Suppression of Narrowband Interference Generated by the Power Supply of the Railway Systems in Public Defibrillators Devices

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Abstract

A specific problem using the public access defibrillators arises at the railway stations. Some countries as Germany, Austria, Switzerland, Norway, Sweden and Slovenia are using AC railroad net power-supply system with rated frequency of 16.6(6) Hz, frequency modulated from 15.69 Hz to 17.36 Hz. The power supply frequency contaminates the electrocardiogram (ECG). It is difficult to be suppressed or eliminated due to the fact that it considerably overlaps the frequency spectra of the ECG. The interference impedes the automated decision of the public access defibrillators whether a patient should be (or should not be) shocked.

The aim of study of this thesis is the suppression of the 16.6(6) Hz interference generated by the power supply of the railway systems in few central european countries. For this purpose, an adaptive filter and a band-stop filter are used and the results obtained are compared in order to get the most suitable solution.
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Chapter 1

Introduction

Cardiac failure and cardiac diseases are among the main causes of death in the world. Therefore, it is necessary to have proper methods to determine the cardiac condition of the patient. Electrocardiography (ECG) is a tool that is widely used to understand the condition of the heart, since it records the electrical activity generated over the cardiac cycle via electrodes positioned at various locations on the body surface [1]. ECG can measure or detect the following features of the heart [2]:

- The rate and rhythm mechanism of the heart.
- How the heart is placed in the chest cavity.
- Evidence of increased thickness (hypertrophy) of the heart muscle.
- Evidence of the occurrence of a prior heart attack (myocardial infarction).
- Adverse effects of various heart diseases or systemic diseases such as high blood pressure, thyroid conditions, etc., on the heart.
- Adverse effects of certain lung conditions such as emphysema, pulmonary embolus (blood clots to lung) on the heart.
- Evidence of abnormal blood electrolytes (potassium, calcium, magnesium).

ECG of a patient is examined visually in time domain. But examining the ECG curve visually is usually inadequate. Signal processing methods are performed to examine the ECG curve accurately. Frequency domain methods, spectrum estimation and filtering are necessary to examine the ECG curve. Suppression of unwanted frequencies is essential and it is required to examine the ECG correctly.
More than 35 years ago external cardiopulmonary resuscitation (CPR) and defibrillation were first described as effective treatments for sudden cardiac arrest. However, survival after out-of-hospital cardiac arrest is still poor. The American Heart Association (AHA) previously addressed this problem by emphasizing the importance of the ‘chain of survival’: early access, early CPR, early defibrillation, and early advanced life support. Because early defibrillation is the single most important intervention, the AHA challenged manufacturers to develop simple, low-cost automatic public access defibrillators for use at locations in which large numbers of people congregate: stadiums, airport, or railway stations, etc.

It has been reported that the chance of survival of a patient at a state of ventricular fibrillation decreases by approximately 10% with each minute that passes after the time of attack. However, response times for paramedics or emergency medical technicians to arrive on site with a defibrillator are often more than ten minutes, resulting in average survival rates of less than 5%. Widespread deployment of automated public access defibrillators is the only feasible method of achieving early defibrillation. The strategy for time reducing is that defibrillators can be used by non-healthcare personnel (for example, untrained bystanders and trained members of the staff at the public place) before the emergency medical services arrive.

1.1 Conduction System of the Heart and the ECG

The distribution of ions across the cell membrane yields a potential difference across the membrane of the cell. This difference is called the transmembrane potential. The transmembrane potential changes during impulse propagation with action potential impulses. An action potential is an essential carrier of the information code that provides the control and coordination of organs like heart. An action potential is a wave of electrical discharge that travels along the membrane of a cell. Depolarization is the rise of the membrane potential, from a negative potential value to a more positive potential. Repolarization is the return of the membrane potential to its resting potential value, as shown in Figure 1.1.
Figure 1.1: Action potential with depolarization, repolarization and the resting phases.

Figure 1.2: Conduction system of the heart and the ECG. [3]
The heart works as a pump that pushes blood to the cells of the body. Atria, ventricles, heart muscle, electrical nodes are the main components of the heart as shown in Figure 1.2. The electrical impulse begins in the sinoatrial (SA) node, located at the top of the right atrium. The SA node is called the heart’s natural pacemaker. When an electrical impulse is released from this natural pacemaker, it causes the atria to contract (systole); this is called the atrial depolarization. The signal then passes from the SA node to atrioventricular (AV) node. The AV node receives the signal and waits for blood to pass through the heart valves into the ventricles. After this delay, AV node sends the signal to the muscle fibers of the ventricles via the “bundle of His” causing the ventricles to contract. The SA node sends electrical impulses at a certain rate, but this rate may still change depending on physical demands, stress, or hormonal factors. The contraction of any muscle is associated with electrical changes, and these changes can be detected by electrodes attached to the surface of the body. ECG is a tool to monitor the electrical activity of the heart by using electrodes positioned on the body surface.

Figure 1.3 illustrates the relation between the heart’s electrical activity and the ECG signal. In this figure, blue regions on the heart and blue coded parts of the ECG signal correspond to depolarization of the heart cells, and pink regions on the heart and pink coded parts of the ECG signal correspond to repolarization of the heart cells. Main components of an ECG signal are the P wave, QRS complex, and the T-wave. P wave corresponds to the depolarization of the atria and during this activity, atrial muscles contract. Formation of the P wave is shown in panels (a-d) of Figure 1.3. When the muscle fibers of the ventricles are excited, contraction of the ventricles starts. QRS complex occurs during the contraction of the ventricles as shown in Figure 1.3 (e-f). T wave represents the repolarization of the ventricles shown in Figure 1.3 (g).
Figure 1.3: Cardiac signal generation detected by ECG.

Figure 1.4 shows a sample ECG recording with more detail. In addition to the P, QRS and T waves that we have explained above, important intervals are also shown in this figure. Among these intervals, P-R interval is the time from the beginning of the P wave to the beginning of the QRS complex. It represents the interval between the activation of the SA node and the beginning of ventricular depolarization. Q-T interval represents the time from the beginning of the QRS complex to the end of the T wave. S-T interval starts from the end of the S wave and ends at the end of the T wave. Finally, R-R interval is the time required for 1 complete cycle, and is measured from one R-wave to the next one. This interval is used to determine heart rate [2].

Figure 1.4: Two cycles of an ECG signal are shown in this figure.
P, QRS and T waves and the indicated intervals are important in diagnosis for clinicians. For instance, dead or injured cells from a myocardial infarction will generally show up as irregularities in the ECG signal as ST segment depression on the S-T interval.

1.2 The 12-Lead Electrocardiogram Signals

In the 19th century it became clear that the heart generated electricity. Systematic approach to the heart from an electrical view started with Augustus Waller. He found that cardiac currents could be recorded by placing surface electrodes on the body. Willem Einthoven invented the first practical ECG in 1903 [4]. Currently, small size 12 lead ECG devices are developed with interface softwares.

![Figure 1.5: Positions of electrodes for 12 lead ECG [5].](image)

Information about heart conditions can be inferred from the ECG recordings by positioning electrical sensing electrodes on the body in standardized locations. The locations of the electrodes for 12 lead ECG are shown in Figure 1.5. Different channels are used because on each channel, electrical activity is monitored from different horizontal and frontal planes. This is like looking at an object from different angles. ECG leads are attached to the body while the patient lies flat on a bed or table. A small amount of gel is applied to the skin, which allows the electrical impulses of the heart to be more easily transmitted to the ECG leads. The numbers 1 to 6 in Figure 1.5 are the positions of electrodes on the chest. R, L, N, and F are the positions of the limb leads where R, L, F and N represent right arm (RA), left arm (LA), left leg (LL), and right leg (RL), respectively.
A sample recording obtained from a 12-lead ECG is shown in Figure 1.6. Channels V1 to V6 are recorded from the chest leads. The other channels are recorded from limb leads and calculated according to Einthoven triangle (I, II, III, aVR, aVL, aVF) [5].

