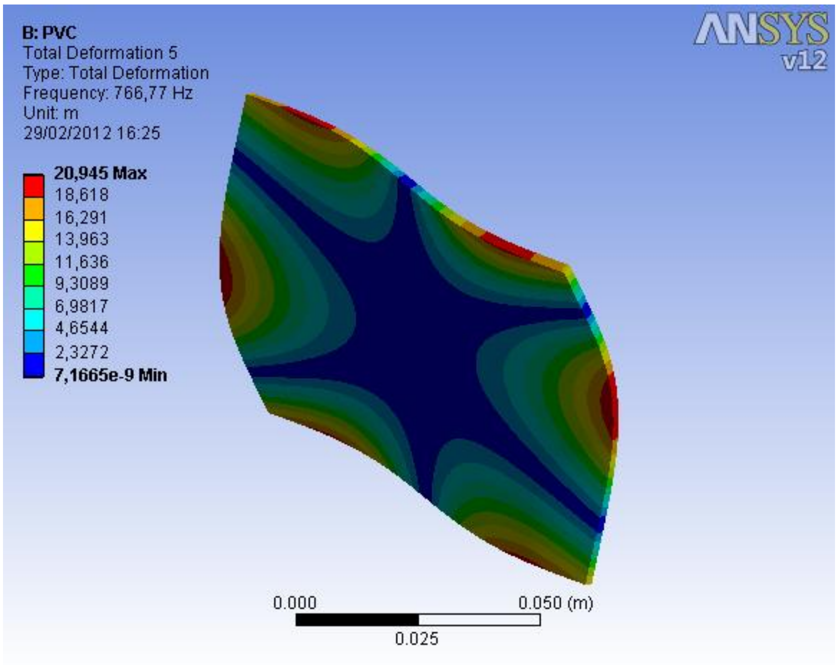

Figure 6: Fifth mode base at 766, 77 Hz

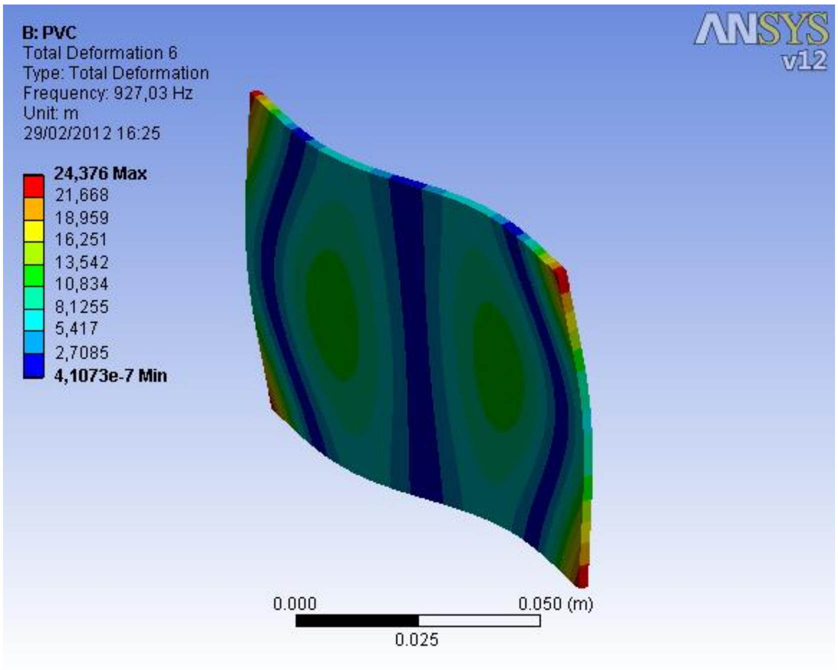

Figure 7: Sixth fashion at 927, 03 Hz

Mode	1	2	3	4	5	6	Volume (m^3)
Frequency (Hz)	265,21	318,58	591,07	626,95	766,77	927,03	$1,65*10^{-5}\ m^3$

Table 4: Frequencies and volume of the base plate

3.2 Optimization

After obtain the results of the natural frequencies. Starting with the base plate, we are going to modified it iteratively plate by adding ribs of different sizes and shapes. Increasingly we ensured that the mass does not exceed the 10% extra. Let us calculate what the mass limit of our plate:

In the following table are the initial data from the mass and volume of our base plate:

Volume	$1.5*10^{-5}\ m^3$
Mass	0,015375 kg

Table 5: Plate initial conditions

After calculating the initial mass, we calculate the 10% of this, and add it to the initial mass to obtain the value of our final mass:

Maximum volume	$1.65*10^{-5}\ m^3$
Maximum mass	0, 0169125 kg

Table 6: Plate final conditions

Therefore we will try to achieve with the optimization that the natural frequency is the highest possible keeping the mass below 0, 0169125 kg. To optimize the natural frequency we will work with the volume, as the value is much more accurate and easier to handle. Therefore our final solid volume will have to be equal to or smaller than 1, $65 \cdot 10^{-5}$ m^3.

Now we will set up the input and output parameters for a geometry created. The input parameters will be the variables that define the geometry of our model as the height and width of the ribs or the diameter of the holes, while the output parameters will be the natural frequency and volume of the plate. Once we have defined everything we can perform optimization.

Finally we will selected the method of optimization (the Screening approach, the MOGA approach, or the NLPQL approach) and we will put as constraints that the first resonance frequency is maximized and the volume is equal to or greater than $1,65*10^{-5}$ m^3.

3.2.1 Model with diagonal ribs

The first modification of our base plate will be the placement of diagonal ribs. The two ribs will have the same width and height. Therefore the input parameters of this new plate will be:

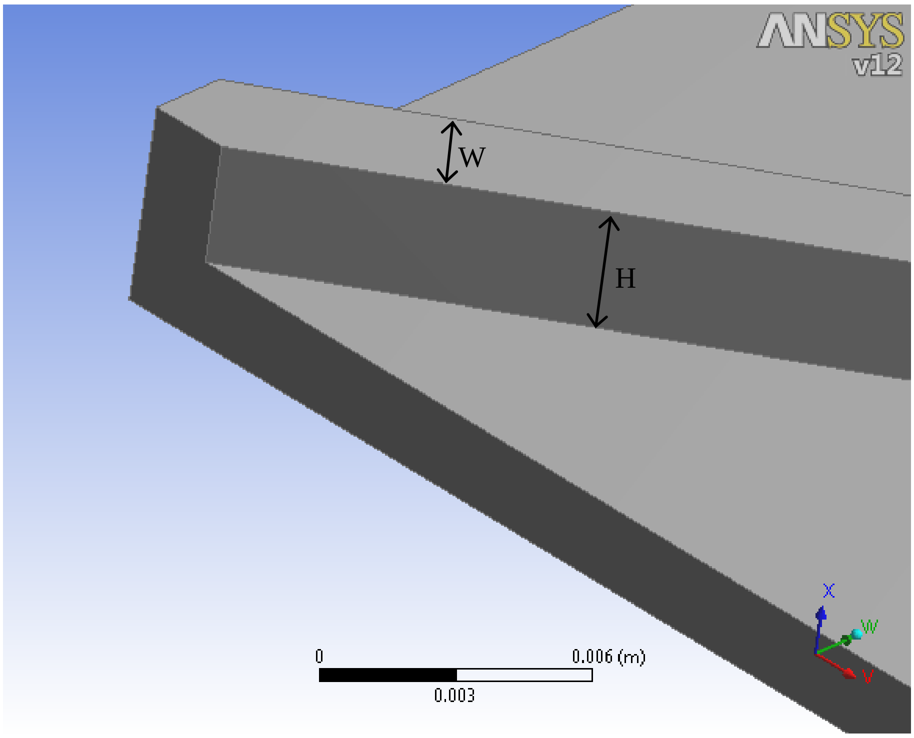

Figure 7: Input parameters of model with diagonal ribs.