Figure 1.6: 12 lead ECG signals [5].
1.3 Defibrillators

Defibrillation is a common treatment for life-threatening cardiac dysrhythmias, ventricular fibrillation, and pulseless ventricular tachycardia. Defibrillation consists of delivering a therapeutic dose of electrical energy to the affected heart with a device called a defibrillator. This depolarizes a critical mass of the heart muscle, terminates the dysrhythmia, and allows normal sinus rhythm to be reestablished by the body’s natural pacemaker, in the sinoatrial node of the heart. Defibrillators can be external, transvenous, or implanted, depending on the type of device used or needed. Some external units, known as automated external defibrillators (AEDs), automate the diagnosis of treatable rhythms, meaning that lay responders or bystanders are able to use them successfully with little, or in some cases no training at all [6].

1.4 Problem Definition and Purpose of the Study

A specific problem using the public access defibrillators arises at the railway stations. Some countries as Germany, Austria, Switzerland, Norway, Sweden and Slovenia are using AC railroad net power-supply system with rated frequency of 16.6(6) Hz. According to the Official journal of the European communities the interference is frequency modulated from 15.69 Hz to 17.36 Hz. The power supply frequency is magnetically induced and contaminates the electrocardiogram (ECG). It is difficult to be suppressed or eliminated due to the fact that it considerably overlaps the frequency spectra of the ECG. The interference impedes the automated decision of the public access defibrillators whether a patient should be (or should not be) shocked. In most of the cases the interference is wrongly considered as ventricular fibrillation.

We could not find any material dealing with 16.6(6) Hz noise suppression generated by the power supply of the railway systems. That is why we referred to some standard real-time methods used for main interference (50 or 60 Hz) suppression, from the point of view whether it is possible to adopt them to the specific problem:
- The QRS suppression introduced by notch filters tuned to reject a band from 15.69 Hz to 17.36 Hz is so high that it makes them unacceptable.
- ‘Comb’ filters averaging in a time interval of the interference from 57 ms to 64 ms are over-smoothing the QRS complexes, making it sometimes undistinguished from the T wave.
- The good performance of the mains interference subtraction method is mostly due to the correlation between the interference and the ECG sampling frequency. In the conditions of huge noise modulation the method seems inapplicable.
- Adaptive filtering described by some authors does not disturb the electrocardiogram (ECG) frequency spectrum, but requires re-adaptation at any change of the normal ECG course, such as arrhythmias or appearance of an extra systolic ectopic beat. The
re-adaptation period is characterized by the appearance of a tail of gradually attenuating unsuppressed interference. The same effect can be observed at any change of the interference frequency.

The aim of this study is the suppression of the 16.6(6) Hz interference generated by the power supply of the railway systems. This problem arises in few Central European countries, but there exists a European Standard complying basic requirements for medical equipment to permit interchange ability and compatibility between rail systems [7].

### 1.5 Outline of the Thesis

The first chapter of this thesis is an introduction to ECG signals, defibrillators, problem definition and purpose of the study. Chapter 2 presents the theoretical basis of this study, speaking about the selectivity in frequency filters, response types in frequency filters, digital filters and adaptive filters. The application of the two filters to solve the problem is given in Chapter 3. Simulation Results and Discussions are introduced in Chapter 4. Conclusions are given in Chapter 5. Finally, the Matlab code is given in the appendix.
Chapter 2

Theory of the study

In this chapter, the theoretical background for the thesis is provided. General information on selectivity in frequency filters with the different types is presented first, next the response types in frequency filters and after that the digital filters. Then, characteristics of FIR filters and IIR filters are explained. Finally, a brief theory on adaptive filters with different types of algorithms can be found.

Discrete-time (or digital) filters are ubiquitous in today’s signal processing applications. Filters are used to achieve desired spectral characteristics of a signal, to reject unwanted signals, like noise or interferers, to reduce the bit rate in signal transmission, etc. The notion of making filters adaptive, i.e., to alter parameters (coefficients) of a filter according to some algorithm, tackles the problems that we might not in advance know, for example, the characteristics of the signal, or of the unwanted signal, or of a systems influence on the signal that we like to compensate. Adaptive filters can adjust to unknown environment, and even track signal or system characteristics varying over time.

2.1 Selectivity in Frequency Filters

The filter’s primary purpose is to differentiate between different bands of frequencies, and therefore frequency selectivity is the most common method of classifying filters. Names such as lowpass, highpass, bandpass, and bandstop are used to categorize filters, but it takes more than a name to completely describe a filter. In most cases a precise set of specifications is required in order to allow the proper design of a filter. There are two primary sets of specifications necessary to completely define a filter’s response, and each of these can be provided in different ways.

The frequency specifications used to describe the passband(s) and stopband(s) could be provided in hertz (Hz) or in radians/second (rad/sec). We will use the frequency variable $f$ measured in hertz as filter input and output specifications because it is a slightly more common way of discussing frequency. However, the frequency variable $\omega$ measured in radians/second will also be used as WFilter’s internal variable of choice as well as for unnormalized frequency responses since most of those calculations will use radians/second [8].
2.1.1 Lowpass Filters

Signals from DC to an upper cutoff frequency are passed and signals at higher frequencies are attenuated. Figure 2.1 shows a typical lowpass filter’s response using frequency and gain specifications necessary for precision filter design. The frequency range of the filter specification has been divided into three areas. The passband extends from zero frequency (dc) to the passband edge frequency $f_{\text{pass}}$, and the stopband extends from the stopband edge frequency $f_{\text{stop}}$ to infinity. These two bands are separated by the transition band that extends from $f_{\text{pass}}$ to $f_{\text{stop}}$. The filter response within the passband is allowed to vary between 0 dB and the passband gain $a_{\text{pass}}$, while the gain in the stopband can vary between the stopband gain $a_{\text{stop}}$ and negative infinity. (The 0 dB gain in the passband relates to a gain of 1.0, while the gain of negative infinity in the stopband relates to a gain of 0.0.) A lowpass filter’s selectivity can now be specified with only four parameters: the passband gain $a_{\text{pass}}$, the stopband gain $a_{\text{stop}}$, the passband edge frequency $f_{\text{pass}}$, and the stopband edge frequency $f_{\text{stop}}$.

Lowpass filters are used whenever it is important to limit the high-frequency content of a signal. We should remember that any filter can differentiate only between bands of frequencies, not between information and noise [8].

![Figure 2.1 Lowpass filter specification.](image)

2.1.2 Highpass Filters

The inverse of a low-pass filter. Signals below a specified cutoff frequency are attenuated and signals at higher frequencies are passed. A highpass filter can be specified as shown in Figure 2.2. Note that in this case the passband extends from $f_{\text{pass}}$ to infinity (for analog filters) and is located at a higher frequency than the stopband which extends from zero to $f_{\text{stop}}$. The transition band still separates the passband and stopband. The passband gain is still specified as $a_{\text{pass}}$ (dB) and the stopband gain is still specified as $a_{\text{stop}}$ (dB). Highpass filters are used when it is important to eliminate low frequencies from a signal [8].
2.1.3 Bandpass Filters

The filter specification for a bandpass filter shown in Figure 2.3 requires a bit more description. A bandpass filter will pass a band of frequencies while attenuating frequencies above or below that band. In this case the passband exists between the lower passband edge frequency $f_{pass1}$ and the upper passband edge frequency $f_{pass2}$. A bandpass filter has two stopbands. The lower stopband extends from zero to $f_{stop1}$, while the upper stopband extends from $f_{stop2}$ to infinity (for analog filters). Within the passband, there is a single passband gain parameter $a_{pass}$ in decibels. However, individual parameters for the lower stopband gain $a_{stop1}$ (dB) and the upper stopband gain $a_{stop2}$ (dB) could be used if necessary [8].
2.1.4 Bandstop Filters

The final type of filter to be discussed in this section is the bandstop filter as shown in Figure 2.4. In this case the band of frequencies being rejected is located between the two passbands. The stopband exists between the lower stopband edge frequency $f_{\text{stop1}}$ and the upper stopband edge frequency $f_{\text{stop2}}$. The bandstop filter has two passbands. The lower passband extends from zero to $f_{\text{pass1}}$, while the upper passband extends from $f_{\text{pass2}}$ to infinity (for analog filters). Within the stopband, the single stopband gain parameter $a_{\text{stop}}$ is used. However, individual gain parameters for the lower and upper passbands, $a_{\text{pass1}}$ and $a_{\text{pass2}}$ (in dB) respectively, could be used if necessary [8].