Where W is the width and H the height of the ribs, that will be modified to achieve the highest natural frequency as possible.

Once defined input parameters, we define the output parameters as explained in Section 3.2.1. Then we will perform the analysis, first we will use Screening approach optimization and then we are going to use NLPQL approach.

Screening approach

This type of optimization is that comes by default in Ansys. As we said the Screening approach is a non-iterative direct sampling method by a quasi-random number generator based on the Hammersley algorithm.

In the figure below shows the results obtained after performing the optimization:

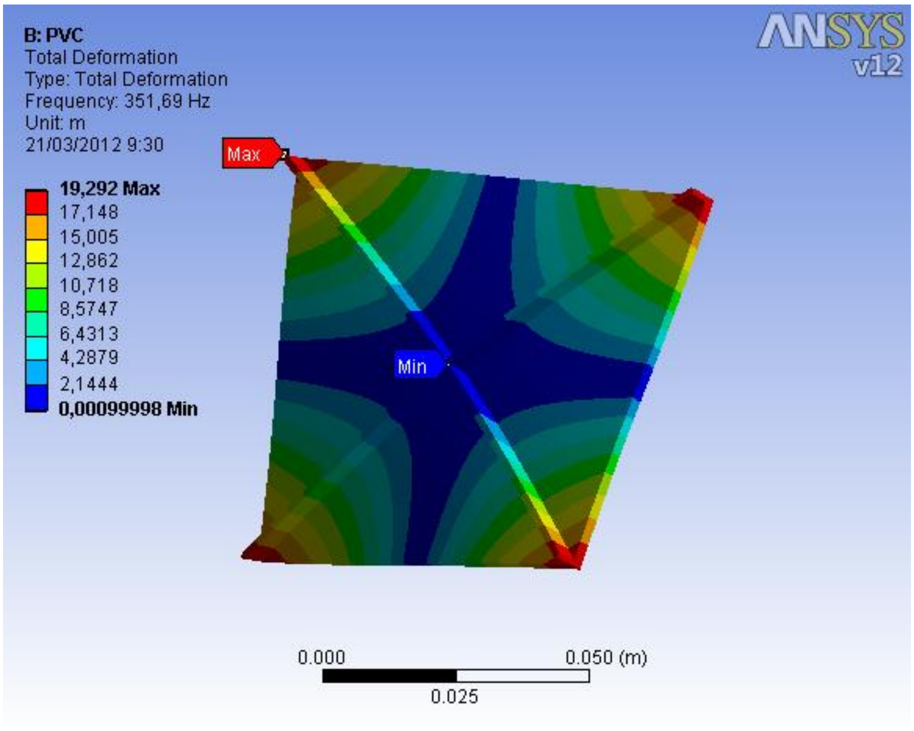

Figure 8: Model with diagonal ribs with Screening approach.

Mode	1	2	3	4	5	6	Volume (m^3)
Frequency (Hz)	351,69	389,1	682,22	778,04	781,31	1305,1	1,6423*10^{-5} m³

Table 7: Frequencies and volume of the base plate with Screening approach.

We see that with this type of optimization results are very good. Respecting the limit of mass, we have achieved that the natural frequency of the plate increase to 351, 69 Hz.

Now let's perform the same procedure, but using the other type of optimization that can be used in Ansys, the NLPQL approach. Let's see if with this method the results are better.

NLPQL approach

For NLPQL approach, first we going to have to make a MOGA approach, and with the best value obtained, we going to make NLPQL approach, because MOGA is to locate the multiple tentative optima and then we have to refine with NLPQL to zoom in on the individual local maximum or minimum value.

NLPQL is a gradient-based algorithm ideally suited for local optimization, so we would have to give better results than screening approach that is a non-iterative direct sampling method.

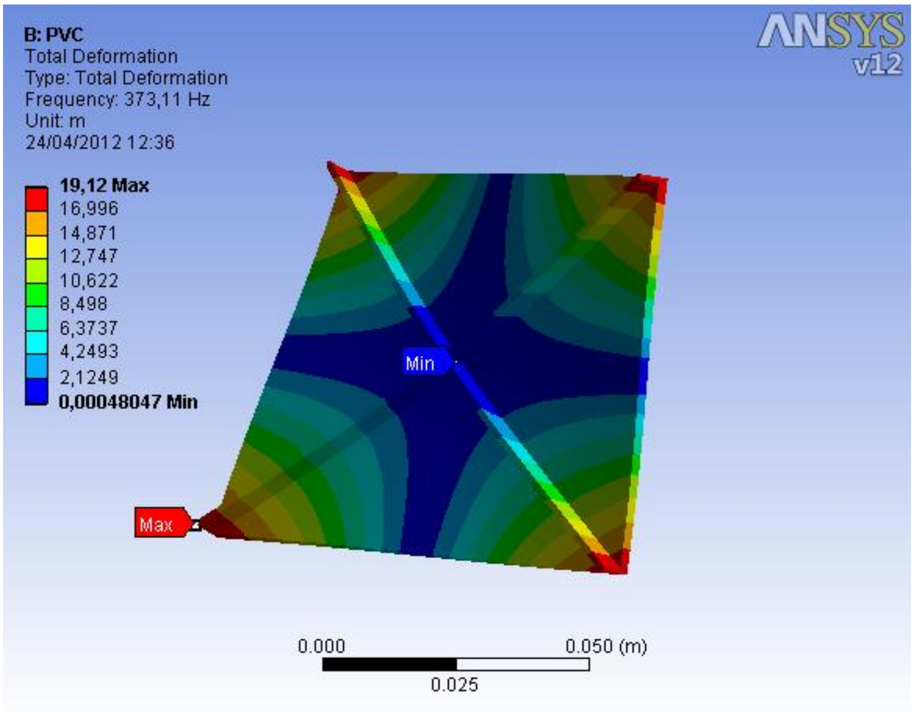

Figure 9: Model with diagonal ribs with MOGA/NLPQL approach.

Mode	1	2	3	4	5	6	Volume (m^3)
Frequency (Hz)	373,11	396,15	711,58	775,29	805,22	1376,7	$1,65 \cdot 10^{-5}$ m³

Table 8: Frequencies and volume of the base plate with MOGA/NLPQL approach.

As shown in the table 7 and the figure 8 the results are better than those obtained with Screening approach table 8 and the figure 9.

From now on all the optimization routines are performed with MOGA /NLPQL as this the optimization that gives better results in maximizing the first frequency.

3.2.2 Model with high ribs

Now we will reduce the thickness of the plate to 0.5 mm so we can do the ribs higher. Apart from the initial ribs, we will insert around our plate, other extra ribs. With this greater height of the ribs and added new ribs, we hope to get higher stiffness and therefore more natural frequency of the plate.

In this case the input parameters concerning the geometry of our new plate will be higher, because we have the height and width of two different ribs. All others input and output parameters remain the same with respect to previous optimization routine described in Section 3.2.1.