![Figure 2.4 Bandstop filter specification.](image)

2.2 Response Types in Frequency Filters

Each type of filter can be implemented in different ways. Each implementation has its own advantages and disadvantages. No implementation has all the good qualities one may want so compromise is necessary. Response types with narrow transition bands in the frequency domain are very frequency selective and have the advantage of requiring fewer components to achieve a given specification but have undesirable overshoot and ringing in their time domain responses. Response types with wide transition bands have little or no overshoot in their time domain responses but require the maximum number of components to achieve a given specification in the frequency domain and may not be able to achieve some specifications at all. The following discusses some of the most common filter response types [9].
2.2.1 Butterworth Response

The Butterworth response is known as the maximally flat response. The response has a smooth roll-off with no ripple. The transition-band is medium in width. The cutoff of the filter is not sharp and the phase response is not too bad for low order filters. This is the easiest filter type to compute pole or zero locations as they fall exactly on a circle whose radius in the s-plane is the cutoff frequency. The phase response is fair and there is moderate overshoot and ringing to a step function. This filter is a good compromise when the time domain response is of medium importance relative to the frequency domain response. A British engineer, S. Butterworth, first used this filter in 1930 [9].

An example of a sixth-order Butterworth low-pass filter is shown in Figure 2.5. This filter is comprised of a cascade of three second-order sections – the response of each is shown in the thin lines. The bold line is the composite response. Note that the three second-order responses range from underdamped to overdamped with a natural frequency of 10 kHz – the cutoff frequency of the filter. The second-order responses fall off at 40 dB per decade while the composite response falls off at 120 dB per decade.

![Figure 2.5: Sixth-order Butterworth Low-pass Filter with 10 kHz cutoff.](image-url)
2.2.2 Chebyshev Response

The Chebyshev response is based on a Chebyshev polynomial that has equal ripple about the desired pass-band response. This concept produces a more accurate approximation to the desired pass-band response. The transition-band is much narrower than that of the Butterworth and the required filter order is generally less than that of a Butterworth type for a given specification – thus saving parts. The cutoff frequency of a Chebyshev filter is defined as the frequency that the magnitude response drops below the specified passband ripple. Pass-band ripple is commonly specified between about 0.01 dB and 1 dB. The phase response is poor and there is significant overshoot and ringing to a step function. This filter is useful only when the time domain response is not important. A Russian mathematician, P. L. Chebyshev, first used Chebyshev polynomials in 1899 while studying the construction of steam engines. Chebyshev polynomials have many other uses besides filter approximations [9].

An example of a sixth-order Chebyshev lowpass filter with 1 dB of ripple is shown in Figure 2.6. Note the very sharp cutoff characteristic. Note also that the three second-order sections have different cutoff frequencies and have lower damping compared to the Butterworth. The low damping is what makes the filter ring so much to a step signal.

![Frequency Response of Filter System](image)

Figure 2.6: Sixth-order Chebyshev 1 dB Low-pass Filter with 10 kHz cutoff.
2.2.3 Elliptical Response

The elliptical response has an extremely narrow transition-band and is very useful when the filter must be very frequency selective. There are transmission zeros in the stop-band and these produce the very high initial cutoff response –thus the narrow transition band. There is ripple in the stop-band response due to the zeros. The key feature of the elliptical response is the very narrow transition-band. Thus, an elliptical response type filter requires the lowest order and therefore the smallest number of components of any of the response types. The price paid is a very poor time domain response with significant overshoot and ringing to a step signal. An elliptical filter would be a very poor choice for pulse signals [9].

2.2.4 Bessel Response (Also Known as Gaussian Response)

The Bessel response emphasizes linear phase response (constant time delay) at the expense of magnitude response (i.e. sharpness of cutoff). The linear phase characteristic causes the time-domain response to have little or no overshoot to a step function thus making this filter very attractive for filtering pulse waveforms. This filter type has a very wide transition-band (even when the order is high) and is very poor at frequency selectivity. A hybrid filter constructed by interpolating the poles of a Bessel and Butterworth filter often offers the best compromise between time and frequency response when both responses are important [9].

2.2.5 Other Response Types

Besides the popular response types listed here, there are many possible response types that can be created by interpolation between these or other methods to achieve particular tradeoffs between the frequency and time domain responses [9].

2.3 Digital Filters

In signal processing, the function of a filter is to remove unwanted parts of a signal, such as random noise, or to specify useful parts of the signal, such as components lying in a certain important frequency range. There are two main kinds of filters; analog and digital. An analog filter uses analog electronic circuits made up from components such as resistors, capacitors and operational amplifiers to produce the required filtering effect. An analog filter can only be changed by redesigning the filter circuit.

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal for filtering. The processor may be a general purpose computer, or a specialized digital signal processor (DSP) chip. A digital filter is realized by a program stored in the processor’s memory. This means the digital filter can simply be changed.
without affecting the circuitry (hardware). Because of flexibility in their design, digital filters are commonly used for filtering. Digital filters are discrete time systems and are characterized by their impulse responses. An impulse response can either have a finite or an infinite duration. A finite impulse response \( h(n) \) has values extending over a finite time interval from 0 to N and it is zero beyond that interval.

\[
h(n) = (h_0, h_1, \ldots, h_N)
\]  

(2.1)

The finite impulse response has values in the interval (0 to N) and it is referred to as a finite impulse response system or filter of order N. So an N’th order digital filter has an impulse response with a size of (N+1) samples. The samples of the impulse response function \( h(n) \) are also called the filter coefficients [10, 11].

The digital filter has linear phase characteristics as shown in Figure 2.7. This means that the phase response is a linear function of frequency and it has a constant group delay response.

![Figure 2.7: Linear phase characteristics of a digital filter. The y-axis is the phase; The x-axis is the frequency axis.](image)

General digital filter design process can be divided into four main steps:

- Approximation (Estimation of the suitable filter parameters for the digital filter).
- Synthesis and realization (Implementation and realization of the filter).
- Performance analysis (Performance test of filter for given specifications and adjusting the filter characteristics in order to design required filter.).
- Implementation (Implementation of the adequate filter on the noisy data and the realization of the suppression process).

Both digital FIR and IIR filters are designed with using this filter design procedure in order to approach the required filter specifications.
2.3.1 Digital FIR Filters

FIR filters are linear discrete time systems, in which the output sequence is related to the input and the impulse response of the filter by the convolution sum:

\[ y(n) = \sum_{m=0}^{N} x(m) \cdot h(n-m) \]  

(2.2)

The summation on the right hand side is a convolution between the input sequence, \( x(n) \), and the impulse response of the filter, \( h(n) \). The frequency response of an \( N \)'th order FIR filter is given by:

\[ H(w) = \sum_{n=0}^{N-1} h(n) \cdot e^{-jwn} \]  

(2.3)

2.3.2 Digital IIR Filters

An IIR filter is one whose impulse response theoretically continues for ever because the recursive terms feedback energy into the filter input and keep it as specified in the following equation:

\[ y(n) = \sum_{k=1}^{N} a(k) \cdot y(n-k) + \sum_{k=0}^{M} b(k) \cdot x(n-k) \]  

(2.4)

\[ H(z) = \frac{\sum_{k=0}^{M} b(k) \cdot z^{-k}}{\sum_{k=0}^{N} a(k) \cdot z^{-k}} \]  

(2.5)
Fig 2.8 shows a direct form implementation of Eq. (2.4). In theory, when a recursive filter is excited by an impulse, the output persists forever. Thus a recursive filter is also known as an Infinite Duration Impulse Response (IIR) filter. Other names for an IIR filter include feedback filters, pole-zero filters and auto-regressive-moving-average (ARMA) filter a term usually used in statistical signal processing literature.

A discrete-time IIR filter has a z-domain transfer function that is the ratio of two z-transform polynomials as expressed in Eq. (2.5); it has a number of poles corresponding to the roots of the denominator polynomial and it may also have a number of zeros corresponding to the roots of the numerator polynomial.