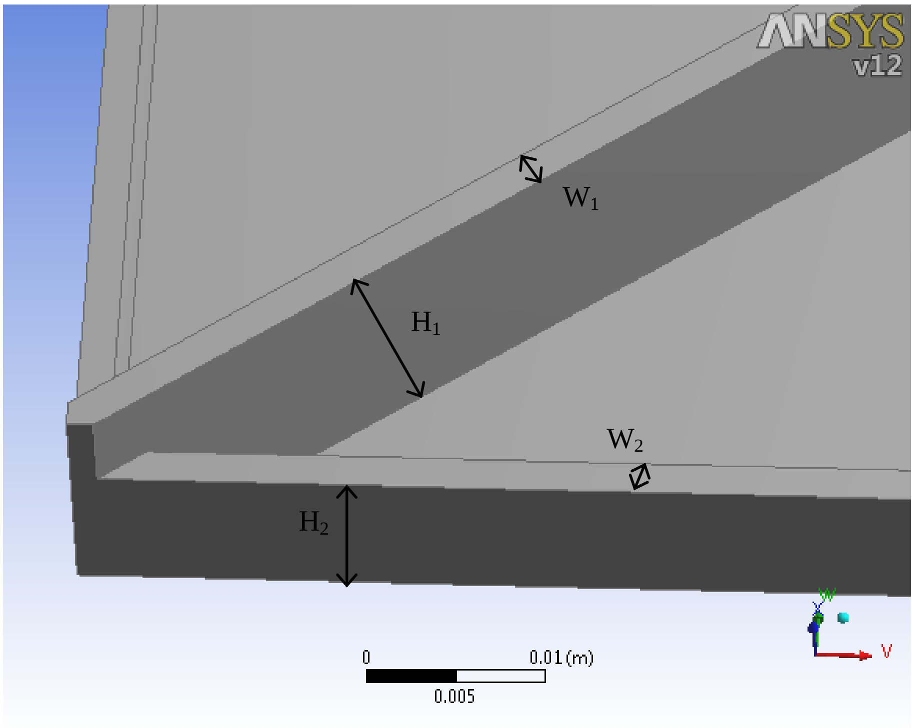

Figure 10: Input parameters of model with high ribs.

Where W_1 is the width and H_1 the height of the diagonal ribs, and W_2 is the width and H_2 the height of the new ribs.

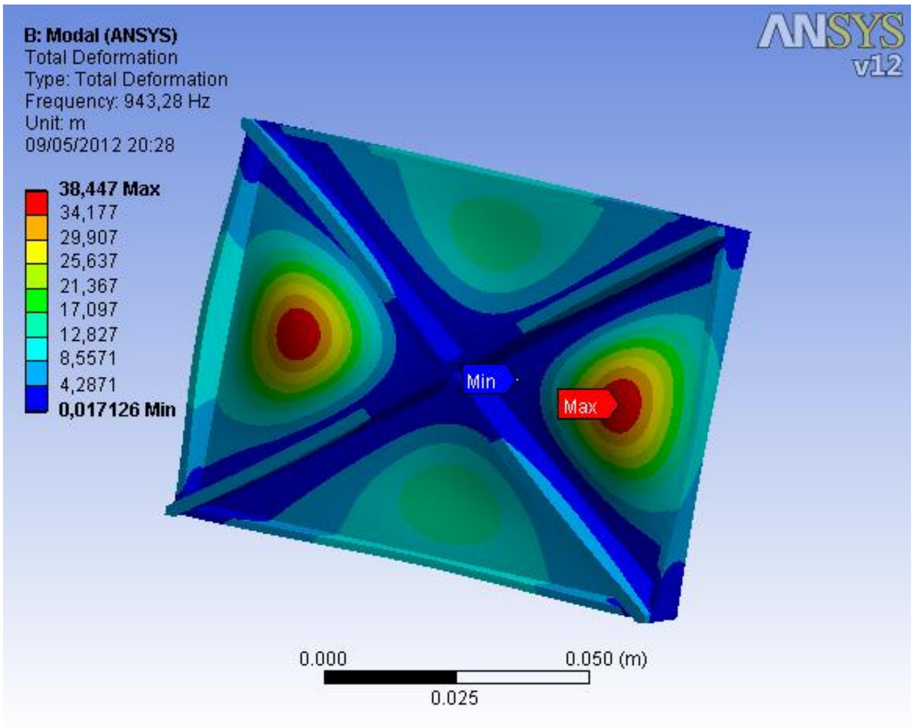

Figure 11: Model with high rib with MOGA/NLPQL approach

Mode	1	2	3	4	5	6	Volume (m^3)
Frequency (Hz)	943,28	949,47	962.85	991,5	1016	1068,9	$1{,}65\cdot10^{-5}$ m³

Table 9: Frequencies and volume of the base plate.