The main difference between IIR filters and FIR filters is that an IIR filter is more compact in that it can usually achieve a prescribed frequency response with a smaller number of coefficients than an FIR filter. A smaller number of filter coefficients imply less storage requirements and faster calculation and a higher throughput. Therefore, generally IIR filters are more efficient in memory and computational requirements than FIR filters. However, it must be noted that an FIR filter is always stable, whereas an IIR filter can become unstable (for example if the poles of the IIR filter are outside the unit circle) and care must be taken in design of IIR filters to ensure stability.
2.4 Adaptive Filters

Adaptive filters are used when the noise is not stationary and the noise is uncorrelated with the signal. When no information is available about the spectral characteristics of the signal and noise, a second source or recording site is available to obtain a reference signal that is strongly correlated with the noise but uncorrelated with the signal. Adaptive filter acts as a fixed filter when the signal and noise are stationary. Least mean square and normalized least mean square algorithms are explained in the following subsections.

Adaptive filters are classified into two main groups: linear and nonlinear. Linear adaptive filters compute an estimate of a desired response by using a linear combination of the available set of observables applied to the input of the filter. Otherwise, the adaptive filter is said to be nonlinear. Adaptive filters may also be classified into [12]:

- **Supervised adaptive filters**, which require the availability of a training sequence that provides different realizations of a desired response for a specified input signal vector. The desired response is compared against the actual response of the filter due to the input signal vector, and the resulting error signal is used to adjust the free parameters of the filter. The process of parameter adjustments is continued in a step-by-step fashion until a steady-state condition is established.

- **Unsupervised adaptive filters**, which perform adjustments of its free parameters without the need for a desired response. For the filter to perform its function, its design includes a set of rules that enable it to compute an input-output mapping with specific desirable properties. In the signal-processing literature, unsupervised adaptive filtering is often referred to as blind deconvolution or blind adaptation.

Gabor was the first to conceive the idea of a nonlinear adaptive filter in 1954 using a Volterra series. The first algorithm used to design a linear adaptive filter is the ubiquitous least-mean-square (LMS) algorithm developed by Widrow and Hoff. The LMS algorithm is often referred to as the **Widrow-Hoff rule**; it was originally derived by Widrow and Hoff in 1959 in their study of a pattern recognition system known as the adaptive linear element (Adaline). The LMS algorithm is closely related to Rosenblatt’s **perceptron** in that they are both built on error-correction learning. They both emerged about the same time in the late 1950s during the formative years of neural networks. The importance of Rosenblatt’s perceptron is largely historical today. On the other hand, the LMS algorithm has survived the test of time.
Adaptive filters find applications in highly diverse fields: channel equalization, system identification, predictive deconvolution, spectral analysis, signal detection, noise cancellation, and beamforming.

2.4.1 Least-Mean-Square (LMS) Algorithm

The objective of the algorithm is to adapt the coefficients of FIR filter, W, to match as closely as possible to the response of unknown system, H. The adaptive filter, W, is adapted using the least mean square algorithm, which is the most widely used adaptive filtering algorithm. As shown in Figure 2.9 d (n) is the desired signal and y (n) is the output of filter. The error signal e (n), is computed as: e (n) = d (n) – y (n) which measures the difference between the output of the adaptive filter and the output of the unknown system. On the basis of this measure, the adaptive filter will change its coefficients in an attempt to reduce the error. In equation 2.6 the filter adaptation equation is given as [13]:

\[ W_{n+1}(k) = W_n(k) + \mu \cdot E(e(n) \cdot X(n)) \]  

(2.6)

In this equation, \( \mu \) is a constant that represents the step size and it controls the gradient information used to update each coefficient. After adjusting each coefficient according to the gradient of the error, the adaptive filter should converge; that is, the difference between the unknown and adaptive systems should get small. The adaptation of the equation is:

\[ W_{n+1}(k) = W_n(k) + \mu \cdot e(n) \cdot X(n-k) \]  

(2.7)

![Figure 2.9: Adaptive filtering block diagram using gradient descent adaptation. Here the W is the coefficient matrix, d (n) is the desired signal, e (n) is the error signal, H is the unknown system.](image)
If $\mu$ is very small, then the coefficients change only a small amount at each update and the filter converges slowly. On the other hand, when the step-size is too large, the coefficients may change too quickly and the filter will diverge. Suitable step size should be selected by adjusting the step size.

LMS filter may not converge due to suitable step size. Since, it controls the gradient information used to update each coefficient. In normalized least mean square (NLMS) filter gradient step factor is normalized and it converges easily.

In ‘normalized’ LMS, the gradient step factor $\mu$ is normalized by the energy of the data vector.

$$\mu(n) = \beta / (x^H(n)x(n) + \varepsilon) \quad (2.8)$$

Where $\beta$ is a normalized step size selected between 0 and 2, and $\varepsilon$ is a small number introduced to prevent division by zero if the denominator zero.

$$W_{n+1}(k) = W_n + \mu(n)*x(n)*e(n) \quad (2.9)$$

When $x(n)$ is a complex number in equation 2.9, the complex conjugate of $x(n)$ is applied.

The LMS algorithm has established itself as the workhorse of adaptive signal processing for two primary reasons [12]:

- Simplicity of implementation and a computational efficiency that is linear in the number of adjustable parameters.
- Robust performance

Basically, the LMS algorithm is a stochastic gradient algorithm, which means that the gradient of the error performance surface with respect to the free parameter vector changes randomly from one iteration to the next. This stochasticity, combined with the presence of nonlinear feedback, is responsible for making a detailed convergence analysis of the LMS algorithm a difficult mathematical task. Indeed, it has attracted research attention for over 25 years.

The LMS algorithm has two major drawbacks: slow rate of convergence, and sensitivity to the eigenvalue spread (i.e., the ratio of the largest eigenvalue to the smallest eigenvalue) of the correlation matrix of the input signal vector. One way of overcoming these limitations is to use projections of the input signal on an orthogonal basis. This desirable objective can be attained by means of transform-domain adaptive algorithms, so called because the adaptation is performed in the frequency domain rather than the original time domain. For a more refined method, we may use a multi-rate adaptive filter which provides better trade-offs between performance improvement, computational complexity and transmission delay.
2.4.2 Recursive Least-Squares (RLS) Algorithm

The LMS algorithm attains simplicity of implementation by using instantaneous estimates of the autocorrelation matrix of the input signal vector, and the cross-correlation vector between the input vector and the desired response. In contrast, the recursive least-squares (RLS) algorithm utilizes continuously updated estimates of these two quantities, which go back to the beginning of the adaptive process. Accordingly, the RLS algorithm exhibits the following properties [12]:

- Rate of convergence that is typically an order of magnitude faster than the LMS algorithm.
- Rate of convergence that is invariant to the eigenvalue spread of the correlation matrix of the input vector.

These desirable properties are however attained at the cost of increased computational complexity. The standard RLS algorithm is built around an FIR filter. For its derivation we may use the matrix inversion lemma, or exploit the correspondence that exists between the variables characterizing the RLS algorithm and those characterizing the celebrated Kalman filter. This latter approach is highly attractive as it provides the basis for deriving the many variants of the RLS algorithm, which include the following:

- **Square-root adaptive filters**, based on the numerically stable QR-decomposition procedure; this class of adaptive filters includes the QR-RLS, extended QR-RLS, and inverse QR-RLS filters, all of which can be implemented using systolic arrays.
- **Order-recursive adaptive filters**, the most important form of which is the QRD-LSL (QR decomposition-based least-squares lattice) filter. An important property of this class of adaptive filtering algorithms is that their computational complexity is linear in the number of adjustable parameters as in the LMS algorithm.
2.4.3 Adaptive Noise Cancelling

When collecting measurements of certain signals or processes, physical constraints often limit our ability to cleanly measure the quantities of interest. Typically, a signal of interest is linearly mixed with other extraneous noises in the measurement process, and these extraneous noises introduce unacceptable errors in the measurements. However, if a linearly related reference version of any one of the extraneous noises can be cleanly sensed at some other physical location in the system, an adaptive filter can be used to determine the relationship between the noise reference \( x(n) \) and the component of this noise that is contained in the measured signal \( d(n) \). After adaptively subtracting out this component, what remains in \( e(n) \) is the signal of interest. If several extraneous noises corrupt the measurement of interest, several adaptive filters can be used in parallel as long as suitable noise reference signals are available within the system.