As shown (Table 9) (Figure 11), the results obtained are better than Model with only diagonals ribs (Table 8) (Figure 9) because the frequency obtained is higher. Therefore, we can affirm that to achieve a higher natural frequency of the plate, it is better to increase the size and number of ribs, although we have to decrease a bit the width of the plate.

3.2.3 Model with holes in the ribs

For the last design, we will take the latest model, model with high rib, and we will make holes in the ribs. Also we add two more ribs one vertically and other one horizontally.

Our goal with this new design is removing material of the ribs we have until now, create these two new horizontal and vertical ribs to increase stiffness, and thus achieved still higher natural frequency of the plate.

Therefore in this case there will be more input parameters related to the geometry since we have the width and height of three different ribs and the diameter of two holes.

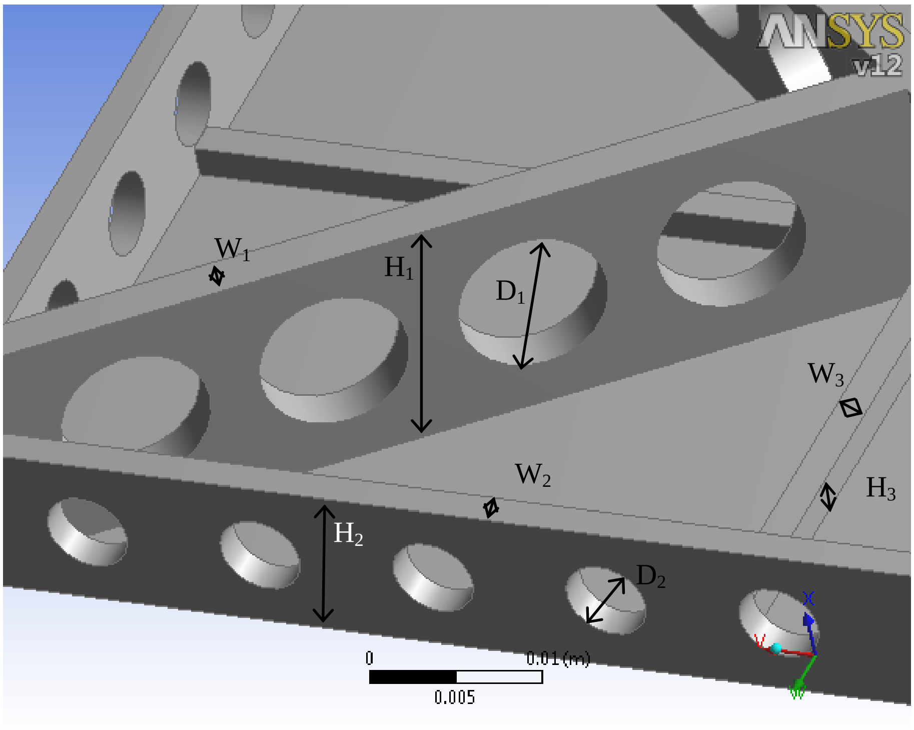

Figure 12: Input parameters of model with high ribs.

Where, W_1 is the width and H_1 the height of the diagonal ribs, W_2 is the width and H_2 the height of the ribs around the plate and W3 is the width and H_3 the height of the new horizontal and vertical ribs. We also have holes which have been defined as D_1 for hole diagonal ribs and D_2 for the other holes.