Adaptive noise cancelling has been used for several applications. One of the first was a medical application that enabled the electroencephalogram (EEG) of the fetal heartbeat of an unborn child to be cleanly extracted from the much-stronger interfering EEG of the maternal heartbeat signal [14].

![Figure 2.10: The general adaptive filtering problem.](image-url)
Chapter 3

Application of the Two Filters to Solve the Problem

This chapter shows how the adaptive filter and the band-stop filters are applied to solve the problem of the interference of 16.6(6) Hz, generated by the power supply of the railway systems, on public defibrillators devices.

3.1 Band-Stop Filters

An ideal band stop filter completely attenuates frequencies above its lower cut-off frequency and below its upper cut-off frequency. Conversely, for frequencies greater than the upper cut-off and frequencies less than the lower cut-off the signal is allowed to pass. This region is referred to as the pass-band. In practice, one finds that the attenuation has a finite slope. Thus, either a shape factor or stop-band width must be specified. In addition, due to spurious modes both the high side and low side of the pass-band will eventually degenerate. This necessitates the inclusion of an outer pass bandwidth. These types of filters are used when some unwanted interfering frequencies be particularly strong or when high attenuation may be needed only at certain frequencies.

This is the reason why this type of filter is used in this study in order to solve the problem. The interference of 16.6 Hz contains a very small band of frequencies that can be removed as the frequency is known.

A block-diagram of band-stop filter is shown in Fig.3.1:

![Figure 3.1. Block-diagram of Band-stop filter.](image-url)
In figure 3.1 you can see what the band-stop filter does:

The input $x(t)$ is a mixture of clear ECG signal $s(t)$ and interference generated by electrical train $n(t)$:

$$x(t) = s(t) + n(t) \tag{3.1}$$

The band-stop filter is applied to the input signal and only a small band of frequencies is removed. The interference $n(t)$ is included in this band, so it is removed.

$$Y(w) = H(w) * X(w) \tag{3.2}$$

The result will be an output similar to the input signal without interference (clear ECG signal $s(t)$):

$$y(t) \approx s(t) \tag{3.3}$$

Three different types of response are used to generate the band-stop filter. Each one of these filters has different characteristics and that can be compared in order to decide which is best for this case.

- The Butterworth response is known as the maximally flat response. The response has a smooth roll-off with no ripple. The transition-band is medium in width. The cutoff of the filter is not sharp and the phase response is not too bad for low order filters. This is the easiest filter type to compute pole or zero locations as they fall exactly on a circle whose radius in the s-plane is the cutoff frequency. The phase response is fair and there is moderate overshoot and ringing to a step function. This filter is a good compromise when the time domain response is of medium importance relative to the frequency domain response.

- The Chebyshev response is based on a Chebyshev polynomial that has equal ripple about the desired pass-band response. This concept produces a more accurate approximation to the desired pass-band response. The transition-band is much narrower than that of the Butterworth and the required filter order is generally less than that of a Butterworth type for a given specification –thus saving parts. The cutoff frequency of a Chebyshev filter is defined as the frequency that the magnitude response drops below the specified passband ripple. Pass-band ripple is commonly specified between about 0.01 dB and 1 dB. The phase response is poor and there is significant overshoot and ringing to a step function. This filter is useful only when the time domain response is not important.
- The elliptical response has an extremely narrow transition-band and is very useful when the filter must be very frequency selective. There are transmission zeros in the stop-band and these produce the very high initial cutoff response – thus the narrow transition band. There is ripple in the stop-band response due to the zeros. The key feature of the elliptical response is the very narrow transition-band. Thus, an elliptical response type filter requires the lowest order and therefore the smallest number of components of any of the response types. The price paid is a very poor time domain response with significant overshoot and ringing to a step signal

### 3.2 Adaptive Filter

The problem of electric train noise suppression could be considered as a particular case of a more common problem of adaptive noise cancellation [14]. We assume that a reference noise signal can be easily picked up and used.

As shown in the figure 3.2, an adaptive noise canceller has two inputs: primary and reference. The primary input receives a signal (s) from the signal source that is corrupted by the presence of noise n1 uncorrelated with the signal. The reference input receives a noise n2 uncorrelated with the signal but correlated in some way with the noise n1. The noise n2 passes through a filter to produce an output (y) that is a close estimate of primary input noise. This noise estimate is subtracted from the corrupted signal to produce an estimate of the signal (e), the Adaptive filter system output.

In noise cancelling systems a practical objective is to produce a system output (e= s + n1 – y) that is a best fit in the least squares sense to the signal (s). This objective is accomplished by feeding the system output back to the adaptive filter and adjusting the filter through an LMS adaptive algorithm to minimize total system output power. In other words the system output serves as the error signal for the adaptive process.
The primary input of the adaptive filter is the active channel $d(i)$, which is a mixture of clear ECG signal $s(i)$ and interference generated by electrical train $n_1(i)$, where $(i)$ is the consecutive number of the sample:

$$d(i) = s(i) + n_1(i)$$  \hspace{1cm} (3.4)

The noise has a sinusoidal waveform and it can be expressed as:

$$n_1(i) = b \cos(2\pi F i + \phi)$$  \hspace{1cm} (3.5)

Where $b$, $F$, and $\phi$ are amplitude, frequency, and phase of the noise signal, respectively.

The reference input of the adaptive filter is a noise signal collected by an antenna:

$$n_2(i) = a \cos(2\pi F i),$$  \hspace{1cm} (3.6)

Which differs from $n_1(i)$ only in amplitude and phase.

The task of the adaptive algorithm depicted in Fig. 1 is to adjust the coefficients $w_1(i)$ and $w_2(i)$ to compensate the amplitude and phase difference between $n_1(i)$ and $n_2(i)$. In other words, $y(i)$ should resemble $n_1(i)$ as close as possible.

We rearrange the reference noise signal in the following manner:

$$x_1(i) = a \cos(2\pi F i)$$  \hspace{1cm} (3.7)

$$x_2(i) = a \sin(2\pi F i),$$  \hspace{1cm} (3.8)
Where $x_2(i)$ is the noise reference signal phase shifted on 90 degrees by Hilbert’s transform. The advantage of this structure compared to the conventional one is that it allows very fine tuning of the signal phase and amplitude which is proved later with the experiments.

Then:

$$y(i) = w_1(i)x_1(i) + w_2(i)x_2(i), \quad (3.9)$$

And the error signal is formed as:

$$e(i) = d(i) - y(i). \quad (3.10)$$

We apply a Least Mean Squares adaptive algorithm with a cost-function $e^2(i)$

In this algorithm, the gradient descent method is used. We take steps proportional to the negative of the gradient of the cost-function in order to walk downwards to the minimum. It is an iterative algorithm in which each is step is closer to the optimal point.

In our case, applying the algorithm to the first coefficient $w_1$, we have:

$$w_1(i+1) = w_1(i) - \mu \left( \frac{\delta e^2}{\delta w_1} \right) \quad (3.11)$$

Where $\mu$ is used to control how fast we want to go towards the minimum of the cost-function. Developing this expression we have:

$$w_1(i+1) = w_1(i) - 2\mu e(i) \frac{\partial e}{\partial w_1} \quad (3.12)$$

Taking into account 3.9 and 3.10:

$$\frac{\partial e}{\partial w_1} = -x_1(i) \quad (3.13)$$

Then:

$$w_1(i+1) = w_1(i) + 2\mu e(i) x_1(i) \quad (3.14)$$

And the same with $w_2$:

$$w_2(i+1) = w_2(i) + 2\mu e(i) x_2(i) \quad (3.15)$$

So finally we have found a system of two equations for adapting the coefficients $w_1(i)$ and $w_2(i)$. As mentioned before, $\mu$ controls the adaptation rate. If $\mu$ is bigger, the adaptation is faster but if $\mu$ is too big, $w_1(i)$ and $w_2(i)$ may oscillate around the optimal or even not to converge.
The elimination of a sinusoidal interference corrupting a signal is typically accomplished by explicitly measuring the frequency of the interference and implementing a fixed notch filter tuned to that frequency. A very narrow notch is usually desired in order to filter out the interference without distorting the signal. However, if the interference is not precisely known, and if the notch is very narrow, the center of the notch may not fall exactly over the interference. This may lead to cancellation of some other frequency components of the signal i.e. distorting the signal, while leaving the interference intact. Thus, it may in fact lead to an increase in the noise level. Also, there are many applications where the interfering sinusoid drifts slowly in frequency. A fixed notch cannot work here at all unless it is designed wide enough to cover the range of the drift, with the consequent distortion of the signal. In situations such as these, it is often necessary to measure in some way the frequency of the interference, and then implement a notch filter at that frequency. However, estimating the frequency of several sinusoids embedded in the signal can require a great deal of calculation.

When an auxiliary reference input for the interference is available, an alternative technique of eliminating sinusoidal interferences is by an adaptive noise canceller. This reference is adaptively filtered to match the interfering sinusoids as closely as possible, allowing them to be filtered out. The advantages of this type of notch filter are:

- It makes explicit measurement of the interfering frequency unnecessary.
- The adaptive filter converges to a dynamic solution in which the time-varying weights of the filter offer a solution to implement a tunable notch filter that helps to track the exact frequency of interference under non-stationary conditions or drifts in frequency.
- It offers easy control of bandwidth as is shown below.
- An almost infinite null is achievable at the interfering frequency due to the close and adjustable spacing of the poles and zeros.
- Elimination of multiple sinusoids is possible by formation of multiple notches with each adaptively tracking the corresponding frequency.
Chapter 4

Simulation Results and Discussions

In this chapter, the adaptive filter and the band-stop filters will be applied to synthetic noisy ECG data, and the performances of these filters will be studied.

The adaptive filtering method and band-stop filtering method are tested with ECG recordings taken from the AHA database. The recording has duration of 30 min, the sampling frequency is 250 Hz and the resolution is 5 mV*bit-1.

The analyses are performed by generating a sinusoidal-like noise $n_1$ having a main frequency of 16.6(6) Hz with amplitude of 1.2, a phase of 45º for 10 s and 20 s.

A signal with the same frequency (16.6(6) Hz) but with different amplitude and phase from $n_1$ is considered as a reference input $n_2$ for the adaptive filtering method.

The first figure (4.1) is a simulation represented in the frequency domain of the next signals: the ECG signal, the 16.6(6) Hz simulated interference generated by the power supply of the railway systems and the ECG signal plus this interference.

As seen, the frequencies of both signals are very close. Therefore the interference will be difficult to be suppressed or eliminated due to the fact that it considerably overlaps the frequency spectra of the ECG.
Figure 4.1: The ECG signal, the interference and the ECG signal plus the interference in the frequency domain.

The second figure (4.2) is the normal ECG signal and the ECG signal plus interference of 16.6 Hz described both in the beginning of chapter.

Figure 4.2: The ECG signal and The ECG signal + interference of 16.6 Hz.
4.1 Band-Stop Filters Results

In this section, the band-stop filters are applied to synthetic noisy ECG data, and the performances of these filters are studied.

The main problem of these filters is that the frequency of the interference signal considerably overlaps the frequency spectra of the ECG, therefore it is very difficult to remove only the interfering frequency without removing frequencies of the ECG signal, which gets distorted. Consequently the filter needs to be as selective as possible to eliminate the fewest frequencies of the ECG signal, but on the other hand, if the noise frequency varies minimally, a band-stop filter so selective will not delete the interference.

The figure 4.3 shows the three types of responses, Butterworth, Elliptic and Chebyshev used like band-stop filters. They are configured to have the lowest possible error between the output of these filters and the initial ECG signal.

The orders of the filters are 1 with a stop band centred at 16.6 Hz with a bandwidth of 0.25 Hz therefore these are very selective.

Another feature configurable elliptic filter is the $R_p$, (decibels of peak-to-peak ripple) and (a minimum stopband attenuation) of $R_s$ decibels, and these values are 1 and 20 respectively.

On the other hand in the Chebyshev filter can oscillate $R$, (decibels of peak-to-peak ripple in the stop-band) and this value is 10.

As seen the elliptical and Butterworth responses are very frequency selective and the results are better than the Chebyshev filter because these remove less frequencies of the ECG signal.
Figure 4.3: The three types of response Simulation to generate the band-stop filters, Butterworth, Elliptic, Chebyshev.

Next figures (4.4, 4.5, 4.6) show the results after applying the previous filters.

When applying the Butterworth filter and the Elliptic filter, the output signal is more noisy at the beginning, where the error is bigger than in the Chebyshev filter. After 4 seconds for the Butterworth filter and 6 seconds for the Elliptic filter are stabilized and its error is very small for the rest of time.

However the Chebyshev filter has a start with less error than the others, but afterwards, the error is bigger.

Therefore the Butterworth filter and Elliptic filter are better, since the response curve of these filters are more selective and flatter than the other filter, eliminating fewer frequencies of the initial ECG signal, consequently the error is smaller.
Figure 4.4: The Initial ECG signal, Butterworth filter output, Elliptic filter output and Chebyshev filter output.

Figure 4.5: Comparison between the initial ECG signal and the Butterworth, Elliptic, Chebyshev filters outputs.
4.2 Adaptive Filter Results

In the second part of this chapter, the adaptive filter is applied to synthetic noisy ECG data, and the performances of this filter are studied.

Next figure (4.7) shows normal ECG signal and the ECG signal plus interference of 16.6Hz. Both signals are described at the beginning of this chapter. Below these plots, the signal of adaptive filter output can be found, that will be explained later.

Figure 4.6: Difference between the initial ECG signal and the Butterworth, Elliptic, Chebyshev filters outputs.
Figure 4.7: The ECG signal, The ECG signal + interference of 16.6 Hz and the Adaptive filter output.

As shown in the figure 4.8, the signal of the adaptive filter output is very similar to the initial ECG signal but it needs about 10 seconds of adaptation time.

These results were obtained after several tests. The filter settings were adjusted as follows:

The initial value of the coefficients are $w_1=0$ and $w_2=0$, these values are 0 because in the reality we probably only know the frequency of the interference but we do not its amplitude and phase, therefore there will be an adaptation time. If we take some starting values of $w_1$ and $w_2$ nearby convergence values, the adaptation time will be shorter or will disappear and with a small $\mu$ (i.e. 0.0001), the adaptive filter output would be almost the same than the initial ECG signal.

With adaptation rate of $\mu=0.001$, this filter reaches stable levels and good results but it needs an adaptation time of around 12 s. The increase of adaptation rate (i.e. 0.01) decreases the adaptation time (1 s) but the filter results get worse after convergence, because the approximations of the coefficients are greater. On the other hand with a smaller adaptation rate (i.e. 0.0001), the adaptation time increases too much, although after it is adapted the results are somewhat better. Therefore, since time is very important to know whether to apply the defibrillator, an adaptation rate of $\mu=0.001$ is correct because it has very good results and the adaptation time is not too much.
The adaptation time could be reduced by a gradual reduction of the adaptation rate, first a second with $\mu = 0.01$ and the rest of the time with $\mu = 0.001$. With this approach, the adaptation time is much less. This can be seen in the figure 4.9.

**Figure 4.9:** Difference between the initial ECG signal and the Adaptive filter output with a gradual reduction ratio adaptation.
4.3 Adaptive Filter versus Band-Stop Filters

As seen in the previous section, the Butterworth filter is slightly better than the others band-stop filters. Therefore it is compared to the adaptive filter to find the better one. Both filters require simple structure and a low level of computational resources.

The adaptive filter has a better result than the other filter. Although the adaptation time is somewhat bigger, after adapting, the error in the adaptive filter is smaller during the rest of the time.
Also, as seen in the previous section, the adaptation time of the adaptive filter could be reduced by a gradual reduction of the adaptation rate.