In the figure below shows the results obtained after performing the optimization:

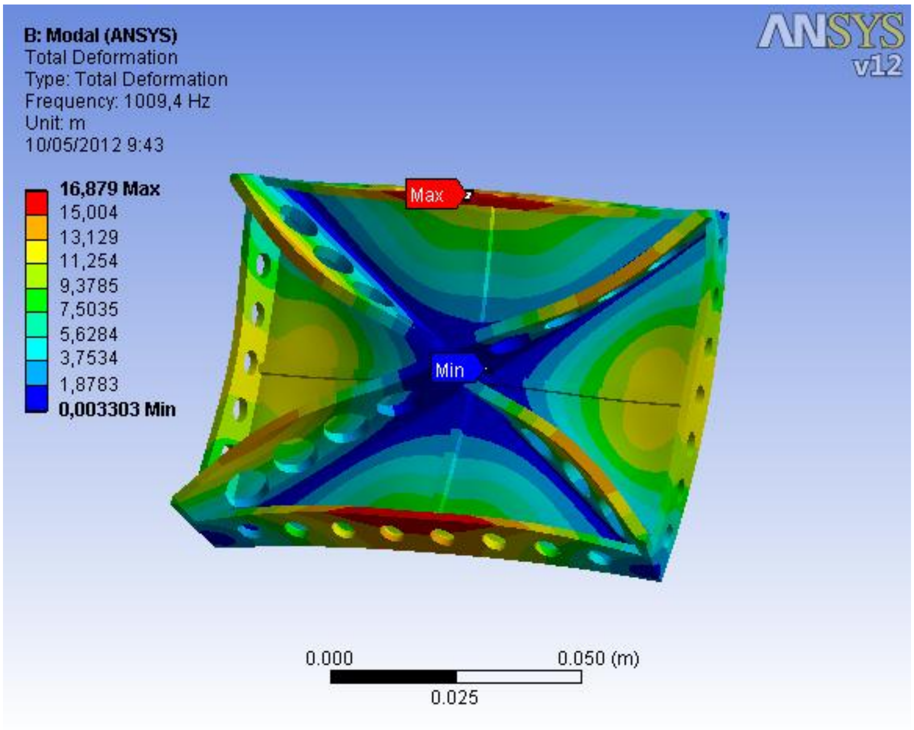

Figure 13: Model with holes in the ribs with MOGA/NLPQL approach

Mode	1	2	3	4	5	6	Volume (m^3)
Frequency (Hz)	1009,4	1024,2	1225,4	1264,6	1275,9	1314,4	$1{,}6497 \cdot 10^{-5}$ m³

Table 10: Frequencies and volume of the base plate.

As noted in the table 10 have been obtained more than 60 Hz more natural frequency than in the table 9, almost with the same volume. Therefore these two new ribs have ensured that the stiffness of our design is greater and therefore the natural frequency of the piece increases.

4 Conclusions

After optimizing the base plate in Ansys, one can conclude that the first resonance frequency is maximized. We observed in the natural frequency equation that if the stiffness increases, the natural frequency also increases, and if the mass increases, the natural frequency decreases. But in our analysis we have verified that increasing the volume by 10% when adding ribs, the natural frequency of our base plate has increased. Also we have observed that with the same mass but with different distribution of it, we have also achieved higher frequencies.

As we have seen in the theoretical framework the natural frequency depends on the stiffness effect and on the mass of the structure. The stiffness of the base plate must be more important than the mass effect, for the natural frequency of our plate to increase. This increased stiffness is achieved by placing the ribs. With these ribs, it is achieved that the stiffness of the plate increases, and so that the deformation amplitude is less.

The same applies when with the same mass, the frequency increases. With the same mass, changing the height of the ribs, making them narrower, and also making holes in the ribs to make more ribs, we achieved that the stiffness of our plate changes, and also the natural frequency. Is to say, changing the amount and form of our ribs, we have achieved that the stiffness increase, thereby obtaining a higher natural frequency.

Finally, one can conclude that applying different optimization algorithm, give different optimization results. The Screening approach is a non-iterative direct sampling method while NLPQL is a gradient based single objective optimizer which is based on quasi-Newton methods; therefore, Screening serves to locate the multiple tentative optima while NLPQL serves to refine, it means to zoom in on the individual local maximum or minimum value. It is normal achieves better results with the second method because it is a more accurate optimization.

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