Another important difference is that if the interference frequency varies a little, the adaptive filter has adaptive capability and removes this interference but a band-stop filter so selective will not delete this interference and it could distort the signal.
Even if the band-stop filter has a very narrow band, in addition to removing the interference it may also remove frequency components of the signal and therefore distort it.

A comparison of these filters can be seen in the figures 4.10 and 4.11

Figure 4.10: The ECG signal, The Butterworth filter output and the Adaptive filter output.
Figure 4.11: Difference between the initial ECG signal with the Butterworth filter output and Adaptive filter output.
Chapter 5

Conclusion

Both the adaptive filter and the band-stop filters can be used for the suppression of the 16.6(6) Hz interference generated by the power supply of the railway because with these filters very good results are obtained and the ECG signal can be distinguished perfectly. Both solutions require a simple structure and a low level of computational resources.

It should be noted that the adaptive filter has some better results because the principal advantages of this solution are its adaptive capability, its low output noise, and its low signal distortion. The adaptive capability allows the processing of inputs whose properties are unknown and in some cases non-stationary.

To conclude, these filters also can be used to remove another interference with different frequency than 16.6 Hz, only by changing the location of the stop band in the new frequency in the band-stop filter and the input reference frequency in the adaptive filter.
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Appendix

Filters Matlab Code

Adaptive Filter Code

%INITIAL SIGNAL
clear all;
fid = fopen('A1001D1.dat', 'r');
s=fread(fid,'int16');
fclose(fid);
Hz=250;
s=s*.005; % ADC values -> mV

s1=s(1:5000); %20 seconds heart signal

t=1/Hz:1/Hz:length(s1)/Hz; %time axis

%interference defibrillation
n=1:length(s1);
b=1.2; %amplitude
f1=16.6; %16.6 Hz
w=2*pi*(f1/Hz); %omega (rad / sample)
n1=b*cos(w*n+(pi/4)); %interference defibrillation

% INITIAL SIGNAL(s1) + interference defibrillation(n1)
d=s1'+n1;

%adaptative filter
u1=0.01; %Controls the adaptation rate
u2=0.001;
v=[0 0]; %initialization coefficients w1 and w2

for k=1:250 %1 second
y=v*[cos(w*k) sin(w*k)]'; %w1 (i) * x1 (i) + w2 (i) * x2 (i),
error(k)=d(k)-y;

v=v+2*u1*error(k)*[cos(w*k) sin(w*k)];
%w1 (i +1) = w1 (i) + 2 * u * e (i) * x1 (i)
%w2 (i +1) = w2 (i) + 2 * u * e (i) * x2 (i)
%coefficients updated
end
for k=251:5000 % 19 seconds
y=v*[cos(w*k) sin(w*k)]; %w1 (i) * x1 (i) + w2 (i) * x2 (i),
error(k)=d(k)-y;

v=v+2*u2*error(k)*[cos(w*k) sin(w*k)];
%w1 (i +1) = w1 (i) + 2 * u * e (i) * x1 (i)
%w2 (i +1) = w2 (i) + 2 * u * e (i) * x2 (i)
%coefficients updated
end

figure(1)
subplot(3,1,1),plot(t,s1),title('Initial ECG signal');
ylabel('[mV]'); xlabel('[Sec]');axis([0 20 -4 4]);
subplot(3,1,2),plot(t,d),title('ECG signal + noise');
ylabel('[mV]'); xlabel('[Sec]');axis([0 20 -4 4]);
subplot(3,1,3),plot(t,error),title('Adaptive filter output');
ylabel('[mV]'); xlabel('[Sec]');axis([0 20 -4 4]);

figure (2)
subplot(2,1,1),plot(t,s1,'b',t,error,'r'),title('Comparison between
Initial ECG signal and adaptive filter output');
ylabel('[mV]'); xlabel('[Sec]');legend('initial ECG signal','adaptive
filter output');axis([0 20 -4 4]);
subplot(2,1,2),plot(t,(s1'-error)),title('Difference between Initial
ECG signal and adaptive filter output');
ylabel('[mV]'); xlabel('[Sec]');axis([0 20 -1 1]);

Band-Stop Filters Code

%INITIAL SIGNAL
clear all;
fid = fopen('A1001D1.dat', 'r');
s=fread(fid,'int16');
close(fid);
Hz=250;
s=s*.005; % ADC values -> mV
s1=s(1:2500);%10 seconds heart signal
t=1/Hz:1/Hz:length(s1)/Hz;%time axis
frec=0:(1/10):Hz-(1/10);%frequency axis

%interference defibrillation
n=1:length(s1);
b=1.2;%amplitude
f1=16.6; %16.6 Hz
n1=b*cos(2*pi*(f1/Hz)*n+(pi/4));

% INITIAL SIGNAL(s1) + interference defibrillation(n1)
d=s1'+n1;
figure(1)
subplot(3,1,1), stem(frec, (fft(s1)));
title('ECG signal frequency');
ylabel('amplitude'); xlabel('frecuency'); axis([0 100 -150 400]);
subplot(3,1,2), stem(frec, (fft(n1)));
title('interference frecuency');
ylabel('amplitude'); xlabel('frecuency'); axis([0 100 0 600]);
subplot(3,1,3), stem(frec, (fft(d)));
title('ECG signal frecuency + interference frecuency');
ylabel('amplitude'); xlabel('frecuency'); axis([0 100 -150 600]);

% bandstop filter (0.1328(16.6 Hz))
Wn1 = 0.1318;
Wn2 = 0.1338; % band stop (0.02(0.25 Hz))
Wn = [Wn1 Wn2]; % The cutoff frequency
[B, A] = butter(1, Wn, 'stop'); % Butterworth filter
[C, D] = ellip(1, 1, 20, Wn, 'stop'); % Elliptic filter
[E, F] = cheby1(1, 10, Wn, 'stop'); % Chebyshev Type I filter

[H1 W1] = freqz(B, A, 125); % frequency response
[H2 W2] = freqz(C, D, 125); % frequency response
[H3 W3] = freqz(E, F, 125); % frequency response

figure(2)
subplot(2,1,1), plot(t, s1), title('ECG signal');
ylabel('[mV]'); xlabel('[Sec]'); axis([0 10 -4 4]);
subplot(2,1,2), plot(t, d), title('ECG signal + noise');
ylabel('[mV]'); xlabel('[Sec]'); axis([0 10 -4 4]);

figure(3)
subplot(3,1,1), plot(abs(H1)); title('response frequency Butterworth filter');
ylabel('amplitude'); xlabel('frecuency');
subplot(3,1,2), plot(abs(H2)); title('response frequency Elliptic filter');
ylabel('amplitude'); xlabel('frecuency');
subplot(3,1,3), plot(abs(H3)); title('response frequency Chebyshev Type I filter');
ylabel('amplitude'); xlabel('frecuency');

e1 = filter(B, A, d); % application the filters in the signal
e2 = filter(C, D, d);
e3 = filter(E, F, d);

e1=filter(B,A,d); % application the filters in the signal
e2=filter(C,D,d);
e3=filter(E,F,d);

figure(4)
subplot(4,1,1), plot(t, s1'), title('Initial ECG signal');
ylabel('[mV]'); axis([0 10 -4 4]);
subplot(4,1,2), plot(t, e1), title('Output Butterworth filter');
ylabel('[mV]'); axis([0 10 -4 4]);
subplot(4,1,3), plot(t, e2), title('Output Elliptic filter');
ylabel('[mV]'); axis([0 10 -4 4]);
subplot(4,1,4), plot(t, e3), title('Output Chebyshev Type I filter');
ylabel('[mV]'); xlabel('[Sec]'); axis([0 10 -4 4]);
Code of Comparison between Band-Stop Filters and Adaptive Filter

%INITIAL SIGNAL
clear all;
fid = fopen('A1001D1.dat', 'r');
s=fread(fid,'int16');
fclose(fid);
Hz=250;
s=s*.005; % ADC values -> mV
s1=s(1:5000);%20 seconds heart signal
t=1/Hz:1/Hz:length(s1)/Hz;%time axis
frec=0:(1/20):Hz-(1/20);

%interference defibrillation
n=1:length(s1);
b=1.2;%amplitude
f1=16.6; %16.6 Hz
w=2*pi*(f1/Hz); %omega (rad / sample)
n1=b*cos(w*n+(pi/4)); %interference defibrillation

%d=s1+n1;% INITIAL SIGNAL(s1) + interference defibrillation(n1)
d=s1+n1;

%bandstop filter (0,1328)
Wn1=0.1318;
Wn2=0.1338; %band stop (0.02(0.25 Hz))

Wn = [Wn1 Wn2]; %The cutoff frequency
[B,A]=butter(1,Wn,'stop');%Butterworth filter

e1=filter(B,A,d);%application the filters in the signal
adaptative filter

u=0.001; %Controls the adaptation rate
v=[0 0]; %initialization coefficients w1 and w2

for k=1:length(s1)
    y=v*[cos(w*k) sin(w*k)]'; %w1 (i) * x1 (i) + w2 (i) * x2 (i),
    error(k)=d(k)-y;
    v=v+2*u*error(k)*[cos(w*k) sin(w*k)];
    %w1 (i +1) = w1 (i) + 2 * u * e (i) * x1 (i)
    %w2 (i +1) = w2 (i) + 2 * u * e (i) * x2 (i)
    %coefficients updated
end

figure (1)
subplot(2,1,1),plot(t,s1'-e1),title('difference between Initial ECG signal and Butterworth filter output');
ylabel('[mV]'); xlabel('[Sec]'); axis([0 20 -1 1]);
subplot(2,1,2),plot(t,s1'-error),title('difference between Initial ECG signal and adaptive filter output');
ylabel('[mV]'); xlabel('[Sec]'); axis([0 20 -1 1]);

figure (2)
subplot(3,1,1),plot(t,s1'),title('Initial ECG signal');
ylabel('[mV]'); axis([0 20 -4 4]);
subplot(3,1,2),plot(t,e1),title('Butterworth filter Output');
ylabel('[mV]'); axis([0 20 -4 4]);
subplot(3,1,3),plot(t,error),title('Adaptive filter Output');
ylabel('[mV]'); axis([0 20 -4 4]);
Suppression of Narrowband Interference Generated by the Power Supply of the Railway Systems in Public Defibrillators Devices

By
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23rd January 2013
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II. Problem Definition and Purpose of the Study
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Cardiac failure and cardiac diseases are among the main causes of death in the world.

An electrocardiogram, also called an ECG, is a simple test that detects and records the electrical activity of the heart.
A defibrillator is a life-saving device that gives the heart an electric shock in some cases of cardiac arrest. This is called defibrillation and it can save lives.

An interference is something which alters, modifies, or disrupts a signal.
A specific problem using the public access defibrillators arises at the railway stations. Some countries as Germany, Austria, Switzerland, Norway, Sweden and Slovenia are using AC railroad net power-supply system with rated frequency of 16.6(6) Hz.

The power supply frequency contaminates the electrocardiogram (ECG). It is difficult to be suppressed or eliminated due to the fact that it considerably overlaps the frequency spectra of the ECG.

The interference impedes the automated decision of the public access defibrillators whether a patient should be (or should not be) shocked.
Problem Definition and Purpose of the Study
The aim of study of this thesis is the suppression of the 16.6(6) Hz interference generated by the power supply of the railway systems in few central European countries.

For this purpose, an adaptive filter and a band-stop filter are used and the results obtained are compared in order to get the most suitable solution.
In signal processing, the function of a filter is to remove unwanted parts of a signal, such as random noise, or to specify useful parts of the signal, such as components lying in a certain important frequency range.

Band-stop Digital Filter: Is a filter that passes most frequencies unaltered, but attenuates those in a specific range to very low levels.
The Theory of the Study

- Band-stop Digital Filter:

Three different types of response are used to generate the band-stop filter. Each one of these filters has different characteristics and that can be compared in order to decide which is best for this case:

- Butterworth response
- Chebyshev response
- Elliptical response
Adaptive Filter:

- An adaptive filter is a filter that self-adjusts its transfer function according to an optimization algorithm driven by an error signal. It uses feedback in the form of an error signal to refine its transfer function to match the changing parameters.

- The system of two equations for adapting the coefficient $w_1(i)$ and $w_2(i)$:
  
  $$w_1(i+1) = w_1(i) + 2\mu e(i)x_1(i)$$

  $$w_2(i+1) = w_2(i) + 2\mu e(i)x_2(i)$$

- where $\mu$ controls the adaptation rate.
The adaptive filter and the band-stop filters are applied to solve the problem of the interference of 16.6(6) Hz, generated by the power Supply of the railway systems, on public defibrillators devices.

Then the results obtained are compared in order to get the most suitable solution.
Simulation Results and Discussions

- **Band-Stop Filters Results:**
  - The main problem of these filters is that the frequency of the interference signal considerably overlaps the frequency spectra of the ECG.
  - Therefore it is very difficult to remove only the interfering frequency without removing frequencies of the ECG signal, which gets distorted.
Consequently the filter needs to be as selective as possible to eliminate the fewest frequencies of the ECG signal.

But on the other hand, if the noise frequency varies minimally, a band-stop filter so selective will not delete the interference.
After the filters are stabilized, the Butterworth filter and Elliptic filter are better, since the response curve of these filters are more selective and flatter than the other filter, eliminating fewer frequencies of the initial ECG signal, consequently the error is smaller.
Adaptive Filter Results:

- The output of this filter varies with the adaptation rate, which is controlled with $\mu$, with $\mu = 0.001$, the filter reaches stable levels and good results but it needs an adaptation time of around 12 s. The increase of adaptation rate (i.e. 0.01) decreases the adaptation time (1 s) but the filter results get worse after convergence, because the approximations of the coefficients are greater. On the other hand with a smaller adaptation rate (i.e. 0.0001), the adaptation time increases too much, although after it is adapted the results are somewhat better.

- Therefore, since time is very important to know whether to apply the defibrillator, an adaptation rate of $\mu = 0.001$ is correct because it has very good results and the adaptation time is not too much.
Simulation Results and Discussions

- **Adaptive Filter Results:**

  - The initial value of the coefficients are $w_1=0$ and $w_2=0$, these values are 0 because in the reality we probably only know the frequency of the interference but we do not its amplitude and phase, therefore there will be an adaptation time. If we take some starting values of $w_1$ and $w_2$ nearby convergence values, the adaptation time will be shorter or will be zero, and with a small $\mu$ (i.e. 0.0001), the adaptive filter output would be almost the same as the initial ECG signal.
The adaptation time could be reduced by a gradual reduction of the adaptation rate, first a second with \( \mu = 0.01 \) and the rest of the time with \( \mu = 0.001 \). With this approach, the adaptation time is much less.
Adaptive Filter versus Band-Stop Filters:

- The adaptive filter has a better result than the other filter. Although the adaptation time is somewhat bigger, after adapting, the error in the adaptive filter is smaller during the rest of the time.

- Also, as seen in the previous slide, the adaptation time of the adaptive filter could be reduced by a gradual reduction of the adaptation rate.

- Another important difference is that if the interference frequency varies a little, the adaptive filter has adaptive capability and removes this interference but a band-stop filter so selective will not delete this interference and it could distort the signal.
Simulation Results and Discussions

➢ Adaptive Filter versus Band-Stop Filters:

- Difference between Initial ECG signal and Butterworth filter output
- Difference between Initial ECG signal and adaptive filter output
Simulation Results and Discussions

- Adaptive Filter versus Band-Stop Filters:

Initial ECG signal

Butterworth filter Output

Adaptive filter Output
Both the adaptive filter and the band-stop filters can be used for the suppression of the 16.6(6) Hz interference generated by the power supply of the railway because with these filters very good results are obtained and the ECG signal can be distinguished perfectly.

Both solutions require a simple structure and a low level of computational resources.

It should be noted that the adaptive filter has some better results because the principal advantages of this solution are its adaptive capability, its low output noise, and its low signal distortion. The adaptive capability allows the processing of inputs whose properties are unknown and in some cases non-stationary.

To conclude, these filters also can be used to remove another interference with different frequency than 16.6 Hz, only by changing the location of the stop band, in the new frequency, in the band-stop filter and the input reference frequency in the adaptive filter.
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